Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	2.0	Transformations on the Coordinate Plane
Lesson Code	2.3	Glide Reflection
Time Limit		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

By the end of this module, the students will have been able to

1. Define and identify glide reflection.
2. Demonstrate glide reflection using graphing tools and software.
3. Illustrate glide reflections on the coordinate plane.

HOOK

Time Allocation: 4 minutes
Actual Time Allocation: _____ minutes

From lessons 2.1 and 2.2, we have learned how to perform translation and reflection on the coordinate plane. These transformations are examples of isometries or rigid transformations wherein length and angle measure are preserved.

Let us explore what will happen when we perform a series of transformations to a figure. Perform the following steps on the coordinate plane below.

Step 1: Translate $\triangle ABC$ 6 units to the right. Label the corresponding vertices A', B', and C'.
Step 2: On the same coordinate plane, reflect $\triangle A'B'C'$ across the $x-$ axis. Label the vertices of the new figure A'', B'', and C''.

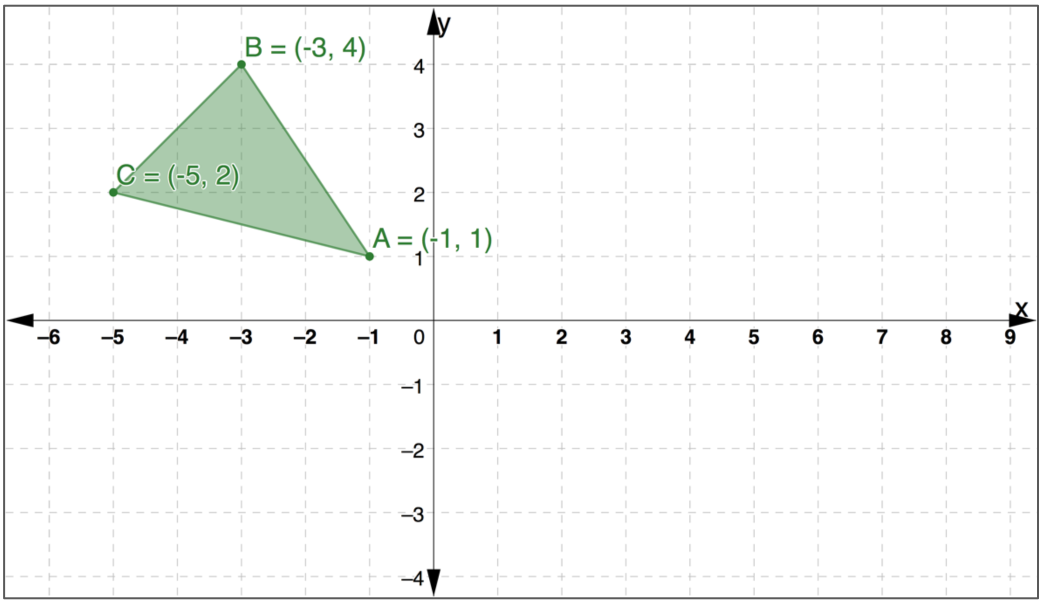


Figure 1. Performing a series of transformations to $\triangle ABC$

We have just performed a <u>composition of transformations</u> to triangle ABC. The result of combining two or more transformations to produce a single transformation is considered a composition of transformations. Glide reflection is an example of this.

One of the most well-known example of glide reflection is M.C. Escher's work entitled *Horseman*.

