Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	2.0	Transformations on the Coordinate Plane
Lesson Code	2.2	Reflections
Time Limit:		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

By the end of this learning guide module, the students should be able to

1. identify and use reflections in a plane.
2. identify relationships between reflections and line symmetry.
3. demonstrate reflections using graphing tools and software.

HOOK

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

You can find reflections everywhere. In water, in a glass, in a mirror, what do you observe?

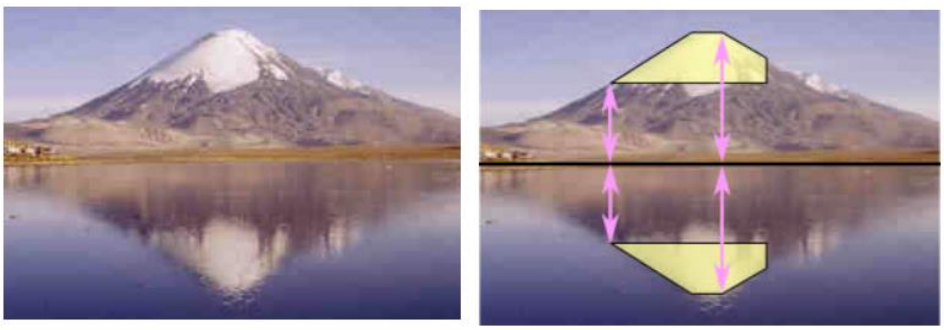


Figure 1.
Retrieved from *https://www.mathsisfun.com/geometry/reflection.html*

In figure 2 shown below, the reflected image has the same size as the original image. Also, every point is the same distance from the mirror line which is also called the **line of reflection**.

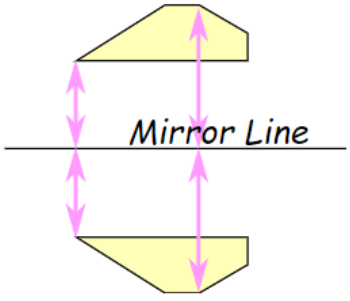


Figure 2.
Retrieved from *https://www.mathsisfun.com/geometry/reflection.html*

Now, can mirror lines be vertical? Can it be in any direction? To answer these questions, we will discuss reflections in detail in this module.

IGNITE

Time Allocation: 15 minutes
Actual Time Allocation: _____ minutes

Another type of geometric transformation is reflection. **Reflection** is a transformation that uses a line of reflection that acts like a mirror, with an image reflected in the line.

When you think about reflection, you think of an object being flipped or folded over the line of reflection. The original object will be the pre-image and the reflection will be called the image. A figure and its image after a reflection have the same shape and size, but the figures face in opposite directions. Also, the lines that connect A to A', B to B', and C to C' is perpendicular to the line of symmetry (*k*).

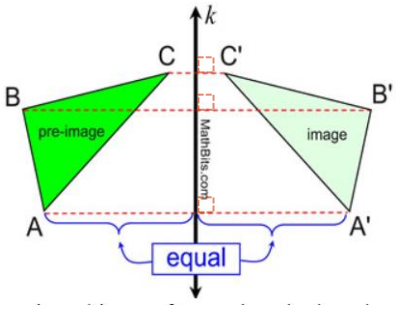


Figure 3. $\Delta A'B'C'$ is the reflection of ΔABC
Retrieved from *https://mathbitsnotebook.com/Geometry/Transformations/TRTransformationReflection.html*

Now that we have a basic understanding of what reflections are, let's learn how to use them on the coordinate plane.

Hands on Activity using Geogebra

1. Using the *Polygon Tool* , form a triangle by plotting the coordinates of its vertices $K(2,-1)$, $L(-2,-3)$, and $M(1,-5)$.

2. Reflect triangle KLM across the line $x = 4$ by using the *Transformation Tool: Reflect about Line*. In the input bar, type the line of reflection. After clicking the GeoGebra icon for reflection , click the polygon, then the line of reflection. Select a different color shade for this figure and label the new image as triangle $K'L'M'$.

3. Write down the coordinates of the vertices of triangle $K'L'M'$ below.
   $R_{x=4}$: $K(2,-1) \rightarrow K'$ (___, ___)
   $R_{x=4}$: $L(-2,-3) \rightarrow L'$(___, ___)
   $R_{x=4}$: $M(1,-5) \rightarrow M'$(___, ___)

4. Now, reflect triangle KLM across the line $y = x$ by using the *Transformation Tool: Reflect about Line*. Select a different color shade for this figure and label the new image as triangle $K"L"M"$. What are the coordinates of the new image?
   $K"$(___, ___), L"(___, ___), $M"$(___, ___)

5. Investigate what will happen if we perform a reflection over the following:
   a) the origin
   b) $y = -x$
   c) $y = x - 1$

   Use the same tools from GeoGebra.

