Mobile robot seminar

Ch4. Mobile robot

학부 연구생 이태연

들어가기 전..

Mobile Robot??

One of the most important functions of

a mobile robot is <u>**to move to some place**</u>

1. move to the light
2. in terms of some geometric coordinate or map reference

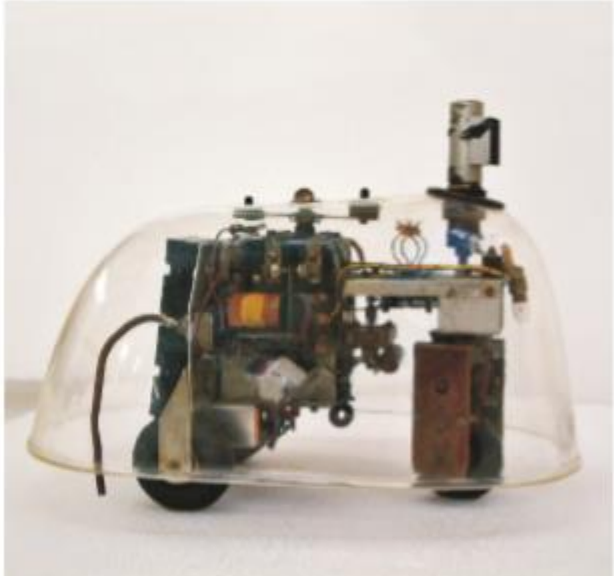
들어가기 전..

Mobile Robot??

Elsie the robotic tortoise

(move to the light)

-> reactive approach



a

b

robot Shakey

(in terms of some geometric coordinate or map reference)

->An alternative to the reactive approach

접근법의 차이

Reactive system

지각하는 시스템은 행동과 연결되어 있어서 빠르고 단순하다.

때문에, 세계를 표현하고 유지할 resource가 필요하지 않고 추론할 능력도 필요하지 않음.

3D perception and created a map

맵을 만들고 그에 대해 추론하는 시스템은 더 많은 resource를 필요로 하지만 더 복잡한 작업을 수행할 수 있다.

These two approaches exemplify opposite ends of the spectrum for mobile robot navigation

들어가기 전..

Mobile Robot??

Mobile Robot 대표 종류

a. Floor

b. Sensor

들어가기 전..

Mobile Robot??

모바일 로봇은 지상에서만 운용되는 것이 아니다.

무인기(UAV), 자율 수중 차량(AUV),

자율 표면 차량(ASV) 로봇보트

들어가기 전..

Mobile Robot??

Mars Science Laboratory (MSL) rover, known as Curiosity/ 항만의 컨테이너 운송 차량/ 광산의 트럭/ 광대한 에이커 농업을 위한 자율주행 트랙터와 같은 현장 로봇 시스템은 이제 다양한 용도에 상업적으로 이용 가능하다.

앞으로 4장에서 다룰 내용들..

Mobile Robot의 기본 원리

Wheeled Mobile Robots

Flying Robot

topic

Advanced Topics

Wheel mobile Robot

Enjoy your stylish business and campus life with BIZCAM

바퀴

이륜카트

사륜 자동차

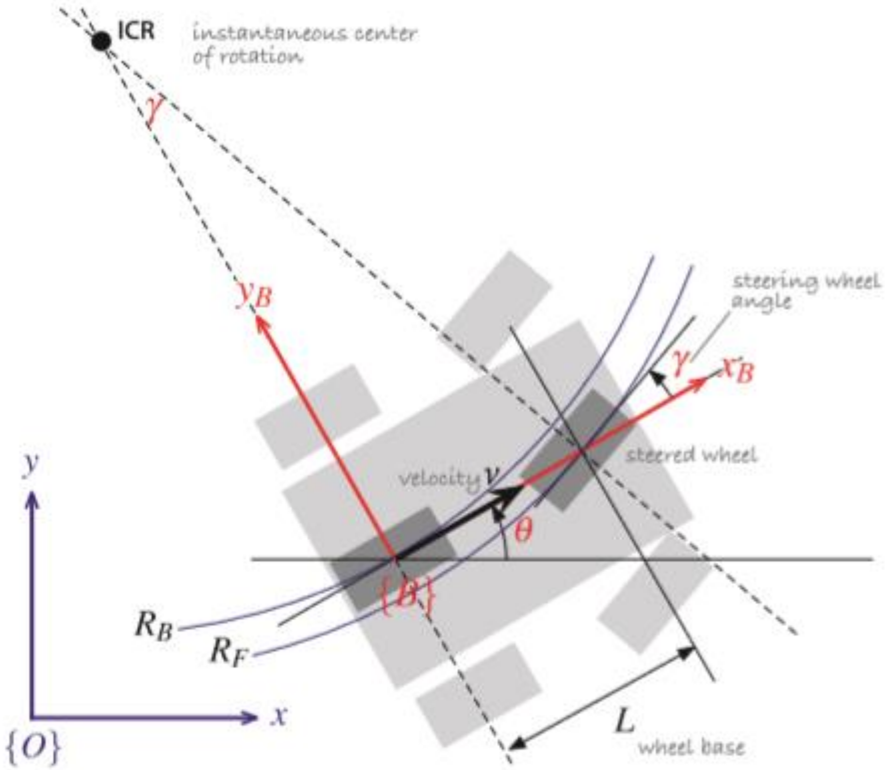
Choice for Robot Platform

It’s not possible to drive sideways.

Can’t rotate a car on the spot.

4.1.1 Car-Like Mobile Robots

Mobile Robot??



$$q = (x, y, \theta) \in \mathcal{C} \text{ where } \mathcal{C} \subset \mathbb{R}^2 \times \mathbb{S}^1.$$

$$^{B}\boldsymbol{v} = (v, 0)$$

{B}:The pose of the vehicle

Symbol	Description	Unit
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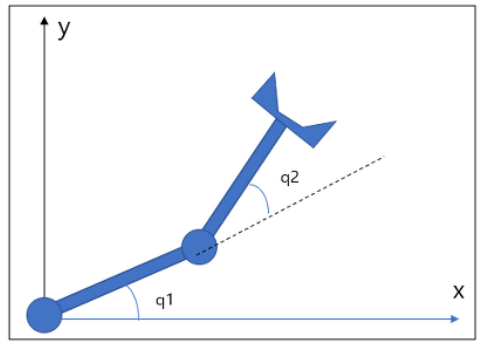
Motion Planning 개념

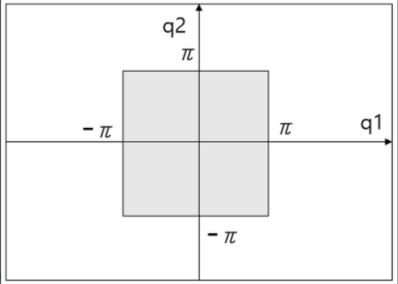
Work Space(or Task Space)

2D 또는 3D 유클리드 공간에서 로봇의 상태를 설명함.
즉, 실제 작업이 이루어 져야하는 x,y,z의 공간을 작업공간(Task Space)라 부르고,



Configuration Space(or Joint Space)

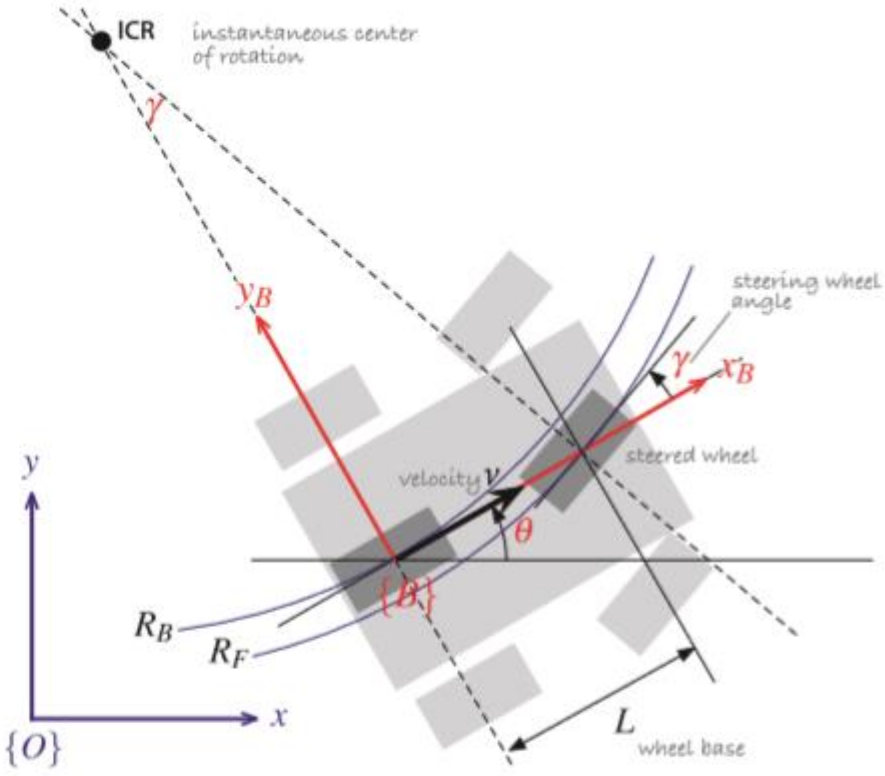
로봇이 제어되는 각 관절 각도(q1,...,qn)에 대한 공간을 관절공간(Joint Space)라고 부릅니다.



결국 로봇의 움직임 생성 문제란 "작업공간에서 잘 움직이도록 관절공간에서 잘 입력을 넣어주는 것"이라고 일반화 시킬 수 있습니다.

4.1.1 Car-Like Mobile Robots

Mobile Robot??



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4.1.1 Car-Like Mobile Robots

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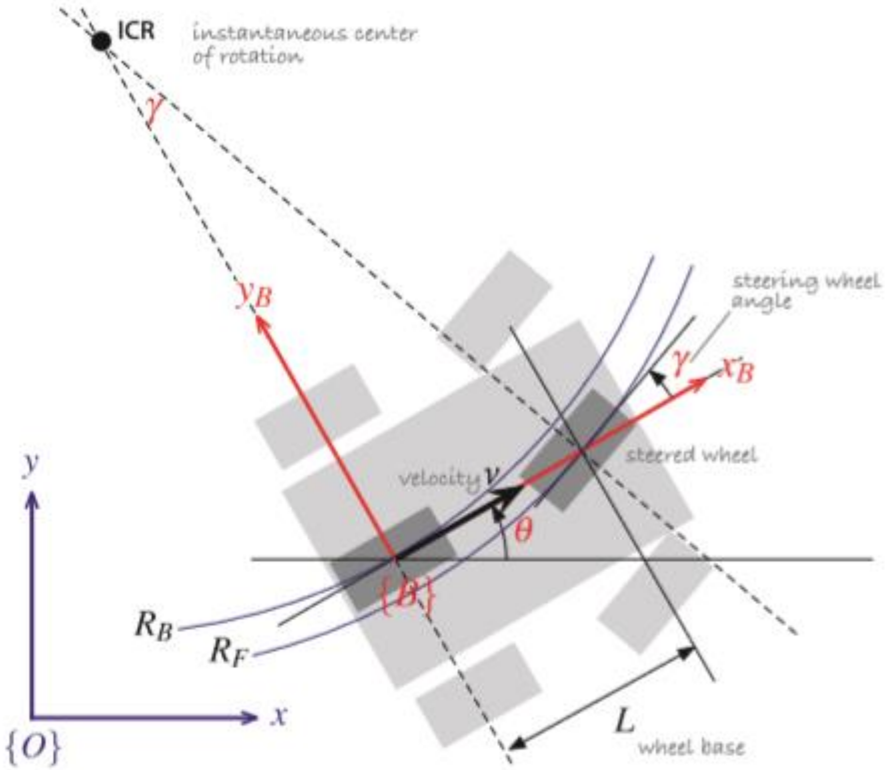

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$^B\boldsymbol{v} = (v, 0)$ **{B}:The pose of the vehicle**

