Differential Equations Plus (Math 286)

H53 Compute the Taylor series of $z \mapsto 1/(z^2+1)$ at $a=1$ and $a=1+\mathrm{i}$.
Hint: Proceed as for $z \mapsto 1/(1-z)$ in the lecture and then use partial fractions.

H54 Using power series, solve each of the following initial-value problems:

a) $t(2-t)y'' - 6(t-1)y' - 4y = 0, \quad y(1) = 1,\ y'(1) = 0;$

b) $y'' + (t^2+2t+1)y' - (4+4t)y = 0, \quad y(-1) = 0,\ y'(-1) = 1.$

H55 a) Find 2 linearly independent solutions of $\quad y'' + t^3y' + 3t^2y = 0.$

b) Find the first 5 terms in the Taylor series expansion about $t=0$ of the solution $y(t)$ of the initial value problem

$$y'' + t^3y' + 3t^2y = \mathrm{e}^t, \quad y(0) = y'(0) = 0.$$

H56 *A Problem from Friday's Lecture*

Suppose (α_n) and (u_n) are sequences of nonnegative real numbers satisfying

$$\begin{aligned}
\alpha_n &\leq \sum_{k=0}^{n-1} \frac{M(k+1)}{n(n-1)}\alpha_k \quad (n \geq 2),\\
u_n &= \sum_{k=0}^{n-1} \frac{M(k+1)}{n(n-1)}u_k \quad (n \geq 2),\\
u_0 &= \alpha_0,\ u_1 = \alpha_1
\end{aligned}$$

for some constant $M > 0$.

a) Show $\alpha_n \leq u_n$ for all n.

b) Show $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = 1$.
Hint: Express u_{n+1} in terms of u_n.

c) Is the sequence (u_n) (and hence (α_n) as well) necessarily bounded from above?

Due on Wed Nov 24, 6 pm

Exercise H56 c) is optional, but should be handed in together with H56 a), b).