Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	1.0	Basic Plane and Coordinate Geometry
Lesson Code	1.1.1	Basic Geometric Terms and Notations 1
Time Limit		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

By the end of this module, the students will have been able to

1. revisit basic geometric terms: points, lines, planes, segments, ray, angles, and triangles;
2. make use of geometric notations in labeling geometric figures

HOOK

Time Allocation: 4 minutes
Actual Time Allocation: _____ minutes

Why does a four-legged chair sometimes wobbles, but a three-legged stool never does? This is a scenario that depicts points and how they lie in a plane. Take note that points make up all geometric shapes.

Figure 1. A four-legged chair and a three-legged stool
(Image from: Halden. (2014, October 20). *Why do chairs sometimes wobble?* Retrieved from https://www.slideserve.com/halden/why-do-chairs-sometimes-wobble)

To answer the initial question, this is because three points make a plane. We need three points to make sure something can only set one way. Notice that no matter where we place the third point, there would only be one plane. Now, imagine we added a fourth point. When a chair had four points, any three of them could make a plane. So, it wobbles going back between groups of three legs. This fourth point could either be on the same plane as the first (a stable chair) or off in another space somewhere (a wobbly chair). This is the reason why a four-legged chair sometimes wobbles but a three-legged stool never does.

In this lesson, we will revisit basic geometric terms such as points, lines, planes, segments, rays, angles, and triangles and the use of geometric notations in labeling geometric figures.

IGNITE

Time Allocation: 15 minutes
Actual Time Allocation: _____ minutes

Formal definitions in geometry are formed using other defined words or terms. However, three words in geometry, namely points, lines, and planes, are not formally defined. We refer to them as the "three undefined terms in geometry".

While these words are "undefined" in the formal sense, we can still "describe" these words.

Point

A point has neither size nor dimension. A point also describes a location. We use dots in pictures and diagrams to represent points. We name or label points with a capital letter.

Line

A line extends in one dimension. A line is made up of an infinite number of points and is considered one-dimensional because they only have length (no thickness or width). We usually represent a line by a straight line with two arrowheads to indicate that the line extends without bound in both directions. A line can be named using a lower-case, italicized letter or by choosing any two points that lie on it.

Plane

A plane extends in two dimensions. Planes are two-dimensional since they have length and width. It is usually represented by a shape that looks like the surface of a table or a sheet of paper. We name a plane by any three points on it or by labeling it with a capital letter.

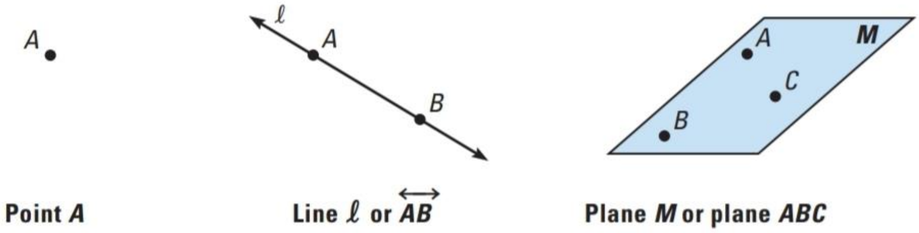

Figure 2. Labeling points, lines, and planes
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry.* McDougal Littell, a division of Houghton Mifflin Company)

Points that lie on the same line are called **collinear points**.

Points that lie on the same plane are called **coplanar points**.

Note: Three coplanar points that are not collinear determines a plane.

<u>Example:</u> Naming collinear and coplanar points

Refer to the figure below

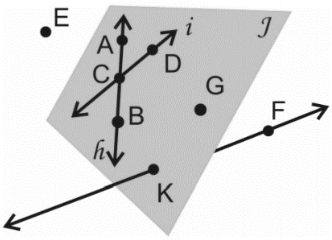

Figure 3. Identifying collinear and coplanar points
(Image from: Dillinger, B. (2011). *Chapter 1- Foundation of Geometry.* CK-12 Foundation Inc.)

A. Are points A, B and K collinear? Are they coplanar?
B. Are points D, B, and G coplanar?
C. Identify four non-collinear points.

<u>Solution</u>

1. Points A, B, and K are non-collinear, but they are coplanar.
2. Points D, B, and G are coplanar. Any three points are coplanar.
3. There are many correct answers. For instance, points B, D, K, and G would be non-collinear.

