Numerical Representation of Planetary Ephemerides

X. X. Newhall, Celestial Mechanics 45:305-310, 1989

Computations

A handy function to compute the derivative of a Chebyshev polynomial.

```
In[1]:= DChebyshevT = Derivative[0, 1][ChebyshevT]

Out[1]= ChebyshevU[-1 + #1, #2] #1 &
```

This function computes matrix *T* from Newhall's equation (5). The parameter degree is the degree of the polynomial (*N* in Newhall), the parameter divisions is the number of subintervals of [-1, 1] (8 in Newhall).

```
In[2]:= NewhallT[degree_Integer, divisions_Integer] :=
         Flatten[
          Table[
           {
            Table[ChebyshevT[j, i], {j, 0, degree}],
            Table[DChebyshevT[j, i], {j, 0, degree}]
           },
           {i, 1, -1, -2 / divisions}
          ], {1, 2}]
```

This function computes matrix W used in Newhall's equation (8). The parameter *w* is the weight of the velocities relative to the positions (0.4 in Newhall).

```
In[3]:= NewhallW[divisions_Integer, w_Rational] :=
         DiagonalMatrix[Flatten[Table[{1, w^2}, {divisions + 1}]]]
```

The following functions compute the four blocks of matrix C1 and assemble them to form C1.

```
In[4]:= NewhallC1UpperLeft[degree_Integer, divisions_Integer, w_Rational] :=
         NewhallT[degree, divisions]^T.NewhallW[divisions, w].NewhallT[degree, divisions]

In[5]:= NewhallC1UpperRight[degree_Integer] :=
         Table[
          {ChebyshevT[i, 1], DChebyshevT[i, 1],
           ChebyshevT[i, -1], DChebyshevT[i, -1]}, {i, 0, degree}
         ]

In[6]:= NewhallC1LowerLeft[degree_Integer] := NewhallC1UpperRight[degree]^T

In[7]:= NewhallC1LowerRight[] := Table[0, {4}, {4}]

In[8]:= NewhallC1[degree_Integer, divisions_Integer, w_Rational] := ArrayFlatten[
           {{NewhallC1UpperLeft[degree, divisions, w], NewhallC1UpperRight[degree]},
            {NewhallC1LowerLeft[degree], NewhallC1LowerRight[]}}
         ]
```

The following functions compute the two blocs of matrix C2 and assemble them to form C2.

```
In[9]:=  NewhallC2Upper[degree_Integer, divisions_Integer, w_Rational] :=
          NewhallT[degree, divisions]^T.NewhallW[divisions, w]

In[10]:= NewhallC2Lower[divisions_Integer] :=
          Drop[IdentityMatrix[2 divisions + 2], {3, 2 divisions}]

In[11]:= NewhallC2[degree_Integer, divisions_Integer, w_Rational] :=
          ArrayFlatten[{{NewhallC2Upper[degree, divisions, w]}, {NewhallC2Lower[divisions]}}]
```

This function computes the matrix C1^{-1}.C2. Newhall doesn't give it a name but calls its elements c_k, so let's use the name *C*.

```
In[12]:= NewhallC[degree_Integer, divisions_Integer, w_Rational] :=
          Inverse[NewhallC1[degree, divisions, w]].NewhallC2[degree, divisions, w]
```

Formatting and Output

Produces a representation of a matrix as an initializer_list containing initializer_lists.

```
In[53]:= BidimMatrixToCDefinition[type_String, variable_String, matrix_List] :=
          type <> " const\r\n    " <> variable <> " =\r\n" <>
           StringReplace[
            ToString[
             CForm[matrix]
            ],
            {
             "List(List(" → "        {{",
             "List(" → "{",
             ")," → "},\r\n        ",
             "," → ",\r\n         ",
             "))" → "}};\r\n\r\n"
            }
           ]
```

Produces a representation of a matrix as a single, flattened initializer list.

```
In[54]:= FlattenedMatrixToCDefinition[type_String, variable_String, matrix_List] :=
          type <> " const\r\n    " <> variable <> " =\r\n" <>
           StringReplace[
            ToString[
             CForm[matrix]
            ],
            {
             "List(List(" → "        {",
             "List(" → "\r\n         ",
             ")," → ",\r\n",
             "," → ",\r\n         ",
             "))" → "};\r\n\r\n"
            }
           ]
```

Produces a representation of a list as an initializer list.

```
In[55]:= ListToCDefinition[type_String, variable_String, list_List] :=
          type <> " const\r\n    " <> variable <> " =\r\n" <>
           StringReplace[
            ToString[
             CForm[list]
            ],
            {
             "List(" → "        {",
             "," → ",\r\n         ",
             ")" → "};\r\n\r\n"
            }
           ]
```

Writes all the Newhall *C* matrices to a single file. Note that we drop the last 4 rows because they correspond to the Lagrange multipliers.

```
In[56]:= file = OpenWrite[FileNameJoin[{DirectoryName[NotebookDirectory[]], "numerics",
              "newhall.mathematica.cpp"}], BinaryFormat → True, PageWidth → Infinity];
         WriteString[
           file,
           "#pragma once\r\n",
           "\r\n",
           "#include <array>\r\n",
           "\r\n",
           "namespace principia {\r\n",
           "namespace numerics {\r\n", "\r\n"
          ];
         Do[
           WriteString[
            file,
            FlattenedMatrixToCDefinition[
             "std::array<double, (" <> ToString[degree] <> " + 1) * (2 * 8 + 2)>",
             "newhall_c_matrix_degree_" <> ToString[degree] <> "_divisions_8",
             Drop[NewhallC[degree, 8, 4 / 10], -4]
            ]
           ],
           {degree, 3, 17}];
         WriteString[
           file,
           "}  // namespace numerics\r\n",
           "}  // namespace principia\r\n"
          ];
         Close[file]

Out[60]= C:\Users\phl\Projects\GitHub\Principia\numerics\newhall.mathematica.cpp
```