Test Exercise 3

(a) $AIC = \log(s^2) + 2k/m$

→ the lower the AIC, the better

• Preference for the smallest model, implies

$$\log(s_1^2) + 2\frac{p_1}{m} > \log(s_0^2) + 2\frac{p_0}{m}$$

$$2\frac{p_1}{m} - \frac{p_0}{m} > \log(s_0^2) - \log(s_1^2)$$

$$2\frac{p_1}{m} - 2\frac{p_0}{m} > \log(s_0^2/s_1^2)$$

$$e^{2/m \cdot (p_1 - p_0)} > s_0^2/s_1^2$$

(b) $$\frac{s_0^2}{s_1^2} < e^{\frac{2}{m}(p_1 - p_0)}$$

• let $\frac{2}{m}(p_1 - p_0) = z$

as $m \to \infty$, $\frac{2}{m}(p_1 - p_0) \to 0$

then $e^z \approx 1 + z$

$$\frac{s_0^2}{s_1^2} < \frac{2}{m}(p_1 - p_0) + 1$$

$$\underbrace{\frac{s_0^2}{s_1^2} - 1}_{} < \frac{2}{m}(p_1 - p_0)$$

$$\frac{s_0^2}{s_1^2} - 1 = \frac{s_0^2 - s_1^2}{s_1^2}$$

$$\frac{s_0^2 - s_1^2}{s_1^2} < \frac{2}{m}(p_1 - p_0)$$

(c) $\dfrac{\underbrace{s_0^2 - s_1^2}{s_1^2}}_{\frac{s_0^2}{s_1^2} - 1} < \dfrac{2}{n}(p_1 - p_0)$

- let $s_0^2 = \dfrac{RSS_0}{n}$, where RSS_0 is the residual sum of squares of the restricted regression.

- let $s_1^2 = \dfrac{RSS_1}{n}$, where RSS_1 is the residual sum of squares of the unrestricted regression

- then

$$\dfrac{\frac{RSS_0}{n}}{\frac{RSS_1}{n}} - 1 < \dfrac{2}{n}(p_1 - p_0)$$

$$\dfrac{RSS_0}{\cancel{n}} \cdot \dfrac{\cancel{n}}{RSS_1} - 1 < \dfrac{2}{n}(p_1 - p_0)$$

$$\dfrac{RSS_0}{RSS_1} - 1 < 2/n(p_1 - p_0)$$

$$\dfrac{RSS_0 - RSS_1}{RSS_1} < 2/n(p_1 - p_0)$$

- from the lectures we know that:

$\rightarrow RSS_0 = e_R' e_R$

$\rightarrow RSS_1 = e_U' e_U$

then

$$\dfrac{e_R' e_R - e_U' e_U}{e_U' e_U} < \dfrac{2}{n}(p_1 - p_0)$$

(d) $\dfrac{e_R'e_R - e_U'e_U}{e_U'e_U} < \dfrac{2}{n}(p_1 - p_0)$

$$\frac{(e_R'e_R - e_U'e_U)n}{e_U'e_U} < 2(p_1 - p_0)$$

- $$\frac{\dfrac{(e_R'e_R - e_U'e_U)n}{e_U'e_U}}{(p_1 - p_0)} < 2$$

- $$\frac{(e_R'e_R - e_U'e_U)n}{e_U'e_U(p_1 - p_0)} < 2$$

- $$F\text{-test} \equiv \frac{\dfrac{(e_R'e_R - e_U'e_U)}{g}}{\dfrac{(e_U'e_U)}{(n-K)}}$$

- rearranging the terms

$$F\text{-test} \equiv \frac{(e_R'e_R - e_U'e_U)(n-K)}{e_U'e_U\,(g)}$$

- $p_1 - p_0 = g$, because this is the number of linear restrictions

→ moreover as $n \to \infty$, $\dfrac{(n)}{(n-K)} \to 1$

- thus, as sample size grows $\dfrac{(e_R'e_R - e_U'e_U)(n)}{e_U'e_U}$ is approximately equal to an F-test with critical value of 2.