Calculus III (Math 241)

W1 a) Find an equational representation $a_1x_1 + a_2x_2 + a_3x_3 = b$ of the plane with parametric representation

$$\mathbf{x} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} + c_1 \begin{pmatrix} 0 \\ 2 \\ -2 \end{pmatrix} + c_2 \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}, \quad c_1, c_2 \in \mathbb{R}.$$

What is the geometric meaning of the vector $\mathbf{a} = (a_1, a_2, a_3)$?

b) Find a parametric representation of the plane $x_1 + x_2 + x_3 = 1$.

c) Explain how to make the equational representation of planes in $\mathbb{R}^3$ *canonical* (i.e., every plane should have a unique associated linear equation of the given form).

W2 a) Compute a parametric representation for the intersection of the two planes in $\mathbb{R}^3$ with equations $x_1 + x_2 - 2x_3 = 4$ and $-2x_1 - x_2 + 5x_3 = 0$, thereby showing that this intersection is a line.

b) Represent the line in $\mathbb{R}^3$ through the two points $(1,2,1)$ and $(3,0,-1)$ as solution set of a system of 2 linear equations.

W3 A plane H in $\mathbb{R}^3$ with equation $a_1x_1 + a_2x_2 + a_3x_3 = 0$ partitions the whole space into 3 sets:

$$\begin{aligned} H^+ &= \{\mathbf{x} \in \mathbb{R}^3; a_1x_1 + a_2x_2 + a_3x_3 > 0\}, \\ H &= \{\mathbf{x} \in \mathbb{R}^3; a_1x_1 + a_2x_2 + a_3x_3 = 0\}, \\ H^- &= \{\mathbf{x} \in \mathbb{R}^3; a_1x_1 + a_2x_2 + a_3x_3 < 0\}, \end{aligned}$$

and similarly for lines in $\mathbb{R}^2$. Can you distinguish the "halfspaces" H^+ and H^- geometrically by a property satisfied by $\mathbf{a} = (a_1, a_2, a_3)$ and the points in H^+, H^- ?

W4 DAVID HILBERT's *Hotel Infinitude* contains infinitely many rooms numbered by $1, 2, 3, \ldots$. The hotel is fully booked during the Midautumn Festival and

a) a new guest arrives;

b) a new guest arrives and insists on being accommodated in Room no. 88;

c) countably many new guests arrive;

d) countably many new tourist groups, each consisting of countably many tourists, arrive;

e) a continuum of new guests arrives.

Explain what you as the hotel manager can do in each case.