Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	3.0	Analysis of Graphs: Polynomial and Rational Functions
Lesson Code	3.4	Increasing, Decreasing, & Constant Functions
Time Frame		30 minutes

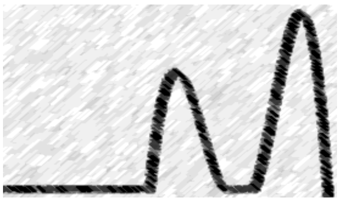
TARGET

After completing this module, you should be able to:
identify the intervals at which a function is constant, increasing, or decreasing.

HOOK

Health & Fitness

After the ECQ, you decided to be healthy & fit so you decided to go out for a mountain climbing to the hill as shown.



When you've learned about the steepness, you were worried about your heart rate.

At what part of the hill will your heart rate increase?

IGNITE

10 MINUTES

Before defining the concepts of the function algebraically, it is instructive to first look at it graphically as shown below.

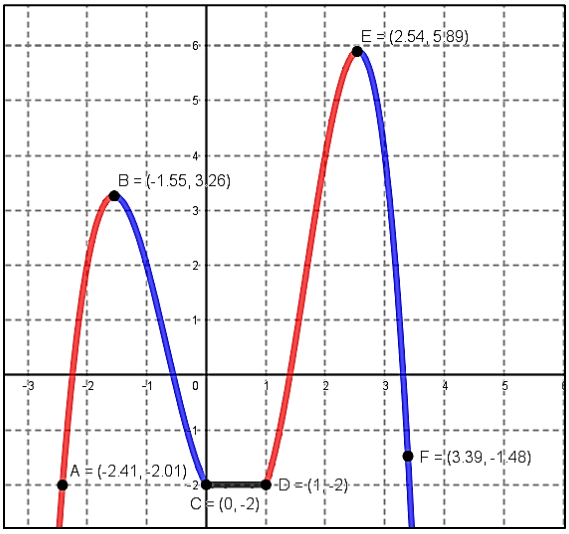


Figure 1. [X:Y=1:1]

We will examine how the function behaves by identifying the intervals at which it is increasing, decreasing, or constant.

If you imagine yourself going up a hill from point A to B, your heart rate ***increases***. When you go down the hill from B to C, it gets easier for you so your heart rate ***decreases***. As you continue to walk from C to D, your heart rate stabilizes and beats **constant**ly.

The behavior of your heart rate as you go up and down the hill illustrates how the function behaves.

Now, that you can visualize the concept, let us identify the intervals over which the graph of the function in some examples increases, decreases, or remains constant.

EXAMPLE 1

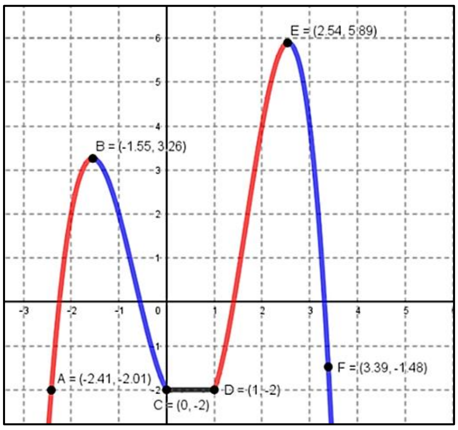


Figure 2. [X:Y=1:1]

Behavior of the graph

Increases over the intervals
from point A to B: [-2.41, -1.55]
from point D to E: [1, 2.54]

Decreases over the intervals
from point B to C:[-1.55, 0]
from point E to F:[2.54, 3.39]

Constant over the interval
from point C to D: [0, 1]

EXAMPLE 2

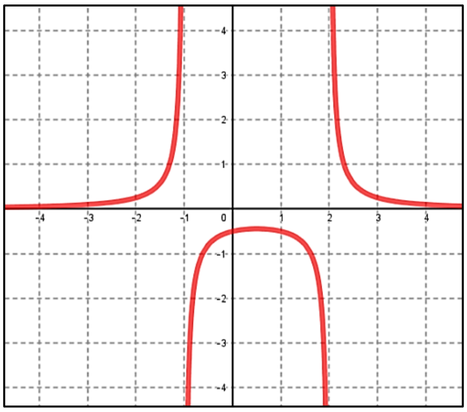


Figure 3. [X:Y=1:1]

Behavior of the graph

Increases over the intervals
$(-\infty, -1) \cup \left(-1, \frac{1}{2}\right)$

Decreases over the intervals
$\left(\frac{1}{2}, 2\right) \cup (2, +\infty)$

There is no interval over which the function is **constant**.