각속도 $\dot{\theta} = \dfrac{v}{R_B}$

회전반경 $R_B = L / \tan\gamma$

경로길이 $R_F > R_B$

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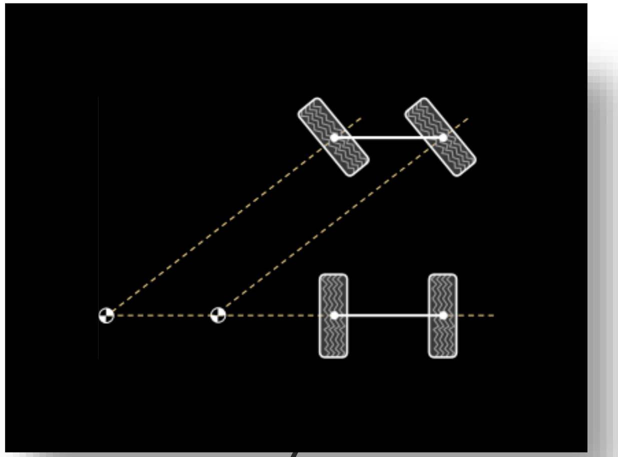
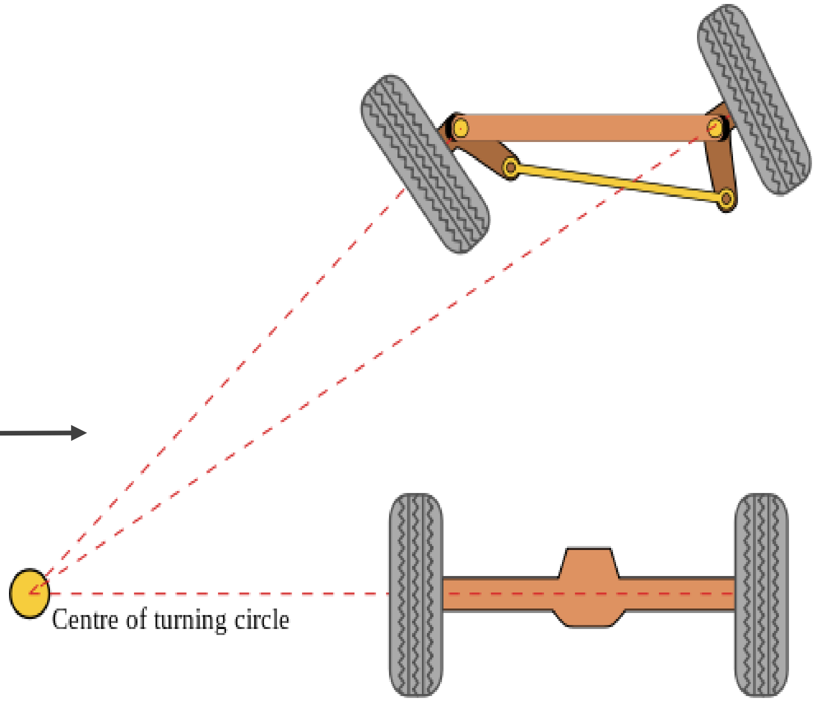
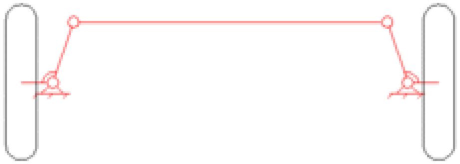
4.1.1 Car-Like Mobile Robots

Mobile Robot??



Solution!!!





Ackermann Steering Geometry?

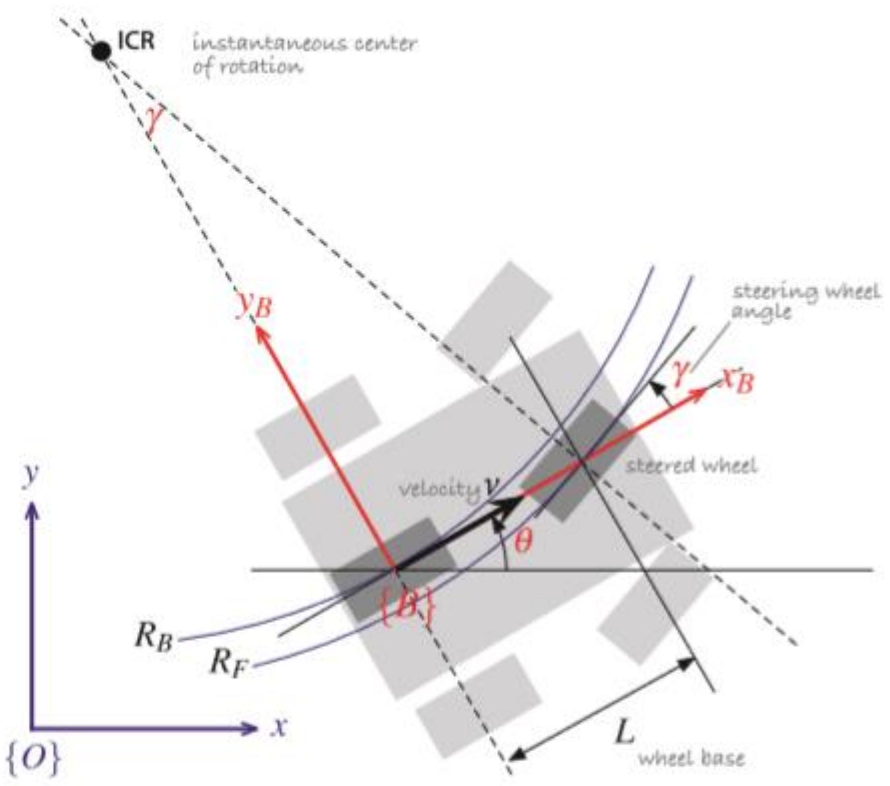
When a four-wheeled vehicle goes around a corner

the two steered wheels follow circular paths of different radii and

therefore the angles of the steered wheels γ L and γ R should be very slightly different.

4.1.1 Car-Like Mobile Robots

Mobile Robot??



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${}^{B}\boldsymbol{v} = (v, 0)$ {B}:The pose of the vehicle

각속도 $\dot{\theta} = \dfrac{v}{R_B}$

회전반경 $R_B = L / \tan\gamma$

경로길이 $R_F > R_B$

속도 및 회전율

$$\dot{x} = v\cos\theta$$
$$\dot{y} = v\sin\theta$$
$$\dot{\theta} = \frac{v}{L}\tan\gamma$$

Y방향 속도 $\dot{y}\cos\theta - \dot{x}\sin\theta \equiv 0$

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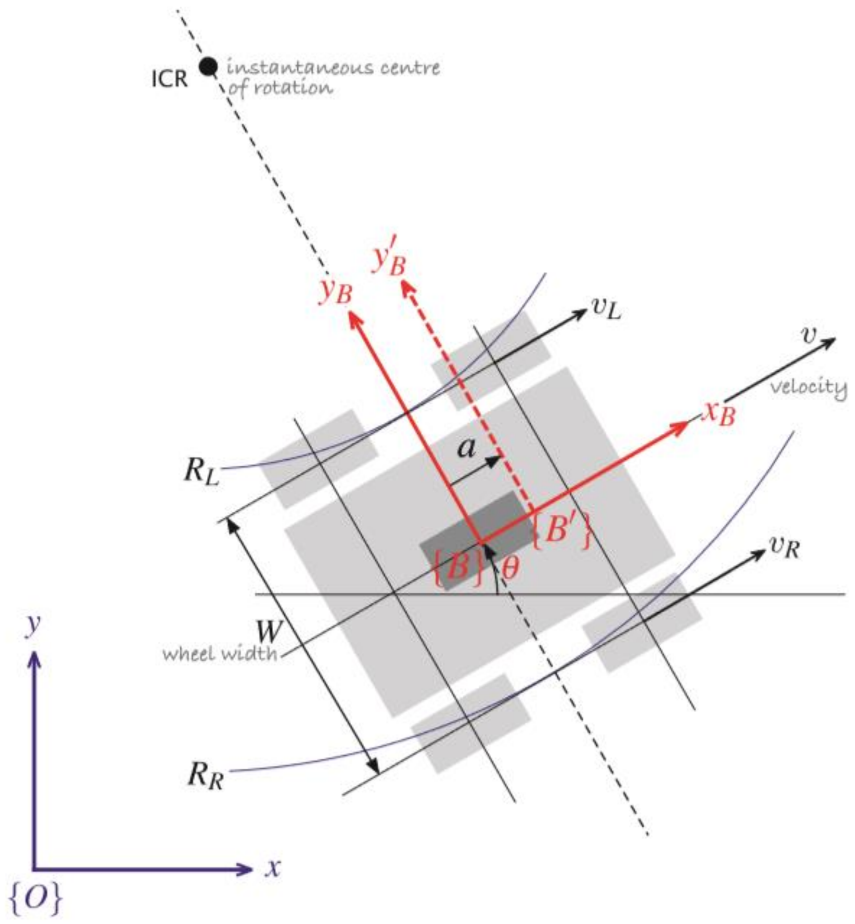
4.1.2 Differentially-Steered Vehicle

Mobile Robot??



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{B}:Frame, pose of the vehicle

{B'}: for trajectory tracking control

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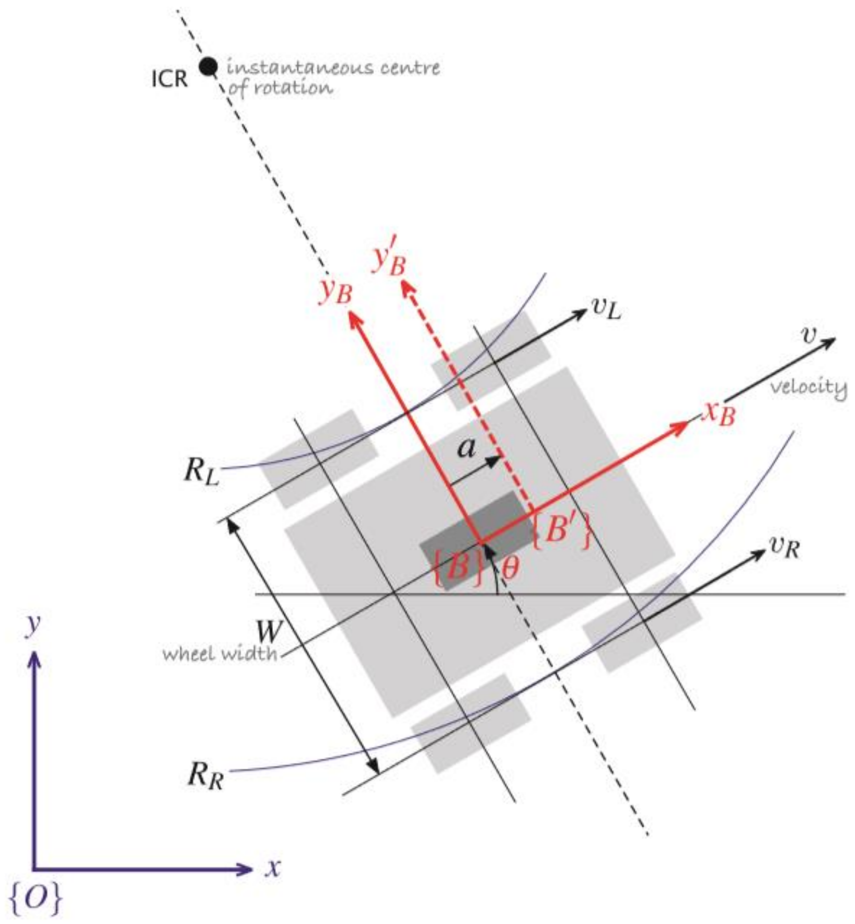

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{B}에서의 각속도 $\dot{\theta} = \frac{v_L}{R_L} = \frac{v_R}{R_R}$

$$R_R = R_L + W$$

이므로

$$\dot{\theta} = \frac{v_R - v_L}{W}$$

$$\dot{x} = v\cos\theta$$
$$\dot{y} = v\sin\theta$$
$$\dot{\theta} = \frac{v_\Delta}{W}$$

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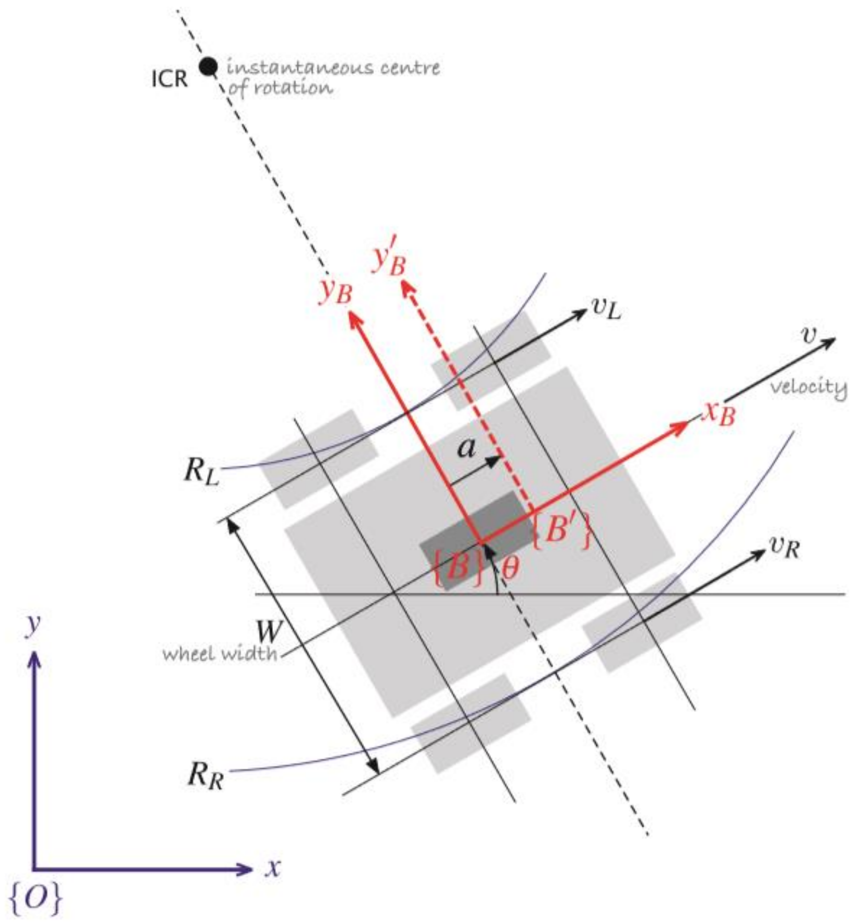

