

A memetic algorithm with adaptive operator selection for graph coloring

AHEAD (Adaptive HEAD) Algorithm

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Studied Problems

GCP - Graph Coloring Problem

Graph Coloring

Objective: find a legal color that minimizes the number of colors

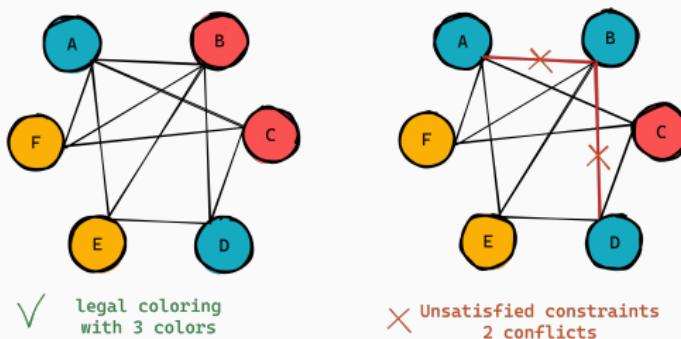
Score :

- Number of colors k (legal)
- Number of conflicts $|C|$ (illegal)
- Number of uncolored vertices $|U|$ (partially legal)

NP-Hard problem

Applications :

- Scheduling problems
- Register allocation
- Sudoku
- ...



WVCP - Weighted Vertex Coloring Problem

Weighted Vertex Coloring

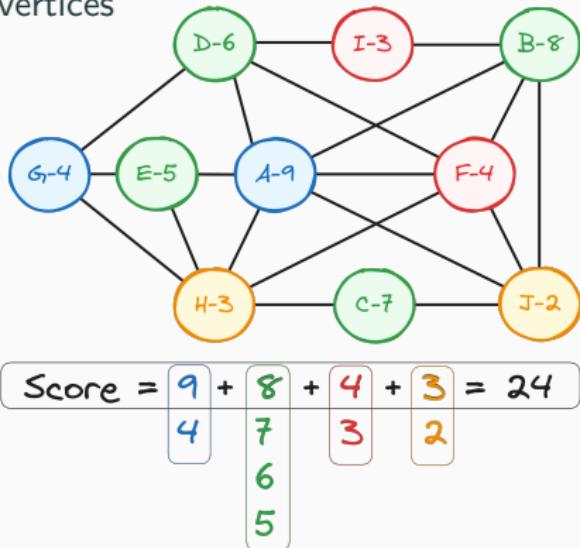
Objective: find a legal coloring that minimizes the sum of the weights of the heaviest vertices in each color

Score : $\sum_{i=1}^k \max_{v \in V_i} w(v)$

NP-Hard problem

Applications :

- Traffic management in satellite communications
- Matrix decomposition problem
- Scheduling batch job in parallel



WVCP - Scheduling Batch Job in Parallel

<p>8 Jobs</p> <p>J1 - 9s J2 - 8s J3 - 8s J4 - 6s J5 - 5s J6 - 5s J7 - 4s J8 - 2s</p> <p>3 Resources</p> <p>1 - Prepare the jobs in a bipartite graph (jobs - resources)</p>	<p>2 - Projection of the bipartite graph onto the resources to obtain a common needs graph</p>	<p>3 - Use the time of each task as a weight for each vertex</p>															
<p>optimal score = $9 + 8 + 6 + 2 = 25$</p> <p>4 - Solve the problem by minimizing the sum of the maximum weights of each color</p>	<p>4 Batches</p> <table border="1"> <tr> <td>B1 - 9s</td> <td>B2 - 8s</td> <td>B3 - 6s</td> <td>B4 - 2s</td> <td>Total : 25s</td> </tr> <tr> <td>J1 - 9s J3 - 8s J5 - 5s</td> <td>J2 - 8s</td> <td>J4 - 6s J6 - 5s J7 - 4s</td> <td>J8 - 2s</td> <td></td> </tr> <tr> <td>R1 R2 R3</td> <td>R1 R2 R3</td> <td>R1 R2 R3</td> <td>R1</td> <td></td> </tr> </table> <p>8 Jobs</p> <p>3 Resources</p>	B1 - 9s	B2 - 8s	B3 - 6s	B4 - 2s	Total : 25s	J1 - 9s J3 - 8s J5 - 5s	J2 - 8s	J4 - 6s J6 - 5s J7 - 4s	J8 - 2s		R1 R2 R3	R1 R2 R3	R1 R2 R3	R1		<p>5 - Prepare the batches according to the color of each job</p>
B1 - 9s	B2 - 8s	B3 - 6s	B4 - 2s	Total : 25s													
J1 - 9s J3 - 8s J5 - 5s	J2 - 8s	J4 - 6s J6 - 5s J7 - 4s	J8 - 2s														
R1 R2 R3	R1 R2 R3	R1 R2 R3	R1														

