Mathematics II

029

09/11/ 2016 8.30am-11.30am

Rwanda Education Board

ADVANCED LEVEL NATIONAL EXAMINATIONS, 2016

SUBJECT: MATHEMATICS II

COMBINATIONS:
- **MATHS-CHEMISTRY-BIOLOGY (MCB)**
- **MATHS-COMPUTER SCIENCE-ECONOMICS (MCE)**
- **MATHS -ECONOMICS-GEOGRAPHY (MEG)**
- **MATHS-PHYSICS-COMPUTER SCIENCE (MPC)**
- **MATHS -PHYSICS-GEOGRAPHY (MPG)**
- **PHYSICS-CHEMISTRY- MATHS (PCM)**
- **PHYSICS-ECONOMICS- MATHS (PEM)**

DURATION: 3 HOURS

INSTRUCTIONS:

1) Do not open this question paper until you are told to do so.
2) Write your names and index number on the answer booklet as written on your registration form, and **DO NOT** write your names and index number on additional answer sheets of paper if provided.
3) This paper consists of **two** sections: **A** and **B**.
   - **Section A**: Attempt **all** questions. **(55marks)**
   - **Section B**: Attempt **only three** questions. **(45marks)**
4) ***Geometrical instruments and silent non-programmable calculators may be used.***
5) Use a **blue or black** pen.

SECTION A : ATTEMPT ALL QUESTIONS. (55 MARKS)

1) Find r,s and t for the parabola of equation $y=rs^2+sx+t$ that has the line of symmetry $d\equiv x=\frac{3}{4}$; cut the $x-axis$ at the point of x- coordinates 2 and $y-axis$ at the point of y- coordinates -2. **(3marks)**

2) The perimeter of a rectangle is 36cm.
   (a) What are the dimensions (length and width) of that rectangle? **(2marks)**
   (b) What is its greatest possible area? **(1mark)**

3) Find the term independent of x in the binomial expansion of $\left(3x-\frac{2}{x^2}\right)^{18}$. **(3marks)**

4) A box has length of *2x + 1 units*, width of *x + 4 units* and height of *x + 4 units*. If you build the box using x^3; x^2;x *and unit* (1) blocks; how many blocks of each will you need? **(3marks)**

5) One solution of the equation $log_4(8a)=1+log_2\left(\frac{a+8}{2+x}\right)$ is 2; calculate the value of a and solve the equation. **(4marks)**

6) Consider two sequences $\{U_n\}$ and $\{V_n\}$ given by

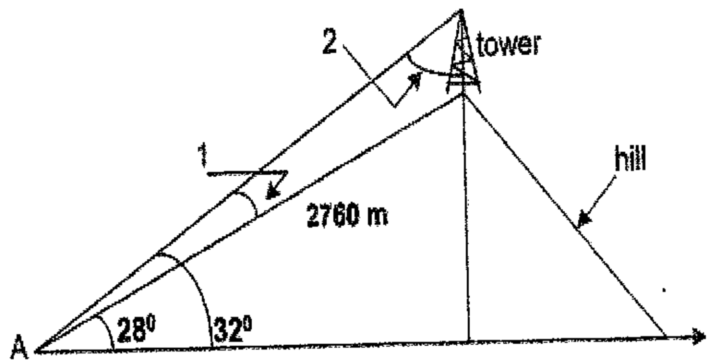
$$U_0=9;\ U_{n+1}=\frac{1}{2}U_n-3\ \ and\ V_n=U_n+6$$

   (a) Show that $\{V_n\}$ is a geometric sequence. **(3marks)**
   (b) Express $S_n=V_0+V_1+V_2+\cdots+V_n$ in terms of n. **(2marks)**

7) Evaluate:
   (a) $\int_0^2 |2x-1|dx$ **(2marks)**
   (b) $\lim_{x\to\infty}\left(\frac{x+3}{x-1}\right)^{x+1}$ **(2marks)**

8) Given that $f(x)=-1+tan^{-1}\left(\frac{4x}{5}\right)$; find the inverse $f^{-1}(x)$. **(3marks)**

9) The distance from point A to the top of the hill is 2760 m. The angle of elevation from A to the base of the tower is 28^0 and the angle of elevation from A to the top of the tower is 32^0.



(a) Find the measures of angles 1 and 2. **(1mark)**

(b) Find the height of the tower. **(2marks)**

10) Verify if $\lim_{x\to 3} f(x)$ exists where $f(x)=\begin{cases}\dfrac{x+2}{2}; x\le 3\\ \dfrac{12-2x}{3}; x\succ 3\end{cases}$ **(3marks)**

11) Let $T:\mathfrak{R}^3 \to \mathfrak{R}^3$ be the linear mapping defined by

$T(x,y,z) = (x+2y-z, y+z, x+y-2z)$.

