Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	3.0	Analysis of Graphs: Polynomial and Rational Functions
Lesson Code	3.2	Even, Odd, and Neither Odd nor Even Functions
Time Frame		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

At the end of this lesson, students should be able to determine whether a function is odd, even, or neither odd nor even by:

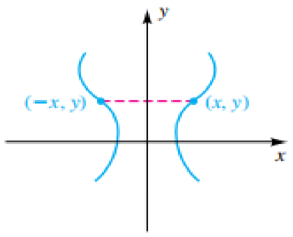
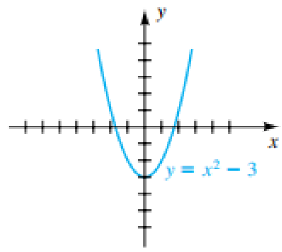
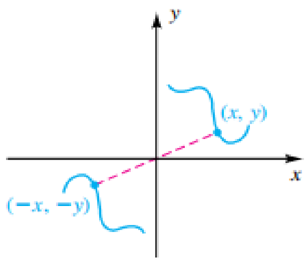
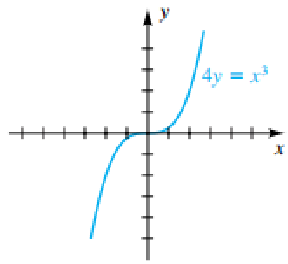
a) examining the graph of the function
b) applying the guidelines in determining whether a function is odd, even, or neither odd nor even.

HOOK

Time Allocation: 4 minutes
Actual Time Allocation: _____ minutes

Launch your graphing calculator on your laptop or on your mobile phone. Now, try graphing the functions $f(x) = x^2 - 14$ and $g(x) = x^3 - 7x$ in your Desmos/GeoGebra graphing app. Share your observations. To which axis line or point is the function $f(x) = x^2 - 14$ symmetric to? To which axis line or point is the function $g(x) = x^3 - 7x$ symmetric to?

Interpret your results using the chart below and classify the given functions as either odd or even.

Terminology	Graphical Interpretation	Test for Symmetry	Illustration
The graph is symmetric with respect to the y-axis		When we evaluate $f(-x)$ by substituting x with $-x$, the result is equal to $f(x)$.	
The graph is symmetric with respect to the origin.		When we evaluate $f(-x)$ by substituting x with $-x$, the result is the negative of $f(x)$ or $-f(x)$.	

Source: Algebra and Trigonometry with Analytic Geometry/ Swokowski & Cole/ p.134

So how was the first exercise? You may try a different function on your app. Did it match any of the ones on the chart above? Okay, then let's keep going!

IGNITE

Time Allocation: 15 minutes
Actual Time Allocation: _____ minutes

From the chart on the previous page, notice that the graphs exhibit symmetry. Even functions are functions whose graphs have symmetry with respect to the y-axis while odd functions are functions whose graphs have symmetry with respect to the origin .

We can formally define it as in the box below.

A function f is called even if $f(-x) = f(x)$ for every x in its domain. This means that when $-x$ is substituted for x, **the equation is not changed**. Using **the symmetry test**, **the graph of an even function is symmetric with respect to the *y*-axis.**

A function f is called odd if $f(-x) = -f(x)$ for every x in its domain. This means that when $-x$ is substituted for x, the result is the negative of $f(x)$. Applying the **symmetry test, the graph of an odd function is symmetric with respect to the origin.**

Going back to the exercises given previously, in

$$f(x) = x^2 - 14,$$
$$f(-x) = (-x)^2 - 14 = x^2 - 14$$

As shown above, since $f(-x) = f(x)$, then, the function $f(x) = x^2 - 14$ is **even**.

Meanwhile, functions are considered odd if substituting x with $-x$ in $f(x)$ gives the negative of the value of the given function. Thus, in

$$f(x) = x^3 - 7x$$
$$f(-x) = (-x)^3 - 7(-x) = -x^3 + 7x = -(x^3 - 7x)$$

As shown, since $f(-x) = -f(x)$, then the function $f(x) = x^3 - 7x$ is **odd**.