State of the Art

State of the Art - GCP

- Local Search:
 - **TabuCol** Hertz et Werra [1987] : illegal, one-move
 - **PartialCol** Blöchliger et Zufferey [2008] : partial legal, grenade
 - **ILS** Chiarandini et Stützle [2002] : perturbations, acceptance criteria
- Memetic Algorithms :
 - **HEA** Galinier et Hao [1999] : GPX, TabuCol
 - **Evo-Div** Porumbel *et al.* [2010] : multi-parents crossover, distances
 - **MACOL** Lü et Hao [2010] : multi-parents crossover, distances
 - **HEAD** Moalic et Gondran [2018] : 2 individuals, GPX, TabuCol
 - **DLMCOL** Goudet *et al.* [2022] : +20 000, NN select crossover
 - **AHEAD** Grelier *et al.* [2024] : Adaptive HEAD
- Learning :
 - **PLSCOL** Zhou *et al.* [2018] : local search, reinforcement learning
 - **TensCol** Goudet *et al.* [2021] : tensor, gradient descent
 - **NRPA** Cazenave *et al.* [2021] : MCTS, sequence, gradient descent

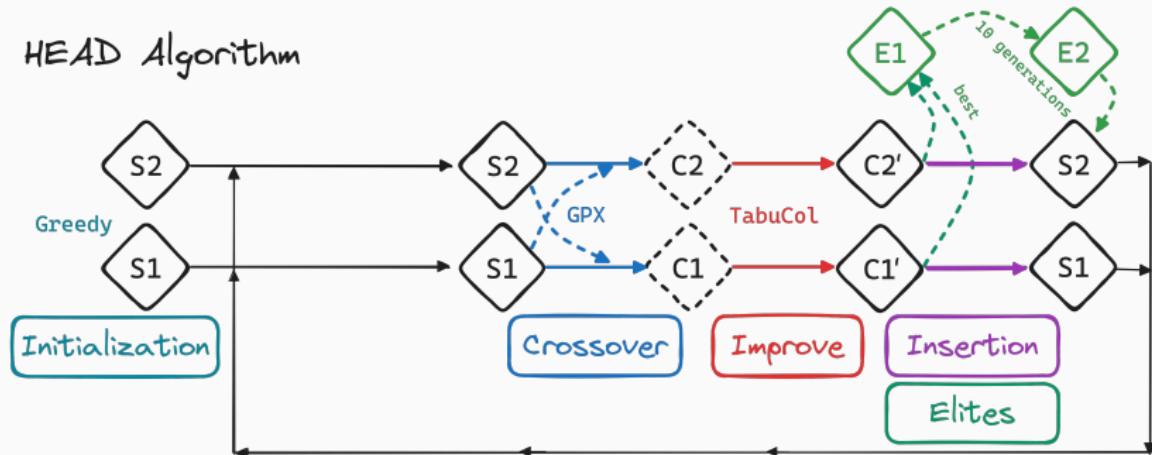
State of the Art - WVCP

- Learning :
 - **MCTS + Local Search** Grelier *et al.* [2022] LS as simulation
 - **MCTS + Hyperheuristics** Grelier *et al.* [2023] : select LS