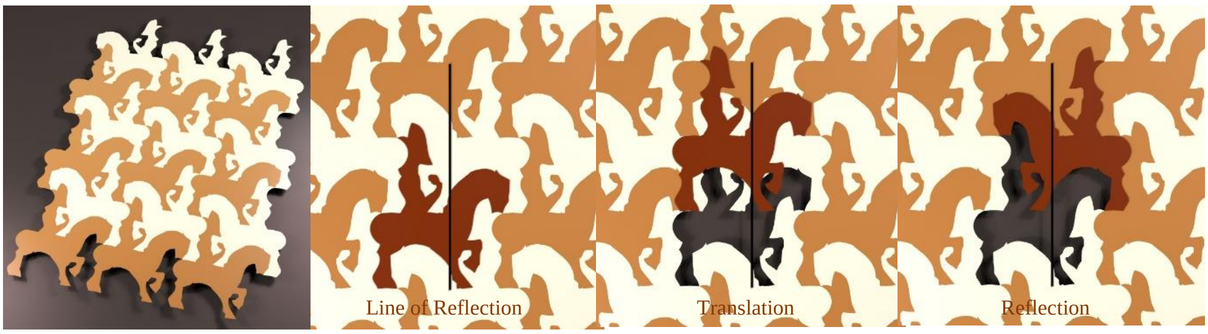


Figure 2. Horseman by M.C. Escher Depicting Glide Reflection
Retrieved from: *http://www.mi.sanu.ac.rs/vismath/roelofs/index.html*

IGNITE

Time Allocation: 15 minutes
Actual Time Allocation: _____ minutes

Hands-On Activity Using GeoGebra

1. Using the *Polygon Tool*, form a triangle by plotting the coordinates of its vertices $K(2,-1)$, $L(-2,-3)$, and $M(1,-5)$.

2. Translate triangle KLM 5 units upward by using the *Transformation Tool*: *Translate by Vector*. (Recall from Lesson 2.1 how to translate a figure using Geogebra.) Select a different color shade and label the figure as triangle $K'L'M'$.

3. Write down the coordinates of the vertices of triangle $K'L'M'$ below.
   $T_{0,5}$: $K(2,-1) \rightarrow K'$ (___, ___)
   $T_{0,5}$: $L(-2,-3) \rightarrow L'$(___, ___)
   $T_{0,5}$: $M(1,-5) \rightarrow M'$(___, ___)

4. Now, reflect triangle $K'L'M'$ across the line $x = 4$ by using the *Transformation Tool: Reflect about Line* (Recall from Lesson 2.2 how to reflect a figure across a line using Geogebra.) Select a different color shade for this figure and label the new image as triangle $K''L''M''$.

5. Write down the coordinates of the vertices of triangle $K''L''M''$ below.
   $R_{x=4}$: K'(___ , ___) $\rightarrow$ K'' (___, ___)
   $R_{x=4}$: L'(___ , ___) $\rightarrow$ L''(___, ___)
   $R_{x=4}$: M'(___ , ___) $\rightarrow$ M''(___, ___)

6. Investigate what will happen if we perform a reflection(reflect across line $x = 4$) before a translation (shift 5 units up). Use the same tools from Geogebra. Does the final image have the same coordinates as your previous result?

7. Let us perform another series of transformations to $\triangle KLM$: a translation of 1 unit to the right and 5 units up, followed by a reflection across $y = x$. If we do the same transformations in reverse order, will we get the same image?

A **glide reflection** is a transformation that maps every point P to a point P'' with a series of transformations namely translation and reflection. We do this with the following steps:

Step 1: P is mapped onto P' with a translation.
Step 2: P' is then mapped onto P'' by a reflection along line h which is parallel to the direction of the translation in Step 1.

A slide and a reflection maps one figure onto another when two figures have the same shape and size but with opposite orientations. These figures are not merely reflections of each other.

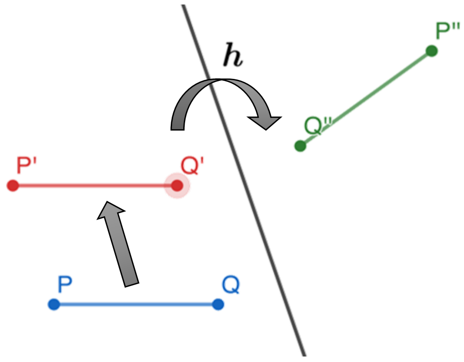


Figure 3. Glide reflection

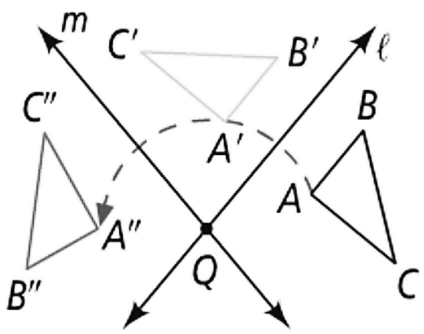
The two component transformations, translation and reflection, of a glide reflection can be performed in either order as long as the line of reflection is parallel to the direction of the translation.

