

Differential Equations Plus (Math 286)

H1 We have considered the ODE $y' = -x/y$ as an example in the lecture. Actually there are four ODE's, viz. $y' = \pm x/y$ and $y' = \pm y/x$, which look very similar. Draw direction fields for the other three ODE's and determine their solutions in both implicit and explicit form (if possible).

H2 Determine all points $(t_0, y_0) \in \mathbb{R}^2$ such that there is a unique solution on $[t_0, \infty)$ of the IVP $y' = \sqrt{|y|}$, $y(t_0) = y_0$ ("the value at time t_0 determines the values at all future times $t > t_0$ ").

H3 Let $t_0, y_0, y_1 \in \mathbb{R}$. Show that the IVP

$$y'' = -y, \quad y(t_0) = y_0, \quad y'(t_0) = y_1$$

has a unique solution.

H4 For each of the following ODE's, determine at least one nonzero solution by using the "Ansatz" $y(x) = a e^{\alpha x}$ or $y(x) = b x^\beta$.

- a) $y'' = y^2$;
- b) $y'' - 5y' + 6y = 0$;
- c) $y'' - 5y' + 6y = e^x$;
- d) $y'' - \frac{1}{2x} y' + \frac{1}{2x^2} y = 0$;
- e) $(2x + 1)y'' + (4x - 2)y' - 8y = 0$;
- f) $x^2(1 - x)y'' + 2x(2 - x)y' + 2(1 + x)y = 0$.

H5 Do two of the three exercises on the pendulum equation in [BDM17], Ch. 1.3 (Exercises 23–25 in the 11th global edition).

Due on Fri Sep 24, 6 pm

Instructions For your homework it is best to maintain 2 notebooks, which are handed in on alternate Fridays. You may also use A4 sheets, provided they are firmly stapled together.

Don't forget to write your name (English and Chinese) and your student ID on the first page.

Homework is handed in on Fridays before the discussion session starts (late homework won't be accepted!) and will be returned on the next Friday.

Answers to exercises must be justified; it is not sufficient to state only the final result of a computation. Answers must be written in English.

For a full homework score it is sufficient to solve ca. 80 % of the mandatory homework exercises. Optional exercises contribute to the homework score, but they are usually more difficult and you should work on them only if you have sufficient spare time.