Deep Reasoning

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References

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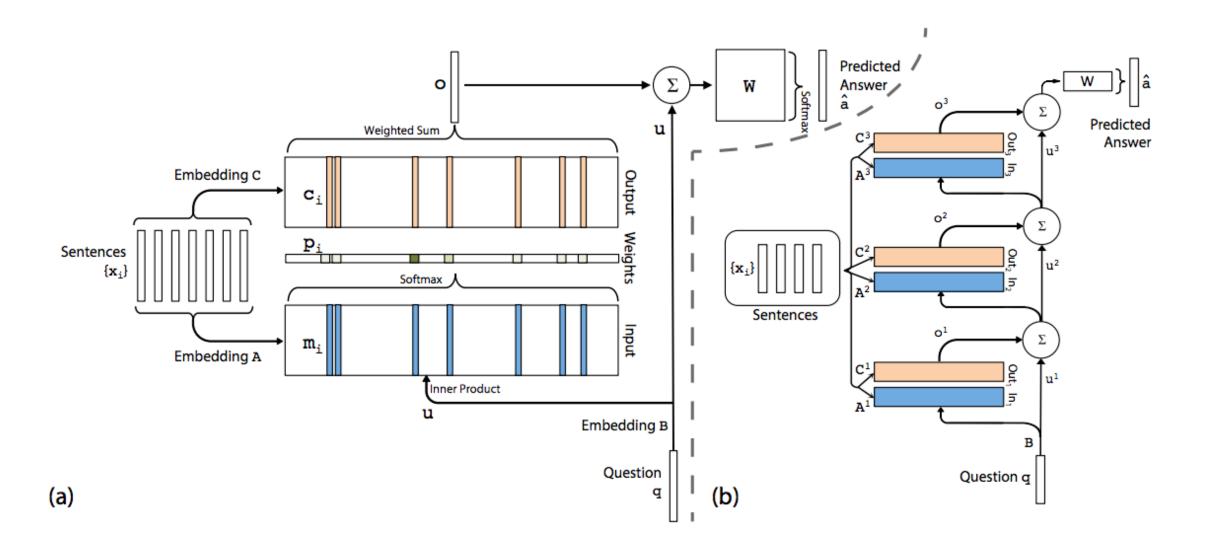
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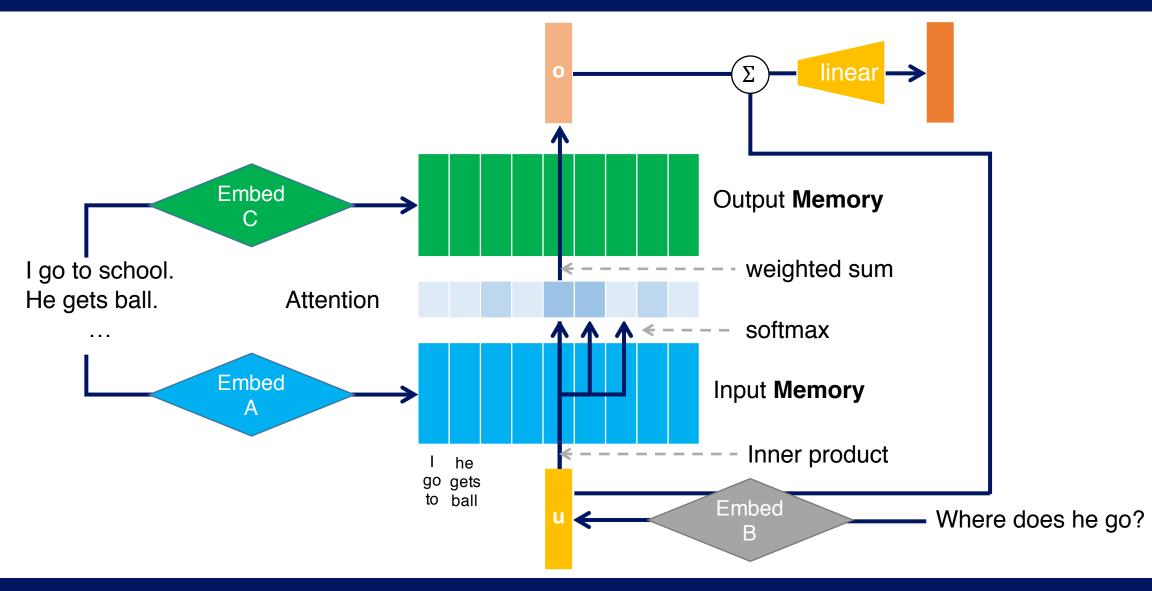
Models

Answer selection (WikiQA)	General QA (CNN)	Considered transitive inference (bAbI)
ABCNN	E2E MN	E2E MN
Variational	Impatient Attentive Reader	DMN
Attentive Pooling	Attentive (Impatient) Reader	ReasoningNet
	Attention Sum Reader	NTM

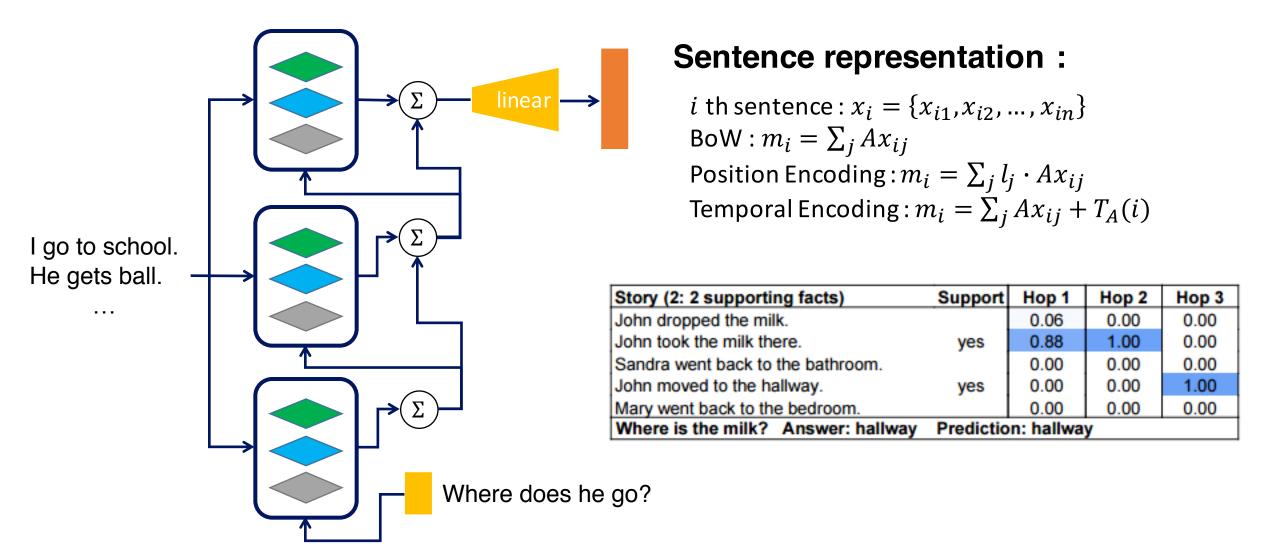
End-to-End Memory Network [Sukhbaatar, 2015]



End-to-End Memory Network [Sukhbaatar, 2015]



End-to-End Memory Network [Sukhbaatar, 2015]



Linear Start (LS) help avoid local minima

- First train with softmax in each memory layer removed, making the model entirely linear except for the final softmax
- When the validation loss stopped decreasing, the softmax layers were re-inserted and training recommenced

RNN-style layer-wise weight tying

- The input and output embeddings are the same across different layers

Learning time invariance by injecting random noise

- Jittering the time index with random empty memories
- Add "dummy" memories to regularize $T_A(i)$