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운동방정식

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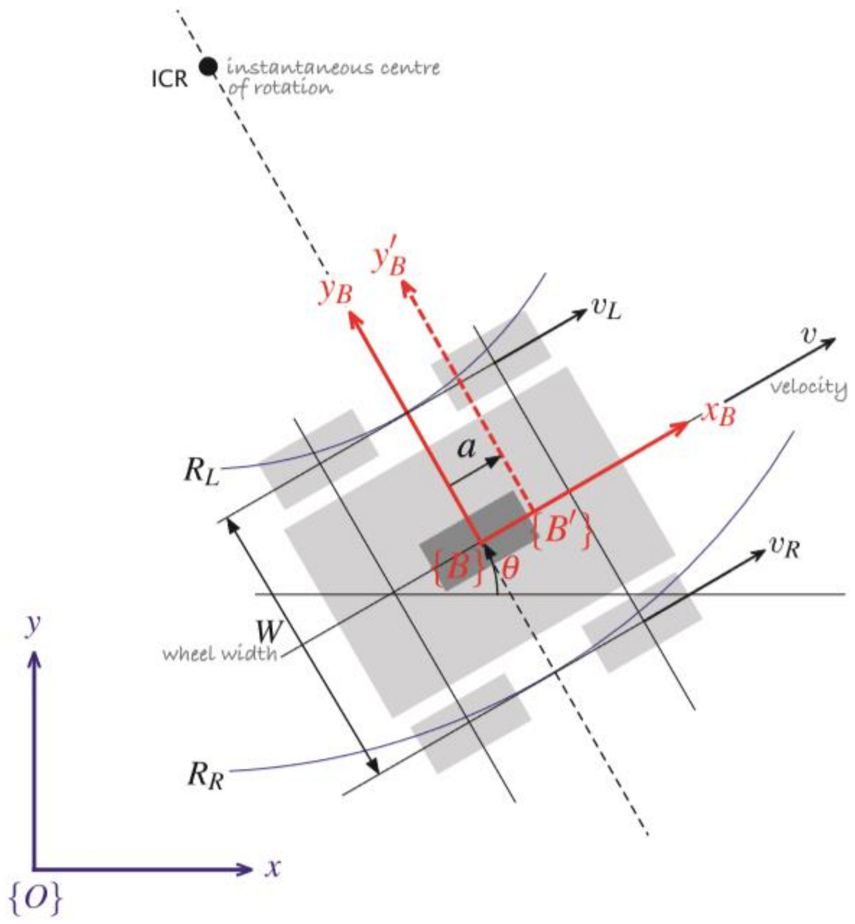

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운동방정식

유사점과 차이점?

$$\dot{x} = v\cos\theta \qquad \dot{y} = v\sin\theta \qquad \dot{\theta} = \frac{v_\Delta}{W}$$

Differentially-Steered Vehicle

$$\dot{x} = v\cos\theta \qquad \dot{y} = v\sin\theta \qquad \dot{\theta} = \frac{v}{L}\tan\gamma$$

Car-Like Mobile Robots

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4.1.2 Differentially-Steered Vehicle

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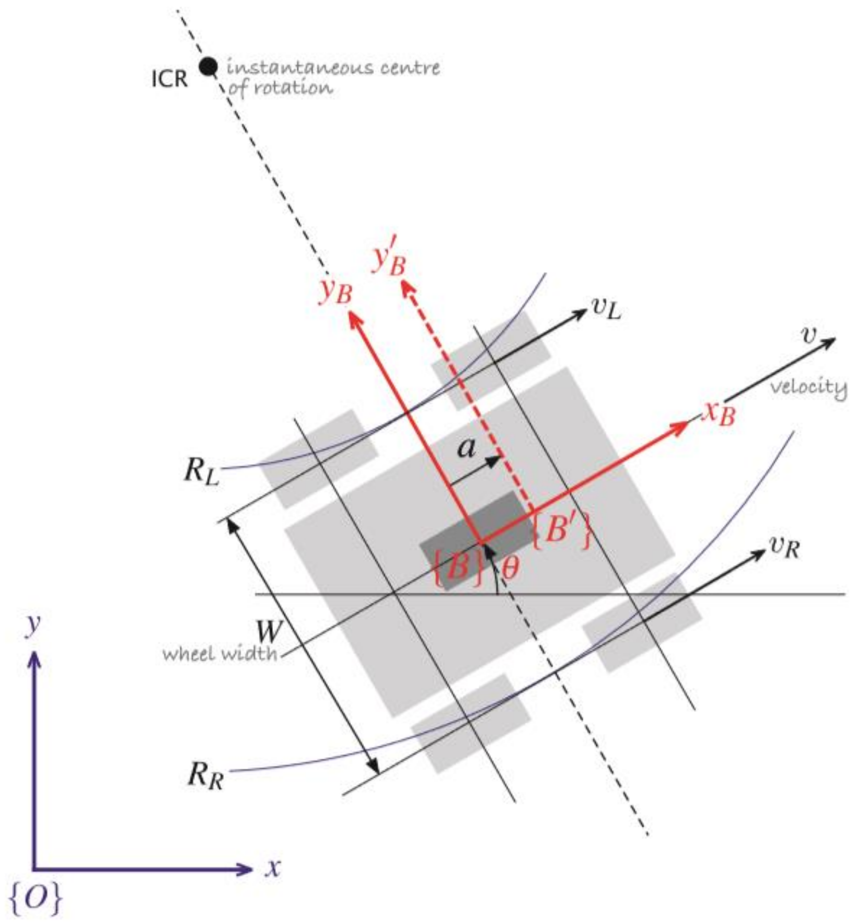

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Differentially-Steered Vehicle

↔

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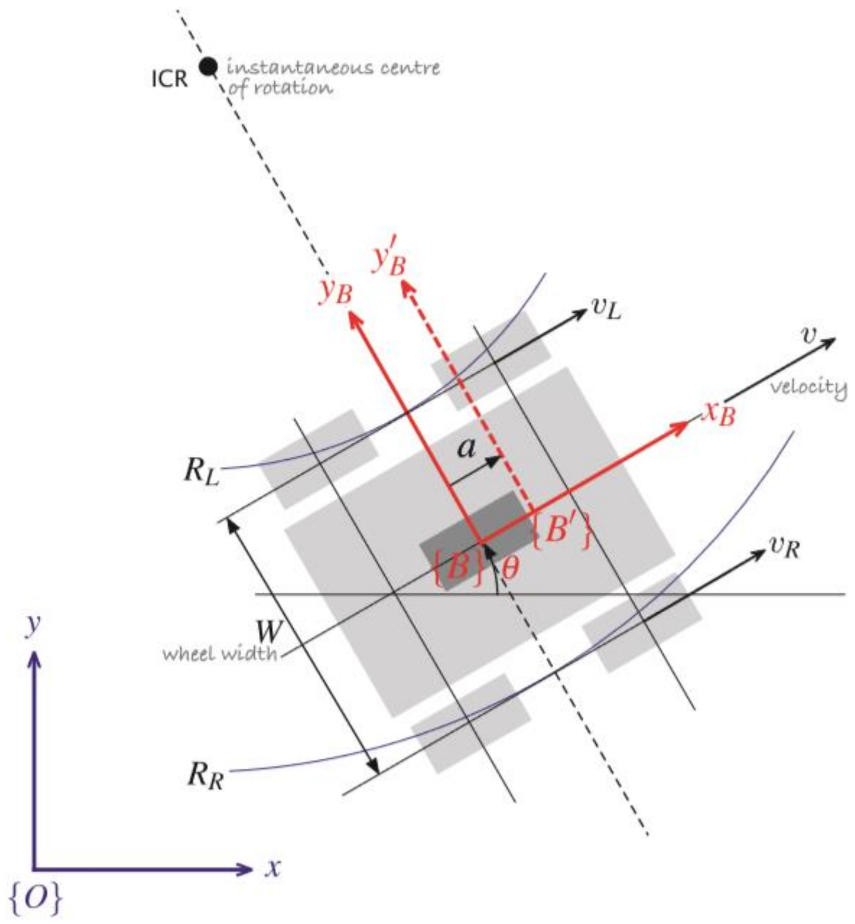
4.1.2 Differentially-Steered Vehicle

Mobile Robot??



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운동방정식

$$\dot{x} = v\cos\theta$$
$$\dot{y} = v\sin\theta$$
$$\dot{\theta} = \frac{v_\Delta}{W}$$

Matrix Form!!

Differentially-Steered Vehicle

If we move the vehicle's reference frame to $\{B'\}$ and ignore orientation we can rewrite Eq. 4.5 in matrix form as

$$\begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \cos\theta & -a\sin\theta \\ \sin\theta & a\cos\theta \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}$$

and if $a \neq 0$ this can be be inverted

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\frac{1}{a}\sin\theta & \frac{1}{a}\cos\theta \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} \tag{4.7}$$

to give the required forward speed and turn rate to achieve an arbitrary velocity $(\dot{x}, \dot{y})$ for the origin of frame $\{B'\}$.

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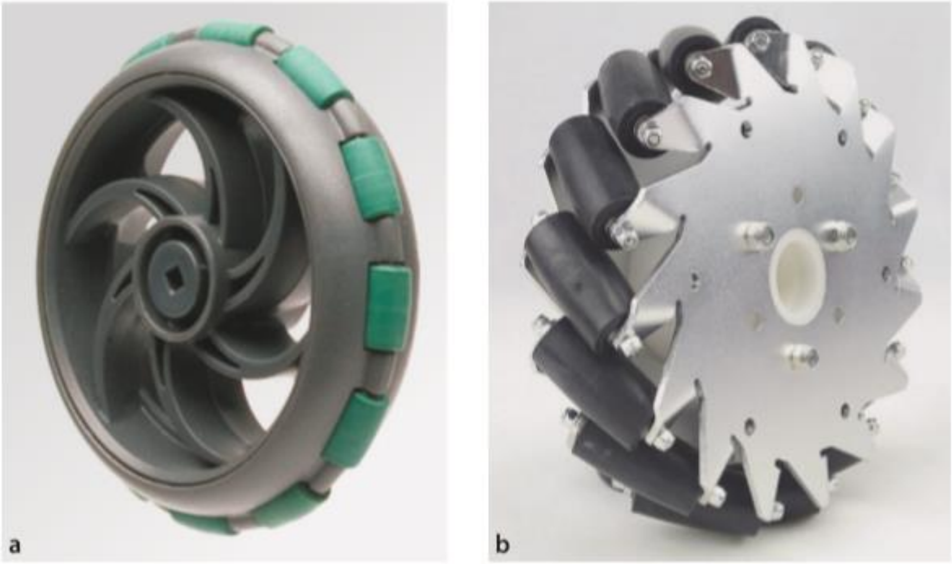
4.1.3 Omnidirectional Vehicle

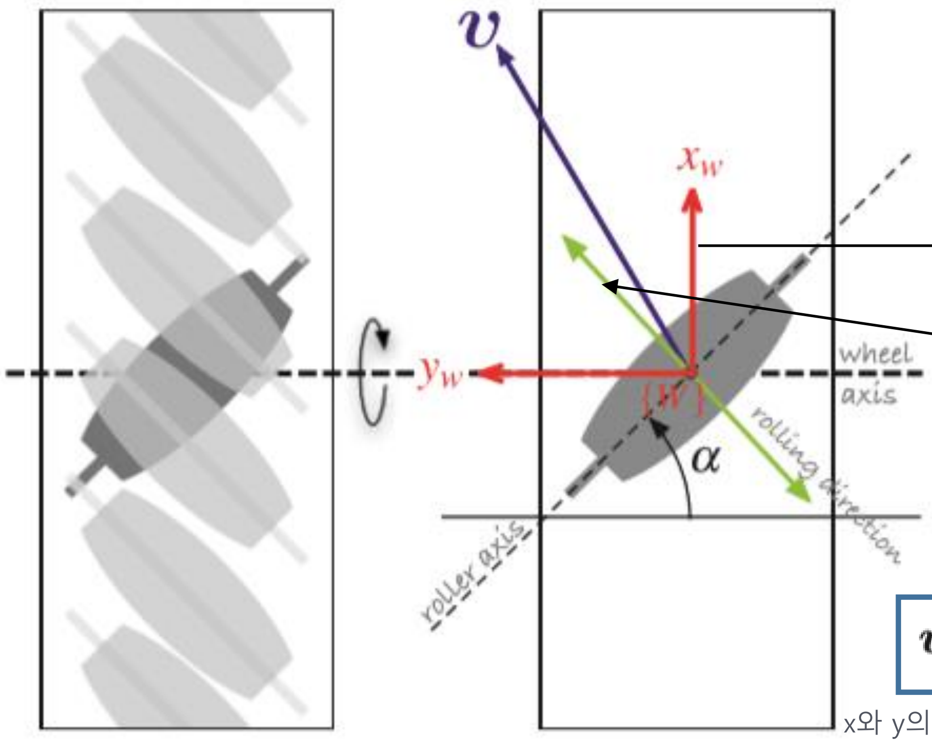
Mobile Robot??



