Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	2.0	Transformations on the Coordinate Plane
Lesson Code	2.5.1	Scale Transformations (Dilation) 1
Time Limit		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: ______ minutes

By the end of this module, the students will have been able to

1. Define and demonstrate scale transformations using graphing tools and software and other manipulative devices
2. Illustrate scale transformations on the coordinate plane
3. Determine the image of a figure under dilation with the origin as the center of dilation

HOOK

Time Allocation: 2 minutes
Actual Time Allocation: ______ minutes

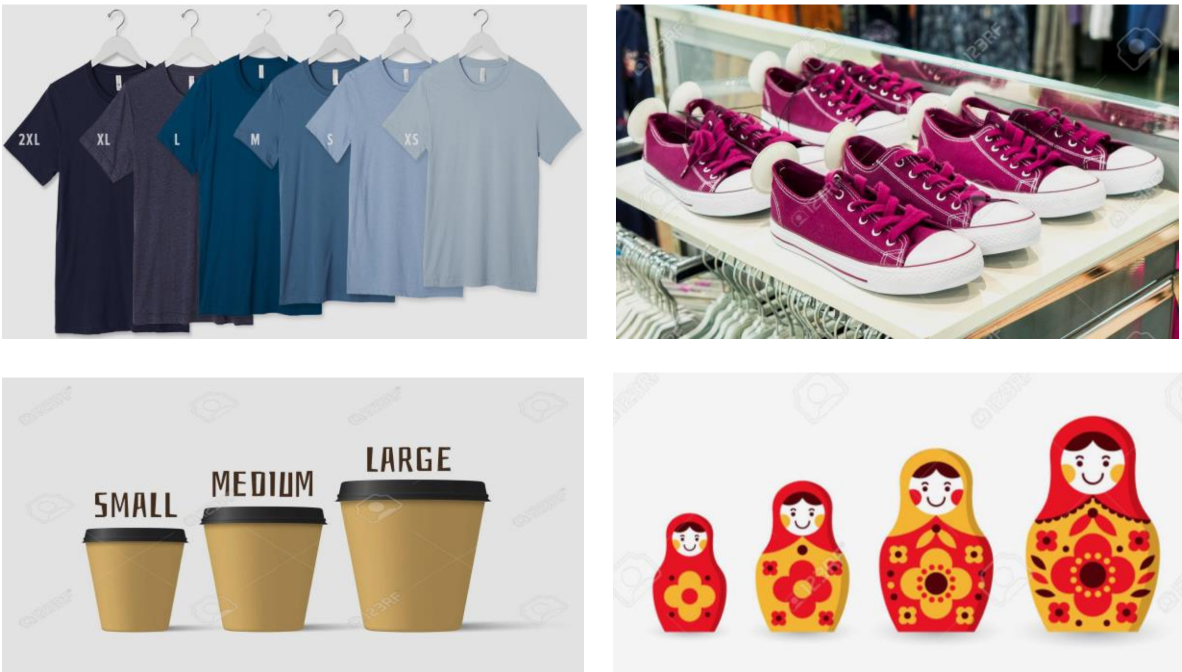


Figure 1. Photograph of T-shirts in different sizes. From *How To Choose The Right T-shirt Size For Your Order* by Bella+Canvas. http://blog.bellacanvas.com/picking-right-size-tees-order/; Photograph of burgundy violet sneakers on display by O. Usik. From 123RF. https://www.123rf.com/photo_72403164_few-rows-of-colorful-burgundy-violet-sneakers-with-such-color-shoe-lines-with-round-security-magnet-.html;Realistic coffee cups of different sizes isolated on a white background by vpif. From 123RF. https://www.123rf.com/photo_90333374_stock-vector-realistic-coffee-cups-of-different-sizes-isolated-on-a-white-background-paper-cups-mockup-vector-ill.html; Set of matryoshka Russian nesting dolls of different sizes, souvenirs from Russia by E. Serechenko. From 123RF. https://www.123rf.com/photo_92940289_stock-vector-set-of-matryoshka-russian-nesting-dolls-of-different-sizes-souvenir-from-russia.html

We see different goods sold in the market that come in the same shape but different sizes. We can think of clothes (S, M. L, XL, XXL), shoes (size 5, 6, 7, 8, 9, etc), drinks (8 oz., 12 oz., 16 oz., etc), food, toys, bags, and many more. Even structures of buildings may have the same design but differ in height.

