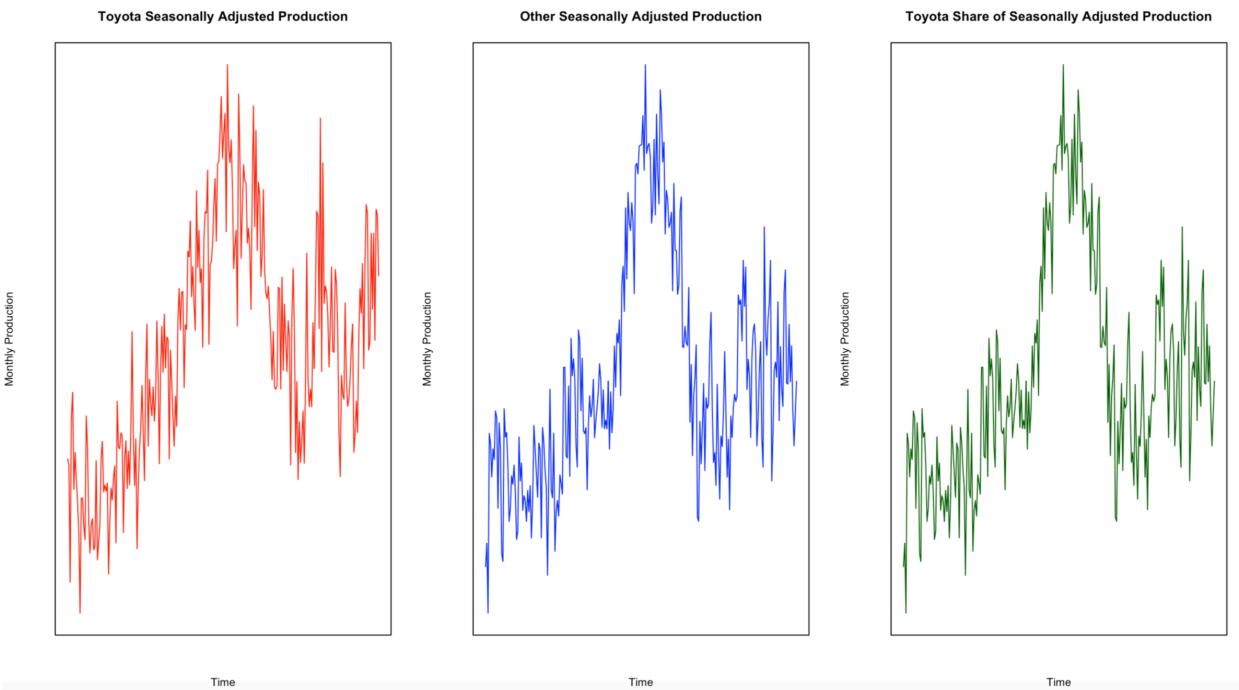
Erasmus Econometrics

Test Exercise 6

Time Series Econometrics

a.) Make time series plots of the variables yt and xt, and also of the share of Toyota in all produced passenger cars, that is yt/(yt + xt). What conclusions do you draw from these plots?



Commentary: The three variables seem to share the same stochastic trend. Moreover, the graphs suggest that the three time-series are not stationary.

b)

(i) Perform the Augmented Dickey-Fuller (ADF) test for yt. In the ADF test equation, include a constant (α) and three lags of Δyt, as well as the variable of interest, yt−1. Report the coefficient of yt−1 and its standard error and t-value, and draw your conclusion.

The following table shows the result for the ADF Test on yt where it includes a time trend.

Parameter	Estimate	Std. Error	t value	P-Value
(Intercept)	21743.4572	8908.5711	2.4407	0.0154
Time Variable	14.5161	16.8972	0.8591	0.3912
Previous TSA (y_{t-1})	-0.1016	0.0426	-2.3865	0.0178
lag1 TSAdiff	-0.5483	0.0720	-7.6179	0.0000
lag2 TSAdiff	-0.3139	0.0755	-4.1591	0.0000
lag3 TSAdiff	-0.0584	0.0653	-0.8932	0.3727

Commentary: The value of the t-statistic (-2.3865) suggests that we cannot reject H_0 that the series is not stationary, because it is smaller than -3.5 critical value for ADF tests with trend.

The following table shows the result for the ADF Test on yt where it DOES NOT include a time trend.