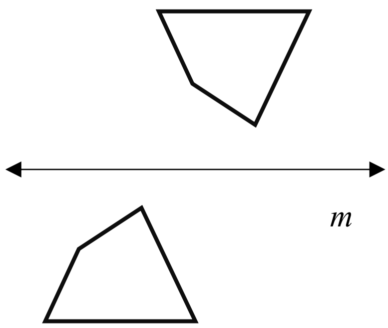
Example 1:

Identify whether the following examples are glide reflections or not.

a) $(2x-1,-x) \rightarrow (2x+3,x)$

c) 

b) 

d) 

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Solution:

(a) is a glide reflection. The mapping shows a translation of 4 units to the right and a reflection across the $x-$ axis.
(b) is not a glide reflection. A composition of two reflections across two intersecting lines was made to triangle ABC. This is equivalent to a rotation which we will be discussing in the next lesson.
(c) and (d) are both glide reflections. The image and pre-image are congruent, opposite in orientation but not completely reflections of each other.

Example 2:

Perform a glide reflection to $\triangle QRS$ with coordinates $Q(-1,-3), R(-5,-2),$ and $S(-8,-5)$.

Translation: $\boldsymbol{T_{11,0}}$

Reflection: $\boldsymbol{r_{y=1}}$

To describe glide translation, we use this notation: $\boldsymbol{T_{11,0} \circ r_{y=1}}$ **or** $\boldsymbol{r_{y=1} \circ T_{11,0}}$ **.**

Solution:

Begin by graphing $\triangle QRS$. Then, translate the figure 11 units to the right. Label the new figure as $\triangle Q'R'S'$. Next, graph the line of reflection $y = 1$ using a broken line. Lastly, reflect $\triangle Q'R'S'$ about $y = 1$ to produce $\triangle Q"R"S"$.

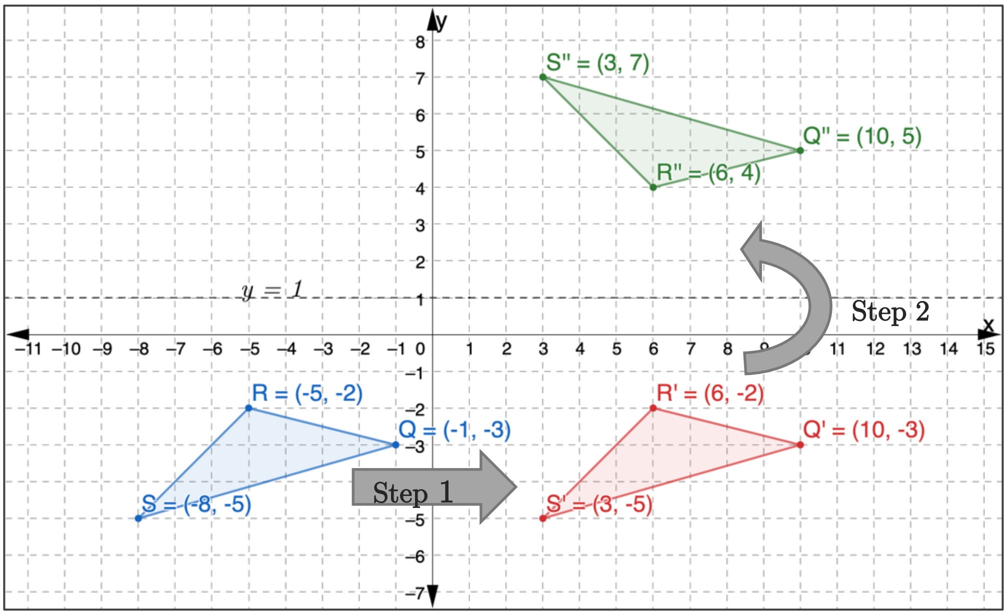


Figure 4. Pre-image and Image of $\triangle QRS$ after a glide reflection

Here is a supplementary link to help you graph glide reflections.