- Memetic Algorithms :
 - **DLMCOL** Goudet *et al.* [2022] : +20000, NN select crossover
 - **AHEAD** Grelier *et al.* [2024] : Adaptive HEAD
- Local Search :
 - **AFISA** Sun *et al.* [2018] : illegal, one-move, adaptive coefficient
 - **RedLS** Wang *et al.* [2020] : illegal, weighted edges, perturbations
 - **ILS-TS** Nogueira *et al.* [2021] : p-legal, 6 neighbors, perturbations
 - **TabuWeight** Grelier *et al.* [2022] : legal, one-move
- Exact Methods :
 - **2-Phase** Malaguti *et al.* [2009] : column generation, ILP
 - **MWSS** Cornaz *et al.* [2017] : MIP, max weight stable set problem
 - **CP** Goudet *et al.* [2023] : 3 CP models, reduction

Memetic Algorithm – AHEAD

HEAD - Hybrid Evolutionary Algorithm in Duet

HEAD Algorithm



Moalic et Gondran [2018] – Variations on memetic algorithms for graph coloring problems

Why HEAD?

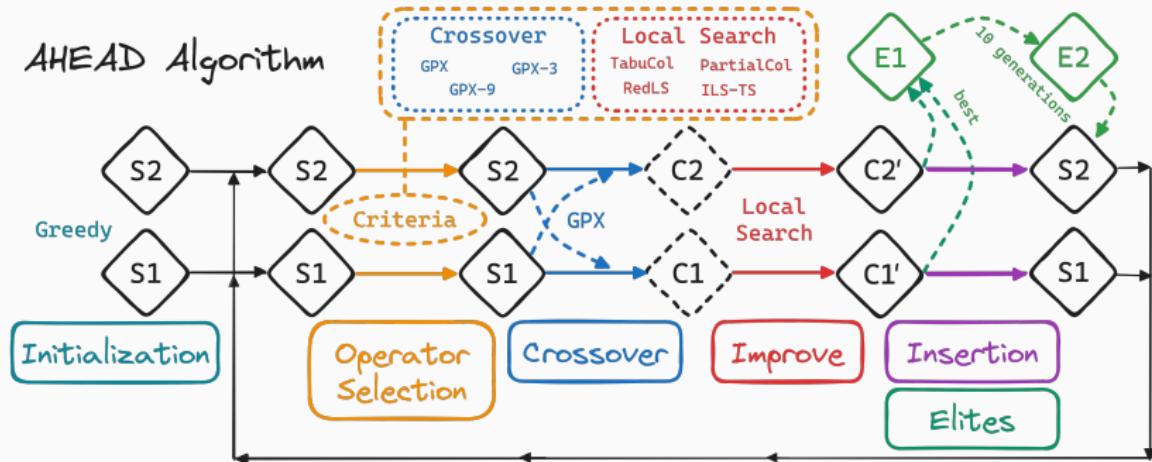
- One of the best algorithm for GCP
- Simple and efficient

How?

- 2 individuals
- GPX
- TabuCol

AHEAD - Adaptive Hybrid Evolutionary Algorithm in Duet

AHEAD Algorithm



Why AHEAD?

