Mathematics II	REPUBLIC OF RWANDA
029	
30 Oct. 2013 8.30-11.30 am	RWANDA EDUCATION BOARD

ADVANCED LEVEL NATIONAL EXAMINATIONS 2013

SUBJECT: MATHEMATICS II

COMBINATIONS: - MATHEMATICS-CHEMISTRY-BIOLOGY (MCB)
- MATHS-COMPUTER SCIENCE-ECONOMICS (MCE)
- MATHEMATICS-ECONOMICS-GEOGRAPHY (MEG)
- MATHS-PHYSICS-COMPUTER SCIENCE (MPC)
- MATHEMATICS-PHYSICS-GEOGRAPHY (MPG)
- PHYSICS-CHEMISTRY-MATHEMATICS (PCM)
- PHYSICS-ECONOMICS-MATHEMATICS (PEM)

DURATION: 3 HOURS

INSTRUCTIONS:

1. Don't open this question paper until you are told to do so.
2. This paper consists of **two** sections: **A** and **B**.
   - **Section A**: Attempt **all** questions.   **(55 marks)**
   - **Section B**: Attempt **only three** questions.   **(45 marks)**
3. *Geometrical instruments and silent non-programmable calculators may be used.*

SECTION A : ATTEMPT ALL QUESTIONS.   (55 marks)

1. Let consider the graph of the function $f(x) = -2x^2 + 12x + 5$

    a) Specify the type of graph and explain why.   *(1 mark)*

    b) Explain the concavity.   *(1mark)*

    c) Find the vertex.   *(2marks)*

2. Two cars start out at the same point. One car start out driving north at 25 km/h. Two hours later the second car starts driving east at 20 km/h. How long after the first car starts travelling does it take for the two cars to be 300 km apart?   *(4 marks)*

3. Find the probability of getting 5 heads and 7 tails in 12 flips of a coin.   *(2 marks)*

4. Solve for x :

    a) $4^{5-9x} = \dfrac{1}{8^{x-2}}$ *(3 marks)*

    b) $log_2(x^2 - 6x) = 3 + log_2(1 - x)$ *(3 marks)*

5. Evaluate the following limit: $\lim_{x \to -\infty} \dfrac{\sqrt{3x^2 + 6}}{5 - 2x}$ *(3 marks)*

6. Find the derivative of the following function using the definition of the derivative:

$$g(t) = \frac{t}{t+1}$$ *(3 marks)*

7. A sphere was measured and its radius was found to be 45 cm with a possible error of no more than 0.01 cm. What is the maximum possible error in the volume if we use this value of the radius?   *(3 marks)*

8. Evaluate the following integral: $\displaystyle\int \frac{2t^3 + 1}{(t^4 + 2t)^3} \, dt$ *(3 marks)*

9. Solve: $\cos x + \sin x > \sqrt{2}$ (Hint: use complex number theory). *(3 marks)*

10. Using integral rules, compute the circumference of a circle centred at the origin. *(6 marks)*

11. Calculate the expected value , the variance and the standard deviation for the following probability density function: *(3 marks)*

x_i	-5	-4	1	2
$f(x_i)$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{8}$

12. Solve for $z: |3 - 2z| \leq 5$ *(3 marks)*

13. Find the slope and the y-intercept of the regression line

    $y = ax + b$ that fits the following data: *(4 marks)*

Marks(x)	5	5	7	7	9	11	13	15	14	13	16	17
Marks(y)	4	8	10	7	10	10	12	13	15	16	17	17

14. Solve: $2(\ln x)^3 + (\ln x)^2 - 5\ln x + 2 = 0$ *(4 marks)*

15. Determine the shape of the graph with the following equation: *(4 marks)*

    $x^2 + y^2 - 3x + 10y - 1 = 0$

    Find its parameters.

SECTION B: Attempt ONLY THREE questions *(45 marks)*

16. a) Solve the following system of equations using the Gauss-Jordan elimination method.

    $$\begin{cases} 3x + y - 2z = 2 \\ x - 2y + z = 3 \\ 2x - y - 3y = 3 \end{cases}$$ *(7 marks)*

    b) Solve: $y'' - y' - 2y = \sin 2x$ *(8 marks)*

17. a) Solve for t: $4\sin^2\left(\frac{t}{3}\right) - 3\sin\left(\frac{t}{3}\right) = 1$ *(7 marks)*

b) Determine if the following vectors are in the same plane: **(8 marks)**
$\vec{a}(1;4;-7)$; $\vec{b}(2;-1;4)$; $\vec{c}(0;-9;18)$.

18. Calculate the area of the surface bounded by the curve :

a) $x = 6(\theta - \sin\theta)$ and $y = 6(1-\cos\theta)$, where $0 \le \theta \le 2\pi$. **(8 marks)**

b) $f(x) = \ln\dfrac{1+x}{1-x}$, where $\dfrac{\pi}{2} \le \theta \le 2\pi$ **(7 marks)**

19. a) Determine the standard form of the following flat shape : **(5 marks)**
$$5x^2 + y^2 - 10x + 4y + 4 = 0$$

b) Which shape is it? **(1 mark)**

c) Find the coordinates of the most points, the focal points, and the equation of the focal line. **(9 marks)**

20. Compute:

a) $\int \sin x \ e^{\cos x} dx$ **(5 marks)**

b) $\int \sin x * \sin 3x \ dx$ (Hint: apply complex numbers). **(10 marks)**