TIP (The Important Point)

Suppose f is a function defined on an interval I. We say f is:

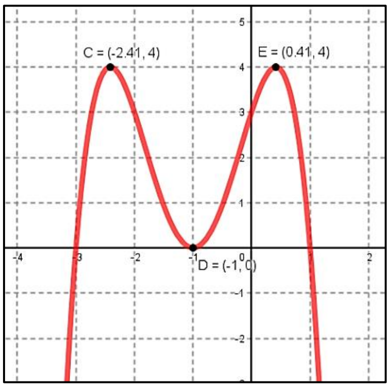
- **Increasing** on I
  if and only if $f(a) < f(b)$ for all real numbers a, b in I with $a < b$.
- **Decreasing** on I
  if and only if $f(a) > f(b)$ for all real numbers a, b in I with $a < b$.
- **Constant** on I
  if and only if $f(a) = f(b)$ for all real numbers a, b in I with $a < b$.

We are confident that you understood the examples. This time take the challenge with yourself.

Identify the intervals over which the function is increasing, decreasing, and constant.

15 MINUTES

1. $f(x) = -x^4 - 4x^3 - 2x^2 + 4x + 3$



C (-2.41, 4)
D (0.41, 4)
E (-1,0)

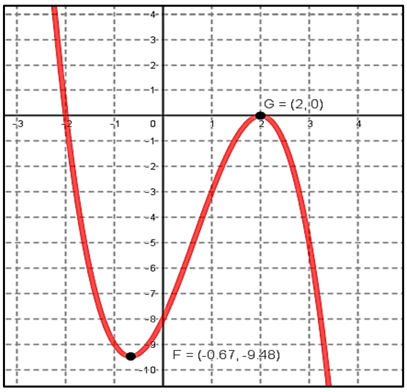
Figure 1. [X:Y=1:1]

Increasing:

Decreasing:

Constant:

2. $f(x) = -x^3 + 2x^2 + 4x - 8$



F (-0.67, -9.48)
G (2, 0)

Figure 2. [X:Y=1:1]

Increasing:

Decreasing:

Constant:

3.

$$f(x) = \frac{x + 1}{x^2 + 2x - 3}$$

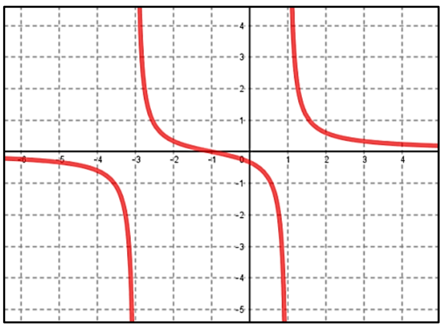


Figure 3. [X:Y=1:1]

Increasing:

Decreasing:

Constant:

4.

$$f(x) = \frac{x^3 + 2x^2 - 4x - 8}{x - 1}$$

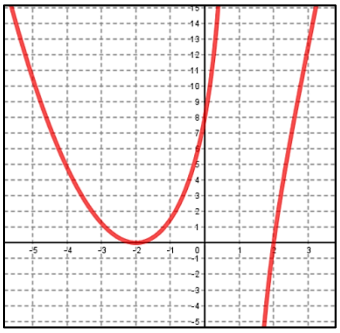


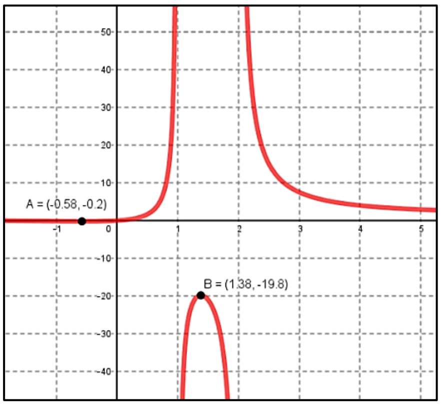
Figure 4. [X:Y=1:1]

Increasing:

Decreasing:

Constant:

5. $$f(x) = \frac{(x^2 + 2x)(x + 1)}{(x^2 - 3x + 2)(x + 1)}$$



A (-0.58, 0.2)
B (1.38, -19.8)