We can encapsulate the guidelines implied above with the steps provided below.

GUIDELINES:
Given $f(x)$,
1. Get $f(-x)$ by substituting x with $-x$ in $f(x)$ and simplify.
2. Compare the result $f(-x)$ to $f(x)$ and $-f(x)$.
3. Decide whether the function is even, odd or neither even nor odd using the following rules:
   a. The function is considered to be even if $f(-x) = f(x)$.
   b. The function is considered to be odd if $f(-x) = -f(x)$.
   c. The function is neither odd nor even if $f(-x)$ cannot be classified as either *a* or *b*.

Let us look at these illustrative examples.

Example 1: Determine whether the function $f(x) = 2x^6 + 3x^2 - 1$ is even, odd, or neither even nor odd .
Solution:
Step 1: Get $f(-x)$ by substituting x with $-x$ in $f(x)$ and simplify.

Given function:

$$f(x) = 2x^6 + 3x^2 - 1$$

Replace x with $-x$:

$$f(-x) = 2(-x)^6 + 3(-x)^2 - 1$$

Simplified form:

$$f(-x) = 2x^6 + 3x^2 - 1$$

Step 2: Compare $f(-x)$ to $f(x)$ and $-f(x)$.

$$f(-x) = 2x^6 + 3x^2 - 1$$

$$f(x) = 2x^6 + 3x^2 - 1$$

$$2x^6 + 3x^2 - 1 = 2x^6 + 3x^2 - 1$$

As shown, $f(x) = f(-x)$. The function $f(-x)$ cannot be both equal to $f(x)$ and $-f(x)$, thus, there is we don't need to compare $f(-x)$ to $-f(x)$ anymore.

Step 3: Decide whether the function is even, odd, or neither even nor odd.

The function is **even** since $f(-x) = f(x)$.

Example 2: Decide whether the function $f(x) = 5x^3 + 6x^2 - x$ is even, odd or neither even nor odd then graph the function and describe the symmetry.

Solution:
Step 1: Get $f(-x)$ by substituting x with $-x$ in $f(x)$ and simplify.

Given function:

$$f(x) = 5x^3 + 6x^2 - x$$

Replace x with $-x$:

$$f(-x) = 5(-x)^3 + 6(-x)^2 - (-x)$$

Simplified Form:

$$f(-x) = -5x^3 + 6x^2 + x$$

$$f(-x) = -(5x^3 - 6x^2 - x)$$

Step 2: Compare $f(-x)$ to $f(x)$ and $-f(x)$.

$$f(-x) = -(5x^3 - 6x^2 - x)$$

$$f(x) = 5x^3 + 6x^2 - x = -(5x^3 - 6x^2 + x)$$

$$-f(x) = -(5x^3 + 6x^2 - x)$$

$$-(5x^3 - 6x^2 - x) \neq -(5x^3 - 6x^2 + x)$$

$$f(-x) \neq f(x)$$

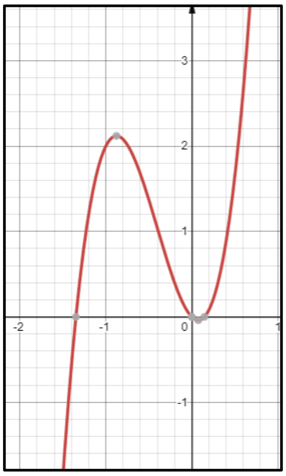
$$-(5x^3 - 6x^2 - x) \neq -(5x^3 + 6x^2 - x)$$

$$f(-x) \neq -f(x)$$

Step 3: Decide whether the function is even, odd, or neither even nor odd.

As shown, $f(-x) \neq f(x) \neq -f(x)$, then the function is neither even nor odd.

Let us observe the graph of this function below.



Looking at the graph of the given function, we can observe that the graph is not symmetric with respect to the $y-axis$ nor it is symmetric to the origin . That being said, it is then safe to say that the given function is neither odd nor even.

Example 3: Let $f(x) = |x|$, determine whether the given function is even, odd or neither even nor odd.