Example of bAbl tasks

Story (1: 1 supporting fact)	Support	Hop 1	Hop 2	Hop 3
Daniel went to the bathroom.		0.00	0.00	0.03
Mary travelled to the hallway.		0.00	0.00	0.00
John went to the bedroom.		0.37	0.02	0.00
John travelled to the bathroom.	yes	0.60	0.98	0.96
Mary went to the office.		0.01	0.00	0.00
Where is John? Answer: bathroom	n Prediction: bathroom			

Story (16: basic induction)	Support	Hop 1	Hop 2	Hop 3
Brian is a frog.	yes	0.00	0.98	0.00
Lily is gray.		0.07	0.00	0.00
Brian is yellow.	yes	0.07	0.00	1.00
Julius is green.		0.06	0.00	0.00
Greg is a frog.	yes	0.76	0.02	0.00
What color is Greg? Answer: yellow	Prediction: yellow			

Story (2: 2 supporting facts)	Support	Hop 1	Hop 2	Hop 3
John dropped the milk.		0.06	0.00	0.00
John took the milk there.	yes	0.88	1.00	0.00
Sandra went back to the bathroom.		0.00	0.00	0.00
John moved to the hallway.	yes	0.00	0.00	1.00
Mary went back to the bedroom.		0.00	0.00	0.00
Where is the milk? Answer: hallway	Prediction: hallway			

Story (18: size reasoning)	Support	Hop 1	Hop 2	Hop 3
The suitcase is bigger than the chest.	yes	0.00	0.88	0.00
The box is bigger than the chocolate.		0.04	0.05	0.10
The chest is bigger than the chocolate.	yes	0.17	0.07	0.90
The chest fits inside the container.		0.00	0.00	0.00
The chest fits inside the box.		0.00	0.00	0.00
Does the suitcase fit in the chocolate? Answer: no Prediction: no				

```
S: 1 Mr. Cropper was opposed to our hiring you .
   2 Not , of course , that he had any personal objection to you , but he is set
   against female teachers , and when a Cropper is set there is nothing on earth can
   change him .
   3 He says female teachers ca n't keep order .
   4 He 's started in with a spite at you on general principles , and the boys know
   5 They know he 'll back them up in secret , no matter what they do , just to prove
   his opinions .
   6 Cropper is sly and slippery , and it is hard to corner him . ''
   7 `` Are the boys big ? ''
   8 queried Esther anxiously .
   9 `` Yes .
   10 Thirteen and fourteen and big for their age .
   11 You ca n't whip 'em -- that is the trouble .
   12 A man might , but they 'd twist you around their fingers .
   13 You 'll have your hands full , I 'm afraid .
   14 But maybe they 'll behave all right after all . ''
   15 Mr. Baxter privately had no hope that they would , but Esther hoped for the
   16 She could not believe that Mr. Cropper would carry his prejudices into a
   personal application .
   17 This conviction was strengthened when he overtook her walking from school the
   next day and drove her home .
   18 He was a big , handsome man with a very suave , polite manner .
   19 He asked interestedly about her school and her work , hoped she was getting on
   well , and said he had two young rascals of his own to send soon .
   20 Esther felt relieved .
Q: She thought that Mr. had exaggerated matters a little .
C: Baxter, Cropper, Esther, course, fingers, manner, objection, opinion, right, spite.
a: Baxter
```

- Context sentences : $S = \{s_1, s_2, ..., s_n\}, s_i : BoW word representation$
- Encoded memory : $m_i = \phi(s) \forall s \in S$
- Lexical memory
 - Each word occupies a separate slot in the memory
 - s is a single word and $\phi(s)$ has only one non-zero feature
 - Multiple hop only beneficial in this memory model
- Window memory (best)
 - s corresponds to a window of text from the context s centered on an individual mention of a candidate s in s $m_i = \{w_{i-(b-1)/2} \dots w_i \dots w_{i+(b-1)/2}\}$
 - Where $w_i \in C$ which is an instance of one of the candidate words
- Sentential memory
 - Same as original implementation of Memory Network

Self-supervision for window memories

- Memory supervision (knowing which memories to attend to) is not provided at training time
- Making gradient steps using SGD to **force** the model to give a **higher score to the supporting** memory \tilde{m} relative to any other memory from any other candidate using:

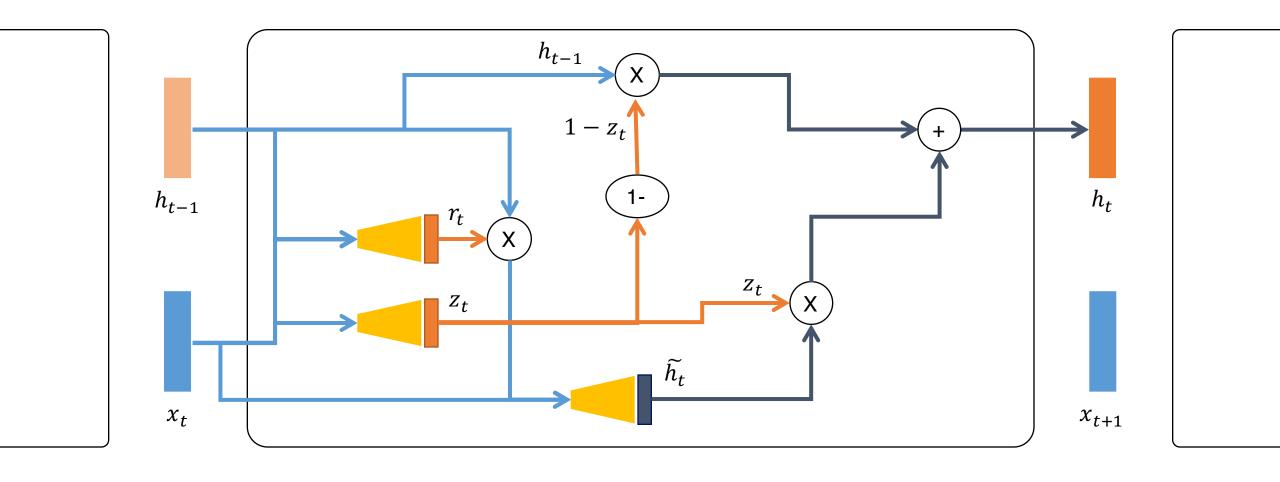
Hard attention (training and testing) :
$$m_{o1} = \underset{i=1,...,n}{\operatorname{argmax}} c_i^T q$$