bit.ly/3f8PPnv

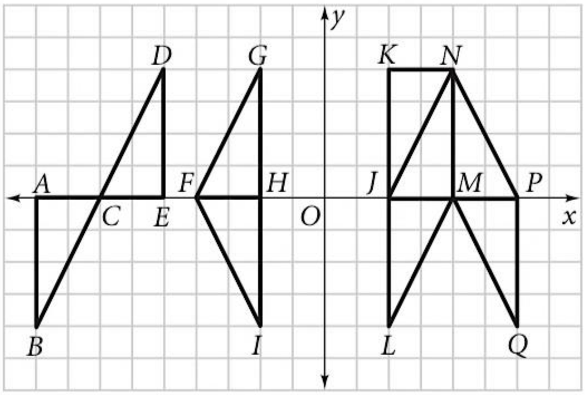
NAVIGATE

Time Allocation: 8 minutes
Actual Time Allocation: _____ minutes

Answer the following questions. Items marked with an asterisk (*) will be graded.

1. Identify whether each mapping is a glide reflection or not. If yes, find the glide reflection rule that maps the pre-image to its image.

   a) $\triangle ABC \rightarrow \triangle EDC$
   b) $\triangle EDC \rightarrow \triangle PQM$ *
   c) $\triangle MNJ \rightarrow \triangle EDC$
   d) $\triangle PQM \rightarrow \triangle JLM$ *
   e) $\triangle PQM \rightarrow \triangle KJN$
   f) $\triangle JLM \rightarrow \triangle MNJ$ *



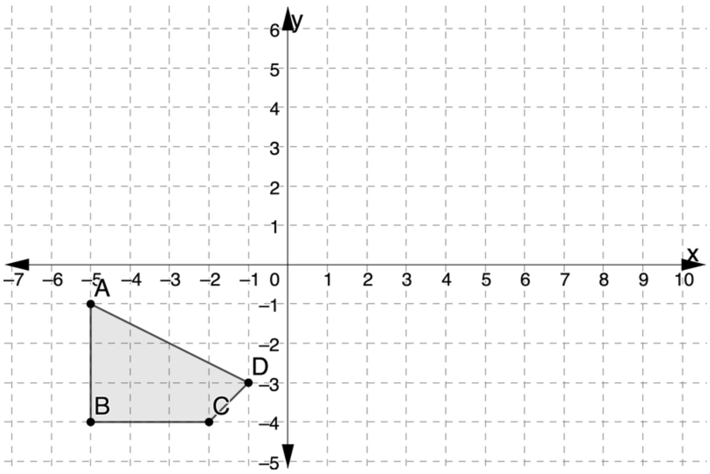
Source: Carter, J., Cuevas, G., Day, R., and Malloy, C., (2012). Glencoe Geometry. USA: The McGraw-Hill Companies, Inc.

2. What is the image of $E(2,-5)$ after the glide transformation $T_{0,-1} \circ r_{y-axis}$? * ________

3. What is the image of $G(-2+\sqrt{3},-\sqrt{2})$ after the glide transformation $T_{-3,0} \circ r_{y=-2}$

   ____

4. Graph the image of the figure below after the glide transformation $T_{0,6} \circ r_{x=2}$. Label the image properly. *



5. The endpoints of $\overline{FG}$ are $F(2,-5)$ and $G(4,1)$. The image of point F after a translation and a reflection across a horizontal line is $(-1,3)$. Give a rule for the performed glide reflection and identify the image of Point G under the same glide reflection. *

   ____

KNOT

Time Allocation: 2 minutes
Actual Time Allocation: _____ minutes

A **composition of transformations** is the combination of two transformations applied to a figure one after another**.** A **glide reflection** is a translation followed by a reflection across a line parallel to the translation. A glide reflection is an isometry or a rigid transformation. The pre-image and image are opposite in orientation but not completely a reflection. In applying a glide reflection, the order of transformation does not matter.

