

Differential Equations Plus (Math 286)

H63 Find the Laplace transforms of

a) $1 + 2t + 3t^2$; b) e^{5t+3} ; c) $\int_0^t \tau \sin \tau \, d\tau$; d) $\sin^3 t$.

H64 Find inverse Laplace transforms of

a) $\frac{5}{s+6}$; b) $\frac{2s-1}{s^2+3}$; c) $\frac{1}{(s^2+1)(s^2+4)}$; d) $\frac{d}{ds} \frac{1-e^{-5s}}{s}$;
e) $\ln \frac{s}{s-1}$; f) $\ln \frac{s^2+1}{(s-1)^2}$; g) $\frac{s+1}{s^2(s^2+1)}$; h) $\frac{e^{-2s} - e^{-4s}}{s}$;
i) $\operatorname{arccot} \frac{s}{\omega}$; j) $\frac{s^2-1}{(s^3+s^2-5s+3)(s^2-4)}$.

Six answers suffice.

H65 Solve the following initial value problems with the Laplace transform:

a) $y'' - 3y' + 2y = 6e^{-t}$, $y(0) = 9$, $y'(0) = 6$;
b) $y'' + 2y' - 3y = 6\sinh(2t)$, $y(0) = 0$, $y'(0) = 4$;
c) $y''' + y'' - 5y' + 3y = 6\sinh(2t)$, $y(0) = y'(0) = 0$, $y''(0) = 4$.

H66 Find the Laplace transform of the Bessel function J_0 in two ways:

- a) From the power series of J_0 and termwise integration of the Laplace integral.
Hint: The power series expansion

$$\frac{1}{\sqrt{1-4x}} = \sum_{n=0}^{\infty} \binom{2n}{n} x^n, \quad \text{valid for } |x| < 1/4,$$

may help (but you should prove it first).

- b) From the Bessel ODE of order $\nu = 0$.

H67 Do Exercise 24 in [BDM17], Ch. 6.3, and use the result to verify that $\mathcal{L}\{|\sin t|\} = \frac{1}{s^2+1} \coth \frac{\pi s}{2}$ for $\operatorname{Re}(s) > 0$; cp. also [BDM17], Ch. 6.3, Ex. 28.

Due on Wed Dec 8, 6 pm