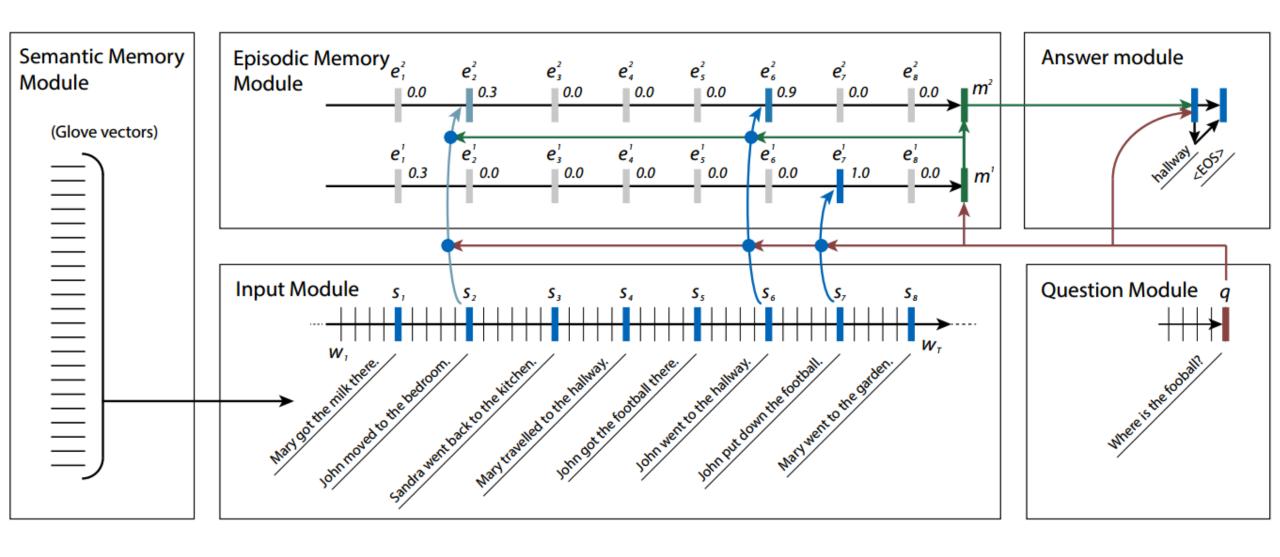
Soft attention (testing) :
$$m_{o1} = \sum_{i=1...n} \alpha_i m_i$$
, with $\alpha_i = \frac{e^{c_i^T q}}{\sum_i e^{c_i^T q}}$

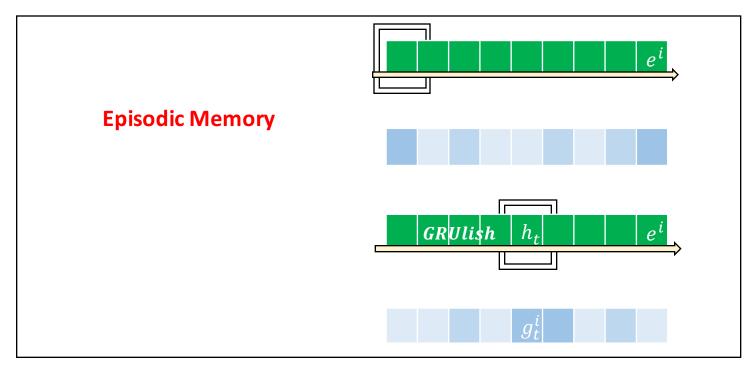
- If m_{o1} happens to be different from \widetilde{m} (memory contain true answer), then model is updated
- Can be understood as **a way of achieving** *hard attention* **over memories** (no need any new label information beyond the training data)

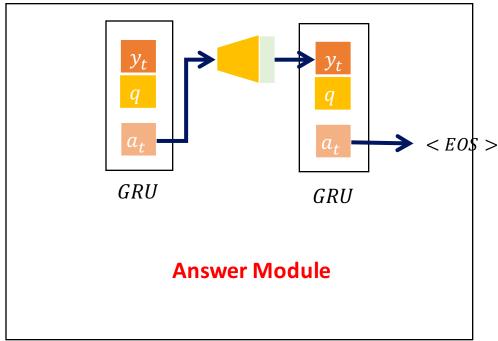
```
S: 1 (He thought that Old)Mr. Toad was trying to fool him .
S: 1 So they had to fall (a long way .
   2 So they got their tails fast (in their mouths .
                                                                                2 Presently Peter Rabbit came along
   3 So they could n't get them out again .
                                                                                3 He found Jimmy Skunk sitting in a brown study .
   4 That 's all . '
                                                                                4 He had guite forgotten to look for fat beetles , and when he (forgets to do
      Thank you , (' said Alice , ` it )'s very interesting .
                                                                               (that you) may make up your mind that Jimmy is doing some hard thinking .
                                                                                    Hello , old Striped-coat , what have you got on your mind this fine
   6 I never knew so much about a whiting before .
                                                                                morning ? ''
   7 I can tell you more than that , if you like
                                                   ' said the Gryphon .
       Do you know why it 's called a whiting ? ''
                                                                                6 cried Peter Rabbit .
   9 I never thought about it (, ' said Alice .
                                                                                     Him (, '' said Jimmy simply), pointing down the Lone Little Path .
        Why ? '
                                                                                8 Peter looked .
   11 TODES THE BOOTS AND SHOES
                                                                                  (Do you mean) Old Mr.) Toad ! ''
   12 the Gryphon replied very solemnly .
                                                                                10 he asked .
   13(Alice was thoroughly)puzzled .
                                                                                11 Jimmy nodded .
      Does the boots and shoes ! '
                                                                                  ``(Do you see)anything)queer about him ? ''
                                                                                13 he asked in his turn .
   15 she repeated in a wondering tone
        Why , what are YOUR shoes done with)?
                                                                                14( Do you see) anything gueer about him? "
   17 said the Gryphon . '
                                                                                15 he asked .
   18 I mean , what makes them so shiny ? '
                                                                                16 Peter stared down the Lone Little Path .
                                                                                     No , ('' he replied , ``) except that he seems in a great hurry . ''
   19 (Alice looked down) at them , and considered a little before she (gave)
                                                                                     That 's just it ,('' Jimmy returned promptly .
   her answer .
       They 're done with blacking , I believe .
                                                                                     Did you ever see him hurry unless (he was frightened ? ''
                                                                                20 Peter confessed that he never had
q: Boots and shoes under the sea , ' the
                                                                            Q: `` Well , he is n't
                                                                                                           now , yet just look at him go '' retorted Jimmy .
                                                   went on in a deep
voice , are done with a whiting
C: Alice, BOOTS, Gryphon, SHOES, answer, fall, mouths, tone, way, whiting.
                                                                            C: Do, came, confessed, frightened, mean, replied, returned, said, see, thought.
MemNNs (window + self-sup.):
                                                                            MemNNs (window +self-sup.): frightened
                                           Gryphon
```

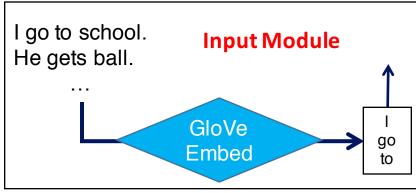
Gated Recurrent Network (GRU)