Consider the **line** AB (symbolized by $\overleftrightarrow{AB}$) in the figure below.

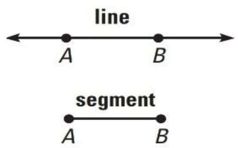

Figure 4. Line and line segment
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry*. McDougal Littell, a division of Houghton Mifflin Company)

The **line segment** or **segment** AB (symbolized by $\overline{AB}$) consists of the **endpoints** A and B, and all points on $\overleftrightarrow{AB}$ that are between A and B.

The **ray** AB (symbolized by $\overrightarrow{AB}$), as shown in the figure below, consists of the initial point A and all points on $\overleftrightarrow{AB}$ that lie on the same side of A as point B.

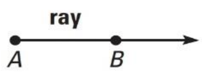

Figure 5. Ray *AB*
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry*. McDougal Littell, a division of Houghton Mifflin Company)

Note that $\overleftrightarrow{AB}$ is the same as $\overleftrightarrow{BA}$, $\overline{AB}$ is the same as $\overline{BA}$. However, $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are not the same. This means that they have different initial points and extends in different directions.

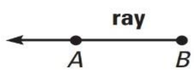

Figure 6. Ray *BA*
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry*. McDougal Littell, a division of Houghton Mifflin Company)

If C is between A and B, then $\overrightarrow{CA}$ and $\overrightarrow{CB}$ are **opposite rays.**

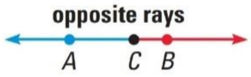

Figure 7. Opposite rays
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry*. McDougal Littell, a division of Houghton Mifflin Company)

Two opposite rays are considered to be collinear like how points, segments and rays are collinear if they lie on the same line. When segments, rays, and lines lie on the same plane, we consider them to be coplanar.

<u>Example:</u> Drawing Lines, Segments, and Rays

Consider three noncollinear points J, K, and L. Draw these points, then draw $\overleftrightarrow{JK}$, $\overline{KL}$, and $\overrightarrow{LJ}$.

<u>Solution:</u>

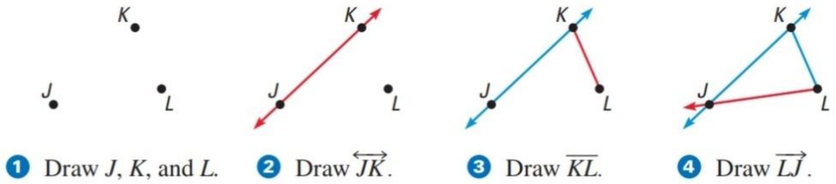

Figure 8. Points J,K and L, $\overleftrightarrow{JK}$, $\overline{KL}$, and $\overrightarrow{LJ}$
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry*. McDougal Littell, a division of Houghton Mifflin Company)

We say that two or more geometric figures **intersect** when they have at least one point in common. The **intersection** of the figures is the set of all points the figures have in common.

<u>Example:</u> Modeling Intersection

Label two index cards as shown in the figure below. Cut slots halfway along each card.

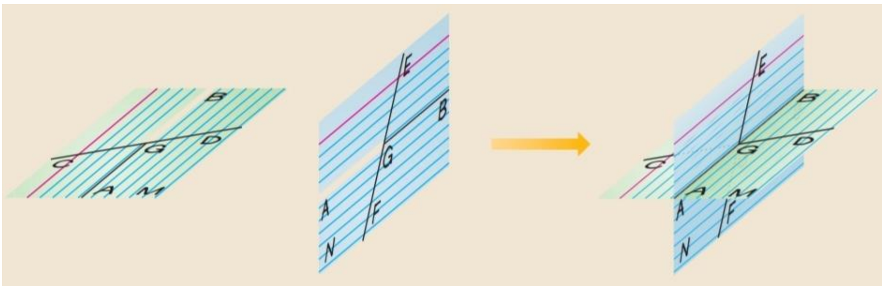

Figure 9. Intersection Activity
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry*. McDougal Littell, a division of Houghton Mifflin Company)

A. Put the cards together along the slots as shown above. What is the intersection of $\overline{AB}, \overline{CD}$ and $\overline{EF}$?
B. Identify the intersection of planes M and N.
C. Are $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ coplanar? Explain.