More Radical Wheel

4.1.3 Omnidirectional Vehicle

Mobile Robot??



전방속도 $R\varpi\hat{\boldsymbol{x}}_w$ **휠 회전율** $\varpi = v_w/R.$

휠 반지름 R **휠 동작 방향** $\hat{\boldsymbol{x}}_w$

휠 회전으로 인한 속도 v_w **Rolling speed** v_r

Roller axis에 수직, 속도 v에 상관없이 잠재적으로 임의의 속도존재

$$\boldsymbol{v} = \underbrace{v_w\hat{\boldsymbol{x}}_w}_{\text{driven}} + \underbrace{v_r\left(\cos\alpha\hat{\boldsymbol{x}}_w + \sin\alpha\hat{\boldsymbol{y}}_w\right)}_{\text{rolling}}$$

$$\boxed{\boldsymbol{v} = v_x\hat{\boldsymbol{x}}_w + v_y\hat{\boldsymbol{y}}_w} = \left(v_w + v_r\cos\alpha\right)\hat{\boldsymbol{x}}_w + v_r\sin\alpha\hat{\boldsymbol{y}}_w$$

x와 y의 구성요소 형태로 원하는 속도 v식 표현

$$\boxed{v_r = \boldsymbol{v}_y/\sin\alpha}$$

휠 속도 term 최종정리

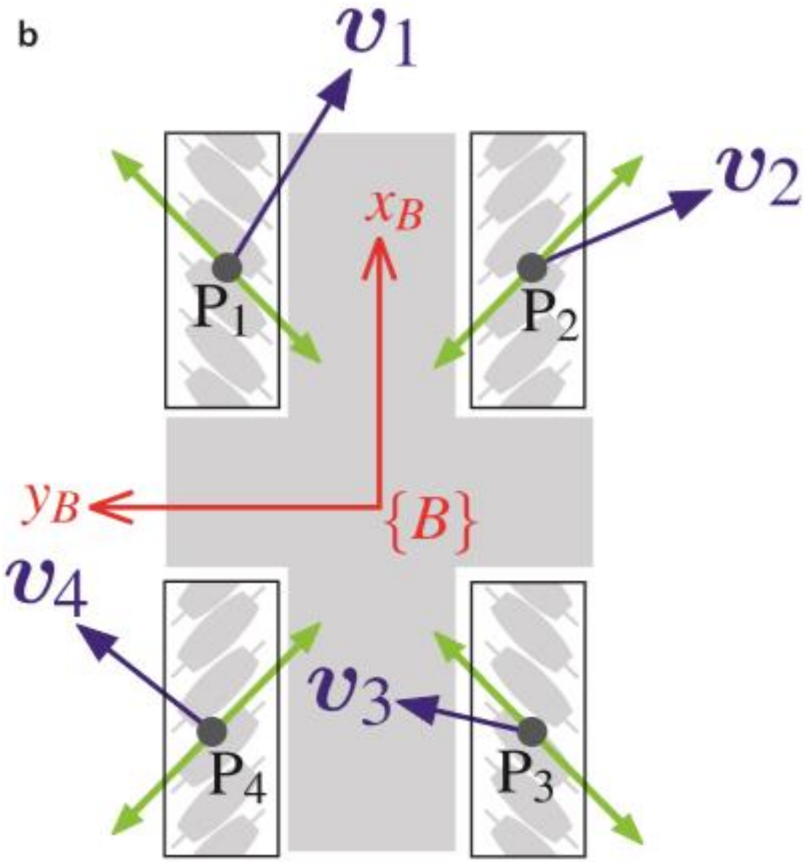
$$\boxed{v_w = \boldsymbol{v}_x - \boldsymbol{v}_y\cot\alpha}$$

4.1.3 Omnidirectional Vehicle

Mobile Robot??

4개의 메카넘 휠이 있는

kuka youbot



kuka youbot 모델링 도식

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4.1.3 Omnidirectional Vehicle

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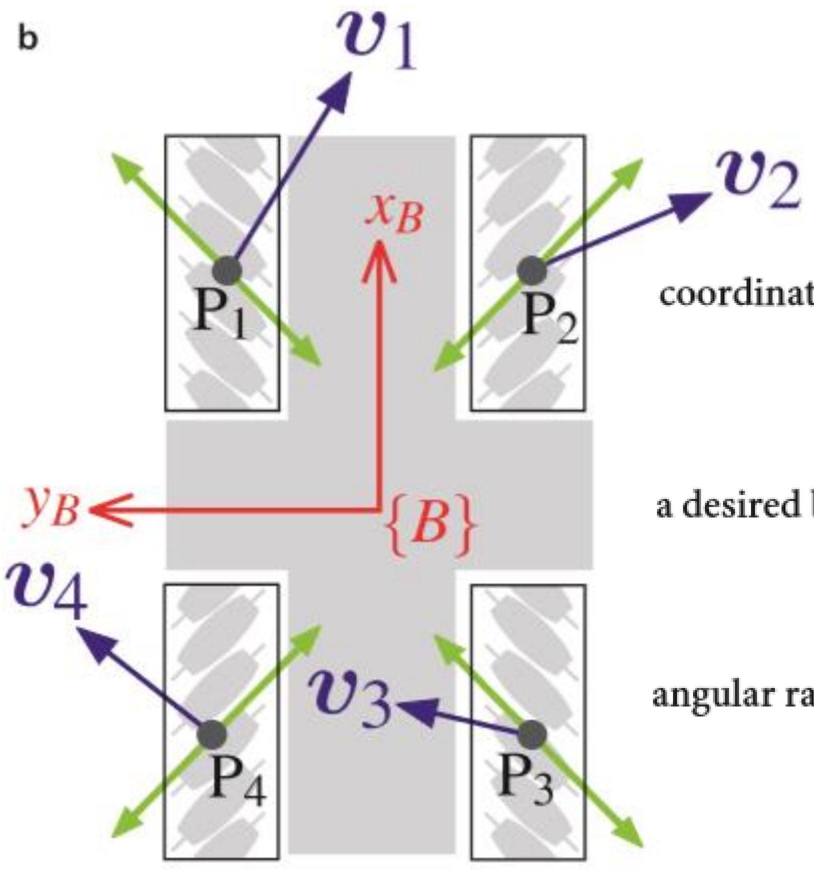


coordinate vectors ${}^B\boldsymbol{p}_i$

a desired body velocity ${}^B\boldsymbol{v}_B$ ⟶ $\boxed{{}^B\boldsymbol{v}_i = {}^B\boldsymbol{v}_B + {}^B\omega\hat{\boldsymbol{z}}_B \times {}^B\boldsymbol{p}_i}$

angular rate ${}^B\omega$

각각의 휠 접촉 지점의 속도

kuka youbot 모델링 도식

$\boldsymbol{q} = (x, y, \theta) \in \mathcal{C}$ where $\mathcal{C} \subset \mathbb{R}^2 \times \mathbb{S}^1$.

4.2 Flying Robots

Mobile Robot??

Flying Robots

1.micro air vehicles or MAVs

2. Fixed wing UAVs

3. Rotorcraft UAVs

4.2 Flying Robots

Mobile Robot??

Flying robots **differ** from ground robots in some important ways

→ 1. configuration

configuration $q \in \mathcal{C}$ where $\mathcal{C} \subset \mathbb{R}^3 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$.

2. Dynamic Model

수중 로봇과 Flying Robots의 유사점과 차이점

→ underwater equivalents (fixed wing aircraft, rotorcraft)

→ buoyancy force, drag forces added mass

4.2 Flying Robots

Mobile Robot??

1.Easy to fly

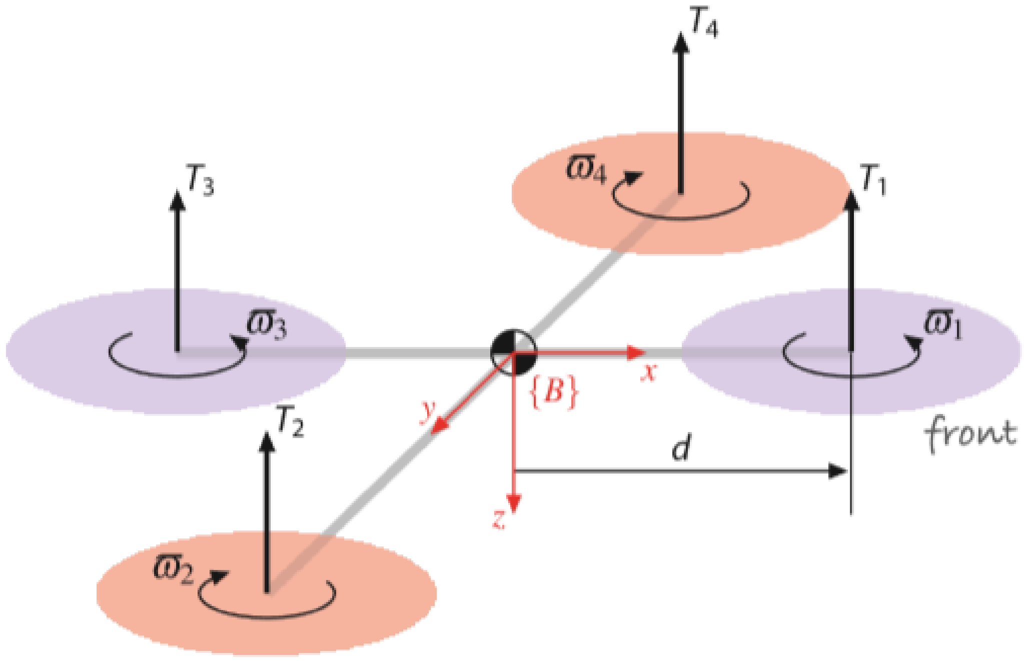
2. does not have the complex mechanism

3. easier to model and control than others

Quadrotor modeling!

4.2 Flying Robots

Mobile Robot??



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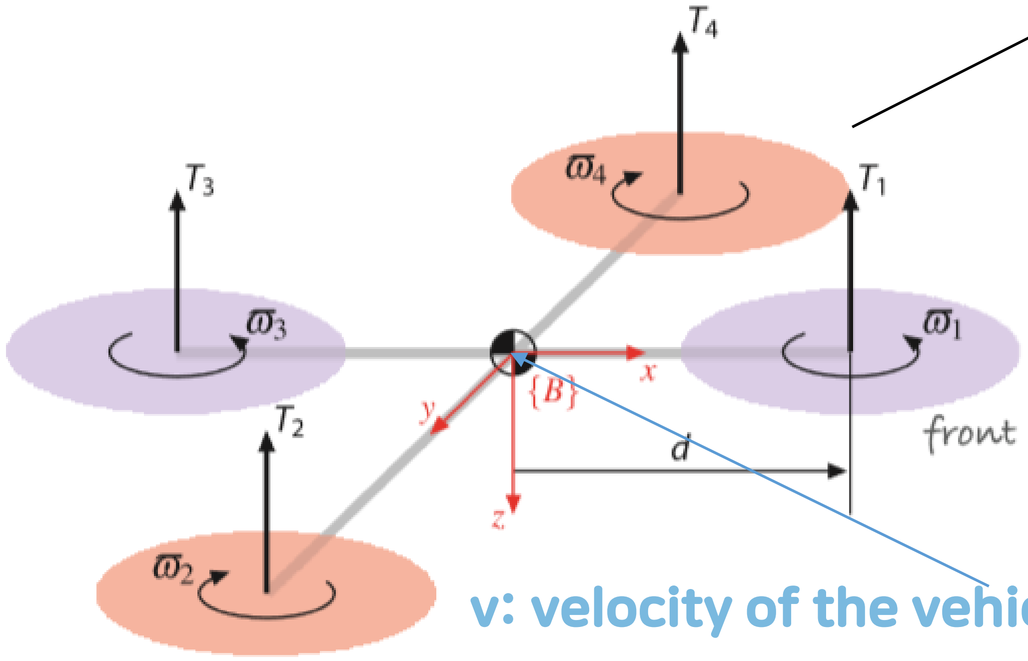
Quadrotor Modeling 도식

configuration $q \in \mathcal{C}$ where $\mathcal{C} \subset \mathbb{R}^3 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$.

