Mathematics II

027

02 Nov. 2011   08.30am - 11.30am

ADVANCED LEVEL NATIONAL EXAMINATIONS 2011

SUBJECT: MATHEMATICS II

COMBINATIONS:
- MATHS-CHEMISTRY-BIOLOGY: MCB
- MATHS-COMPUTER SCIENCE-ECONOMICS: MCE
- MATHS-ECONOMICS-GEOGRAPHY: MEG
- MATHS-PHYSICS-COMPUTER SCIENCE: MPC
- MATHS-PHYSICS-GEOGRAPHY: MPG
- PHYSICS-CHEMISTRY-MATHS: PCM
- PHYSICS-ECONOMICS-MATHS: PEM

TIME: 3 HOURS

INSTRUCTIONS:

- This paper consists in **two** sections: **A** and **(B)**

Section A : Attempt **all** questions.	**(55 marks)**
Section B: Attempt any **three** questions.	**(45 marks)**

Geometrical instruments and silent non-programmable calculators may be used.

SECTION A: Attempt all questions.   **(55 marks)**

1. What values (real numbers) of x satisfying the following condition:

   (a) $4(x+5)-6(2x+3)=3(x+14)-2(5-x)+9$ **(2.5 marks)**

   (b) $|6-3x| \succ 14$ where $|a|$ stands for absolute value of a *defined*

   $as \ |a| = \begin{cases} a \ if \ a \geq 0 \\ -a \ if \ a \leq 0 \end{cases}$ **(2 marks)**

2. If (u,v,w) is a basis of the real vector space $\Re^3$ determine
   whether or not $(u+v, u+2w, u-w)$ is also a basis of $\Re^3$.   **(3 marks)**

3. Let $f:\Re \to \Re : f(x) = \dfrac{x^2-1}{|x-1|}$

   (a) Find $\lim\limits_{x \to 1\succ} f(x)$ *and* $\lim\limits_{x \to 1\prec} f(x)$.   **(3.5 marks)**

   (b) Discuss the limit of *f(x)* as *x* approaches 1.   **(0.5 mark)**

   (c) Sketch the graph of *f(x)*.   **(1 mark)**

4. Express $f(x) = \dfrac{3x^3-x^2-13x-13}{x^2-x-6}$ in partial fractions.

   Then find antiderivative of *f(x)*.   **(4.5marks)**

5. Find the number of ways that 6 teachers can be assigned to
   4 sections of mathematics course if no teacher is assigned to
   more than one section.   **(2.5 marks)**

6. In Euclidian space, find an equation for the plane consisting
   of all points that are equidistant from the points (-4, 2, 1)
   and (2, -4, 3).   **(3 marks)**

7. Find any asymptotes of the function f if $f(x) = \dfrac{x-1}{x^2-1}$ **(3marks)**

8. Find a second degree polynomial P (x) such that P(2)=5,
   P'(2)=3 and P''(2)=2 where P' and P'' are first and second
   derivatives of P respectively .   **(4 marks)**

9. (a) Evaluate the derivative of $f(x) = \ln(x^3+7x^2)$
   where **ln** stands for natural logarithm function;   **(1.5 marks)**

   (b) and evaluate the integral $\displaystyle\int \dfrac{x^3}{x^4+7} dx$ **(1.5 marks)**

10. How many distinct permutations can be made from the
    letters of the word **"infinity"**?   **(2marks)**

11. Find the domain and the derivative of the numerical

   function f, if $f(x) = \dfrac{x}{1 - \ln(x-1)}$. **(4marks)**

12. The probability that a patient recovers from a delicate heart
    operation is 0.8. What is the probability that
    (a) Exactly 2 of the next 3 patients who have this operation
       survive? **(1.5 marks)**
    (b) All of the next 3 patients who have this operation survive?**(1.5 marks)**

13. Find the value of the complex number $Z = \left(-\dfrac{1}{2} + i\dfrac{\sqrt{3}}{2} \right)^{2010}$.

    Leave your answer in standard form $Z = a + b\,i$. **(4marks)**

14. Sketch the plane region bounded by $y = x$ and $y = x^2 - 6$.
    Then estimate the volume generated by this region when
    revolved about $x = -3$. **(5.5 marks)**

15. The numbers of incorrect answers on a true-false competency
    test for a random sample of 15 students were recorded as
    follows: 2, 1, 3, 0, 1, 3, 6, 0, 3, 3, 5, 2, 1, 4, and 2.
    Find the mean, the median, the mode and the sample standard
    deviation. **(4 marks)**

SECTION B: Attempt any three questions. **(45 marks)**

16. Let $f : \Re \to \Re : f(x) = |x - 2| - 1 + \dfrac{1}{x^2}$
    (a) Determine the domain of $f(x)$. **(2 marks)**
    (b) Write $f(x)$ without signs of absolute value. **(2 marks)**
    (c) Study the derivability of $f(x)$ at $x=-2$. **(4 marks)**
    (d) Evaluate the limit of $f(x)-(-x+1)$ when x approaches $-\infty$
       and the limit of $f(x)-(x-3)$ when x approaches $+\infty$.
       Is there any relationship between lines y=x-3
       and y=-x+1, and the graph of the function f? **(3 marks)**

    (e) Evaluate $\int_1^3 f(x)dx$. **(4 marks)**

17. (a) Solve the equation ${}^{n-1}C_{n-5} = 3\ {}^{n-3}C_{n-7}$ $\left(\text{or} \begin{pmatrix} n-1 \\ n-5 \end{pmatrix} = 3\begin{pmatrix} n-3 \\ n-7 \end{pmatrix}\right)$

      in the set of positive integers. **(6.5marks)**

(b)   Consider the quadratic polynomial $z^2 - 6z + c$ where c is real. For what values of c does this polynomial have real roots? **(2.5 marks)**

(c) Multiply out the expression $(z + 7)(z^2 - 6z + 25)$ and hence find all roots (real or complex) of the polynomial $z^3 + z^2 - 17z + 175$. **(6 marks)**

18. (a) In Euclidian space, find vector, parametric and symmetric equations for
(i)   the line through origin and the point (1,2,3) **(4marks)**
(ii) The line through (0,2,-1) and parallel to the line with

parametric equations $\begin{cases} x = 1 + 2t \\ y = 3t \\ z = 5 - 7t \end{cases}$

(4 marks)

(b) Find all cube roots of the complex number $W = -1 + i$.
Leave your answer in polar form and trigonometric form. **(7 marks)**

19. (a) Compute the sixth degree Taylor polynomial generated by $f(x) = \ln x$ about $x=1$.

Using this result, evaluate $\lim\limits_{x \to 1} \dfrac{\ln x}{x - 1}$. **(6marks)**

(b) Find all values of x that satisfy

(i) the inequality $2\cos(x) + 1 \geq 0$ in the interval $[0, 2\pi]$; **(2.5 marks)**

(ii) the equality $\log_2 \dfrac{x^2 - 1}{x + 1} = 1$ **(3.5 marks)**

(c) If 3 books are picked at random from a shelf containing 5 novels, 3 books of poems, and one dictionary, what is the probability that
(i) The dictionary is selected? **(1.5 marks)**
(ii) 2 novels and 1 book of poems are selected? **(1.5 marks)**

20. (a) Let * be a binary operation defined on the set Z of all integers by $x * y = x + y + 3$.
Determine whether the operation is commutative, and whether there is an identity element.
Can you find a symmetric (inverse) of any integer? **(8 marks)**

(b) Sketch and estimate the area of the region bounded by the curves $y = x$ and $y = x^2 - 2$. **(7 marks)**