Solution

A. The intersection of $\overline{AB}, \overline{CD}$ and $\overline{EF}$ is point G.
B. The intersection of planes M and N is the line segment AB or $\overline{AB}$.
C. $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ are not coplanar since they do not lie on the same plane. $\overleftrightarrow{CD}$ lies in plane M while $\overleftrightarrow{EF}$ lies in plane N.

Angle

An angle consists of two different rays that have the same initial point. The **sides** of the angle are the rays and the common initial point is the **vertex** of the angle.

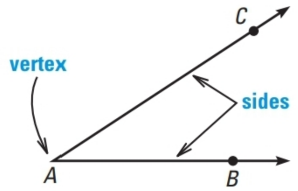

Figure 10. Angle
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry*. McDougal Littell, a division of Houghton Mifflin Company)

We can denote an angle that has sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in three ways: $\angle BAC, \angle CAB,$ or $\angle A$. The vertex of this angle is point A. In writing the name of an angle, we make sure that the vertex is in the middle of the three points.

<u>Example:</u> Naming Angles

Name the angles in the figure below.

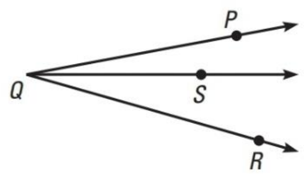

Figure 11. Naming Angles
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry*. McDougal Littell, a division of Houghton Mifflin Company)

Solution

There are three different angles.

- $\angle PQS$ or $\angle SQP$
- $\angle SQR$ or $\angle RQS$
- $\angle PQR$ or $\angle RQP$

Be careful not to name any of these angles as $\angle Q$ because all three angles share the same vertex, Q. The name $\angle Q$ would not distinguish one angle from the others.

The **measure** of any $\angle A$ is denoted by $m\angle A$. We approximate the measure of an angle, in units called **degrees** (°), by using a protractor. Let's take the example shown in the figure below. $\angle BAC$ has a measure of 50°, which can be written as: $m\angle BAC = m\angle A = 50°$.

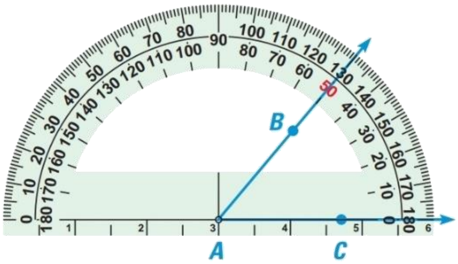

Figure 12. Measuring Angles
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry*. McDougal Littell, a division of Houghton Mifflin Company)

Congruent angles are angles that have the same measure. For example, both $\angle BAC$ (figure 12) and $\angle DEF$ (figure 13) have a measure of $50°$, so they are congruent.

Measures are equal: $m\angle BAC = m\angle DEF$ "is equal to"

Angles are congruent: $\angle BAC \cong \angle DEF$ "is congruent to"

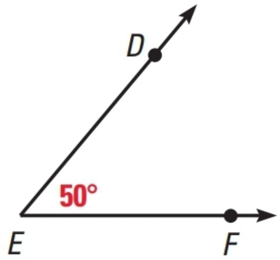

Figure 13. $m\angle DEF = 50°$
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry*. McDougal Littell, a division of Houghton Mifflin Company)

A point is in the **interior** of an angle if it is between points that lie on each side of the angle.

A point is in the **exterior** of an angle if it is not on the angle or in its interior.

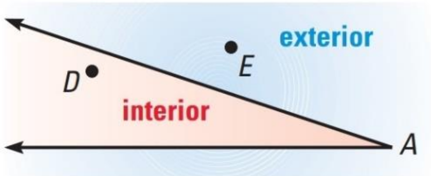

Figure 14. Interior and Exterior point
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry*. McDougal Littell, a division of Houghton Mifflin Company)

Angle Addition Postulate
If P is in the interior of $\angle RST$, then $m\angle RSP + m\angle PST = m\angle RST$

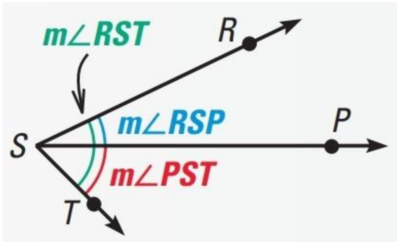

Figure 15. Angle Addition Postulate
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry*. McDougal Littell, a division of Houghton Mifflin Company)

Example:

In the figure below, if $m\angle ABD = 107°$, find the value of x and $m\angle ABC$ and $m\angle CBD$.