4.2 Flying Robots

Mobile Robot??



$$T_i = b\varpi_i^2, \quad i = 1, 2, 3, 4$$

Lift constant $b > 0$

Newton's second law

$$m\dot{v} = \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} - {}^0R_B \begin{pmatrix} 0 \\ 0 \\ T \end{pmatrix} - Bv$$

g:중력 가속도

m:vehicle 질량

B:aerodynamic friction

$T = \Sigma T_i$ is the total upward thrust

v: velocity of the vehicle's center of mass

Quadrotor Modeling 도식

configuration $\boldsymbol{q} \in \mathcal{C}$ where $\mathcal{C} \subset \mathbb{R}^3 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$.

terms 분석

4.2 Flying Robots

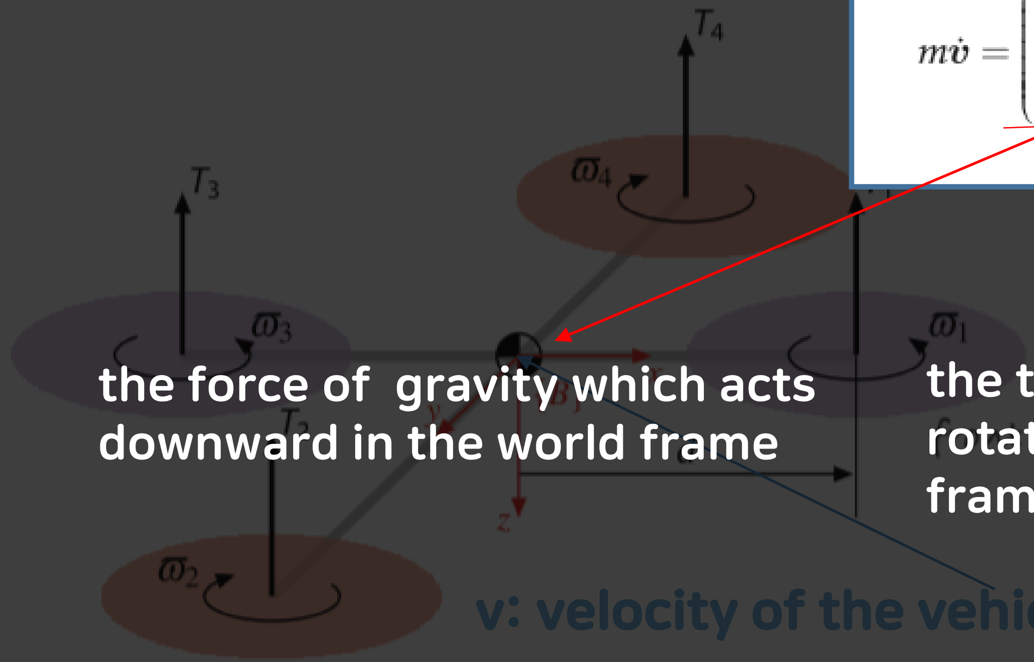
Mobile Robot??

$$m\dot{v} = \begin{pmatrix}0\\0\\mg\end{pmatrix} - {}^0R_B\begin{pmatrix}0\\0\\T\end{pmatrix} - Bv$$

$i = 1, 2, 3, 4$

aerodynamic drag

Newton's second law



the force of gravity which acts downward in the world frame

the total thrust in the vehicle frame rotated into the world coordinate frame

g:중력 가속도

m:vehicle 질량

B:aerodynamic friction

$T = \Sigma T_i$ is the total upward thrust

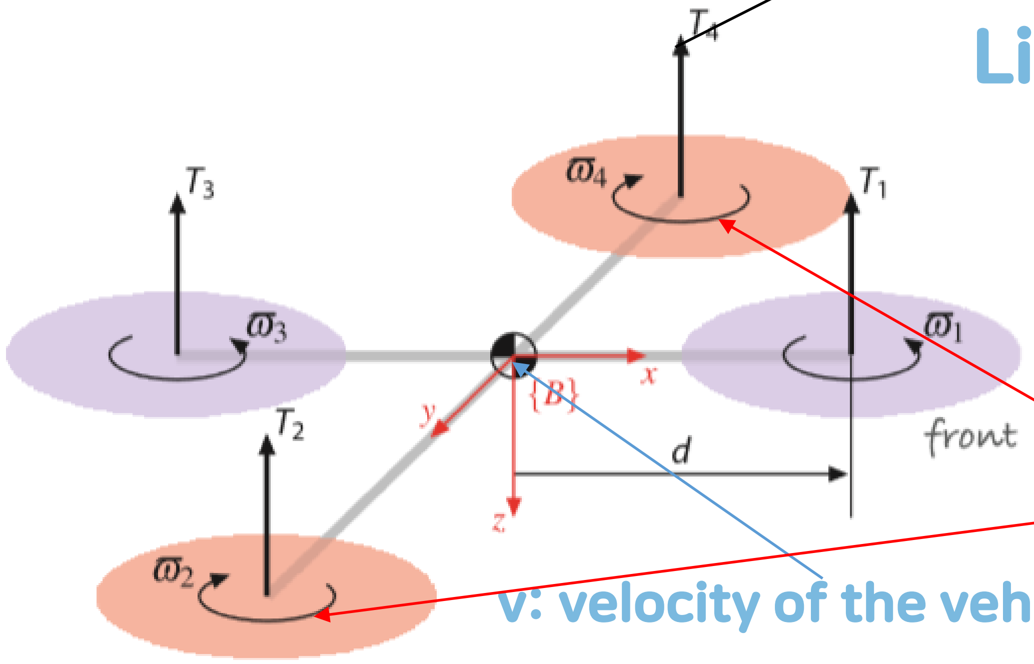
v: velocity of the vehicle's center of mass

Quadrotor Modeling 도식

configuration $q \in \mathcal{C}$ where $\mathcal{C} \subset \mathbb{R}^3 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$.

4.2 Flying Robots

Mobile Robot??



$$T_i = b\varpi_i^2, \quad i = 1,2,3,4$$

Lift constant $b > 0$

Newton's second law

$$m\dot{v} = \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} - {}^0R_B \begin{pmatrix} 0 \\ 0 \\ T \end{pmatrix} - Bv$$

g:중력 가속도

m:vehicle 질량

B:aerodynamic friction

$T = \Sigma T_i$ is the total upward thrust

v: velocity of the vehicle's center of mass

Rotor thrust의 차이가 vehicle을 회전하게 만든다.

$$\tau_x = dT_4 - dT_2$$

&

$$T_i = b\varpi_i^2, \quad i = 1,2,3,4$$

=>

$$\tau_x = db\left(\varpi_4^2 - \varpi_2^2\right)$$

Quadrotor Modeling 도식

configuration $q \in \mathcal{C}$ where $\mathcal{C} \subset \mathbb{R}^3 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$.

4.2 Flying Robots

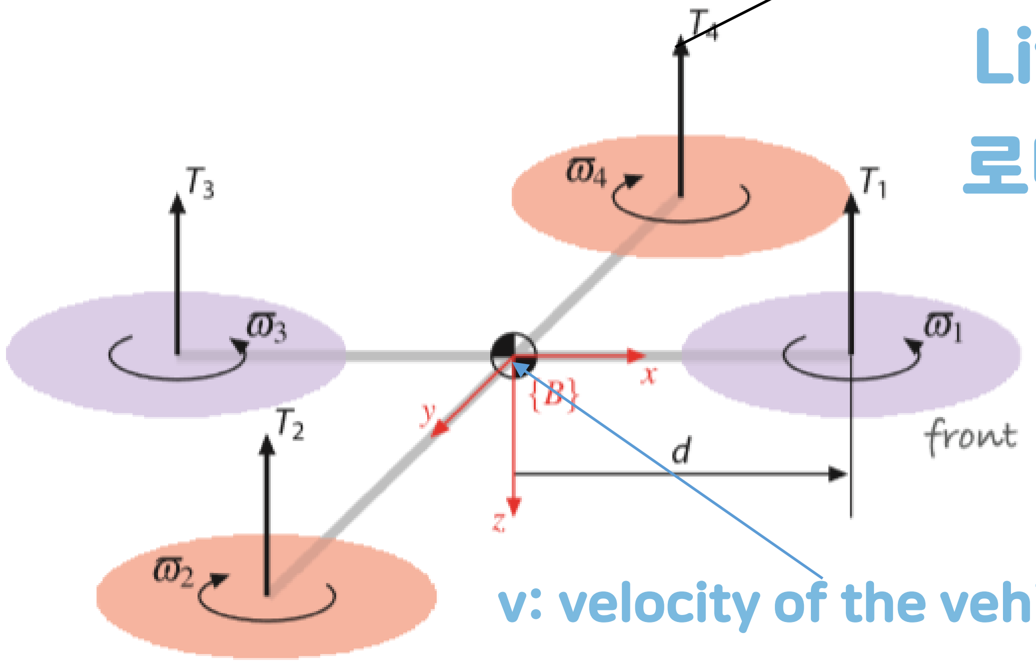
Mobile Robot??

$$T_i = b\varpi_i^2, \quad i = 1, 2, 3, 4$$

Lift constant $b > 0$

로터 속도의 관점에서 토크식(x축,y축)

$$\tau_x = db\left(\varpi_4^2 - \varpi_2^2\right) \Rightarrow \tau_y = db\left(\varpi_1^2 - \varpi_3^2\right)$$



v: velocity of the vehicle's center of mass

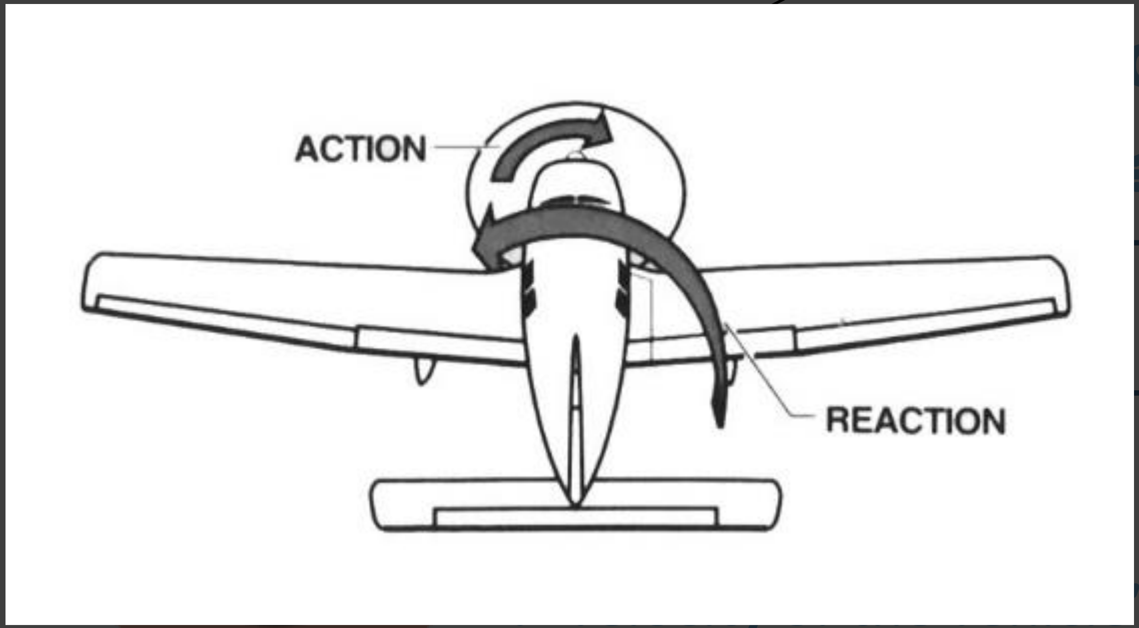
Reaction Torque???

$$Q_i = k\varpi_i^2$$

Quadrotor Modeling 도식

configuration $q \in \mathcal{C}$ where $\mathcal{C} \subset \mathbb{R}^3 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$.

Reaction Torque?



