Math 286 Differential Equations (Prof. Honold) **Midterm Exam** 2020/05/14

Name: Student No.: Group 0

For each of the following problems, find the correct answer (tick as appropriate!). No justifications are required. Each problem has exactly one correct solution, which is worth 1 mark. Incorrect solutions (including no answer, multiple answers, or unreadable answers) will be assigned 0 marks; there are no penalties.

1. The (real or complex) solution space of $y - 2y' + t\,y^{(3)} = 0,\quad t > 0,$ has dimension

   1 2 3 4 5

2. The sequence $\phi_0, \phi_1, \phi_2, \dots$ of Picard-Lindelöf iterates for the IVP $y' = 2y,\ y(1) = 1$ has $\phi_2(t)$ equal to

   $-1+2t-2t^2$ $1-2t+2t^2$ $1-t+t^2$ $-1+t-t^2$
   $1+t+t^2/2$

3. For the solution $y(t)$ of the IVP $y' = (y-1)\cos t,\ y(0) = 2$ the value $y(\pi/2)$ is equal to

   e $1+\mathrm{e}$ $1+2\mathrm{e}$ $1-\mathrm{e}$ $2+\mathrm{e}$

4. Which of the following ODE's has distinct solutions $y_1, y_2\colon [0,1) \to \mathbb{R}$ satisfying $y_1(0) = y_2(0)$?

   $y' = y^2$ $y' = y\sqrt{t}$ $y' = t\sqrt{y}$ $y' = ty$ $y' = |y|$

5. $\mathrm{e}^x(x+1)\,\mathrm{d}x + (y\mathrm{e}^y - x\mathrm{e}^x)\,\mathrm{d}y = 0$ has the integrating factor

   0 1 e^{-x} e^{-y} e^{-x-y}

6. For the solution $y(t)$ of the IVP $y' = y^3 - 4y,\ y(0) = 3$ the limit $\lim\limits_{t\to+\infty} y(t)$ is equal to

   0 2 -2 $+\infty$ $-\infty$

7. For the solution $y(t)$ of the IVP $y' = (\cos t)/y,\ y(0) = 1$ the value $y(\pi/2)$ is equal to

   0 1 $\frac{1}{2}$ $\sqrt{3}$ $\frac{1}{2}\sqrt{3}$

8. For which of the following ODE's does the set of solutions $\phi\colon \mathbb{R} \to \mathbb{R}$ not form a (linear) subspace of $\mathbb{R}^{\mathbb{R}}$?

   $y' = |t|\,y$ $yy' = 0$ $ty' = y$ $y' = t(y+1)$ $y'' = t^2(y'-y)$

9. For which choice of $f_n(x)$ does $\sum\limits_{n=1}^{\infty} f_n(x)$ converge uniformly on $[0, +\infty)$?

   $f_n(x) = \sin(x)/n$ $f_n(x) = \mathrm{e}^{-nx}/n$ $f_n(x) = x/n^4$
   $f_n(x) = 1/(n+x^2)$ $f_n(x) = 1/(n^2+x)$

10. If $y = y(x)$ solves $y' = x/y$ then $z = y/x$ solves

    $z' = z$ $z' = (1-z^2)/(xz)$ $z' = xz/(1-z^2)$ $z' = 0$
    $z' = 1/z$

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11. For the solution $y\colon (0,+\infty)\to\mathbb{R}$ of the IVP $\quad t^2y''-2y=0, \quad y(1)=y'(1)=1 \quad$ the value $y(2)$ is equal to

$\frac{2}{6}$ $\qquad$ $\frac{7}{6}$ $\qquad$ $\frac{12}{6}$ $\qquad$ $\frac{17}{6}$ $\qquad$ $\frac{22}{6}$

12. Any solution $y(t)$ of $\quad y''+4y=0 \quad$ satisfying $y(0)=0$ also satisfies

$y(\pi/4)=0$ $\qquad$ $y(\pi/2)=0$ $\qquad$ $y'(0)=0$ $\qquad$ $y'(0)=1$ $\qquad$ $y'(0)=2$

13. $y''-2y'+y=\mathrm{e}^t-2 \quad$ has a particular solution $y_p(t)$ of the form

$c_0+c_1\mathrm{e}^t$ $\qquad$ $c_0+c_1t\mathrm{e}^t$ $\qquad$ $c_0+c_1t^2\mathrm{e}^t$ $\qquad$ $(c_0+c_1t)\mathrm{e}^t$

$(c_0+c_1t^2)\mathrm{e}^t$

14. The function $\quad f(x)=\sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)^a}\cos\bigl((2k+1)x\bigr), \quad a\in\mathbb{Z}, \quad$ is continuous on $\mathbb{R}$ if and only if

$a\geq 0$ $\qquad$ $a\geq 1$ $\qquad$ $a\geq 2$ $\qquad$ $a\geq 3$ $\qquad$ $a\geq 4$

15. Maximal solutions of $\quad y'=y^2-y+1 \quad$ are defined on an interval of the form

(a,b) $\qquad$ $[a,b]$ $\qquad$ $(a,+\infty)$ $\qquad$ $(-\infty,b)$ $\qquad$ $(-\infty,+\infty)$

with $a,b\in\mathbb{R}$.

Time allowed: 45 min $\qquad$ CLOSED BOOK $\qquad$ ***Good luck!***