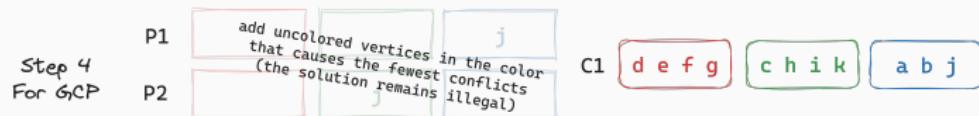
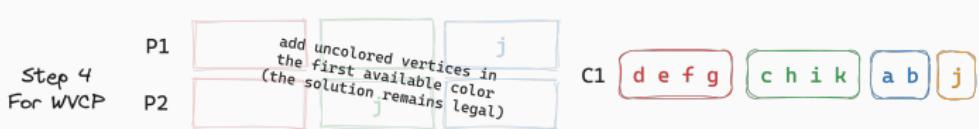
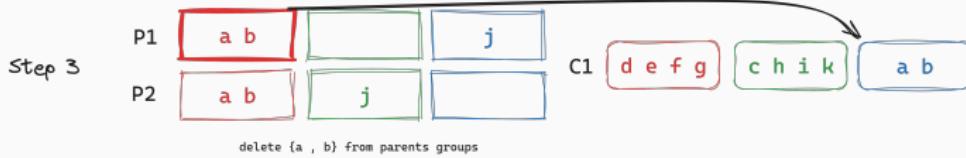
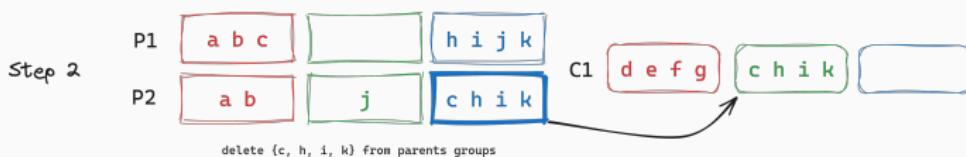
- Attempt to improve HEAD
- Adapt to the instance

How?

- Hyperheuristics
- Multiple crossover
- Multiple local search

GPX - Galinier et Hao [1999]

GPX crossover - Greedy Partition Crossover



AHEAD - Operators

Crossover

- **GPX** : 1 color in P1 for 1 color in P2
- **GPX-3** : 3 colors in P1 for 1 color in P2
- **GPX-9** : 9 colors in P1 for 1 color in P2

Local Search - GCP

- **TabuCol** : Hertz et Werra [1987]
- **PartialCol** : Blöchliger et Zufferey [2008]

Local Search - WVCP

- **RedLS** : Wang *et al.* [2020]
- **ILS-TS** : Nogueira *et al.* [2021]

Why?

- Diversification during the search
- No best operator on all instances
- Adapt to the instance

How?

- Selection criteria : Learn to select the best operator
- Reward : Score of the solution after the local search
- Selection : pair <crossover, local search>
- Exception : NN : Generate all crossovers and select the best one

Hyperheuristics - Criteria

Selection criteria

- **Random** Uniform random choice
- **Deleter** Delete the least performing operators (o)
- **Roulette** Goëffon *et al.* [2016] Random selection weighted by rewards (r)

$$proba[o] = p_{min} + (1 - |O| * p_{min}) * \frac{r[o]}{\sum r}$$

- **Pursuit** Goëffon *et al.* [2016] Selection in favor of the best operator (b)