Parameter	Estimate	Std. Error	t value	P-Value
(Intercept)	19281.8945	8430.4101	2.2872	0.0231
Previous TSA (y_{t-1})	-0.0832	0.0368	-2.2623	0.0246
lag1 TSAdiff	-0.5630	0.0699	-8.0565	0.0000
lag2 TSAdiff	-0.3243	0.0745	-4.3546	0.0000
lag3 TSAdiff	-0.0639	0.0650	-0.9835	0.3264

Commentary: The value of the t-statistic (-2.2623) suggests that we cannot reject H_0 that the series is not stationary, because it is smaller than -2.9 critical value for ADF tests without a trend.

(ii) Perform a similar ADF test for xt.

The following table shows the result for the ADF Test on xt where it includes a time trend.

Parameter	Estimate	Std. Error	t value	P-Value
Intercept	32506.3542	15240.8757	2.1328	0.0340
Time	7.5434	27.1691	0.2776	0.7815
Previous OSA (x_{t-1})	-0.0738	0.0364	-2.0279	0.0437
lag1 OSAdiff	-0.5074	0.0690	-7.3521	0.0000
lag2 OSAdiff	-0.3584	0.0713	-5.0290	0.0000
lag3 OSAdiff	-0.1012	0.0649	-1.5591	0.1204

Commentary: The value of the t-statistic (-2.0279) suggests that we cannot reject H_0 that the series is not stationary, because it is smaller than -3.5 critical value for ADF tests with trend.

The following table shows the result for the ADF Test on yt where it DOES NOT include a time trend.

Parameter	Estimate	Std. Error	t value	P-Value
(Intercept)	31540.3617	14808.7693	2.1298	0.0342
Previous OSA (x_{t-1})	-0.0696	0.0331	-2.1057	0.0363
lag1 OSAdiff	-0.5112	0.0675	-7.5701	0.0000
lag2 OSAdiff	-0.3614	0.0703	-5.1387	0.0000
lag3 OSAdiff	-0.1030	0.0645	-1.5961	0.1118

Commentary: The value of the t-statistic (-2.1057) suggests that we cannot reject H_0 that the series is not stationary, because it is smaller than -2.9 critical value for ADF tests without a trend.

c) Perform the two-step Engle-Granger test for cointegration of the time series yt and xt. In step 1, regress yt on a constant and xt. In step 2, perform a regression of the residuals et in the model $\Delta et = \alpha + \rho et{-}1 + \beta 1 \Delta et{-}1 + \beta 2 \Delta et{-}2 + \beta 3 \Delta et{-}3 + \omega t$. What is your conclusion?

The following table shows the results for the first step of the Engle-Granger test for cointegration:

Parameter	Estimate	Std. Error	t value	P-Value
Intercept	26786.431	8451.526	3.169	0.002
Other SA	0.452	0.019	23.919	0.000

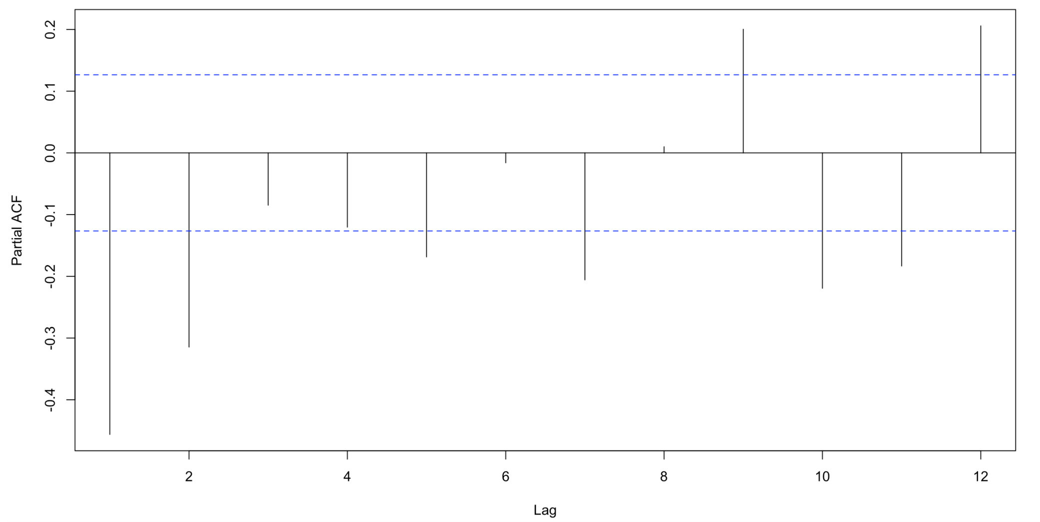
This second table shows the results for the second step of the Engle-Granger test for cointegration (an ADF test on the residuals of the first equation):

