Mooc Econometrics - Test Exercise 5

$$Pr[resp_i = 1] = \frac{\exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 Age_i + \beta_4 (age_i/10)^2)}{1 + \exp(\beta_0 + \beta_1 male_i + \beta_2 active_i + \beta_3 Age_i + \beta_4 (age_i/10)^2)}$$

for $i = 1, \ldots, 925$

(a)

Marginal effect

$$\frac{\partial Pr[resp_i = 1]}{\partial active_i} = Pr[resp_i = 1]\, Pr[resp_i = 0]\, \beta_2$$

Elasticity

$$\frac{\partial Pr[resp_i]}{\partial active_i} \cdot \frac{active_i}{Pr[resp_i = 1]} = Pr[resp_i = 0] . active_i . \beta_2$$

50 yo, active, male

$$Pr[resp_i = 1] = \frac{\exp(-2.48 + (0.954)(1) + (0.914)(1) + (0.07)(50) + (-0.069)(25))}{1 + \exp(-2.48 + (0.954)(1) + (0.914)(1) + (0.07)(50) + (-0.069)(25))}$$

$Pr[resp_i = 1] = 0.7611$

$Pr[resp_i = 0] = 1 - 0.7611 = 0.2389$

Elasticity $= 0.2389(1)(0.914) \approx 0.2183$ //

50 yo, inactive, male

$$Pr[resp_i = 1] = \frac{\exp(-2.48 + (0.954)(1) + (0.914)(0) + (0.07)(50) + (-0.069)(25))}{1 + \exp(-2.48 + (0.954)(1) + (0.914)(0) + (0.07)(50) + (-0.069)(25))}$$

$Pr[resp_i = 1] = 0.5610$

$Pr[resp_i = 0] = 1 - 0.5610 = 0.4398$

Elasticity $= 0.4398(0)(0.914) = 0$ //

(b) $\dfrac{Pr[resp_i=1 \mid Active_i=1] - Pr[resp_i=1 \mid Active_i=0]}{Pr[resp_i=1 \mid Active_i=0]}$

i) $\dfrac{Pr[resp_i=1 \mid Active_i=0]}{Pr[resp_i=1 \mid Active_i=0]} = 1$

ii) $\dfrac{Pr[resp_i=1 \mid Active_i=1]}{Pr[resp_i=1 \mid Active_i=0]}$

$$= \dfrac{\dfrac{\exp(\beta_0+\beta_1+\beta_2+\beta_3+\beta_4)}{1+\exp(\beta_0+\beta_1+\beta_2+\beta_3+\beta_4)}}{\dfrac{\exp(\beta_0+\beta_1+\beta_3+\beta_4)}{1+\exp(\beta_0+\beta_1+\beta_3+\beta_4)}}$$

$$= \frac{\exp(\beta_0+\beta_1+\beta_2+\beta_3+\beta_4)(1+\exp(\beta_0+\beta_1+\beta_3+\beta_4))}{\exp(\beta_0+\beta_1+\beta_3+\beta_4)(1+\exp(\beta_0+\beta_1+\beta_2+\beta_3+\beta_4))}$$

$$= \frac{\exp(\beta_2)(1+\exp(\beta_0+\beta_1+\beta_3+\beta_4)}{(1+\exp(\beta_0+\beta_1+\beta_2+\beta_3+\beta_4))}$$

$$= \frac{\exp(\beta_2)+\exp(\beta_0+\beta_1+\beta_2+\beta_3+\beta_4)}{1+\exp(\beta_0+\beta_1+\beta_2+\beta_3+\beta_4)}$$

$$= \underbrace{\frac{\exp(\beta_2)}{1+\exp(\beta_0+\beta_1+\beta_2+\beta_3+\beta_4)}}_{(a)} + \underbrace{\frac{\exp(\beta_0+\beta_1+\beta_2+\beta_3+\beta_4)}{1+\exp(\beta_0+\beta_1+\beta_2+\beta_3+\beta_4)}}_{(b)}$$

(b) $\exp(\beta_2) \cdot \dfrac{1}{1+\exp(\beta_0+\beta_1+\beta_2+\beta_3+\beta_4)}$

$\exp(\beta_2) \cdot Pr[resp_i=0 \mid Active_i=1]$

(a) $\dfrac{\exp(\beta_0+\beta_1+\beta_2+\beta_3+\beta_4)}{1+\exp(\beta_0+\beta_1+\beta_2+\beta_3+\beta_4)} = Pr[resp_i=1 \mid Active_i=1]$

$$\Rightarrow \frac{Pr[resp_i = 1 \mid Active_i = 1] - Pr[resp_i = 1 \mid Active_i = 0]}{Pr[resp_i = 1 \mid Active_i = 0]}$$

$$= \exp(\beta_2)\, Pr[resp_i = 0 \mid Active_i = 1] - \underbrace{Pr[resp_i = 1 \mid Active_i = 1] - 1}$$

$$Pr[resp_i = 1 \mid Active_i = 1] - 1 = -Pr[resp_i = 0 \mid Active_i = 1]$$

$$= \exp(\beta_2)\, Pr[resp_i = 0 \mid Active_i = 1] - Pr[resp_i = 0 \mid Active_i = 1]$$

$$= (\exp(\beta_2) - 1)\, Pr[resp_i = 0 \mid Active_i = 1]$$

(c) $\dfrac{Pr[respi=1 \mid active\ i=1] - Pr[respi=1 \mid Active\ i=0]}{Pr[respi=1=1 \mid Active\ i=0]}$

- $Pr[respi=1 \mid Active\ i=1]$ for 50yo / Male / Active
$= 0.7611$ (from exercise (a))

- $Pr[respi=1 \mid Active\ i=0]$ for 50yo / Male / Active
$= 0.5610$ (from exercise (a))

$= \dfrac{0.7611 - 0.5610}{0.5610} = 0.3566845 \approx 35.66\%$