Reflections on a Coordinate Plane

Reflections over the x-axis: When we reflect a point (x, y) over the *x*-axis, the *x*-coordinate remains the same, but the *y*-coordinate is changed into its opposite. So, the image of the point (x, y) is the point $(x, -y)$ when reflected across the *x*-axis.

Example 1. Δ ABC is reflected over the *x*-axis resulting to the image ΔA'B'C'.

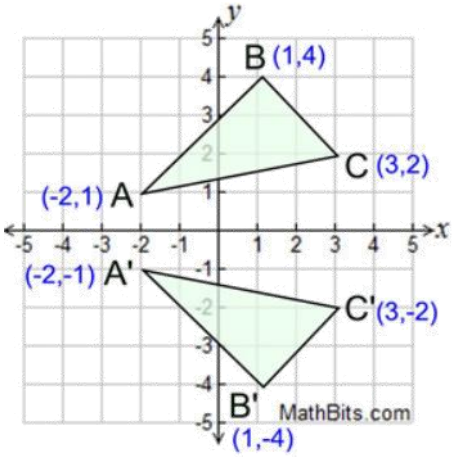


Figure 5. Reflecting Δ ABC over the x-axis.
Retrieved from: *https://mathbitsnotebook.com/Geometry/Transformations/TRTransformationReflection.html*

Reflection over the y-axis: When a point is reflected across the *y*-axis, the *y*-coordinate remains the same, but the *x*-coordinate is changed to its opposite. So, the image of the point (x, y) is the point $(-x, y)$ when reflected across the *y*-axis.

Example 2. Δ ABC is reflected over the y-axis resulting to the image ΔA'B'C'.

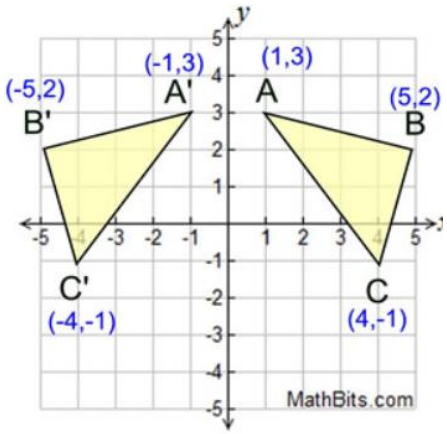


Figure 6. Reflecting Δ ABC over the y-axis.
Retrieved from *https://mathbitsnotebook.com/Geometry/Transformations/TRTransformationReflection.html*

Reflection over a line $x = k$ or $y = k$: Take note that when we reflect a figure, the distance of each point in the image from the line of reflection is equal to the distance of each point in the pre-image from the line of reflection. If an image is reflected over a line, $x = k$ or $y = k$ where k is a constant, the line of reflection will lie directly in the middle between the preimage and its image. Also, the line that connects a point to its image is *perpendicular* to the line of symmetry.

Example 3. Δ ABC is reflected over the line $x = -2$ resulting to the image ΔA'B'C'.

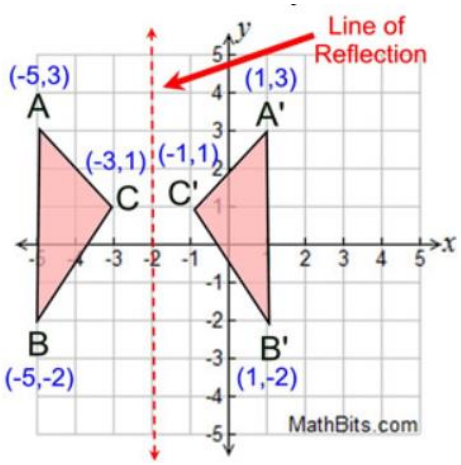


Figure 7. Reflecting Δ ABC over the line $x = -2$.
Retrieved from *https://mathbitsnotebook.com/Geometry/Transformations/TRTransformationReflection.html*

Reflection in the origin (0,0): The image of the point (x, y) is the point $(-x, -y)$ when a point is reflected in the origin.

Example 4. Δ ABC is reflected in the origin resulting to the image ΔA'B'C'.