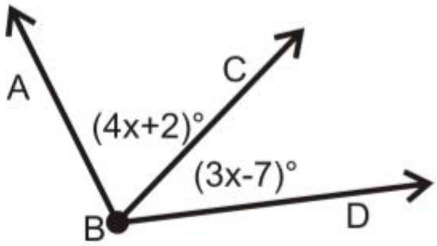

Figure 17. Algebra Connection of Angle Addition Postulate
(Image from: Dillinger, B. (2011). *Chapter 1- Foundation of Geometry.* CK-12 Foundation Inc.)

Solution:

From the Angle Addition Postulate, $m\angle ABD = m\angle ABC + m\angle CBD$. Substitute in what you know and solve the equation:

$$112° = (4x + 2)° + (3x - 7)°$$
$$107° = (7x - 5)°$$
$$x = 16°$$

So, $m\angle ABC = 4(16°) + 2° = 66°$ and $m\angle CBD = 3(16°) - 7° = 41°$.

We classify angles as **acute, right, obtuse,** and **straight,** according to their measures as shown in the figure below.

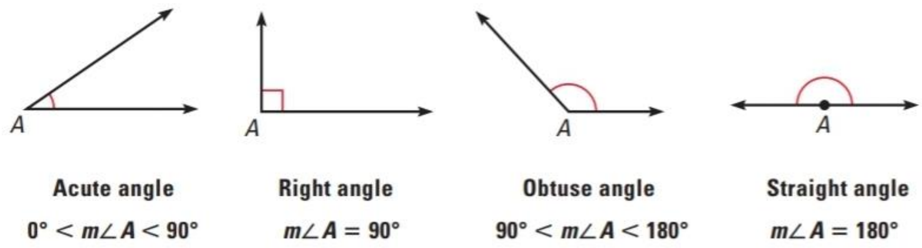

Figure 18. Classification of Angles
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry*. McDougal Littell, a division of Houghton Mifflin Company)

Example: Classifying angles in a coordinate plane

Plot the points $L(-4,2), M(-1,-1), N(2,2), Q(4,-1),$ and $P(2,-4)$. Then measure and classify each angle listed below as acute, right, obtuse or straight.

A. $\angle LMN$
B. $\angle LMP$
C. $\angle NMQ$
D. $\angle LMQ$

Solution

Begin by plotting the points. Then you may use a protractor to measure each angle.

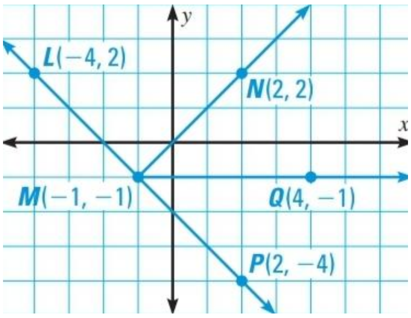

Figure 19. Classification of Angles in a coordinate plane
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry.* McDougal Littell, a division of Houghton Mifflin Company)

A. $m\angle LMN = 90°$, so $\angle LMN$ is a right angle
B. $m\angle LMP = 180°$, so $\angle LMP$ is a straight angle
C. $m\angle NMQ < 90°$, so $\angle NMQ$ is an acute angle
D. $90° < m\angle LMQ < 180°$ so $\angle LMQ$ is an obtuse angle

Adjacent angles are two angles that share a common vertex and side but have no common interior points.

In the example above, we can say that $\angle LMN$ and $\angle NMP$ are adjacent angles since their common vertex is point M and the common side is $\overrightarrow{MN}$ and they have no common interior points.

Triangle

A triangle is a figure formed by three segments joining three noncollinear points. A triangle can be classified by its sides and by its angles, as shown in the figure and definitions below.