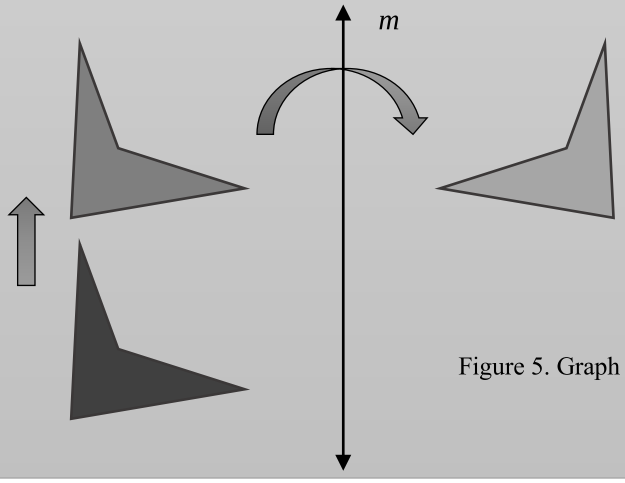
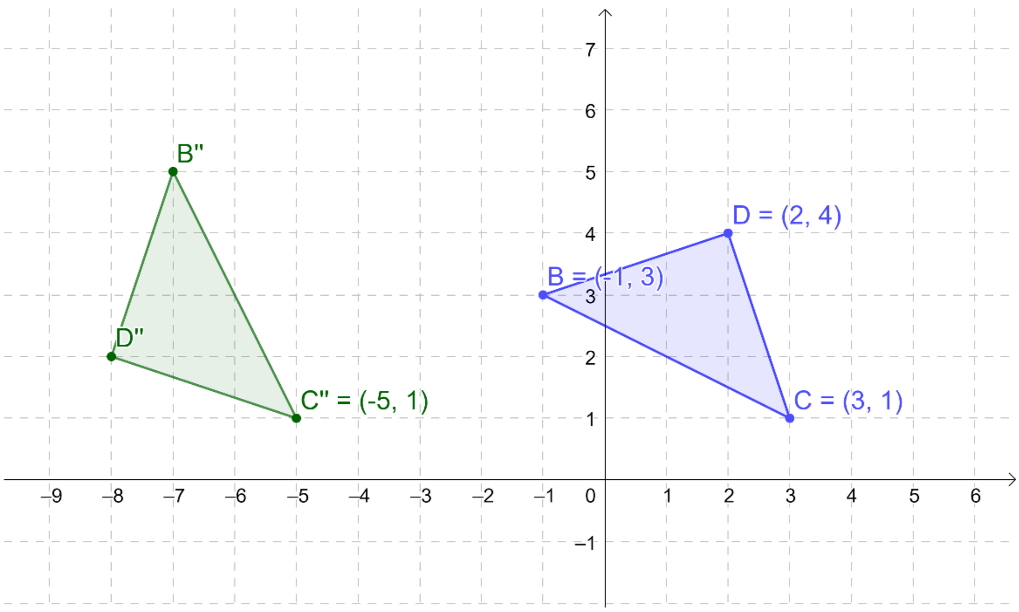


Figure 5. Graph of a Glide Reflection

Supplementary Exercises:

1. What is the image of $P(-3,5)$ after the glide reflection $T_{2,2} \circ r_{y=x}$? ________________
2. The line $y = 3x - 2$ is shifted 2 units down and reflected across the line $x = 1$. What is the equation of the image? ________________
3. Describe the transformation shown in the figure below.