뉴턴의 운동 법칙 때문에,
엔진이 프로펠러를 회전시키기 위해 가하는
힘(토크)은 엔진에 다시 가해지는 반응적인
reactive force를 유발하고
이때 이 force는 전체 비행기에 가해지게
된다는 개념

4.2 Flying Robots

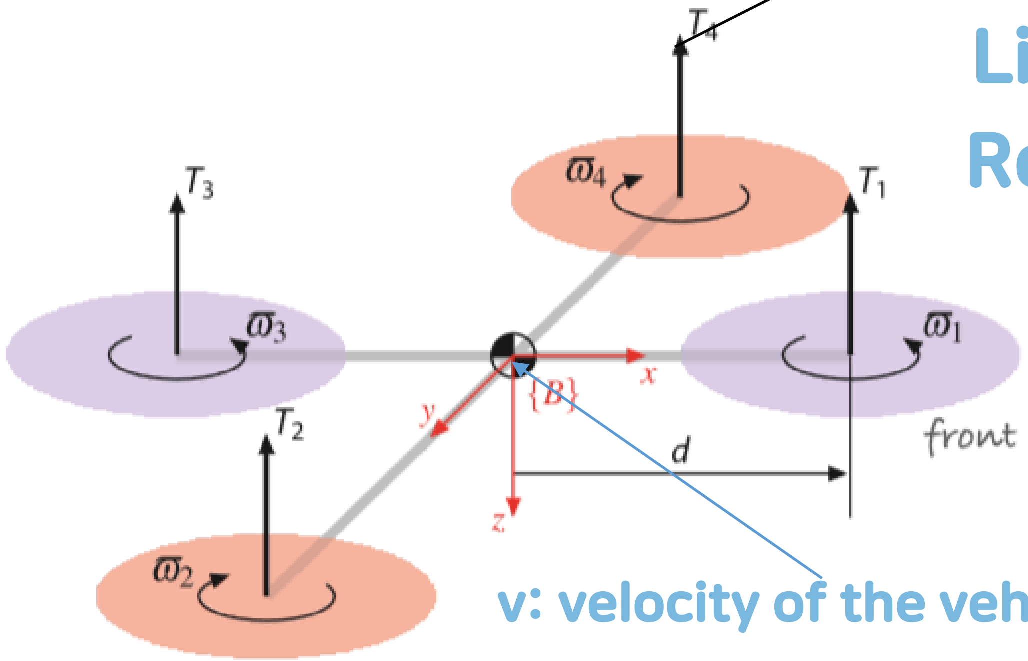
Mobile Robot??

$$T_i = b\varpi_i^2, \quad i = 1, 2, 3, 4$$

Lift constant $b > 0$

Reaction Torque

$$Q_i = k\varpi_i^2$$



v: velocity of the vehicle's center of mass

Total Reaction Torque

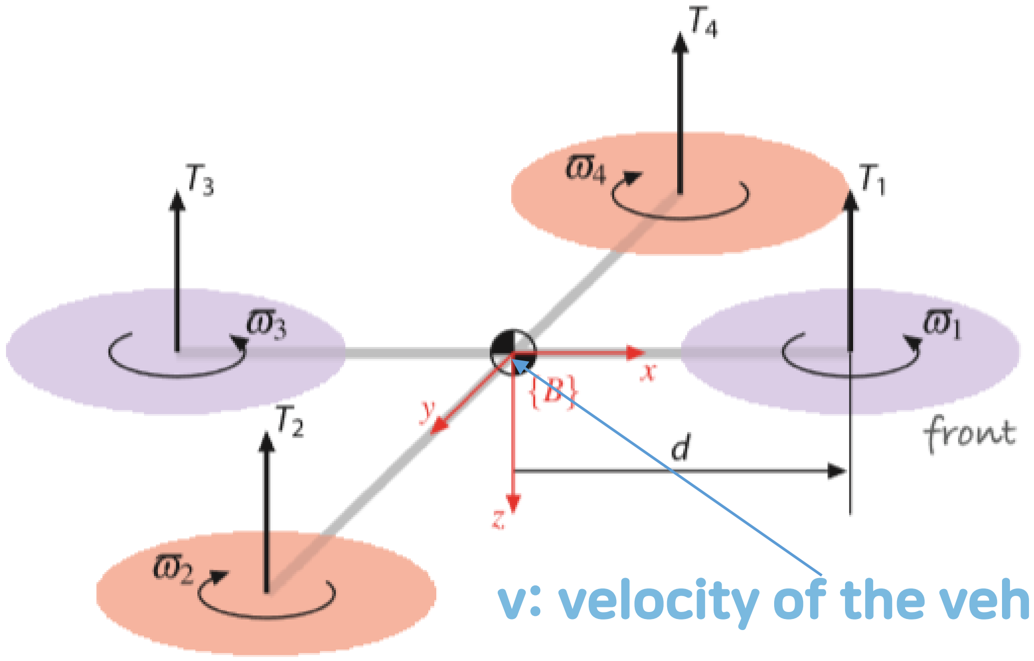
$$\begin{aligned}\tau_z &= Q_1 - Q_2 + Q_3 - Q_4 \\ &= k\left(\varpi_1^2 + \varpi_3^2 - \varpi_2^2 - \varpi_4^2\right)\end{aligned}$$

Quadrotor Modeling 도식

configuration $\boldsymbol{q} \in \mathcal{C}$ where $\mathcal{C} \subset \mathbb{R}^3 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$.

4.2 Flying Robots

Mobile Robot??



v: velocity of the vehicle's center of mass

Quadrotor Modeling 도식

configuration $q \in \mathcal{C}$ where $\mathcal{C} \subset \mathbb{R}^3 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$.

로터 속도의 관점에서 토크식(x축,y축)

$\tau_x = db\left(\varpi_4^2 - \varpi_2^2\right)$ => $\tau_y = db\left(\varpi_1^2 - \varpi_3^2\right)$

Total Reaction Torque(z축)

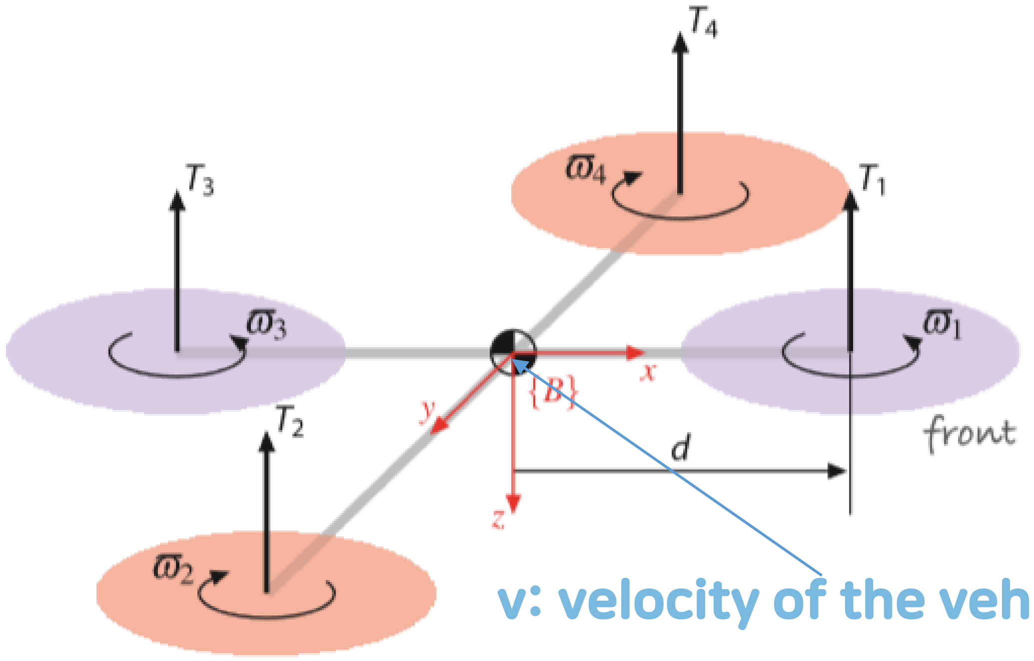
$$\begin{aligned}\tau_z &= Q_1 - Q_2 + Q_3 - Q_4 \\ &= k\left(\varpi_1^2 + \varpi_3^2 - \varpi_2^2 - \varpi_4^2\right)\end{aligned}$$

Total Torque

$\boldsymbol{\tau} = (\tau_x, \tau_y, \tau_z)^T$

4.2 Flying Robots

Mobile Robot??



v: velocity of the vehicle's

center of mass

Quadrotor Modeling 도식

configuration $\boldsymbol{q} \in \mathcal{C}$ where $\mathcal{C} \subset \mathbb{R}^3 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$.

Total Torque

$$\boldsymbol{\tau} = (\tau_x, \tau_y, \tau_z)^T$$

Euler's equations of motion

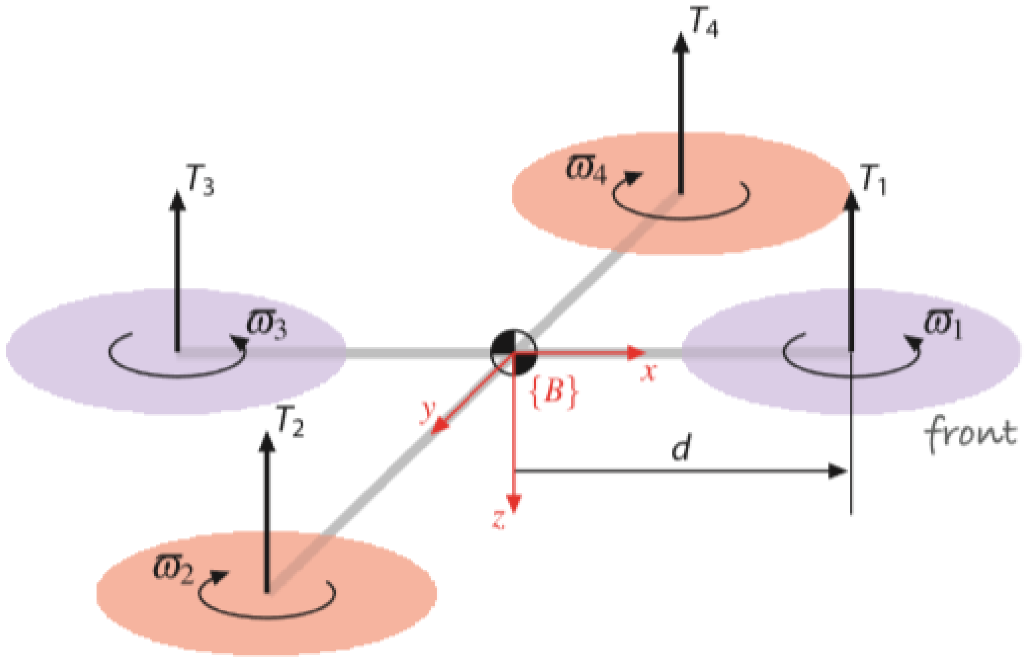
$$^B\boldsymbol{J}\,^B\dot{\boldsymbol{\omega}} + {}^B\boldsymbol{\omega} \times \left({}^B\boldsymbol{J}\,^B\boldsymbol{\omega}\right) = {}^B\boldsymbol{\tau}$$

→ **Total Torque**

$$J\dot{\omega} = -\omega \times J\omega + \tau$$

4.2 Flying Robots

Mobile Robot??



Total Torque

$$J\dot{\omega} = -\omega \times J\omega + \tau$$

\+

Dynamics by Newton's second law

$$m\dot{v} = \begin{pmatrix} 0 \\ 0 \\ mg \end{pmatrix} - {}^{0}R_B \begin{pmatrix} 0 \\ 0 \\ T \end{pmatrix} - Bv$$

Quadrotor Modeling 도식

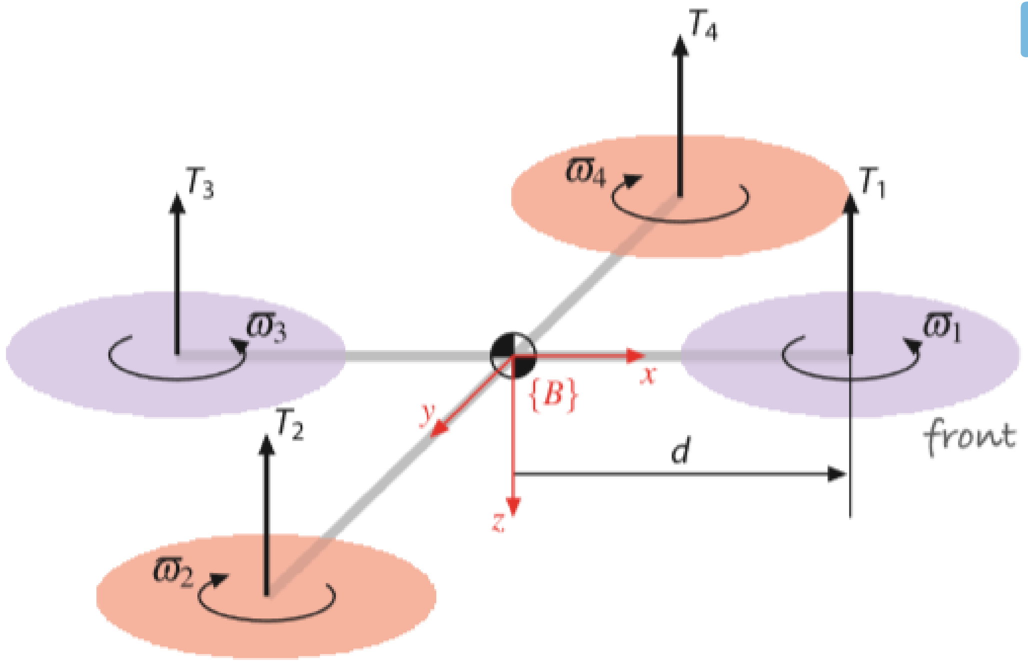
configuration $q \in \mathcal{C}$ where $\mathcal{C} \subset \mathbb{R}^3 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$.