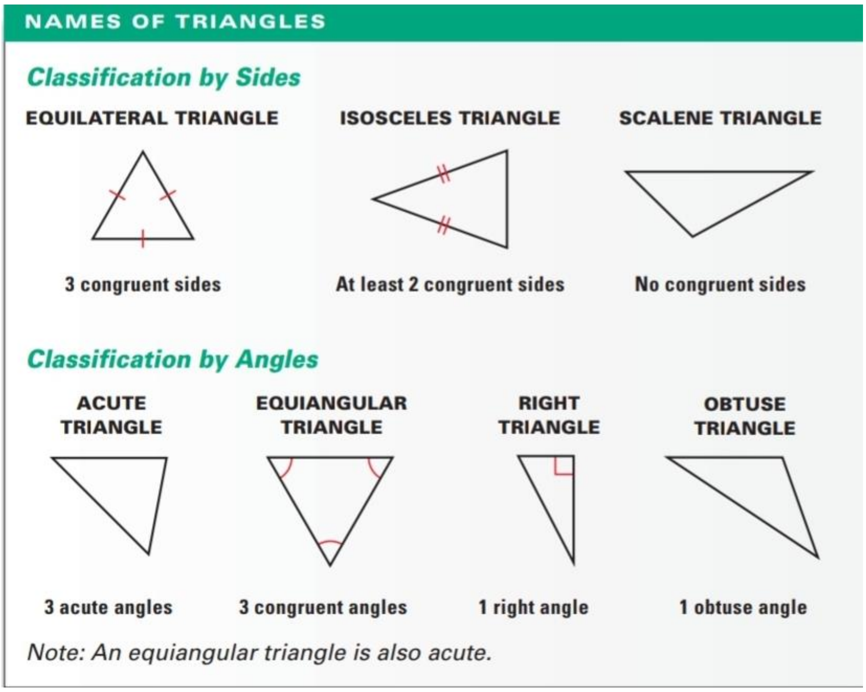

Figure 20. Classification of Triangles by sides and angles
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry.* McDougal Littell, a division of Houghton Mifflin Company)

<u>Example:</u> Classifying Triangles

Classify the triangle in the figure below.

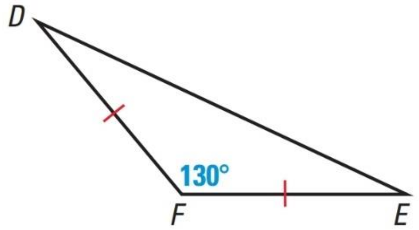

Figure 21. Classifying Triangles
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry*. McDougal Littell, a division of Houghton Mifflin Company)

<u>Solution</u>

For the figure above, Triangle DEF or ΔDEF is an obtuse isosceles triangle since there is an obtuse angle and there are two sides that are congruent (tick marks would mean that the lengths of the sides are equal, hence congruent sides).

Each of the three points joining the sides of a triangle is a **vertex**. For examples, in ΔDEF above, points D, E, and F are vertices (plural of vertex).

When two sides of a triangle share a common vertex, they are considered to be **adjacent sides.** In ΔABC below, $\overline{CA}$ and $\overline{BA}$ are adjacent sides. The third side, $\overline{BC}$, is the sides opposite $\angle A$.

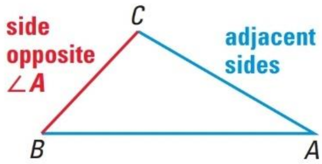

Figure 22. Adjacent and Opposite sides of a triangle
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry*. McDougal Littell, a division of Houghton Mifflin Company)

Right and Isosceles triangles

In a right triangle, the sides that form the right angle are the **legs** of the right triangle. The **hypotenuse** of the triangle is the side opposite the right angle.

An isosceles triangle can have two or three congruent sides. In an isosceles triangle, the two sides are the **legs** of the isosceles triangle. The third side is the **base** of the isosceles triangle.

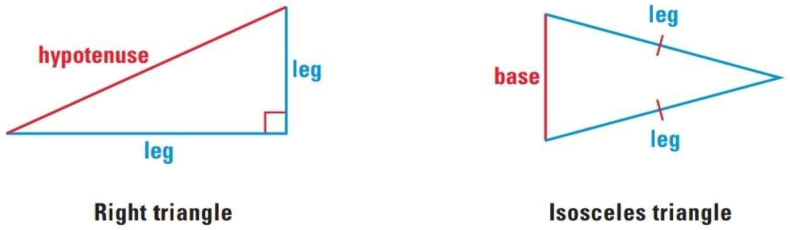

Figure 23. Right and Isosceles Triangle
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry*. McDougal Littell, a division of Houghton Mifflin Company)

References:

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