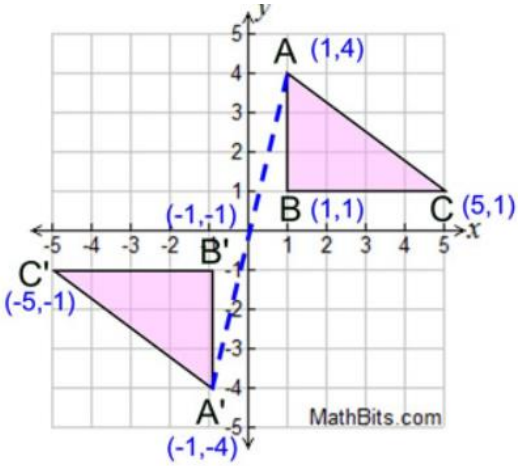


Figure 8. Reflecting Δ ABC in the origin.
Retrieved from *https://mathbitsnotebook.com/Geometry/Transformations/TRTransformationReflection.html*

To understand better about reflections on the coordinate plane, let us take a look at the following examples.

Example 5. Graph each of the following reflections.

a. $H\ (2, 2)$ reflected over the x-axis.

b. $G\ (5, 4)$ over the line $y = 4$.

Solution:

a. Since point H is to be reflected across the x −axis $(2,2)$, then the x − coordinate will remain the same but the y- coordinate will be changed to its opposite. Hence, H' should have the coordinate $(2, -2)$. Notice that point H is two units above the *x*-axis and H' is also two units below the *x*-axis.

b. Graph the line $y = 4$ first and plot G. You can see that G is on the line. Thus implies $G = G'$.

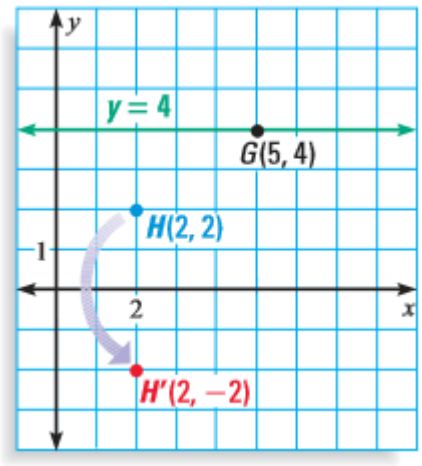


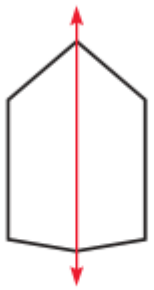
Figure 9: Reflections of points G and H to G' and H', respectively.
Retrieved from: GEOMETRY by Boswell, Laurie, Larson, Ron, Stiff, Lee. McDougal Littell 2004.

Reflections and Line Symmetry

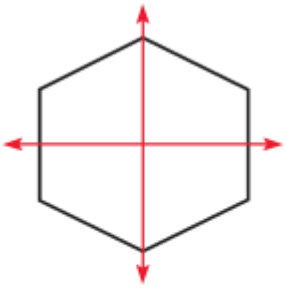
A figure in the plane has a *line of symmetry* if a figure can be mapped onto itself by a reflection across a certain line.

Example 6.

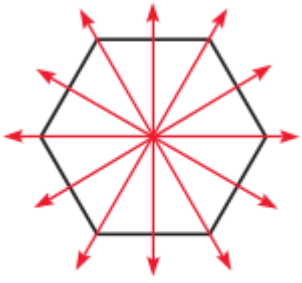
Figures below show the different lines of symmetry of a hexagon. Depending on their shape, hexagon can have different lines of symmetry.

a. 

This hexagon has only one line of symmetry.

b. 

This hexagon has two lines of symmetry.

c. 

This hexagon has six lines of symmetry.

Figure 11: Hexagons with lines of symmetry
Retrieved from GEOMETRY by Boswell, Laurie, Larson, Ron, Stiff, Lee. McDougal Littell 2004.

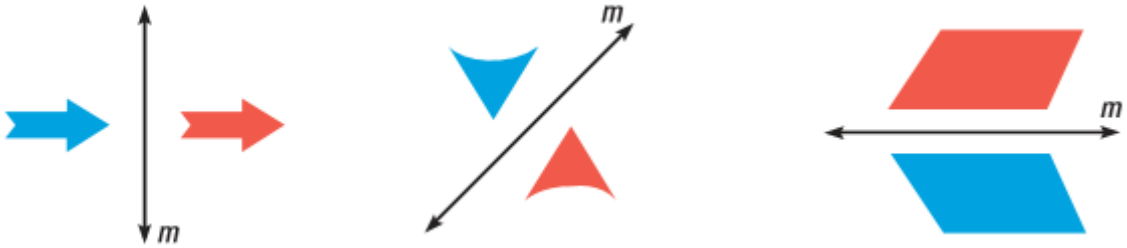
NAVIGATE

Time Allocation: 9 minutes
Actual Time Allocation: _____ minutes

Answer the following questions.

