

**Mathematics II**

**029**

**14 Nov. 2012      8.30-11.30 am**

**REPUBLIC OF RWANDA**



**RWANDA EDUCATION BOARD (REB)**

**ADVANCED LEVEL NATIONAL EXAMINATIONS 2012**

**SUBJECT: MATHEMATICS II**

**COMBINATIONS:**

- **MATHEMATICS-CHEMISTRY-BIOLOGY (MCB)**
- **MATHS-COMPUTER SCIENCE-ECONOMICS (MCE)**
- **MATHEMATICS-ECONOMICS-GEOGRAPHY (MEG)**
- **MATHS-PHYSICS-COMPUTER SCIENCE (MPC)**
- **MATHEMATICS-PHYSICS-GEOGRAPHY (MPG)**
- **PHYSICS-CHEMISTRY-MATHEMATICS (PCM)**
- **PHYSICS-ECONOMICS-MATHEMATICS (PEM)**

**DURATION: 3 HOURS**

**INSTRUCTIONS:**

This paper consists of **two** sections: **A** and **B**.

**Section A:** Attempt **all** questions.

**(55 marks)**

**Section B:** Attempt any **three** questions.

**(45 marks)**

**Geometrical instruments and silent non-programmable calculators may be used.**

**SECTION A : Attempt all questions. (55 marks)**

01. Show that  $C(n-1, p-1) + C(n-1, p) = C(n, p)$ . (4 marks)

02. Find the total number of diagonals that can be drawn in a decagon. (3 marks)

03. Determine the continuity of  $f(x) = \frac{\ln x + \tan^{-1} x}{(x-1)(x+1)}$ . (3 marks)

04. Find the value of  $x$  if  $\sqrt{3} \tan x = 2 \sin x$  (3 marks)

05. The matrix  $M(\alpha)$  is define by

$$M(\alpha) = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}.$$

Verify that  $M(\alpha)M(\beta) = M(\alpha + \beta)$ .

(2 marks)

06. A person, standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is  $60^\circ$ ; when he retreats 40 meters from the bank, he finds the angle to be  $30^\circ$ . Find the breadth of the river and the height of the tree. (5 marks)

07. If  $T_p, T_q$  and  $T_r$  are the  $p^{th}, q^{th}$  and  $r^{th}$  terms of an arithmetic

progression, then find the value of  $\begin{vmatrix} T_p & T_q & T_r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$  (3 marks)

08. For what value of  $k$ , the points  $(1,5)$ ,  $(k,1)$  and  $(11,7)$  are collinear? (3 marks)

09. Evaluate  $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$  (3 marks)

10. From the following data of marks in Mathematics and Physics obtained by four students out of thirty. Calculate the correlation coefficient : (5 marks)

Mathematics:	14	45	27	38
Physics	: 35	40	20	21

11. In Euclidian space  $\mathbb{R}^2$ , the sphere with  $M(2,-1,3)$  as center passes through the point  $T(1,2,-3)$ . Write the equation of the sphere and parametric equations of a line which is tangent through  $T$ . (4 marks)

12. A tank is the form of an inverted cone having height 8 meters and radius 2 meters. Water is flowing into the tank at the rate of  $\frac{1}{8}m^3/\text{minute}$ . How fast is the water level rising when the water is 2.5 meters deep? **(4 marks)**
13. Calculate :
- a)  $\int \frac{\sin x}{1 + \sin x} dx$  **(3 marks)**
- b)  $\int_0^2 \frac{5x+1}{x^2+4} dx$  **(3 marks)**
14. a) In a single throw of two dice, determine the probability of getting a total of 2 or 4. **(2 marks)**
- b) The letters of the word **"DIVORCE"** are arranged at random. Find the probability that the vowels may occupy the even places. **(2 marks)**
15. Find the sum of  $1 + \frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \dots$  **(3 marks)**

**SECTION B: Attempt ONLY THREE questions (45 marks)**

16. Consider a real valued numerical function defined as  $f: \mathbb{R} \rightarrow \mathbb{R}$
- $$x \rightarrow \frac{1}{2}x^2 e^{x+1}.$$
- a) Find the domain of function  $f(x)$  **(1 mark)**
- b) Find the intersection with axis of coordinates. **(2 marks)**
- c) Find the asymptotes **(5 marks)**
- d) Discuss the first and second derivative of  $f(x)$  **(3 marks)**
- e) Sketch the graph of  $f(x)$  **(2 marks)**
17. The sides of perfect die are colored as follows: three sides are orange, two sides are green and one side is red. A player bets 200 RWF is refunded for each throw. When red face of the die is up, a player is refunded 10 % of 200 RWF, when orange face is up, a player is refunded 30 % of 200 RWF and when green face is up, a player is given 500 RWF. If  $X$  is the difference between the refunded money and the betted money,
- a) determine the sets of values of  $X$  and the distribution probability of  $X$ . **(5.5 marks)**

- b) calculate the mathematical expectation  $E(X)$  of  $X$  and interpret the obtained values. (4 marks)
- c) calculate the variance and the standard deviation of  $X$ . (5.5 marks)
18. A straight line passes through points  $A(-1, -5)$ ,  $B(0, -8)$  and  $2y + 16 = 4x^2$  is the equation of the curve  $C$ .
- a) Find the equation on the straight line  $AB$ . (1 mark)
- b) In the same Cartesian plane, draw the straight line  $AB$  and the curve  $C$ . (3 marks)
- c) Calculate the area between the curve  $C$  and the straight line  $AB$ . (6 marks)
- d) Calculate the volume of solid of revolution about the  $x$ -axis of the surface area in c) above. (5 marks)
19. a) Suppose  $f$  and  $g$  are linear transformations on real vector space  $\mathbb{R}^2$  with their respective representative matrices  $F = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$  and  $G = \begin{bmatrix} 3 & 0 \\ -2 & 1 \end{bmatrix}$  relative to the basis  $B$ . (3 marks)
- Find the matrix that represents  $g \circ f$ .
- b) Find a vector  $u$  such that  $f(u) = 2u$  and vector  $v$  such that  $f(v) = v$ . (4 marks)
- c) Prove that  $B = (u, v)$  is a basis of the vector space  $\mathbb{R}^2$ . (2 marks)
- d) Write the matrix  $T$  that represents  $f$  relative to the basis  $B$ . (4 marks)
- e) Find a relationship between  $F$  and  $T$ . (2 marks)
20. a) For what point of the parabola  $y^2 = 18x$ , is the ordinate equal to three times the abscissa? (5 marks)
- b)  $S$  and  $T$  are the foci of the ellipse  $\frac{x^2}{a} + \frac{y^2}{b^2} = 1$  and  $B$  is the end of the minor axis. If  $STB$  is an equilateral triangle, find the eccentricity of that ellipse. (5 marks)
- c) A variable circle passes through a fixed point  $(2, 0)$  and touches the  $y$ -axis. Find the locus of its centre. (5 marks)