

Name: _____

Student No.: _____

Group A

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.

1. Which of the following ODE's has distinct solutions $y_1, y_2: I \rightarrow \mathbb{R}$ satisfying $y_1(t_0) = y_2(t_0)$ for some $t_0 \in I$?

☐ $y' = \sin(ty^2)$
☐ $yy' = 0$
☐ $y' = |ty|$
☐ $y' = y\sqrt{t}$
☒ $y' = \sqrt{|ty|}$

2. $(x^4y^2 - y)dx + (x^2y^4 - x)dy = 0$ has the integrating factor

☐ $1/(xy)$
☐ 1
☒ $1/(xy)^2$
☐ $1/(xy^2)$
☐ $1/(x^2y)$

3. The family of curves $y = c/x^2$, $c \in \mathbb{R}$ satisfies the ODE

☐ $dy = x^{-2}dx$
☐ $dy = 2x^{-3}dx$
☒ $2xydx + x^2dy = 0$
☐ $dx = dy$
☐ $2yx^{-3}dx - x^{-2}dy = 0$

4. For the solution $y(t)$ of the IVP $y' = y^3 - 7y + 6$, $y(0) = 0$ the limit $\lim_{t \rightarrow +\infty} y(t)$ equals

☐ -3
☐ -2
☐ -1
☒ 1
☐ 2

5. For the solution $y(t)$ of the IVP $y' = (y/t) - 1$, $y(1) = \ln 2$ the value $y(2)$ is equal to

☒ 0
☐ 1
☐ 2
☐ $\ln 2$
☐ $2\ln 2$

6. For the solution $y(t)$ of the IVP $y' = y^2 e^{-t}$, $y(0) = -1$ the value $y(-1)$ is equal to

☐ e
☒ $1/(e-2)$
☐ $e+2$
☐ $1/(e+2)$
☐ $e-2$

7. For the solution $y: (0, +\infty) \rightarrow \mathbb{R}$ of the IVP $t^2y'' + 2ty' - 2y = 1$, $y(1) = 0$, $y'(1) = 1$ the value $y(2)$ is equal to

☐ $\frac{13}{8}$
☐ 1
☐ $\frac{13}{24}$
☐ $\frac{19}{8}$
☒ $\frac{19}{24}$

8. The power series $\sum_{k=1}^{\infty} 2^k z^{k^2}$ has radius of convergence

☐ 0
☐ $\frac{1}{2}$
☒ 1
☐ 2
☐ ∞

9. The smallest integer s such that $f_s(x) = \sum_{k=1}^{\infty} \frac{\cos(k^2x)}{k^s}$ is differentiable on \mathbb{R} is equal to

☐ 0
☐ 1
☐ 2
☐ 3
☒ 4

10. If $y(t)$ solves $y' = t^2y + ty^2$ then $z = 1/y(t)$ solves

☐ $z' = -t^2z$
☐ $z' = -z^2 + t$
☐ $z' = t^2/z + t/z^2$
☐ $z' = -z^2$
☒ $z' = -t^2z - t$

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11. The sequence $\phi_0, \phi_1, \phi_2, \dots$ of Picard-Lindelöf iterates for the IVP $y' = y + 2t$, $y(0) = -2$ has $\phi_2(t)$ equal to

☐ $-2 + 2t + \frac{1}{3}t^3$

☐ $-t^2 + \frac{1}{3}t^3$

☒ $-2 - 2t + \frac{1}{3}t^3$

☐ $t^2 + \frac{1}{3}t^3$

☐ $1 + t + \frac{3}{2}t^2 + \frac{1}{3}t^3$

12. $y'' - 4y' + 4y = 2t + e^{2t}$ has a particular solution $y_p(t)$ of the form

☐ $c_0 + c_1 t + c_2 e^{2t}$

☒ $c_0 + c_1 t + c_2 t^2 e^{2t}$

☐ $c_0 t + c_1 t^2 e^{2t}$

☐ $c_0 + c_1 t$

☐ $c_0 t + c_1 e^{2t}$

13. Maximal solutions of $y' = y^2 + y$ satisfying $y(0) > 0$ are defined on an interval of the form

☐ (a, b)

☐ $[a, b]$

☐ $(a, +\infty)$

☒ $(-\infty, b)$

☐ $(-\infty, +\infty)$

with $a, b \in \mathbb{R}$.

14. For $\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$, the matrix $e^{\mathbf{A}t}$ is equal to

☐ $\begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix}$

☐ $\begin{pmatrix} e^t & e^{-t} \\ e^t & e^{-t} \end{pmatrix}$

☐ $\begin{pmatrix} e^t & e^t \\ e^{-t} & e^{-t} \end{pmatrix}$

☐ $\begin{pmatrix} 1+t & t \\ -t & 1-t \end{pmatrix}$

☒ $\begin{pmatrix} 1+t & -t \\ t & 1-t \end{pmatrix}$

15. The matrix norm of $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ (subordinate to the Euclidean length on \mathbb{R}^2) is equal to

☐ 0

☐ 1

☒ $\sqrt{2}$

☐ 2

☐ 4

Time allowed: 60 min

CLOSED BOOK

Good luck!