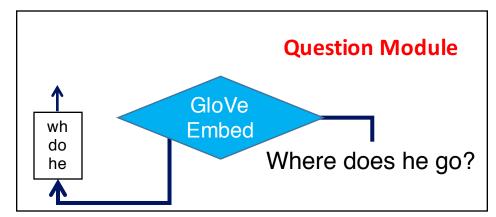


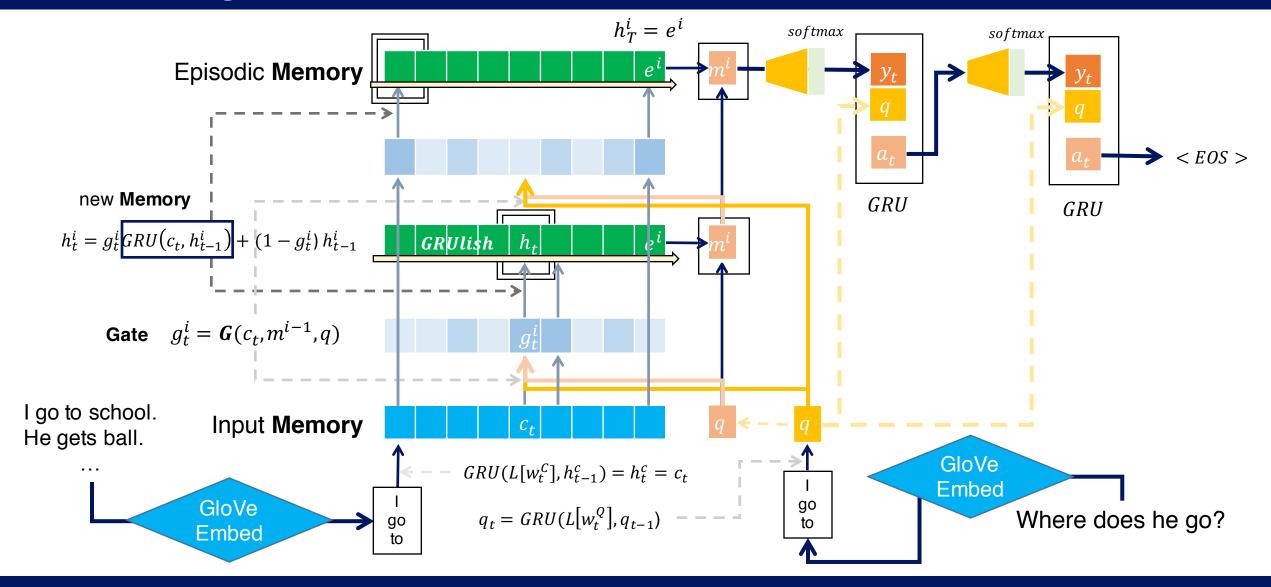


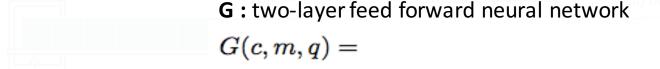












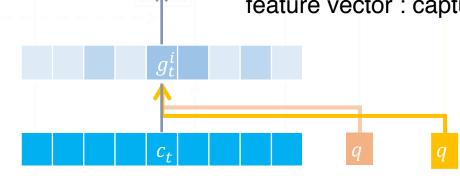
$$\sigma\left(W^{(2)}\tanh\left(W^{(1)}z(c,m,q)+b^{(1)}\right)+b^{(2)}\right)$$

$$\left[c,m,q,c\circ q,c\circ m,|c-q|,|c-m|,c^TW^{(b)}q,c^TW^{(b)}m
ight]$$

feature vector : captures a similarities between c, m, q



Gate $g_t^i = G(c_t, m^{i-1}, q)$

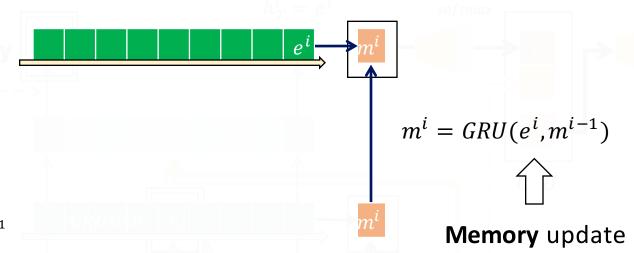




$$h_t^i = g_t^i \boxed{GRU(c_t, h_{t-1}^i) + (1 - g_t^i) h_{t-1}^i}$$

$$e^i = h_{T_C}^i$$
Gate

Episodic memory update

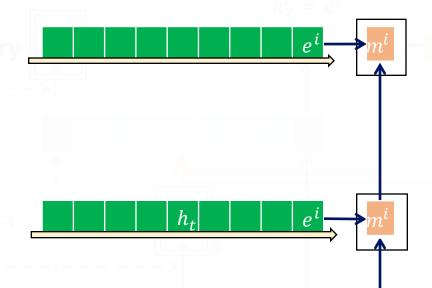


Episodic Memory Module

- **Iterates** over **input** representations, while updating episodic memory $oldsymbol{e^i}$
- Attention mechanism + Recurrent network ightarrow Update memory $m{m^i}$

Max	task 3	task 7	task 8	sentiment
passes	three-facts	count	lists/sets	(fine grain)
0 pass	0	48.8	33.6	50.0
1 pass	0	48.8	54.0	51.5
2 pass	16.7	49.1	55.6	52.1
3 pass	64.7	83.4	83.4	50.1
5 pass	95.2	96.9	96.5	N/A

Table 4. Effectiveness of episodic memory module across tasks. Each row shows the final accuracy in term of percentages with a different maximum limit for the number of passes the episodic memory module can take. Note that for the 0-pass DMN, the network essential reduces to the output of the attention module.



Criteria for Stopping

- Append a special end-of-passes representation to the input $oldsymbol{c}$
- Stop if this representation is chosen by the gate function
- Set a maximum number of iterations
- This is why called **Dynamic** MM

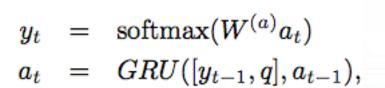
Multiple Episodes

- Allows to attend to different inputs during each pass
- Allows for a type of transitive inference, since the first pass may uncover the need to retrieve additional facts.

Q: Where is the football?

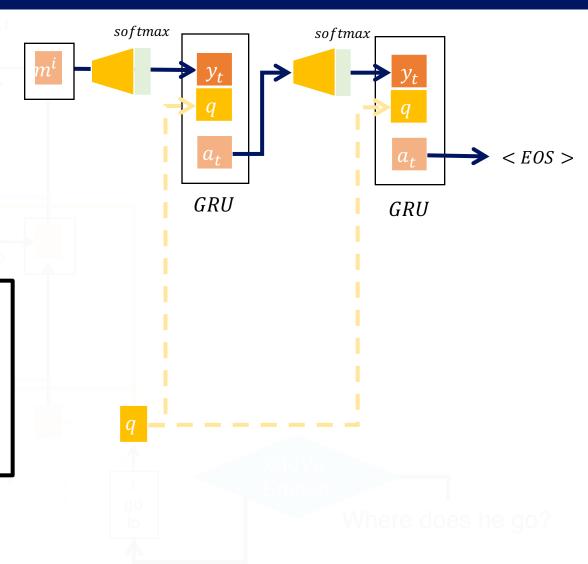
C1: John put down the football.

Only once the model sees C1, John is relevant, can reason that the second iteration should retrieve where John was.



Answer Module

- Triggered once at the end of the episodic memory or at each time step
- Concatenate the last generated word and the question vector as the input at each time step
- Cross-entropy error



- Adam optimization
- L_2 regularization, dropout on the word embedding (GloVe)

bAbl dataset

- Objective function : $J = \alpha E_{CE}(Gates) + \beta E_{CE}(Answers)$
- Gate supervision aims to select one sentence per pass
 - Without supervision : GRU of $\, c_{
 m t}$, h^i_t and $e^i = h^i_{T_C}$
 - With supervision (simpler): $e^i = \sum_{t=1}^T softmax(g_t^i)c_t$, where $softmax(g_t^i) = \frac{\exp(g_t^i)}{\sum_{j=1}^T \exp(g_j^i)}$ and g_t^i is the value before sigmoid
 - Better results, because softmax encourages sparsity & suited to picking one sentence

Stanford Sentiment Treebank (Sentiment Analysis)

- Use all full sentences, subsample 50% of phrase-level labels every epoch
- Only evaluated on the full sentences
- Binary classification, neutral phrases are removed from the dataset
- Trained with GRU sequence models

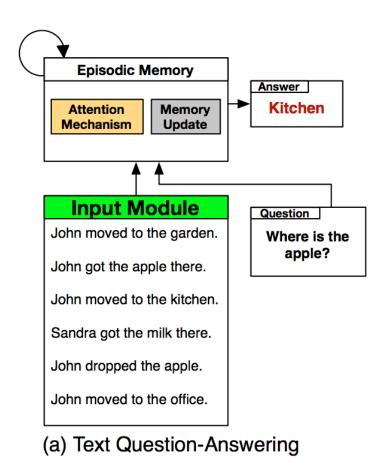
Task	Binary	Fine-grained
MV-RNN	82.9	44.4
RNTN	85.4	45.7
DCNN	86.8	48.5
PVec	87.8	48.7
CNN-MC	88.1	47.4
DRNN	86.6	49.8
CT-LSTM	88.0	51.0
DMN	88.6	52.1