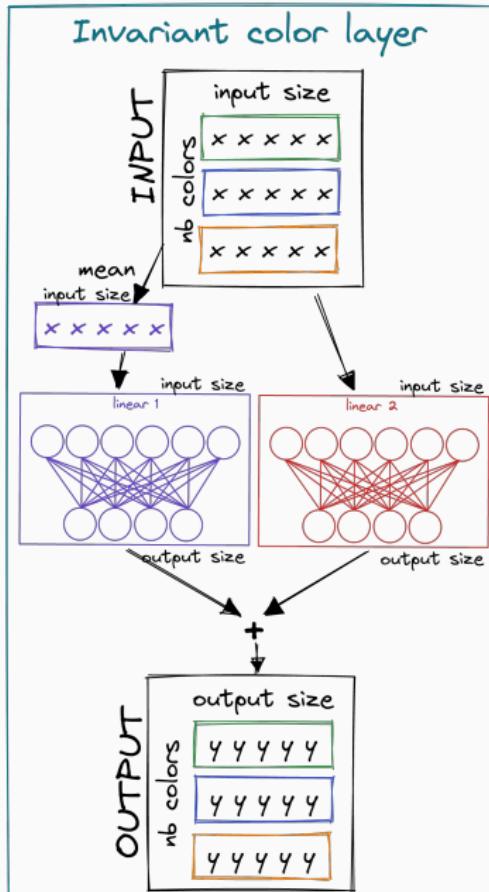
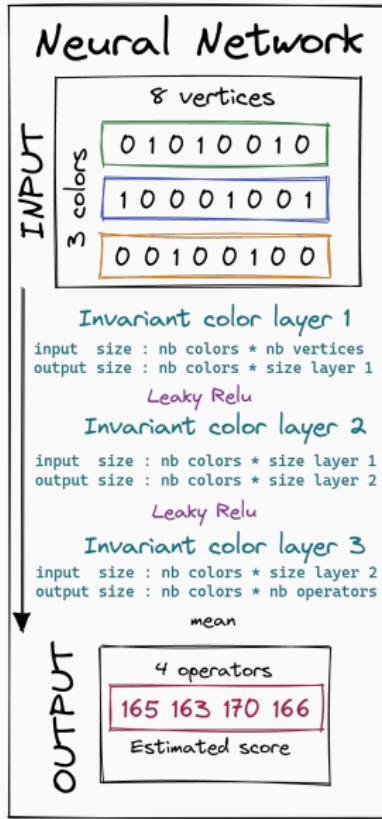
$$\begin{cases} proba[b] = proba[b] + \beta(p_max - proba[b]) \\ proba[o] = proba[o] + \beta(p_min - proba[o]) \end{cases}$$

- **UCB** : Focusing on the best while encouraging exploration

$$score[o] = r[o] + c * \sqrt{2 * \frac{\log(\sum visits)}{visits[o]}}$$

- **NN** : Recommendation of a neural network on a raw solution with Deep Sets (Zaheer *et al.* [2017])

Neural Network - NN - Deep sets



AHEAD - Results WVCP

1 point/instance if the average is significantly better for the method on the line compared to the one in the column (Wilcoxon signed-rank, p-value < 0.001)

/48	MCTS+UCB	RedLS	ILS-TS	HEAD+RedLS	HEAD+ILS-TS	Random Roulette Deleter UCB Pursuit NN						# BKS	# Best	# Best Avg		
						MCTS+UCB	RedLS	ILS-TS	HEAD+RedLS	HEAD+ILS-TS	Random	Roulette	Deleter	UCB	Pursuit	NN
MCTS+UCB	-	25	15	3	20	0	0	0	0	0	0	0	0	0	0	20
RedLS	11	-	10	9	14	9	9	9	9	9	9	9	9	9	9	15
ILS-TS	10	27	-	8	19	3	6	5	4	1	3	23	25	21	23	25
HEAD+RedLS	16	26	15	-	25	1	1	0	0	1	0	19	19	11	19	19
HEAD+ILS-TS	5	20	6	5	-	0	0	0	0	0	0	18	19	13	18	19
Random	19	27	20	10	25	-	0	0	0	0	0	21	22	19	21	22
Roulette	17	26	20	9	26	0	-	0	0	0	0	22	22	17	22	22
Deleter	20	26	19	9	26	3	0	-	0	0	0	24	28	19	24	28
UCB	20	26	20	9	26	1	1	0	-	0	0	23	23	19	23	23
Pursuit	19	26	23	11	26	1	0	0	0	-	0	24	26	22	24	26
NN	20	27	21	10	27	0	1	0	0	0	-	21	23	19	21	23

AHEAD - Results GCP

1 point/instance if the average is significantly better for the method on the line compared to the one in the column (Wilcoxon signed-rank, p-value < 0.001)