A. Determine whether the blue figure maps onto the red figure with a reflection in line m.

1. 2. 3.



Retrieved from: GEOMETRY by Boswell, Laurie, Larson, Ron, Stiff, Lee. McDougal Littell 2004.

B. Short Answer Questions.

4. What are the coordinates of the image of point Q $(-8, -2)$ when reflected across the x- axis?

5. What is the image of the point $(5, -3)$ when reflected across the line $y = x$?

6. What are the coordinates of the image of point M with coordinates $(-8, -6)$ when reflected across the y-axis?

C. Do as indicated.

For nos. 7 and 8, use the diagram below to name the image of $\overline{AB}$ after the reflection.

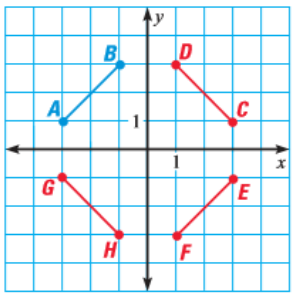


Image retrieved from: GEOMETRY by Boswell, Laurie, Larson, Ron, Stiff, Lee. McDougal Littell 2004.

7. Reflection in the *y*-axis.

8. Reflection in the *y*-axis, followed by a reflection in the *x*-axis.

9. Sketch an octagon with exactly two lines of symmetry.

10. If the figure below shows a reflection across the line n, then find the value of the variables $u, v \text{ and } w$.

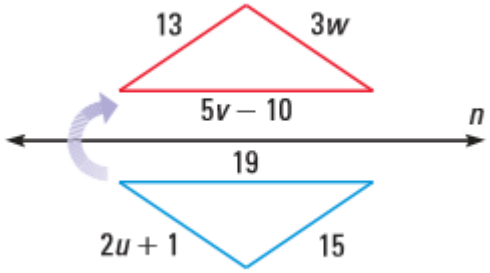


Image retrieved from: GEOMETRY by Boswell, Laurie, Larson, Ron, Stiff, Lee. McDougal Littell 2004.

KNOT

Time Allocation: 4 minutes
Actual Time Allocation: _____ minutes

In summary,

Reflection is a transformation that uses a line of reflection that acts like a mirror, with an image reflected in the line.

Reflections over the x-axis:
The image of the point (x, y) when reflected in the *x*-axis is the point $(x, -y)$.

Reflection over the y-axis:
The image of the point (x, y) when reflected in the *y*-axis is the point $(-x, y)$.

Reflection over a $x = k$ or $y = k$:
If an image is reflected over a line $x = k$ or $y = k$, the line of reflection will lie directly in the middle between the original figure and its image.

Reflection over the line $y = x$:
The image of the point (x, y) when reflected in the line $y = x$ is the point (y, x).

Reflection in the origin (0,0):
The image of the point (x, y) when reflected in the origin is the point $(-x, -y)$.

Reflections and Line Symmetry
If a figure can be mapped onto itself by a reflection in the line then this figure in the plane has a line of symmetry.

Do as indicated.

A. True or False. Explain your reasoning.

1. The image of point $M\ (6, -2)$ is $M'\ (0, -2)$ when reflected across the line $x = 3$.

2. We will get the image $U'(-3,\ 3)$ when we reflect point $U\ (5,\ 3)$ across the line $x = 1$.

B. Find the value of each variable given that the diagram shows a reflection in a line.

3.

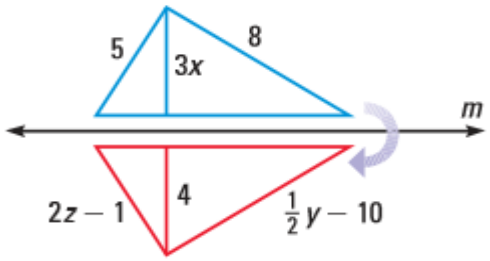


Image retrieved from: GEOMETRY by Boswell, Laurie, Larson, Ron, Stiff, Lee. McDougal Littell 2004.

