Calculus III (Math 241)

W5 a) Explain the following observation, which at the first glance seems somewhat paradoxical: When drawing the line connecting two distinct points $\mathbf{a}, \mathbf{b}$ on a whiteboard, we can add the midpoint $\frac{1}{2}(\mathbf{a}+\mathbf{b})$ without knowing the position of the origin, but not the sum $\mathbf{a}+\mathbf{b}$.

b) Give an analytic geometry proof of the following well-known theorem from plane geometry: The three medians (lines connecting a vertex of a triangle to the midpoint of the opposite side) are concurrent in a point, which divides each median in the ratio $2:1$.

W6 Solve [Ste16], Section 12.5, Exercise 77.

W7 Consider a linear equation

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n = b, \quad a_1, \dots, a_n, b \in \mathbb{R} \tag{LE}$$

a) Show that the solution set of (LE) forms an affine subspace of $\mathbb{R}^n$, and in the special case $b = 0$ a linear subspace.

b) Show that a) generalizes to the solution of a system of $m \geq 1$ linear equations.

W8 Compute the following matrix product:

$$\begin{pmatrix} -2 & 2 & 6 & 1 \\ -4 & 6 & -1 & 1 \\ 1 & 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 \\ -7 & -10 & -1 \\ 0 & 6 & 3 \\ 1 & 1 & 1 \end{pmatrix}$$

W9 Compute the powers $\mathbf{A}^2$, $\mathbf{A}^3$, ... for the following 2×2 matrices $\mathbf{A}$. Can you explain the emerging patterns?

a) $\begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$, $a \in \mathbb{R}$ b) $\begin{pmatrix} 1 & 0 \\ b & 1 \end{pmatrix}$, $b \in \mathbb{R}$ c) $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

d) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ e) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ f) $\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$