/31	PartialCol	TabuCol	HEAD+PC	HEAD+TC	Random	Roulette	Deleter	UCB	Pursuit	NN	# BKS	# Best	# Best Avg
PartialCol	-	2	3	2	1	2	1	2	2	2	5	8	11
TabuCol	14	-	11	2	2	1	0	2	0	1	8	14	7
HEAD+PC	8	6	-	1	0	0	1	0	0	0	6	10	7
HEAD+TC	18	12	20	-	4	2	1	2	2	2	7	17	15
Random	17	11	19	1	-	0	1	1	0	0	9	17	9
Roulette	17	11	19	1	0	-	0	0	0	0	11	19	12
Deleter	19	15	20	5	8	3	-	5	1	1	13	24	20
UCB	19	11	20	1	1	0	0	-	0	0	10	18	10
Pursuit	19	13	20	3	5	2	0	1	-	0	11	20	14
NN	19	12	20	2	4	0	0	0	0	-	12	23	16

AHEAD - Conclusion

About the selection criteria

- No changes in the selections
- More conservative GPX Crossover often preferred but without very large differences
- Local Search more important

To summarize

- Local Search alone keeps some instances with better results
- Better results with AHEAD than other methods in general
- Deleter often better with new best scores :
 - **GCP** : C2000.9 (404)
 - **WVCP** : le450_15a (211), le450_15b (215), queen14_14 (214)

Thank you for your attention !

Questions ?

Source code, results tables, articles :



<https://cyril-grelier.github.io/>

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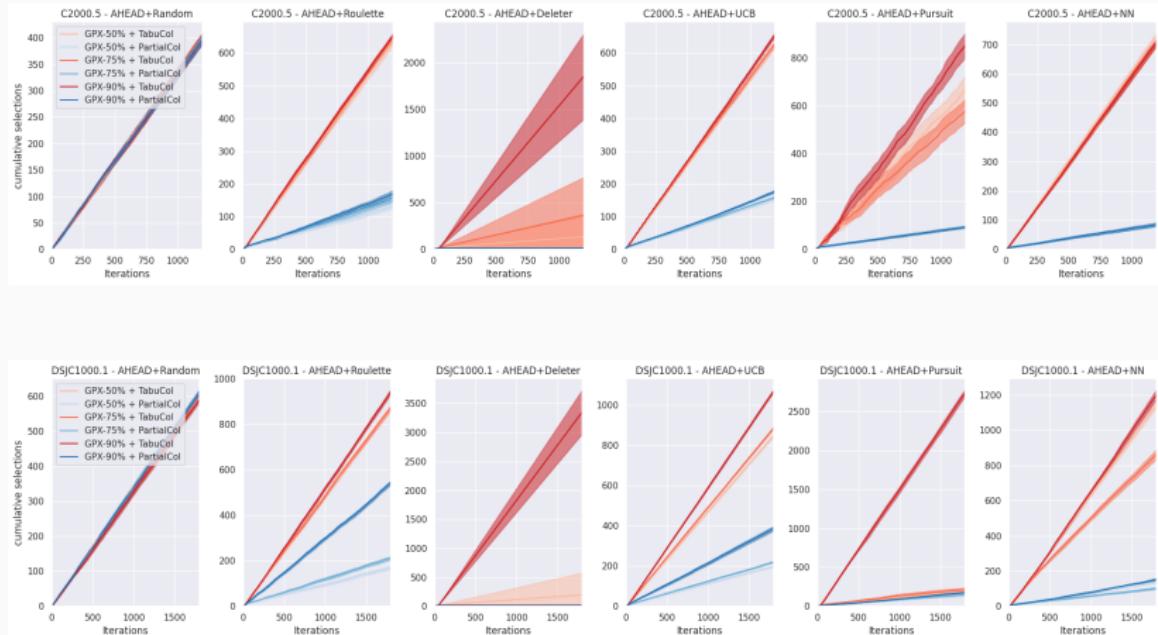
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AHEAD