4. What is the reflection image of $(x\ , y)$ across the line $y\ =\ -2$?

References:

Albarico, J.M. (2013). THINK Framework. Based on Science LINKS by E.G. Ramos and N. Apolinario. Quezon City: Rex Bookstore Inc.

Larson, R., Boswell, L., & Stiff, L. (2004). *Geometry.* McDougal Littell, a division of Houghton Mifflin Company.

Geometric Transformations Reflections. Retrieved from https://mathbitsnotebook.com/Geometry/Transformations/TRTransformationReflection.html

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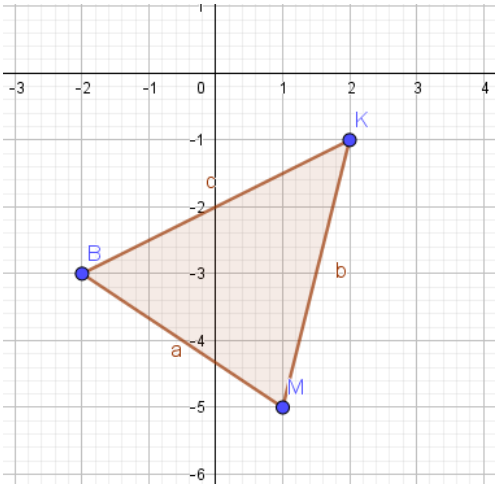
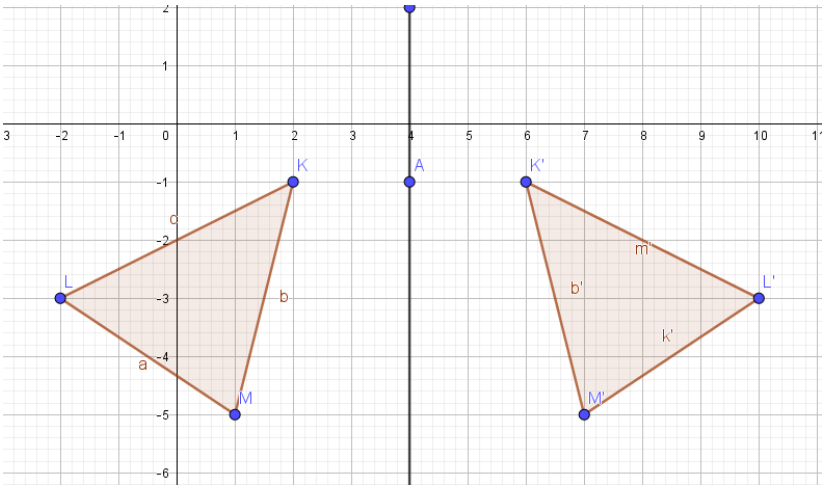
Answer Key

Hands-on Activity:

1.

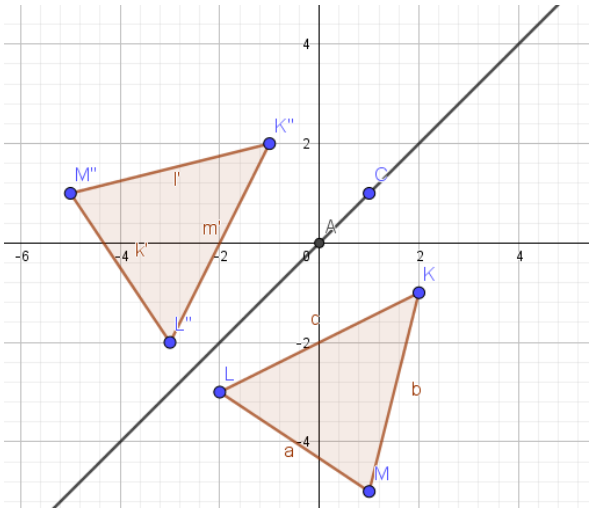


2.