Quadrotor Motion

$$\begin{pmatrix} T \\ \tau \end{pmatrix} = \begin{pmatrix} -b & -b & -b & -b \\ 0 & -db & 0 & db \\ db & 0 & -db & 0 \\ k & -k & k & -k \end{pmatrix} \begin{pmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{pmatrix} = A \begin{pmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{pmatrix}$$

4.2 Flying Robots

Mobile Robot??



Quadrotor Modeling 도식

configuration $q \in \mathcal{C}$ where $\mathcal{C} \subset \mathbb{R}^3 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$.

Quadrotor Motion

$$\begin{pmatrix} T \\ \tau \end{pmatrix} = \begin{pmatrix} -b & -b & -b & -b \\ 0 & -db & 0 & db \\ db & 0 & -db & 0 \\ k & -k & k & -k \end{pmatrix} \begin{pmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{pmatrix} = A \begin{pmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{pmatrix}$$

full rank if $b, k, d > 0$

역행렬!

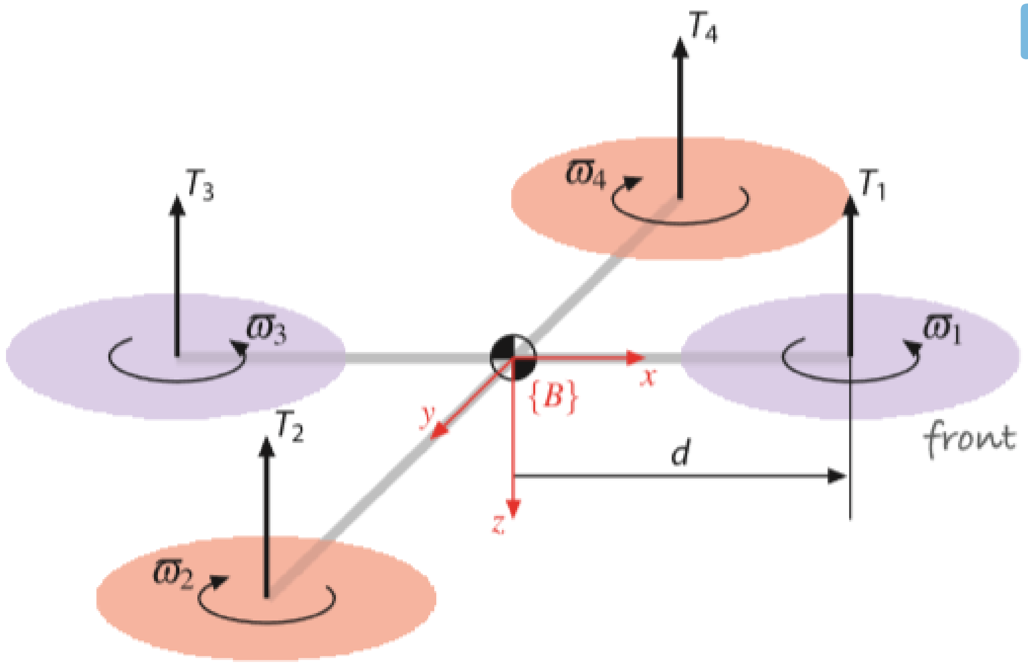
Quadrotor Speed

$$\begin{pmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{pmatrix} = A^{-1} \begin{pmatrix} T \\ \tau_x \\ \tau_y \\ \tau_z \end{pmatrix}$$

cf) Control of motor velocity is discussed in Sect. 9.1.6.

4.2 Flying Robots

Mobile Robot??



Quadrotor Modeling 도식

configuration $q \in \mathcal{C}$ where $\mathcal{C} \subset \mathbb{R}^3 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$.

Quadrotor Motion

$$\begin{pmatrix} T \\ \tau \end{pmatrix} = \begin{pmatrix} -b & -b & -b & -b \\ 0 & -db & 0 & db \\ db & 0 & -db & 0 \\ k & -k & k & -k \end{pmatrix} \begin{pmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{pmatrix} = A \begin{pmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{pmatrix}$$

full rank if $b, k, d > 0$

역행렬!

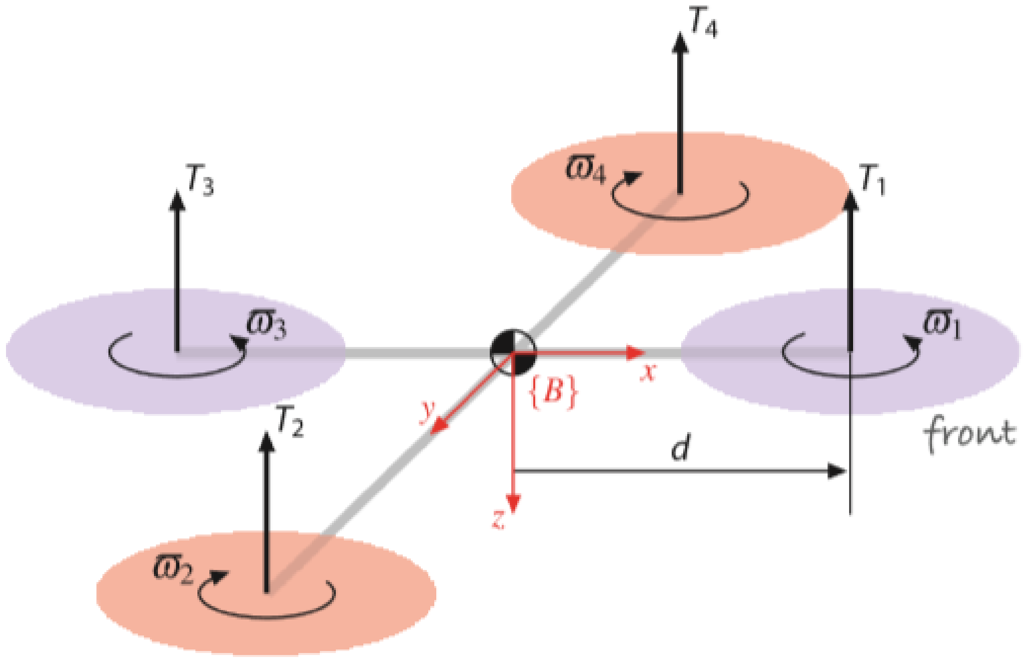
Quadrotor Speed

$$\begin{pmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{pmatrix} = A^{-1} \begin{pmatrix} T \\ \tau_x \\ \tau_y \\ \tau_z \end{pmatrix}$$

cf) Control of motor velocity is discussed in Sect. 9.1.6.

4.2 Flying Robots

Mobile Robot??



Quadrotor Modeling 도식

configuration $q \in \mathcal{C}$ where $\mathcal{C} \subset \mathbb{R}^3 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$.

Quadrotor Speed

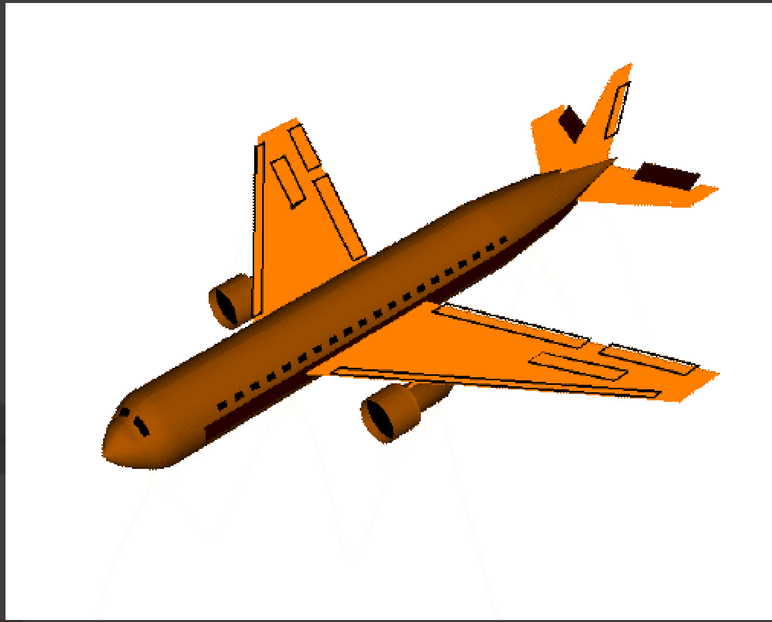
$$\begin{pmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{pmatrix} = \boldsymbol{A}^{-1} \begin{pmatrix} T \\ \tau_x \\ \tau_y \\ \tau_z \end{pmatrix}$$

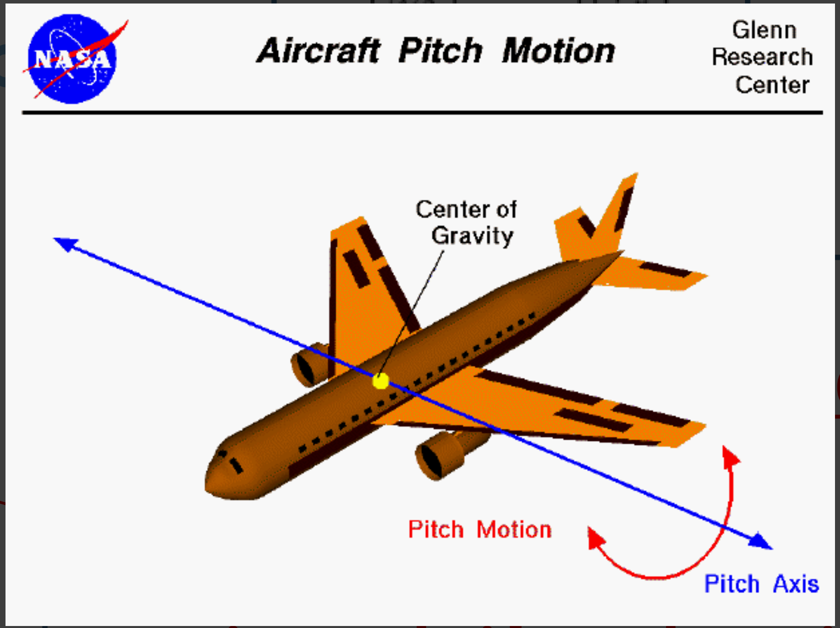
<u>a nested control structure</u>

for pitch and

x-translational motion

Pitch motion?





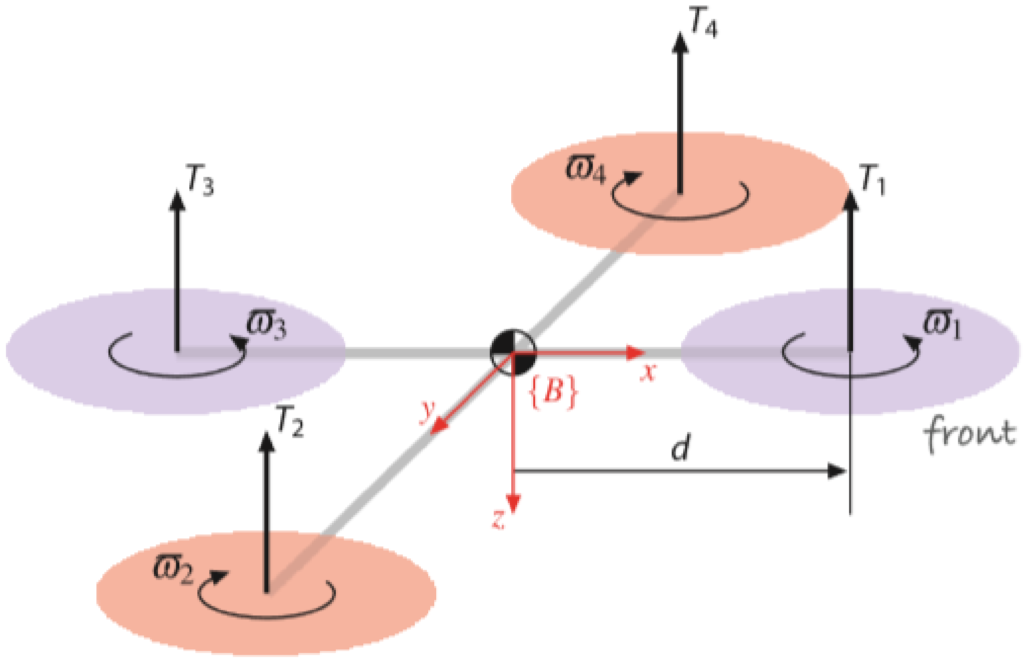
항공기는 비행중 중량의 평균 위치인 무게중심을 중심으로 회전한다
이때 이 좌표계를 3차원으로 정의 한다면 각각의 주요 축을 따라
항공기 부품들의 회전량으로 항공기의 방향을 정의할 수 있다.
(엘리베이터나 엔진, 의자 등을 포함한 모든 요소가 항공기 전체의 회전량에 영향을 미친다는 것)
피치 축은 항공기 중심선에 수직이며 날개 평면에 놓여 있다.
이때 Pitch motion은 그림과 같이 항공기의 코부분의 위아래 이동을 의미한다.