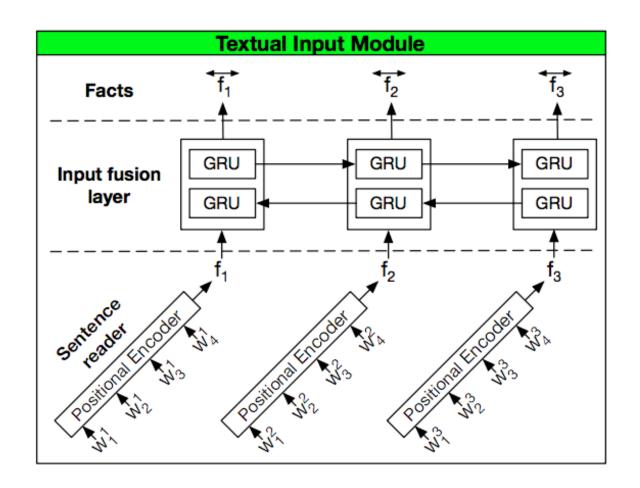
Question: Where was Mary before the Bedroom?

Answer: Cinema.

Facts	Episode 1	Episode 2	Episode 3
Yesterday Julie traveled to the school.			
Yesterday Marie went to the cinema.			
This morning Julie traveled to the kitchen.			
Bill went back to the cinema yesterday.			
Mary went to the bedroom this morning.			
Julie went back to the bedroom this afternoon.		•	
[done reading]			

Dynamic Memory Networks for Visual and Textual Question Answering [Xiong 2016]





Several design choices are motivated by intuition and accuracy improvements

Input Module in DMN

- A single GRU for embedding story and store the hidden states
- GRU provides temporal component by allowing a sentence to know the content of the sentences that came before them
- Cons:
 - GRU only allows sentences to have context from sentences before them, but not after them
 - Supporting sentences may be too far away from each other
- Here comes Input fusion layer

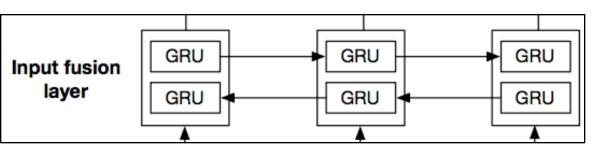
Input Module in DMN+

Replacing a single GRU with two different components

- 1. Sentence reader: responsible only for encoding the words into a sentence embedding
- Sentence Sentence Lincodes Min Position Min

- Use positional encoder (used in E2E) : $f_i = \sum_j l_j \cdot Ax_{ij}$
- Considered GRUs LSTMs, but required more computational resources, prone to overfitting
- 2. Input fusion layer: interactions between sentences, allows content interaction between sentences
 - bi-directional GRU to allow information from both past and future sentences
 - gradients do not need to propagate through the words between sentences
 - distant supporting sentences can have a more direct interaction

$$\overrightarrow{f_i} = GRU_{fwd}(f_i, \overrightarrow{f_{i-1}})$$
 $\overleftarrow{f_i} = GRU_{bwd}(f_i, \overleftarrow{f_{i+1}})$
 $\overleftarrow{f_i} = \overleftarrow{f_i} + \overrightarrow{f_i}$



Input Module for DMN+

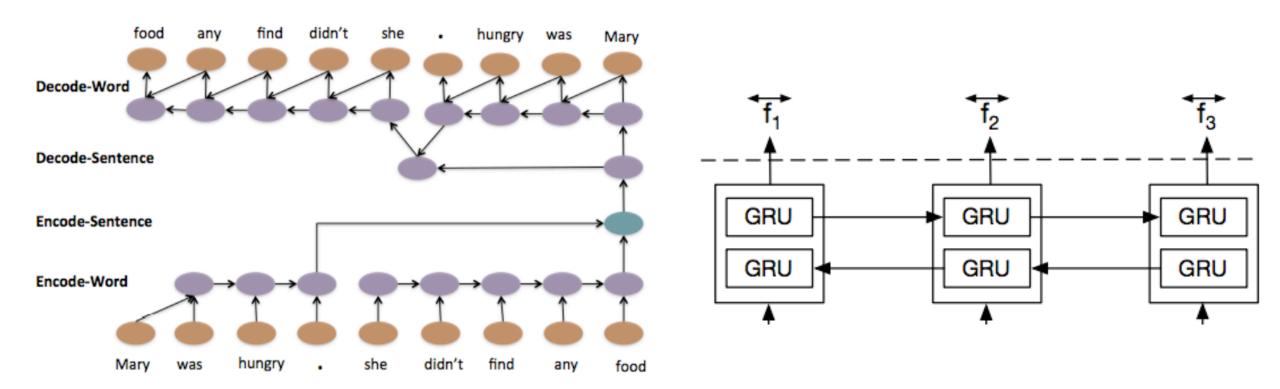


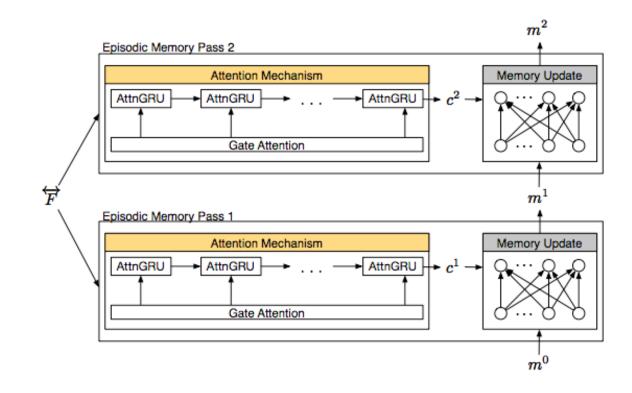
Figure 2: Hierarchical Sequence to Sequence Model.

Referenced paper: A Hierarchical Neural Autoencoder for Paragraphs and Documents [Li, 2015]

Episodic Memory Module in DMN+

- $\overrightarrow{F} = [\overrightarrow{f_1}, \overrightarrow{f_2}, ..., \overrightarrow{f_N}]$: output of the input module
- Interactions between the fact $\overleftarrow{f_i}$ and both the question q and episode memory state m^t

$$\begin{aligned} z_i^t &= [\overrightarrow{f_i} \circ q; \overrightarrow{f_i} \circ m^{t-1}; |\overrightarrow{f_i} - q|; |\overrightarrow{f_i} - m^{t-1}|] \\ Z_i^t &= W^{(2)} \tanh\left(W^{(1)} z_i^t + b^{(1)}\right) + b^{(2)} \\ g_i^t &= \frac{\exp(Z_i^t)}{\sum_{k=1}^{M_i} \exp(Z_k^t)} \end{aligned}$$



Attention Mechanism in DMN+

Use attention to extract contextual vector c^t based on the current focus

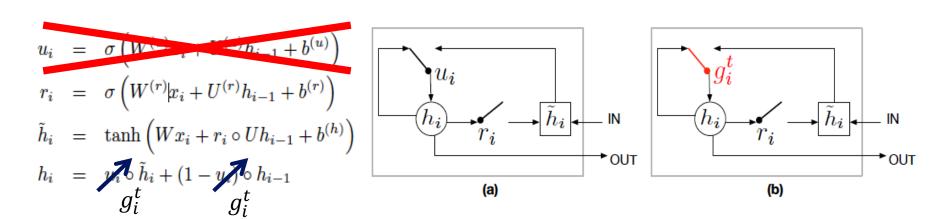
1. Soft attention

- A weighted summation of \vec{F} : $c^t = \sum_{i=1}^N g_i^t \vec{f}_i$
- Can approximate a hard attention by selecting a single fact $\overrightarrow{f_i}$
- Cons: losses positional and ordering information
 - Attention passes can retrieve some of this information, but inefficient