Solution:
Step 1: Get $f(-x)$ by substituting x with $-x$ in $f(x)$ and simplify.

Given function:

$$f(x) = |x|$$

Replace x with $-x$:

$$f(-x) = |-x|$$

Simplified Form:

$$f(-x) = x$$

Step 2: Compare $f(-x)$ to $f(x)$ and $-f(x)$.

$$f(-x) = x$$
$$f(x) = x$$
$$x = x$$

Step 3: Decide whether the function is even, odd, or neither.

Since $f(x) = f(-x)$, and the function $f(-x)$ cannot be both equal to $f(x)$ and $-f(x)$. Thus, we can say that the function is even and its graph is symmetric with respect to the $y - axis$.

NAVIGATE

Time Allocation: 5 minutes
Actual Time Allocation: _____ minutes

Now, it is your turn to work on the following functions to check your skills and understanding. Guided by the illustrative examples above, determine whether f is **even**, **odd** or **neither even nor odd.**

1. $f(x) = 3x^4 - 2x^2 + 5$
2. $f(x) = 2x^5 - 7x^3 + 4x$
3. $f(x) = x^3 + x^2$

KNOT

Time Allocation: 5 minutes
Actual Time Allocation: _____ minutes

The guidelines in determining whether a given function is even, odd or neither even nor odd can be summarized in the table below.

Terminology	Definition	Illustration	Symmetry
f is an even function	$f(-x) = f(x)$ for every x in the domain	$y = f(x) = x^2$	With respect to the y-axis
f is an odd function	$f(-x) = -f(x)$ for every x in the domain	$y = f(x) = x^3$	With respect to the origin

As a final assessment search and explore any real-life applications of odd or even functions. Write one of the many applications you may have read about and elaborate your answer in not less than five (5) sentences.

SYNTHESIS JOURNAL (PSHS System, 2020)

What are the things I have learned about even and odd functions?

____
____
____
____
____
____
____
____
____

What were the difficulties that I encountered throughout the lessons on odd and even functions?

____
____
____
____
____
____
____
____
____

How did I overcome these difficulties?

____
____
____
____
____
____
____
____
____

References:

1. Albarico, J.M. (2013). THINK Framework. Based on Ramos, E.G. and N. Apolinario. (n.d.) Science LINKS. Quezon City: Rex Bookstore Inc
2. Larson, R., Hostetler, R., and Edwards, B. (2005). *College Algebra: A graphing Approach 4th Edition*. Boston, New York: Houghton Mifflin Company.
3. Swokowski, E., and Cole, J. (2010). *Algebra and Trigonometry with Analytic Geometry. Classic 12th Edition.* Belmont, CA: Cengage Learning
4. PSHS System. (2020). *Math 1 Chapter 1 Module Version 2* [PDF]. Philippines: PSHS System.
5. PSHS CBZRC. (2020). *Template-Editable-1* [DOC]. Batangas: PSHS CBZRC.

Prepared by: Maria Cecilia C. Bastian
Position: Special Science Teacher (SST) V
Campus: PSHS- CAR

Reviewed by: Arlene C. Agosto
Position: Special Science Teacher (SST) V
Campus: PSHS-CVisC

Answer Key

NAVIGATE:
In each case the domain of f is R. To determine whether f is even or odd, we begin by examining $f(-x)$, where x is any real number.

1. $f(x) = 3x^4 - 2x^2 + 5$

$$f(-x) = 3(-x)^4 - 2(-x)^2 + 5$$

$$f(-x) = 3x^4 - 2x^2 + 5$$

$$f(-x) = f(x)$$

Since $f(-x) = f(x)$, f is an even function.

3. $f(x) = x^3 + x^2$

$$f(-x) = (-x)^3 + (-x)^2$$

$$f(-x) = -x^3 + x^2$$

$$f(-x) \neq f(x)$$

$$f(-x) \neq -f(x)$$

Since $f(-x) \neq f(x)$, and $f(-x) \neq -f(x)$, f is neither an even nor an odd function.

KNOT:
Answers may vary since the task requires the students to explore/search on any application of even and odd functions. Following this, is the synthesis journal.