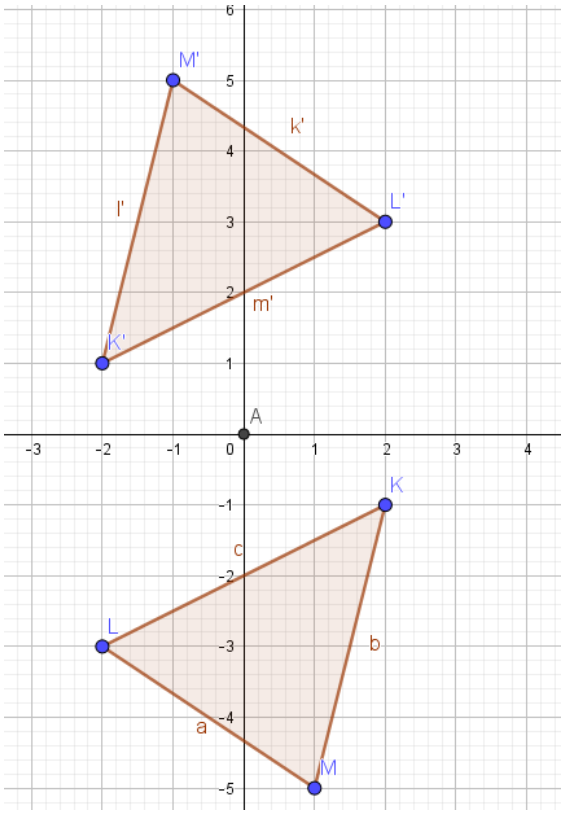


3. $R_{x=4}$: $K(2,-1) \rightarrow K'(6,-1)$
   $R_{x=4}$: $L(-2,-3) \rightarrow L'(10,-3)$
   $R_{x=4}$: $M(1,-5) \rightarrow M'(7,-5)$

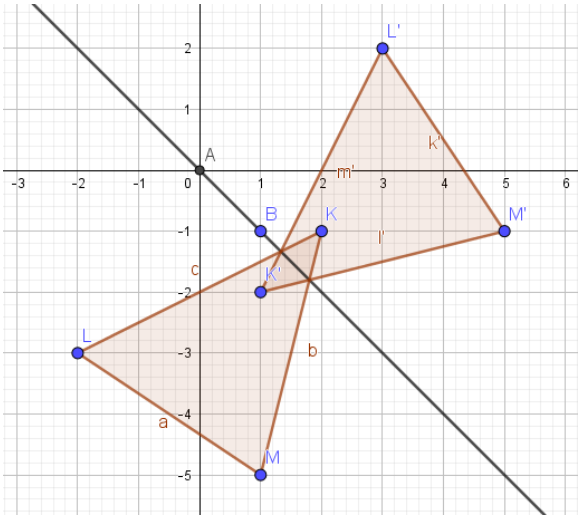
4. $K"(-1,2), L"(-5,1), M"(-3,-2)$



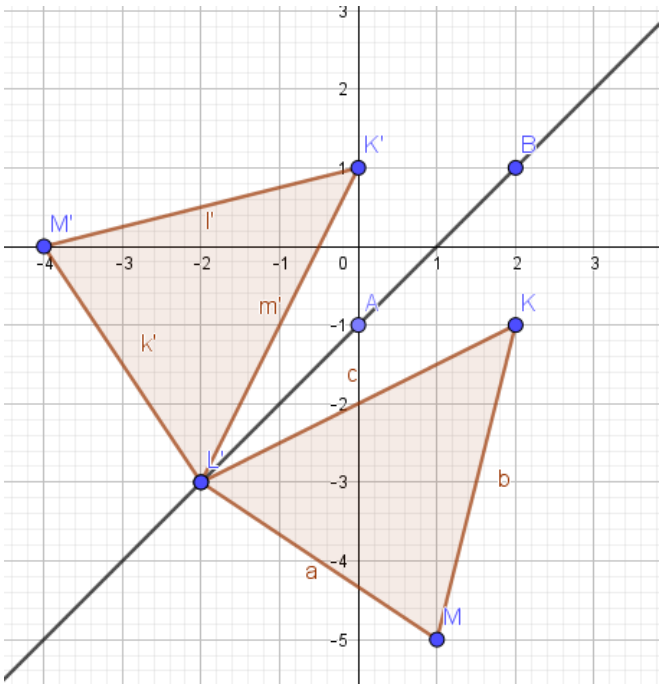
5. a) reflection about the origin



b) reflection about the line $y = -x$

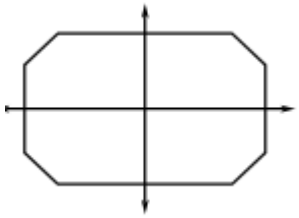


c) reflection about the line $y = x - 1$



Navigate:

1. not a reflection
3. reflection
5. (-5, 3)
7. $\overline{CD}$
9.



Knot:

1. True; *M* is 3 units to the right of line $x = 3$ so its image is 3 units to the left of the line.

3. $x = \frac{4}{3}, y = 36, and\ z = 3$