Attention Mechanism in DMN+

2. Attention based GRU (best)

- position and ordering information : RNN is proper but can't use g_i^t
- u_i : update, r_i : how much retain from h_{i-1}
- Replace u_i (vector) to g_i^t (scalar)
- Allows us to easily visualize how the attention gates activate
- Use final hidden state as c_t , which is used to update episodic memory m^t



Episode Memory Updates in DMT+

1. Untied and **Tied** (better) GRU

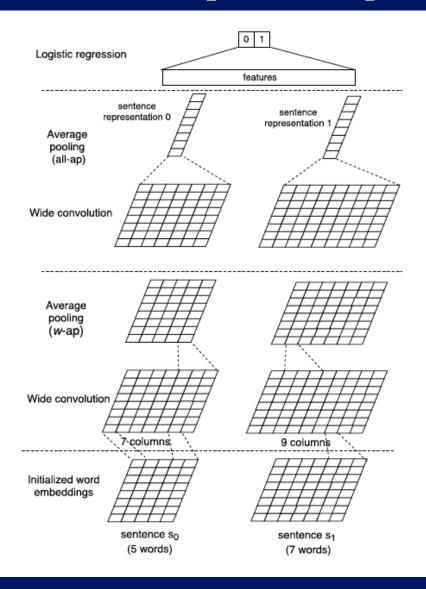
$$m^t = GRU(C^t, m^{t-1})$$

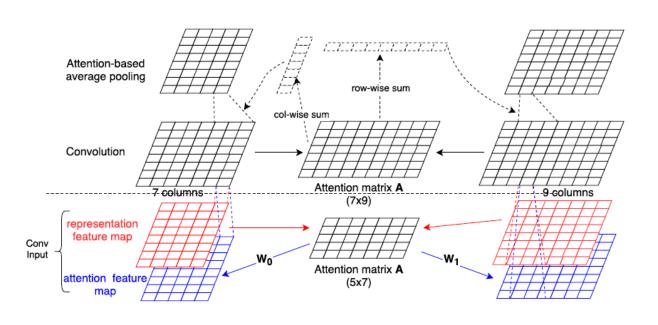
2. Untied ReLU layer (best)

$$m^{t} = ReLU(W^{t}[m^{t-1}; c^{t}; q] + b)$$

- Adam optimization
- Xavier initialization is used for all weights except for the word embeddings
- L_2 regularization on all weights except bias
- Dropout on the word embedding (GloVe) and answer module with p=0.9

ABCNN: Attention-Based Convolutional Neural Network for Modeling Sentence Pairs [Yin 2015]





ABCNN: Attention-Based Convolutional Neural Network for Modeling Sentence Pairs [Yin 2015]

- Most prior work on answer selection model each sentence separately and neglects mutual influence
- Human **focus on key parts** of s_0 by extracting parts from s_1 related by identity, synonymy, antonym etc.
- **ABCNN**: taking into account the interdependence between s_0 and s_1
- Convolution layer: increase abstraction of a phrase from words

ABCNN: Attention-Based Convolutional Neural Network for Modeling Sentence Pairs [Yin 2015]

- 1. Input embedding with word2vec
- 2-1. Convolution layer with wide convolution
- To make each word v_i to be detected by all weights in W
- 2-2. Average pooling layer
- all-ap: column-wise averaging over all columns
- **w-ap**: column-wise averaging over windows of w
- 3. Output layer with logistic regression
- Forward all-ap to all non-final ap layer + final ap layer

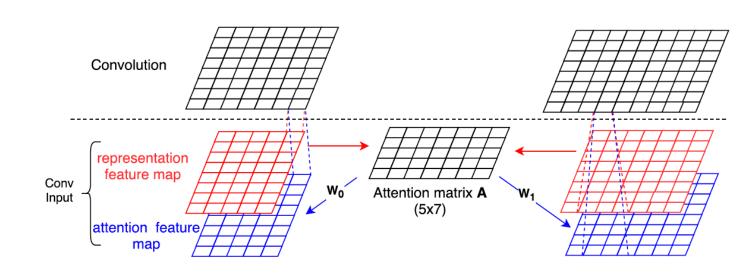
Attention on feature map (ABCNN-1)

- Attention values of row i in A: attention distribution of the i-th unit of s_0 with respect to s_1
- $A_{i,j} = matchscore(F_{0,r}[:,i],F_{1,r}[:,j])$
- matchscore = 1/(1 + |x y|)
- Generate the attention feature map $F_{i,a}$ for s_i

$$\mathbf{F}_{0,a} = \mathbf{W}_0 \cdot \mathbf{A}^{\mathsf{T}}$$

 $\mathbf{F}_{1,a} = \mathbf{W}_1 \cdot \mathbf{A}$

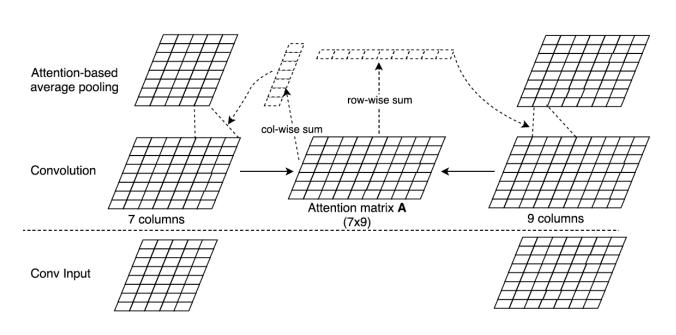
Cons : need more parameters

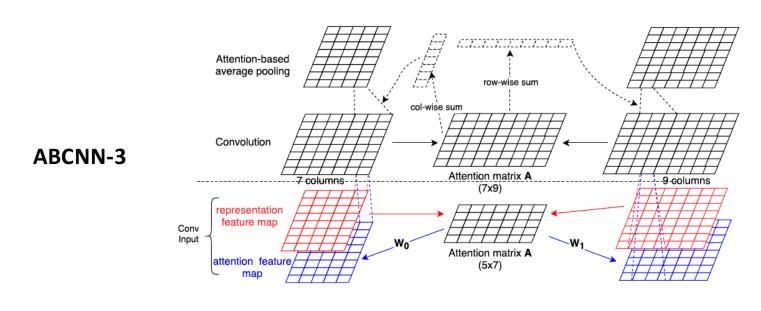


Attention after convolution (ABCNN-2)