Parameter	Estimate	Std. Error	t value	P-Value
Intercept	24.992	847.826	0.029	0.977
Previous Residual	-0.293	0.068	-4.306	0.000
Residual Lag 1	-0.286	0.079	-3.64	0.000
Residual Lag 2	-0.142	0.075	-1.879	0.061
Residual Lag 3	-0.096	0.066	-1.461	0.145

Commentary: The value of the t-statistic (-4.306) suggests that we can reject H_0 that the series is not stationary, because it is smaller than -2.9 critical value for ADF tests without a trend. Therefore, the results show evidence that the two time-series are, in fact, cointegrated (they share the same stochastic trend).

(d) Construct the first twelve sample autocorrelations and sample partial autocorrelations of Δyt and use the outcomes to motivate an AR (12) model for Δyt. Check that only the lagged terms at lags 1 to 5, 10, and 12 are significant, and estimate the following model: $\Delta yt = \alpha + P5\ j{=}1\ \beta j \Delta yt{-}j + \beta 6 \Delta yt{-}10 + \beta 7 \Delta yt{-}12 + \varepsilon t$.

Partial Autocorrelation Graph



The next table shows the results of the proposed OLS model:

Parameter	Estimate	Std. Error	t value	P-Value
Intercept	561.613	919.511	0.611	0.542
lag1 TSAdiff	-0.598	0.062	-9.700	0.000
lag2 TSAdiff	-0.263	0.076	-3.468	0.001
lag3 TSAdiff	-0.227	0.075	-3.035	0.003
lag4 TSAdiff	-0.23	0.072	-3.194	0.002
lag5 TSAdiff	-0.152	0.061	-2.494	0.013
lag10 TSAdiff	-0.268	0.052	-5.124	0.000
lag12 TSAdiff	0.246	0.055	4.512	0.000

(e) Extend the model of part (d) by adding the Error Correction (EC) term (yt −0.45xt), that is, estimate the ECM Δyt = α + γ(yt−1 − 0.45xt−1) + P5 j=1 βjΔyt−j + β6Δyt−10 + β7Δyt−12 + εt (estimation sample is Jan 1980 - Dec 1999). Check that the EC term is significant at the 5% level, but not at the 1% level.

The table below shows the results of the OLS model with the error correction term:

Parameter	Estimate	Std. Error	t value	P-Value
(Intercept)	4728.007	2133.703	2.216	0.028
lag1 TSAdiff	-0.522	0.071	-7.401	0.000
lag2 TSAdiff	-0.187	0.083	-2.24	0.026
lag3 TSAdiff	-0.158	0.081	-1.955	0.052
lag4 TSAdiff	-0.185	0.074	-2.486	0.014
lag5 TSAdiff	-0.133	0.061	-2.179	0.030
lag10 TSAdiff	-0.274	0.052	-5.265	0.000
lag12 TSAdiff	0.252	0.054	4.64	0.000
Error Correction Term	-0.15	0.070	-2.16	0.032

The p-value is equal to 0.032, meaning that if we assume that the null hypothesis that the error correction term parameter is equal to 0, we would obtain a value as extreme as -0.15, around 3% of the time. Therefore, the EC term is significant at the 5% level, but not at the 1% level.

(f) Use the models of parts (d) and (e) to make two series of 12 forecasts of monthly changes in production of Toyota passenger cars in 2000. At each month, you should use the data that are then available, for example, to forecast production for September 2000 you can use the data up to and including August 2000. However, do not re-estimate the model and use the coefficients as obtained in parts (d) and (e). For each of the two-forecast series, compute the values of the root mean squared error (RMSE) and of the mean absolute error (MAE). Finally, give your interpretation of the outcomes.

Metric	No EC Term	EC Term Included
RMSE Errors	16991.80	17523.51
MAE Errors	14703.23	15246.59

Commentary: It appears that the inclusion of the error correction model does not improve the forecast for the dependent variable. The simpler auto-regressive model performed better than the error-correction term both in terms of RMSE and of MAE. One of the reasons why this might be the case is that, perhaps, the more complicated model is overfitting the estimation dataset. It is capturing not only the actual relationship between the variables but also some of the “noise”. Therefore, it fails to generalize to unseen data as well as the simpler model.