AHEAD - Results GCP

instance	BKS	PartialCol			TabuCol			HEAD+TC			AHEAD+Random			AHEAD+Deleter		
		best	mean	time	best	mean	time	best	mean	time	best	mean	time	best	mean	time
C2000.5	145	164	165.2	5313	162	162.8	4628	148	149.2	3330	150	150.7	3101	149	150.7	3152
C2000.9	408	420	420.8	5171	411	412.5	4786	<u>405</u>	406.4	2328	<u>405</u>	407.7	2956	404	405.6	2988
C4000.5	259	304	305.6	6690	303	304.2	5567	278	279.6	3580	280	281.6	3651	279	280.8	3404
DSJC500.1	12	12	128	12		75		12		86	12		80	12		56
DSJC500.5	47	50	50.1	2227	49		460	48		819	48		1258	48		850
DSJC500.9	126	128		975	126	126.3	2988	126		1027	126	126.1	1379	126		632
DSJC1000.1	20	21		1	21		0	21		0	21		1	20	20.9	2391
DSJC1000.5	82	90	90.5	3516	88		1760	83	83.3	2290	83	83.5	2372	83	83.5	2511
DSJC1000.9	222	227	228.4	3630	224	224.9	3345	223		224	223	224.2	2734	223	223.8	1589
DSJR500.5	122*	125	126.2	1666	124	127	1155	123		124	123	124.2	2245	123	123.8	2289
flat300_28_0	28*	28		896	28	29.5	3220	30	30.8	1916	28	28.5	702	28	30.4	5
flat1000_50_0	50*	50		44	50		69	50		28	50		8	50		8
flat1000_60_0	60*	60		213	60		233	60		54	60		28	60		29
flat1000_76_0	76*	89	89.1	2845	86	87	3096	82	82.3	1905	82	82.8	2775	82	82.8	1969
latin_square_10	97	107	110.2	4875	100	100.8	4377	102	103.7	93	103	103.8	1996	99	100.7	1729
le450_25c	25*	27		69	26		0	26		0	25	25.9	1407	25	25.3	1022
le450_25d	25*	27		50	26		0	26		0	26		0	25	25.3	1537
r250.5	65*	67		134	66	67.2	462	65	66	3378	65	66	1638	66		549
r1000.1c	98	141	149.1	61	134	155.2	77	100	101.6	264	100	101.6	1674	100	101.6	1621
r1000.5	234	247	248.1	5638	244	245.6	3622	246	247.6	1479	246	247.4	2134	245	245.5	2009
wap01a	41*	42		1088	42	43	2160	42		137	42		143	41	42	1958
wap02a	40*	41	41.7	4275	40	41.1	6499	41		15	41		15	40	40.8	1634
wap03a	43	44		91	44	45.9	4342	45		261	45		87	43	44.3	2387
wap04a	41	43		61	42	43.1	4869	43		880	43		1186	43		293
wap06a	40*	41		98	40	41.3	4248	40		909	40	40.8	1549	40		246
wap07a	41	44		41	41	42.3	5046	42	42.1	1771	42	43	2526	42	42.1	494
wap08a	40*	43	43.2	2750	41	41.5	2967	42		48	42		365	41	41.9	2146
#BKS		5/31			8/31			7/31			9/31			13/31		
#Best		8/31			14/31			17/31			17/31			24/31		
#Best Avg		11/31			7/31			15/31			9/31			20/31		