4.2 Flying Robots

Mobile Robot??

Quadrotor Speed

$$\begin{pmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{pmatrix} = \boldsymbol{A}^{-1} \begin{pmatrix} T \\ \tau_x \\ \tau_y \\ \tau_z \end{pmatrix}$$



a nested control structure

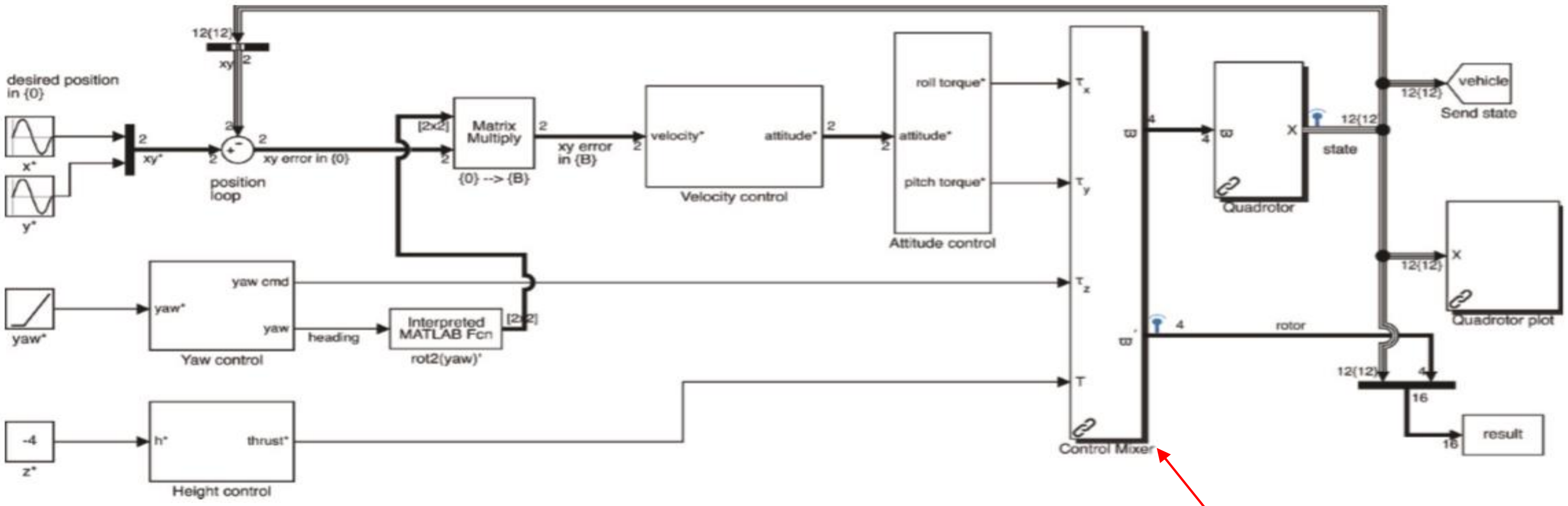
for pitch and

x-translational motion

Quadrotor Modeling 도식

configuration $\boldsymbol{q} \in \mathcal{C}$ where $\mathcal{C} \subset \mathbb{R}^3 \times \mathbb{S}^1 \times \mathbb{S}^1 \times \mathbb{S}^1$.

4.2 Flying Robots



Transfer Function

$$\Theta_y(s) / \tau_y(s) = 1 / (Js^2 + Bs)$$

The innermost loop uses a PD-Controller to compute the required pitching torque on the airframe

4.2 Flying Robots

Mobile Robot??

PID 제어기

$$G_c(s) = K_p + \frac{K_I}{s} + K_D s$$

$K_I = 0$

PD 제어기

$$G_c(s) = K_p + K_D s$$

$\theta_p^{\#}$ would be estimated by an inertial navigation system $\dot{\theta}_p^{\#}$ would be derived from gyroscopic sensors.

Pitching torque

$$\tau_y^* = K_{\tau,p}\left(\theta_p^* - \theta_p^{\#}\right) + K_{\tau,d}\left(\dot{\theta}_p^* - \dot{\theta}_p^{\#}\right)$$

The term $\dot{\theta}_p^*$ is commonly ignored.

Quadrotor Speed

$$\begin{pmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{pmatrix} = \boldsymbol{A}^{-1} \begin{pmatrix} T \\ \tau_x \\ \tau_y \\ \tau_z \end{pmatrix}$$

4.2 Flying Robots

Mobile Robot??

Pitching torque

$$\tau_y^* = K_{\tau,p}\left(\theta_p^* - \theta_p^\#\right) + K_{\tau,d}\left(\dot{\theta}_p^* - \dot{\theta}_p^\#\right)$$

The term $\dot{\theta}_p^*$ is commonly ignored.

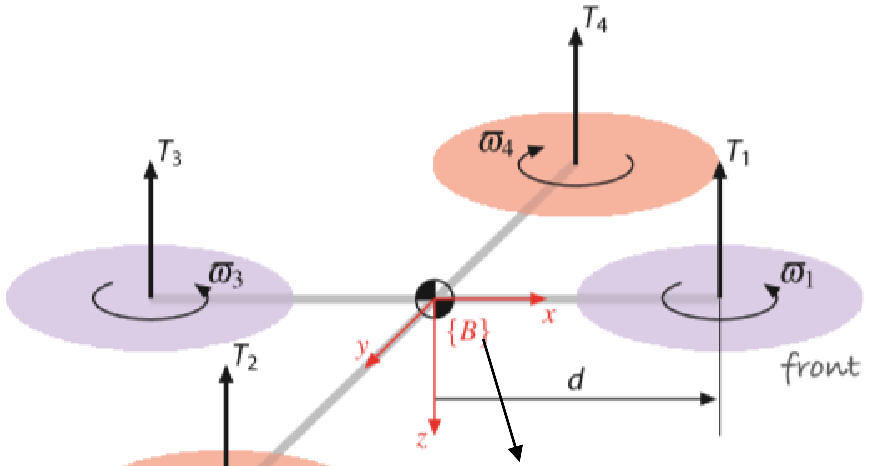
Quadrotor

Speed

$$\begin{pmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{pmatrix} = \boldsymbol{A}^{-1} \begin{pmatrix} T \\ \tau_x \\ \tau_y \\ \tau_z \end{pmatrix}$$

$\theta_p^\#$ would be estimated by an inertial navigation system

$\dot{\theta}_p^\#$ would be derived from gyroscopic sensors.



{B'} 가정!

Pitching을 구해보자!

$${}^{B'}\boldsymbol{f} = \mathscr{R}_y(\theta_p) \cdot \begin{pmatrix} 0 \\ 0 \\ T \end{pmatrix} = \begin{pmatrix} T\sin\theta_p \\ 0 \\ T\cos\theta_p \end{pmatrix}$$

$${}^{B'}f_x = T\sin\theta_p \approx T\theta_p$$

we have assumed that θ_p is small.

4.2 Flying Robots

Pitching을 구해보자! {B'}-x방향의 pitching 속도 조절

$$^{B'}\boldsymbol{f} = \mathscr{R}_y(\theta_p) \cdot \begin{pmatrix} 0 \\ 0 \\ T \end{pmatrix} = \begin{pmatrix} T\sin\theta_p \\ 0 \\ T\cos\theta_p \end{pmatrix}$$

$$^{B'}f_x = T\sin\theta_p \approx T\theta_p$$

we have assumed that θ_p is small.

$$^{B'}f_x^* = mK_f\left(^{B'}v_x^* - {}^{B'}v_x^\#\right)$$

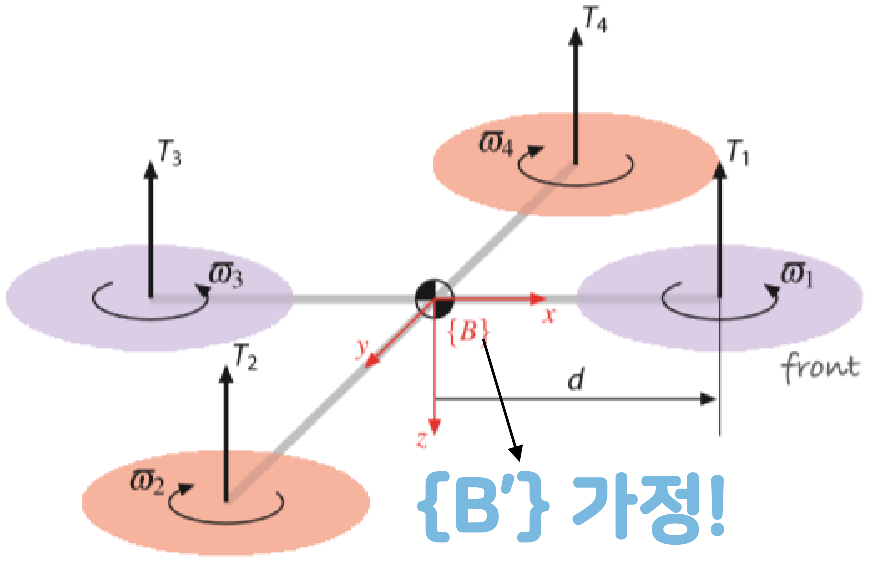
where $K_f > 0$ is a gain.

desired pitch angle

$$\theta_p^* \approx \frac{m}{T}K_f\left(^{B'}v_x^* - {}^{B'}v_x^\#\right)$$

Quadrotor Speed

$$\begin{pmatrix} \varpi_1^2 \\ \varpi_2^2 \\ \varpi_3^2 \\ \varpi_4^2 \end{pmatrix} = A^{-1}\begin{pmatrix} T \\ \tau_x \\ \tau_y \\ \tau_z \end{pmatrix}$$



{B'} 가정!

Pitching torque

$$\tau_y^* = K_{\tau,p}\left(\theta_p^* - \theta_p^\#\right) + K_{\tau,d}\left(\dot{\theta}_p^* - \dot{\theta}_p^\#\right)$$

Yaw?

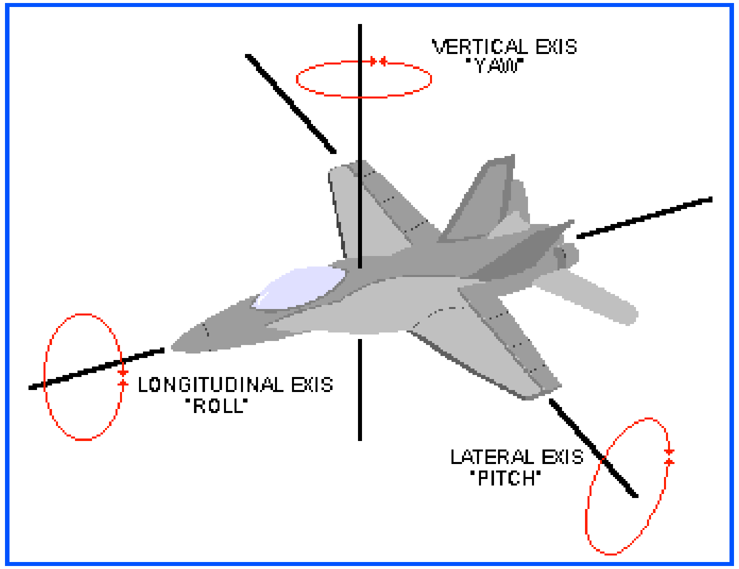


Figure 5-1 The axis of rotation.

which is a function of the yaw angle θ_y

$$\begin{pmatrix} {}^{B'}v_x \\ {}^{B'}v_y \end{pmatrix} = \begin{pmatrix} \cos\theta_y & -\sin\theta_y \\ \sin\theta_y & \cos\theta_y \end{pmatrix}^T \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

Yaw는 이동방향에 대해 수직면에 있는 축주위로의 회전을 의미합니다.
Pitch motion과 마찬가지로 제어모델링을 할 때 꼭 고려를 해줘야 하는 요소입니다.
오른쪽 그림은 Yaw angle에 대한 식입니다.

4.3 Advanced Topics

4.3.1 Nonholonomic and Under-Actuated Systems

홀로노믹(Holonomic)

Controllabe DOF = Total DOF

논 홀로노믹(NonHolonomic)

Controllabe DOF < Total DOF

리던던트(Redundant)

Controllabe DOF > Total DOF

Fully-actuated : 모든 관절에 대해서 제어를 할 수 있는 상태

DOF = Total actuator

Under-actuated : 적은 구동기로 모든 관절을 제어 할 수 있는 상태

DOF > Total actuator

Mobile robot seminar

Ch4. Mobile robot

Thanks for watching