a) $X_1 = \log(RPK_1)$ of company 1
$X_2 = \log(RPK_2)$ of company 2

For $X_1$:

$$\Delta X_{1t} = \alpha + \beta_1 \Delta X_{1t-1} + \beta_2 \Delta X_{1t-2} + \gamma_1 \Delta X_{2t-1} + \gamma_2 \Delta X_{2t-2} + \varepsilon_t$$

$H_0$: $\Delta X_2$ not granger causal for $\Delta X_1$, i.e. $\gamma_1 = \gamma_2 = 0$

$$\Delta X_{1t} = 0.01 + 0.87 \Delta X_{1t-1} - 0.42 \Delta X_{1t-2} + 0.35 \Delta X_{2t-1} - 0.19 \Delta X_{2t-2} + e_t$$

t-values: $\Delta X_{2t-1} \Rightarrow 1.74$
$\Delta X_{2t-2} \Rightarrow -1.27$

Need F-test!

Under $H_0$ $\Delta X_{1t} = \alpha + \beta_1 \Delta X_{1t-1} + \beta_2 \Delta X_{1t-2}$

$$F = \frac{(R_1^2 - R_0^2)/g}{(1 - R_1^2)/(n-k)}$$

$g = 2$
$n = 38$
$k = 5$

$R_1^2 = 0.476$
$R_0^2 = 0.408$

$F = 2.15$

critical value 5% of $F(2, 33) = 3.3$ As $F = 2.15 < 3.3 \rightarrow$ Do not reject $H_0$

For $X_2$:

$$\Delta X_{2t} = \alpha + \beta_1 \Delta X_{2t-1} + \beta_2 \Delta X_{2t-2} + \gamma_1 \Delta X_{1t-1} + \gamma_2 \Delta X_{1t-2} + \varepsilon_t$$

$H_0$: $\Delta X_1$ not Granger causal for $\Delta X_2$, i.e. $\gamma_1 = \gamma_2 = 0$

Under $H_0$: $\Delta X_{2t} = c + \beta_1 \Delta X_{2t-1} + \beta_2 X_{2t-2}$

$g = 2$, $n = 38$, $k = 5$

$R_1^2 = 0.569$ and $R_0^2 = 0.021$

$$F = \frac{(0.569 - 0.021)/2}{(1 - 0.569)/33} = 20.98 > 3.3 \ (5\% \text{ c.v. } F(2,33))$$

$\rightarrow$ Reject $H_0$

b) i) $\Delta X_{1,t} = \alpha + \beta t + \rho X_{1,t-1} + \gamma \Delta X_{1,t-1} + \varepsilon_t$

$\hat{\rho} = -0.181$ with S.E. $= 0.065$ and t-value $= -2.764$

ADF crit. value $= -3.5$

As $t = -2.764 > -3.5$ $\Rightarrow$ $X_1$ is not stationary

ii) $\hat{\rho} = -0.150$ with S.E. $= 0.125$ and t-value $= -1.207$

ADF crit. value $= -3.5$

As $t = -1.207 > -3.5$ $\Rightarrow$ $X_2$ is not stationary

c) $X_{2t} = 0.012 + 0.919 X_{1t} + e_t$

Test stationarity of $e_t$ by ADF on $e_t$

Result:

$\Delta e_t = 0.000 - 0.496 e_{t-1} + 0.304 \Delta e_{t-1}$ + residual

t-value of $-0.496$ is $-3.5$

critical value for cointegration test in equation without trend $= -3.4$

As $t = -3.5 < -3.4 \Rightarrow$ we reject that $e_t$ is non-stationary

so that $X_{1t}$ and $X_{2t}$ are cointegrated

d) ECM for $X_1$:

$$\Delta X_{1,t} = -0.004 + 1.022\,\Delta X_{1,t-1} + 0.463\,(X_{2,t-1} - 0.92\,X_{1,t-1}) + e_t$$

Note: if $X_{2,t-1}$ is 'too large' as compared to equilibrium, so that $X_{2,t-1} - 0.92X_{1,t-1} > 0$, then $\Delta X_{1,t} > 0$ and hence $X_{1,t} \uparrow$ thereby reducing the gap from the equilibrium

ECM for $X_2$:

$$\Delta X_{2t} = 0.018 + 0.056\,\Delta X_{2t-1} - 0.443\,(X_{2t-1} - 0.92\,X_{1,t-1}) + e_t$$

Note: if $X_{2,t-1}$ is 'too large' as compared to equilibrium, so that $X_{2,t-1} - 0.92\,X_{1,t-1} > 0$, then $\Delta X_{2t} < 0$ and hence $X_{2t} \downarrow$ thereby reducing the gap from the equilibrium