AHEAD GCP - Selections



AHEAD - Results WVCP

instance	BKS	RedLS			ILS-TS			HEAD+RedLS			AHEAD+Random			AHEAD+Deleter		
		best	mean	time	best	mean	time	best	mean	time	best	mean	time	best	mean	time
C2000.5	2144	2131	2155.7	18367	2244	2264.4	6423	2244	2257.9	7453	2220	2236.8	12962	2218	2236.3	1782
C2000.9	5477	5439	5455.1	23137	5847	5910.1	23014	5732	5748.2	12980	5732	5783.9	12491	5717	5758.8	12327
DSJC1000.1	300	303	306.9	5839	305	306.2	5819	304	305.6	7380	302	303.8	9348	300	302.2	12874
DSJC1000.5	1185	1190	1206.9	12204	1241	1267.7	21935	1225	1229.7	7011	1222	1228.2	5371	1224	1230.5	1476
DSJC1000.9	2836	2828	2841.8	22796	3004	3035.9	25345	2909	2926.5	820	2911	2928.7	12633	2907	2926.8	2379
DSJC500.1	184	187	194	702	185	187.3	7107	186	186.9	6594	185	186.5	10290	184	185.9	8022
DSJC500.5	685	707	712.5	27147	711	721.2	9150	709	712.6	2534	706	711.5	12516	709	713.5	5838
DSJC500.9	1662	1667	1671	9925	1709	1725.3	24351	1680	1683.5	4053	1678	1684.2	12644	1676	1682.8	8149
DSJC250.1	127	129	131.4	56	127	127.1	11901	127	4516	127	3729	127	127.2	3235		
DSJC250.5	392	399	400.8	2602	392	393.9	10722	395	396.2	8349	393	395.2	9592	392	396.6	6028
DSJC250.9	934*	934	935	9679	934	935.1	14740	934	935.1	6741	934	934.2	8097	934	935	5011
flat1000_50_0	924	1152	1165.7	6259	1213	1230.5	570	1181	1187.7	7544	1179	1186.3	4428	1180	1186.8	2952
flat1000_60_0	1162	1196	1204.8	1877	1247	1263.8	25765	1216	1227.2	10824	1213	1223.7	11726	1217	1224.5	9840
flat1000_76_0	1165	1163	1183.2	28084	1228	1242.2	16513	1192	1204	2214	1187	1203	10742	1196	1204	8938
latin_square_10	1480	1505	1515.3	14189	1555	1575	18924	1523	1532.5	11286	1510	1526.2	13987	1517	1527.8	8732
le450_15a	212	213	215.4	54	211	213.6	11684	212	212.8	6777	212	212.8	8819	211	212.4	10557
le450_15b	216	218	219.9	41	217	217.1	10346	216	217	3204	216	217.1	2736	215	216.5	11124
le450_15c	275	282	285.4	82	279	281.7	16288	277	279.4	8360	277	278.8	7220	278	279.4	4788
le450_15d	272	277	280.6	325	275	277.6	8456	274	276.1	6004	274	275.6	8759	273	275.2	13299
le450_25a	306	306	306.6	2881	306	306	142	306	161	306	169	306	131			
le450_25b	307*	307	307.6	95	307	307	23	307	53	307	28	307	19			
le450_25c	342	348	352.8	583	348	349.1	16413	347	348.1	180	346	347.8	5652	346	348	588
le450_25d	330	335	339.4	232	337	338.7	14212	333	334.4	5904	333	334.2	6282	333	334.2	9648
queen14_14	215	218	223.8	568	215	216.4	9862	216	216.6	7956	215	216.2	6384	214	215.3	8624
wap01a	545	557	577	995	547	550.1	20531	552	559.1	8178	549	553.6	14094	549	552.8	8874
wap02a	538	554	572.1	16183	536	541	21912	550	557.1	13884	541	546.1	7654	541	545.5	12994
wap03a	562	569	575.5	17878	572	575.5	22637	577	579.7	6992	573	576.3	8096	573	575.9	2944
wap04a	563	567	578.9	13939	567	570.5	7346	573	575.6	3152	570	573.2	1970	569	572.5	13790
wap05a	541	542	543.8	7719	542	542.2	11809	542	542.9	4471	542	543	12056	542	543.2	2772
wap06a	516	519	526.1	1575	516	519.5	6264	519	520.7	12180	518	521	9100	520	521.2	5978
wap07a	555	554	573	8460	565	569.2	16299	557	559.4	3360	558	559.8	12040	557	559.2	12460
wap08a	529	536	543.7	19557	543	546.9	19271	539	540.8	7452	539	541.2	1800	538	540.1	10608
#BKS		15/48			23/48			19/48			21/48			24/48		
#Best		24/48			25/48			19/48			22/48			28/48		
# Best Avg		11/48			21/48			11/48			19/48			19/48		

AHEAD WVCP - Selections