Figure 5. [X:Y=1:10]

Increasing:

Decreasing:

Constant:

6. $$f(x) = \frac{2x^2 + 10x - 48}{x^2 + 2x - 15}$$

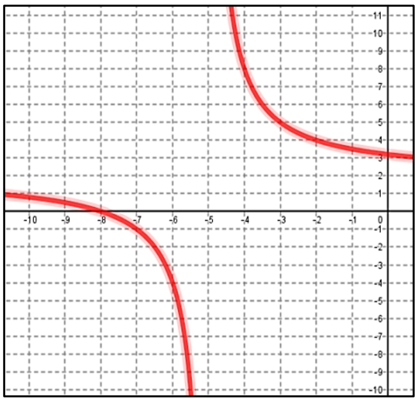


Figure 6. [X:Y=1:1]

Increasing:

Decreasing:

Constant:

KNOT

Before this lesson ends, *Keep* *Notice* of these *Outstanding* *Thoughts*:

The graph of a function can be described by going up a hill, down the hill, and walking through a horizontal path.

- Going up a hill or going up from left to right indicates the graph is increasing
- Going down a hill going down from right to left indicates the graph is decreasing
- Walking through a horizontal path indicates the graph is constant

References:

[1] Swokowski, E., & Cole, J. (2008). *Algebra and Trigonometry with Analytic Geometry, 12th Edition.* Thomson Learning, Inc.
[2] International Geogebra Institute. (2020). *GeoGebra.* https://www.geogebra.org/
[3] Albarico, J.M. (2013). THINK Framework. (Based on Ramos, E.G. and N. Apolinario. (n.d.) *Science LINKS*. Rex Bookstore, Inc.)

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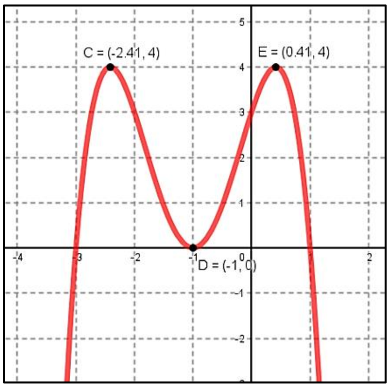
ANSWER KEY:

1. $f(x) = -x^4 - 4x^3 - 2x^2 + 4x + 3$



C (-2.41, 4)
D (0.41, 4)
E (-1,0)

Figure 1. [X:Y=1:1]

Increasing: $(-\infty, -2.41] \cup [-1, 0.41]$

Decreasing: $[-2.41, -1] \cup [0.41, \infty)$

Constant: **none**

3. $$f(x) = \frac{x+1}{x^2 + 2x - 3}$$

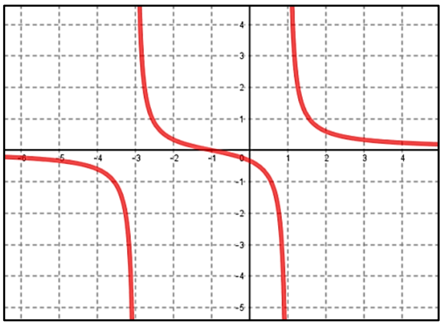


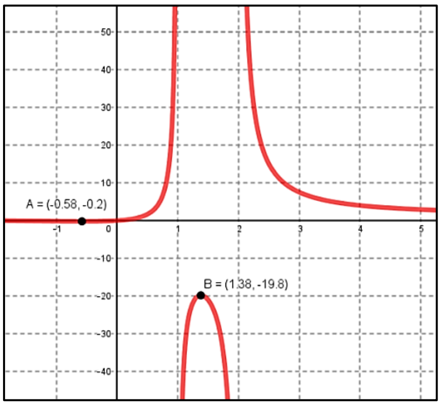
Figure 3. [X:Y=1:1]

Increasing: **none**

Decreasing: $(-\infty, -3) \cup (-3, 1) \cup (1, \infty)$

Constant: **none**

5. $$f(x) = \frac{(x^2 + 2x)(x+1)}{(x^2 - 3x + 2)(x+1)}$$



A (-0.58, 0.2)
B (1.38, -19.8)

Figure 5. [X:Y=1:10]

Increasing: $(-\infty, 1) \cup (1, 1.38]$

Decreasing: $[1.38, 2) \cup (2, \infty)$

Constant: **none**