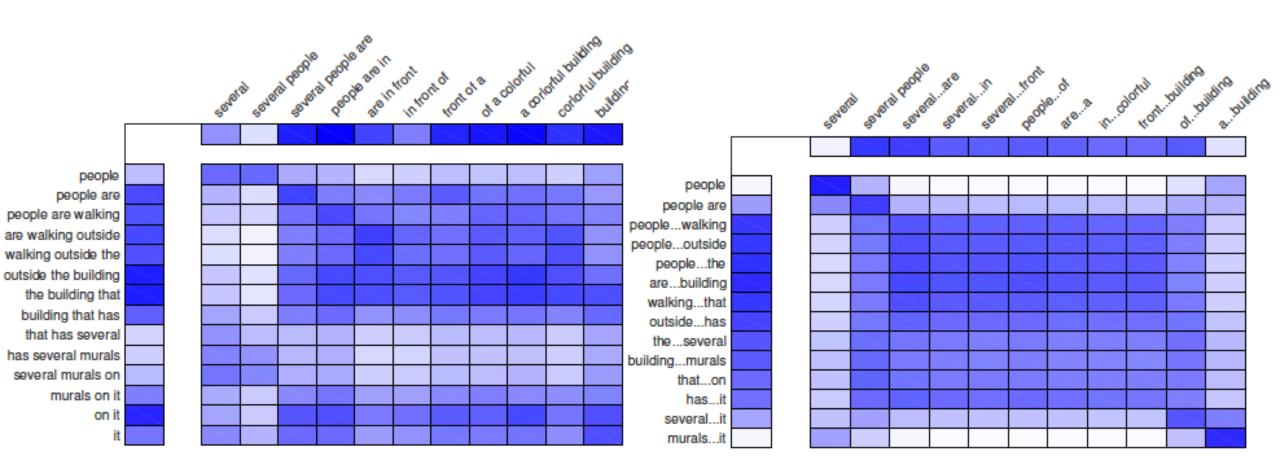
- Attention weights directly on the representation with the aim of improving the features computed by convolution
- $a_{0,j} = \sum A[j,:] \rightarrow \text{col-wise, row-wise sum}$
- w-ap on convolution feature

$$\mathbf{F}_{i,r}^p[:,j] = \sum_{k=j:j+w} a_{i,k} \cdot \mathbf{F}_{i,r}^c[:,k], \quad j = 1 \dots s_i$$

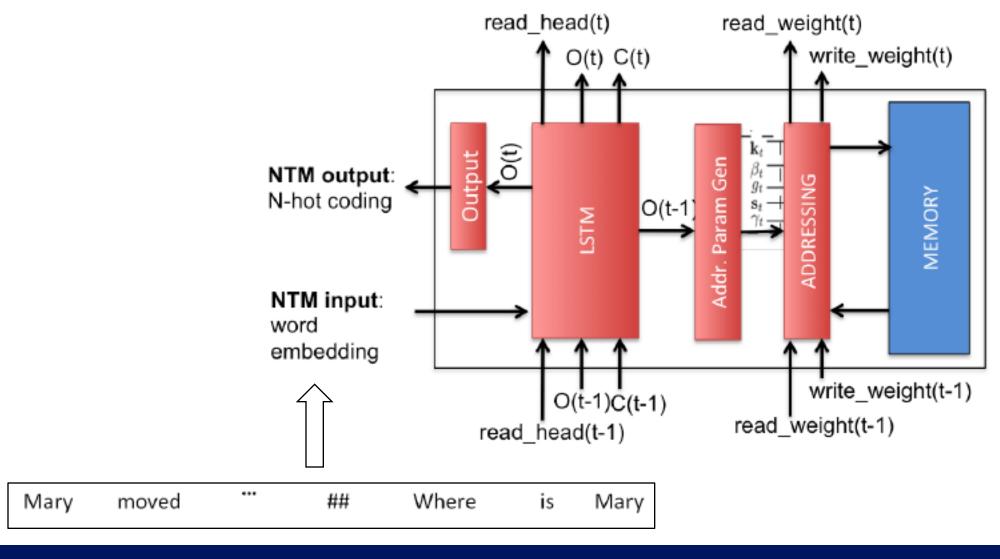




ABCNN-1	ABCNN-2	
Indirect impact to convolution	Direct convolution via pooling (weighted attention)	
Need more features Vulnerable to overfitting	No need features	
handles smaller-granularity units (ex. Word level)	handles larger-granularity units (ex. Phrase level, phrase size = window size)	



Empirical Study on Deep Learning Models for QA [Yu 2015]



Empirical Study on Deep Learning Models for QA [Yu 2015]

The first to examine **Neural Turing Machines** on QA problems

Split QA into two step

- 1. search supporting facts
- 2. Generate answer from relevant pieces of information

NTM

- Single-layer LSTM network as controller
- Input: word embedding
 - 1. Support fact only
 - 2. Fact highlighted: user marker to annotate begin and end of supporting facts
- Output: softmax layer (multiclass classification) for answer

_						
	(ii) Support fact only		(iii) Sup. fact highlighted			
	d	e	f	g		
	NMT	NTM	NMT	NTM		
	100	100	100	100		
	100	100	99.6	100		
	100	100	99.5	100		
	99.1	100	97.5	100		
	99.3	79.2	90.6	73.7		
	100	100	99.8	100		
	98.5	100	96.6	100		
	99	100	92.7	98		
	100	100	99.7	100		
	98.9	94.6	96.8	85.9		
	100	100	100	100		
	100	100	100	100		
	100	100	100	100		
	99.8	100	97.5	100		
	100	100	92.7	100		
	100	100	88.1	100		
	64.2	69.3	58	61.2		
	97.8	93	91.8	93		
	80.7	100	29.7	100		
	100	100	93.3	100		
_	96.9	96.7	91.2	95.6		

Teaching Machines to Read and Comprehend [Hermann 2015]

Original Version	Anonymised Version
Context	
The BBC producer allegedly struck by Jeremy Clarkson will not press charges against the "Top Gear" host, his lawyer said Friday. Clarkson, who hosted one of the most-watched television shows in the world, was dropped by the BBC Wednesday after an internal investigation by the British broadcaster found he had subjected producer Oisin Tymon "to an unprovoked physical and verbal attack."	the ent381 producer allegedly struck by ent212 will not press charges against the "ent153" host, his lawyer said friday. ent212, who hosted one of the most - watched television shows in the world, was dropped by the ent381 wednesday after an internal investigation by the ent180 broadcaster found he had subjected producer ent193" to an unprovoked physical and verbal attack."
Query Producer X will not press charges against Jeremy Clarkson, his lawyer says.	producer X will not press charges against $\underbrace{\it ent212}$, his lawyer says .
Answer	
Oisin Tymon	ent 193

Table 3: Original and anonymised version of a data point from the Daily Mail validation set. The anonymised entity markers are constantly permuted during training and testing.

Teaching Machines to Read and Comprehend [Hermann 2015]

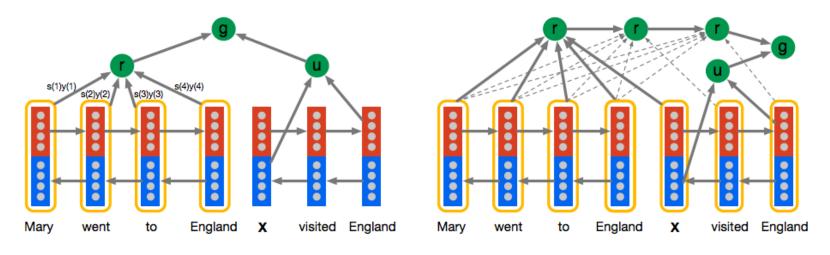
$$u = \overrightarrow{y_q}(|q|) || \overleftarrow{y_q}(1)$$

$$\begin{split} m(t) &= \tanh \left(W_{ym} y_d(t) + W_{um} u \right), \\ s(t) &\propto \exp \left(\mathbf{w}_{ms}^\intercal m(t) \right), \end{split}$$

$$r = \sum_{i} s_{i} f_{i}$$

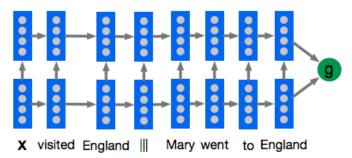
where $f_{i} = y_{d}(t)$

s(t): degree to which the network attends to a particular token in the document when answering the query (soft attention)



(a) Attentive Reader.

(b) Impatient Reader.



(c) A two layer Deep LSTM Reader with the question encoded before the document.