Find a basis and the dimension of the

(a) image $U\ of\ T$ **(2marks)**

(b) kernel $W\ of\ T$ **(2marks)**

12) Find the derivative of $f(x) = ln\left(\frac{1+sinx}{1-sinx}\right)$ **(3marks)**

13) Prove that the circles $x^2+y^2-6x-12y+40=0$ and $x^2+y^2-4y=16$ are orthogonal. **(4marks)**

14) Suppose that a game is to be played with a single die assumed fair. In this game a player wins \$20 if a 2 turns up, \$40 if a 4 turns up; loses \$30 if a 6 turns up; while the player neither wins nor loses if any other face turns up.

Find the expected sum of money to be won. **(4marks)**

15) (a) Find the modulus and argument of $Z = \frac{(1+i)^2}{1-i}$ **(2marks)**

(b) Show that the representative points in the Argand diagram of the complex numbers $1+6i$; $3+10i$; $4+12i$ are collinear. **(4marks)**

SECTION B: ATTEMPT THREE QUESTIONS ONLY. (45 MARKS)

16) (a) Construct a cumulative frequency curve for the following data: **(3marks)**

Test marks	Frequency
1-20	4
21-40	25
41-60	71
61-80	38
81-100	12

Use your graph to estimate

(i) the median score. **(1mark)**

(ii) the inter-quartile range. **(1mark)**

(iii) the pass marks if 60% of the candidates passed. **(1mark)**

(iv) the smallest mark required to obtain an A (*first upper*) grade if 10% of the candidates received an A grade. **(1mark)**

(b) In a physics experiment, a bottle of milk was brought from a cool room into a warm room. Its temperature $y°C$, was recorded at t minutes after it was brought in, for 11 different values of t. The results are summarized as:

$$\sum t = 44, \ \sum y = 205, \ \sum t^2 = 180.4, \ \sum ty = 824.5$$

(i) Calculate the equation of the line of regression of y on t in the form

$y = a + bt$. **(4marks)**

(ii) Explain the practical significance of the value of a. **(2marks)**

(iii) Use your equation to estimate the values of y at $t = 4.5$ **(2marks)**

17) (a) Find the volume of the solid formed by revolving the region bounded by the graph of $y = x^2 + 1$, $y = 0$, $x = 0$ *and* $x = 1$ about the $y - axis$. **(7marks)**

(b) Solve (*by the method of undetermined coefficients*), the differential equation $y'' + y' - 2y = 3\cos 2x$; $y(0) = -1$ *and* $y'(0) = 2$ **(8marks)**

18) Determine the values of m such that the system in unknowns x, y *and* z

$$\begin{cases} x + (m-1)y + (2m-3)z = 1 \\ mx + 2(m-1)y + 2z = 2 \\ (m+1)x + 3(m-1)y + (m^2-1)z = 3 \end{cases}$$

has a unique solution, no solution and more than one solution. **(15marks)**

19) (a) (i) Sketch the graph of $f(x) = \sqrt{x}$ over the interval $[1,9]$. **(2marks)**

(ii) Find an equation of the secant line to the graph of f passing through the points $(1, f(1))$ and $(9, f(9))$. **(1mark)**

(iii) Sketch the graph of the secant line on the same axes as the graph of f. **(1mark)**

(iv) Find the value of the constant c in the interval $]1,9[$ such that

$f'(c) = \dfrac{f(9) - f(1)}{9 - 1}$. **(2marks)**

(v) Find the equation of the tangent line to the graph of f at the point $(c, f(c))$. **(1mark)**

(vi) Sketch the graph of the tangent line on the same axes as the graph of f. **(1mark)**

(b) A logistic model for the data on AIDS cases is given by:

$$N = \frac{948{,}000}{1+17.8e^{-0.317t}}$$

Where N is the number of AIDS cases diagnosed by year t with $t = 0$ representing 1985.

(i) Use the model to predict the number of AIDS cases diagnosed by 2010. **(4marks)**

(ii) Compare the actual number of AIDS cases diagnosed by 2003 to be 929,985 with the number given by the model. **(3marks)**

20) The coordinates of the point A *and* B are $(0,2,5)$ *and* $(-1,3,1)$ respectively and the equation of the line L is $\frac{x-3}{2}=\frac{y-2}{-2}=\frac{z-2}{-1}$.

(a) Find the equation of the plane π which contains A and is perpendicular to L and verify that B lies in π. **(5marks)**

(b) Show that the point C in which L meets π, is $(1,4,3)$ and find the angle between CA *and* CB. **(6marks)**

(c) Find the coordinates of the point P on L which is such that the volume of the tetrahedral $PABC$ *is* 9. **(4marks)**

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