4. Solve for the value of the variables in the glide reflection of $\triangle PQR$ described below.

$$\begin{array}{l} P(-3,-2)\\ Q(-2a,2)\\ R(4,b+5)\end{array} \xrightarrow{\quad T_{0,4} \quad} \begin{array}{l} P'(-3,c-8)\\ Q'(-6,2d-8)\\ R'(-4e,-2)\end{array} \xrightarrow{\quad r_{x=0} \quad} \begin{array}{l} P''(-f,2)\\ Q''(g-4,6)\\ R''(-4,h-3)\end{array}$$

References:

Albarico, J.M. (2013). THINK Framework. Based on *Science LINKS* by E.G. Ramos and N. Apolinario. Quezon City: Rex Bookstore Inc.

Bass, L., Charles, R., Hall, B., Johnson, A., and Kennedy, D., (2007) *Prentice Hall Mathematics Geometry Teacher's Edition.* Pearson Education, Inc.

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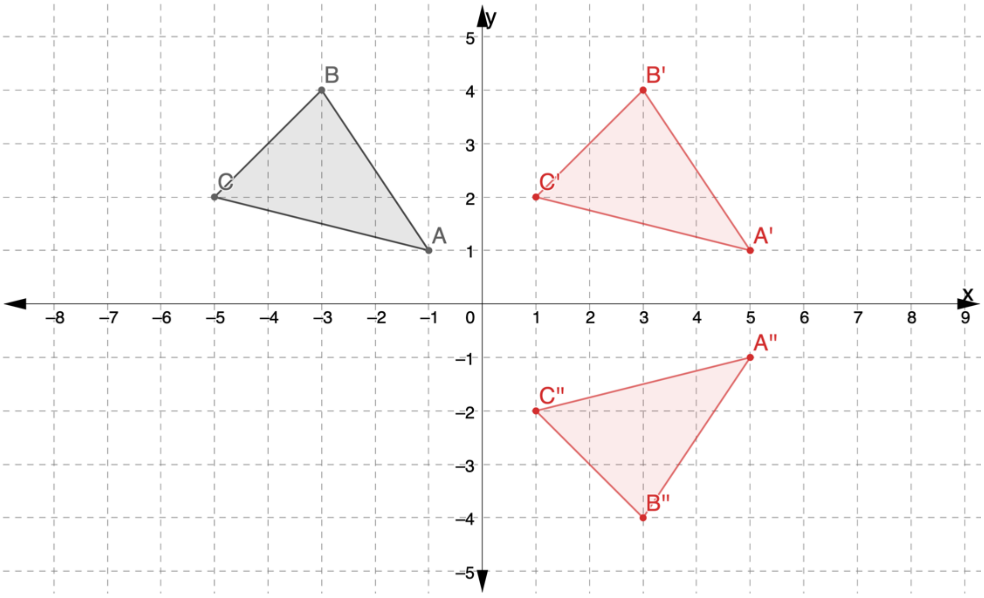
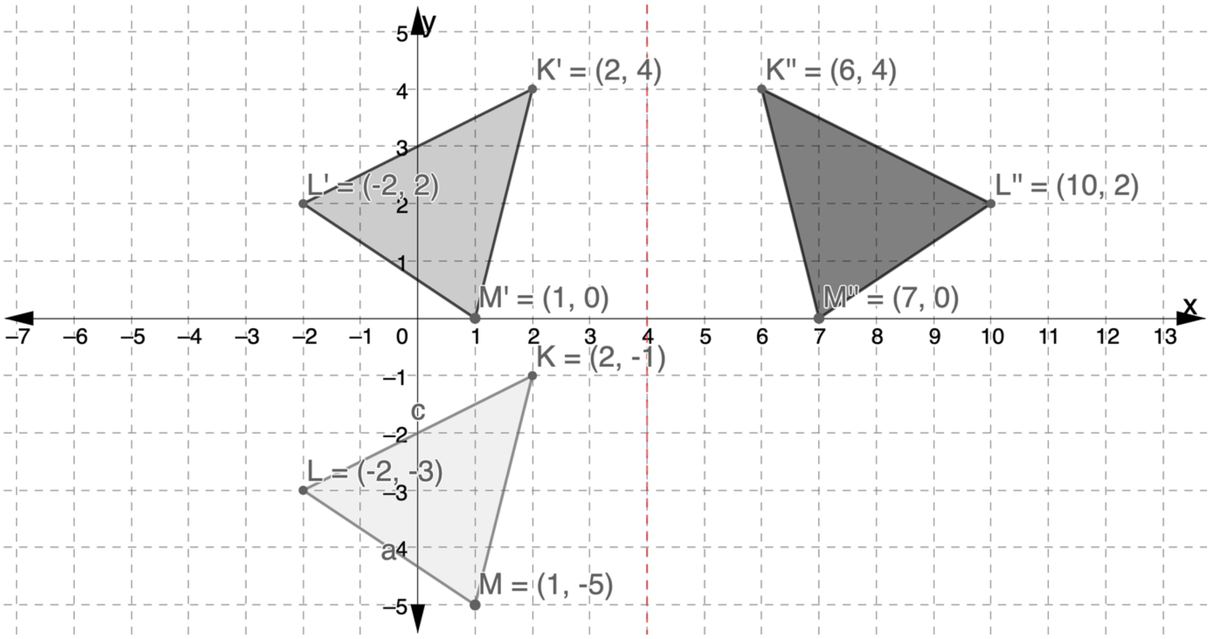
Answer Key:

Hook

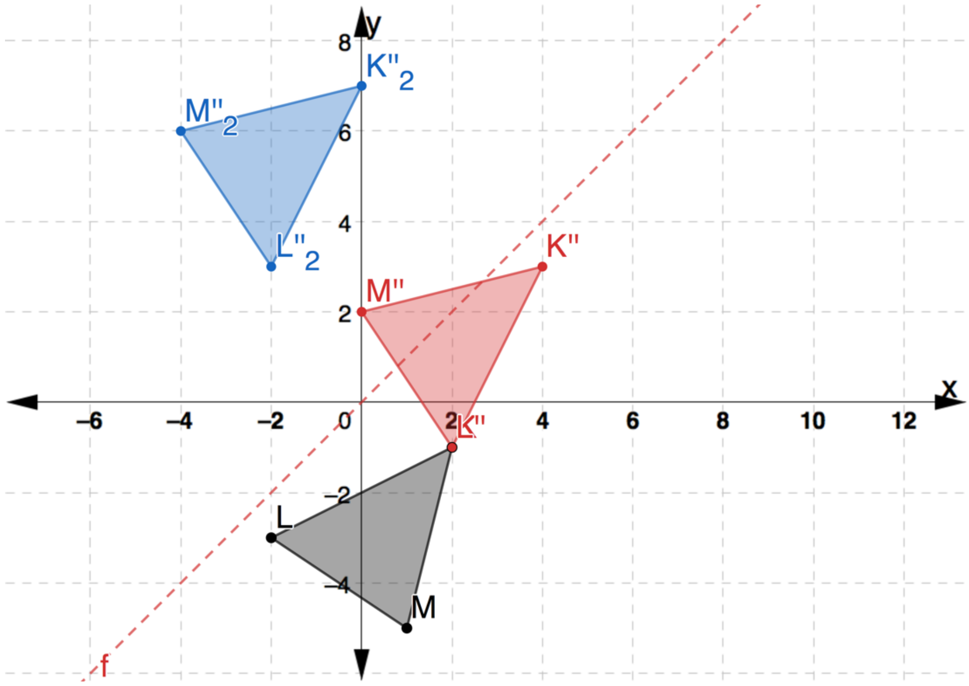


Ignite: Hands-On Activity



3. $K'(2,4), L'(-2,2), M'(1,0)$
5. $K"(6,4)$, L"(10,2), $M"(7,0)$
6. The coordinates will still be the same as the previous result.

7.



No, they don't have the same images. The direction of translation is not parallel to the line of reflection.

Navigate

1. a) not a glide reflection
   c) not a glide reflection
   e) yes, $T_{0,4} \circ r_{x=4}$

3. $\left(-5+\sqrt{3}, -4+\sqrt{2}\right)$