Text Understanding with the Attention Sum Reader Network [Kadlec 2016]

Answer should be in context

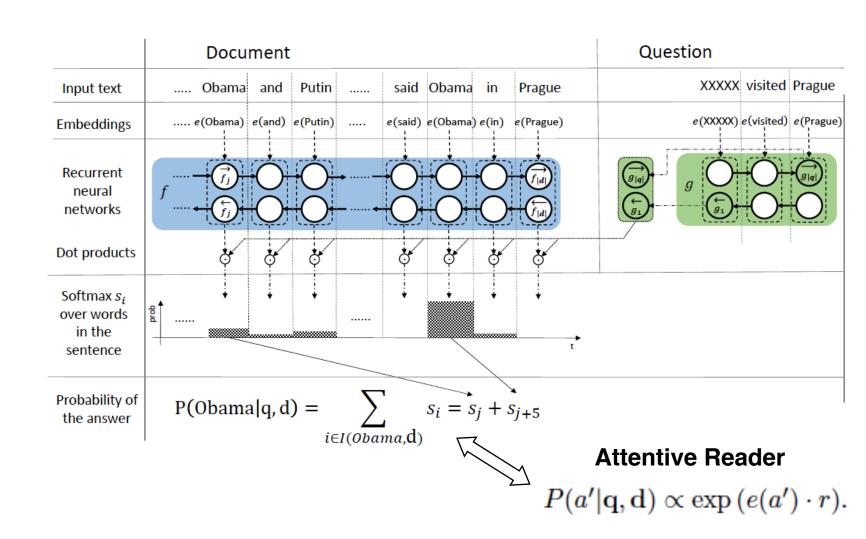
$$s_i \propto \exp\left(f_i(\mathbf{d}) \cdot g(\mathbf{q})\right)$$

$$P(w|\mathbf{q}, \mathbf{d}) = \sum_{i \in I(w, \mathbf{d})} s_i$$

Inspired by Pinter Network

Contrast to Attentive Reader:

 We select answer from context directly using weighted sum of individual representation



Stochastic Latent Variable

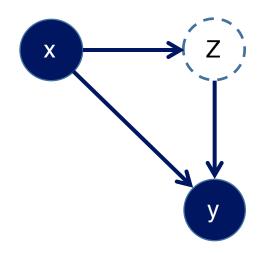
Generative Model



$$p(x) = \sum_{z} p(x,z) = \sum_{z} p(x|z)p(z)$$

$$p(x) = \int_{Z} p(x,z) = \int_{Z} p(x|z)p(z)$$

Conditional Generative Model



$$p(y|x) = \sum_{z} p(y|z, x)p(z|x)$$

$$p(y|x) = \int_{z} p(y|z,x)p(z|x)$$

Variational Inference Framework

$$p(x,z) = p(x|z)p(z) = \sum_{h} p(x|h)p(h|z)p(z)$$

$$\log p_{\theta}(x,z) = \log \int_{h} \frac{q(h)}{q(h)} p(x|h) p(h|z) p(z) dh \ge \int_{h} q(h) \log \frac{p(x|h) p(h|z) p(z)}{q(h)} dh$$

$$= \int_h q(h) \log \frac{p(x|h)p(h|z)}{q(h)} dh + \int_h q(h) \log \frac{p(z)}{q(h)} dh$$

$$= E_{q(h)}[\log p(x|h)p(h|z) - \log q(h)] - D_{KL}(q(h)||p(z))$$

$$= E_{q(h)}[\log p(x|h)p(h|z)p(z) - \log q(h)]$$

Variational Inference Framework

$$p_{\theta}(x,z) = p_{\theta}(x|z)p(z) = \sum_h p_{\theta}(x|h)p_{\theta}(h|z)p(z)$$
 Jensen's Inequality

$$\log p_{\theta}(x,z) = \log \int_{h} \frac{q(h)}{q(h)} p_{\theta}(x|h) p_{\theta}(h|z) p(z) dh \ge \int_{h} q(h) \log \frac{p_{\theta}(x|h) p_{\theta}(h|z) p(z)}{q(h)} dh$$

$$= \int_{h} q(h) \log \frac{p_{\theta}(x|h)p_{\theta}(h|z)}{q(h)} dh + \int_{h} q(h) \log \frac{p(z)}{q(h)} dh$$

$$= E_{q(h)}[\log p_{\theta}(x|h)p_{\theta}(h|z) - \log q(h)] - D_{KL}(q(h)||p(z))$$

$$=E_{q(h)}[\log p_{\theta}(x|h)p_{\theta}(h|z)-\log q(h)]$$
 a tight lower bound if $q(h)=p(h|x,z)$

Conditional Variational Inference Framework

$$p_{\theta}(y|x) = \sum_{z} p_{\theta}(y,z|x) = \sum_{z} p_{\theta}(y|x,z)p_{\pi}(z|x)$$

$$\text{Jensen's Inequality}$$

$$\log p(y|x) = \log \int_{z} \frac{q(z)}{q(z)} p(y|z,x)p(z|x)dz \geq \int_{z} q(z) \log \frac{p(y|z,x)p(z|x)}{q(z)} dz$$

$$= \int_{z} q(z) \log \frac{p(y|z,x)}{q(z)} dz + \int_{h} q(z) \log \frac{p(z|x)}{q(z)} dz$$

$$= \int_{z} q(z) \log p(y|z,x) dz - \int_{z} q(z) \log q(z) dz + \int_{h} q(z) \log \frac{p(z|x)}{q(z)} dz$$

$$= E_{q(z)}[\log p(y|z,x) - \log q(z)] - D_{KL}(q(z) \parallel p(z|x))$$

$$= E_{q(z)}[\log p(y|z,x) - \log q(z)] \quad \text{a tight lower bound if } q(z) = p(z|x)$$

Neural Variational Inference Framework

$$\log p_{\theta}(x, z) \ge E_{q(z)}[\log p(y|z, x) - \log q(z)] - D_{KL}(q(z) || p(z|x)) = \mathcal{L}$$

1. Vector representations of the observed variables

$$u = f_z(z), v = f_x(x)$$

2. Joint representation (concatenation)

$$\pi = g(u, v)$$

3. Parameterize the variational distribution

$$\mu = l_1(\pi), \sigma = l_2(\pi)$$

Neural Variational Document Model [Miao, 2015]

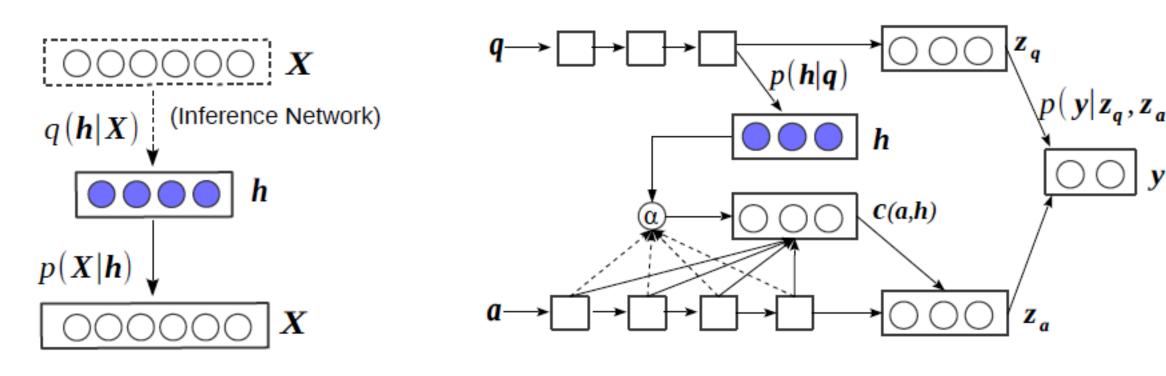


Figure 1: NVDM for document modelling. Figure 2: NASM for question answer selection.