Calculus III (Math 241)

W15 Do Exercises 22, 24, 26 in [Ste16], Ch. 13.1.

W16 Do Exercise 50 in [Ste16], Ch. 13.1.

W17 Do Exercises 24 and 28 in [Ste16], Ch. 13.2.

W18 Consider the vectors $\mathbf{v}_1 = (1,1,1)$, $\mathbf{v}_2 = (1,-1,0)$, $\mathbf{v}_3 = (0,1,-1)$. Define $\mathbf{A} \in \mathbb{R}^{3\times 3}$ as the matrix with columns $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3 - \mathrm{proj}_{\mathbf{v}_2}(\mathbf{v}_3)$ in this order.

a) Compute $\mathbf{A}$ and verify that the columns of $\mathbf{A}$ are mutually orthogonal.

b) Compute the inverse matrix $\mathbf{A}^{-1}$ using the algorithm presented in the lecture.

c) The matrix $\mathbf{A}^{-1}$ arises from $\mathbf{A}$ by a simple transformation. Try to find this transformation and, if successful, prove your discovery.

W19 Consider the matrices

$$R(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix}, \quad S(\phi) = \begin{pmatrix} \cos\phi & \sin\phi \\ \sin\phi & -\cos\phi \end{pmatrix}, \quad \phi \in [0, 2\pi).$$

a) Show that $\mathbb{R}^2 \to \mathbb{R}^2$, $\mathbf{x} \to R(\phi)\mathbf{x}$ is the rotation around $(0,0)^\mathsf{T}$ with angle ϕ.

b) Show that $\mathbb{R}^2 \to \mathbb{R}^2$, $\mathbf{x} \to S(\phi)\mathbf{x}$ is a reflection, and find its axis.

c) Show that $R(\phi)^{-1} = R(-\phi) = R(\phi)^\mathsf{T}$ and $S(\phi)^{-1} = S(\phi) = S(\phi)^\mathsf{T}$.

d) Determine all rotations/reflections of $\mathbb{R}^2$ that preserve the equilateral triangle Δ with vertices $(1,0)$, $\left(-\frac{1}{2}, \pm\frac{1}{2}\sqrt{3}\right)$.

Hint: There are six such symmetries, three rotations and three reflections.