In the previous modules, we discussed different types of geometric transformations that preserve the shape and size of the object being transformed. Such transformations (translation, reflection, rotation, and glide reflection) are called rigid transformation.

This type of transformation produces *congruent* figures. Two figures are said to be congruent when their corresponding sides and angles have the same measures.

There is another geometric transformation that we will talk about in this module. Unlike the first ones, this type of transformation that produces *similar figures*. Two figures are said to be similar when their corresponding sides are proportional. They have the same shape, but their sizes are different. Real-life examples of *similar figures* are shown in the pictures at the beginning of this module.

IGNITE

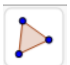
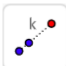
Time Allocation: 20 minutes
Actual Time Allocation: _____ minutes

Definition of Scale Transformation
A scale transformation (dilation) is a transformation that produces an image that has the same shape but differs in size. In this transformation, there is a fixed point, we call the *center of dilation*, and a *scale factor*.

We will also refer to the figure to be dilated as the *preimage* of the transformation.

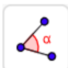
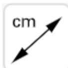
Using a graphing software, let us explore what happens when an object undergoes dilation.

Hands-on Activity:
Objective: To investigate scale transformations of geometric figures on the coordinate plane.
Graphing Software/Tool: GeoGebra

1. Mark the origin as point O. This will be the center of dilation.
2. Plot a triangle using the Polygon Tool  with vertices at $(6,8)$, $(8,4)$ and $(4,0)$.
3. Using the Transformation tool: Dilate from a point , construct the image of the triangle under dilation. Select the object to dilate, then the center of dilation, then type in the scale factor. Investigate what happens to the image as you differ the scale factors. Complete the table below by filling in the coordinates of the image of the triangle given your choice of scale factor.

Vertices	When $k > 1$	When $0 < k < 1$	When $k = 1$	When $k < 0$
	Scale factor: k = ___	Scale factor: k= ____		Scale factor: k = ____
$(6,8)$				
$(8,4)$				
$(4,0)$				

4. Explore other properties of the image such as measure of angles, lengths of the sides, area, alignment of vertices at its corresponding images, etc. Write down your observations.

   (GeoGebra Tip: The measures of the angles and the sides, as well as the area of the polygon can be determined using the function tools , , and , respectively.)

5. Investigate what happens when the center of dilation is *not at the origin*. You may mark any point on the Cartesian plane other than the origin. You may position the fixed point outside or inside the triangle and see what happens. Make this the center of dilation and do the same procedure. You may also vary the geometric figure and take note of your observations.

Guide Questions:

1. When does *enlargement* of the original figure happen?
2. When is the size of the original figure *reduced*?
3. When does the size of the image the same as the original figure?
4. What happens when the scale factor is a negative number? What additional transformation takes place?
5. How do we determine the coordinates of the image under dilation with the origin as the center of dilation?
6. What can be said about the angles of the image versus the angles of the original figure?
7. What can be said about the measure of the lengths of the sides of the image versus that of the original figure?
8. What can be said about the perimeter of the two figures? the area?
9. Are these same observations (nos. 1-8) still true when the center of dilation is not at the origin?

Discussion/Processing:

Let us identify general observations about scale transformations (dilation) that we learned from the Hands-on Activity. (Note: This is also to verify if your answers in the activity are correct.)

Let us focus first on scale transformations where the center of dilation is the origin. We will use the notation D_k, where k is the scale factor, to refer to scale transformations with the origin as the center of dilation. For instance, D_2 refers to dilation about the origin with a scale factor $k = 2$. (Note: Unless otherwise stated, we will refer to the origin as the center of dilation.)

Under dilation, a figure *enlarges* when the scale factor k is greater than 1. On the other hand, it *reduces* (or shrinks in size) when the scale factor k is between 0 and 1.

Let's us refer to triangle ABC from the Hands-on Activity.

When $k = 2$, the vertices of the preimage were mapped to its image as follows:

$$\begin{aligned} D_2\text{: } (6,8) &\rightarrow (12,16) \\ (8,4) &\rightarrow (16,8) \\ (4,0) &\rightarrow (8,0) \end{aligned}$$

In other words, each vertex (x, y) is mapped to $(2x, 2y)$. In symbols, D_2: $(x, y) \rightarrow (2x, 2y)$.

Figure 2 shows the output when the Transformation tool: Dilate from a point  in GeoGebra is used. Notice that the image is bigger than the preimage.

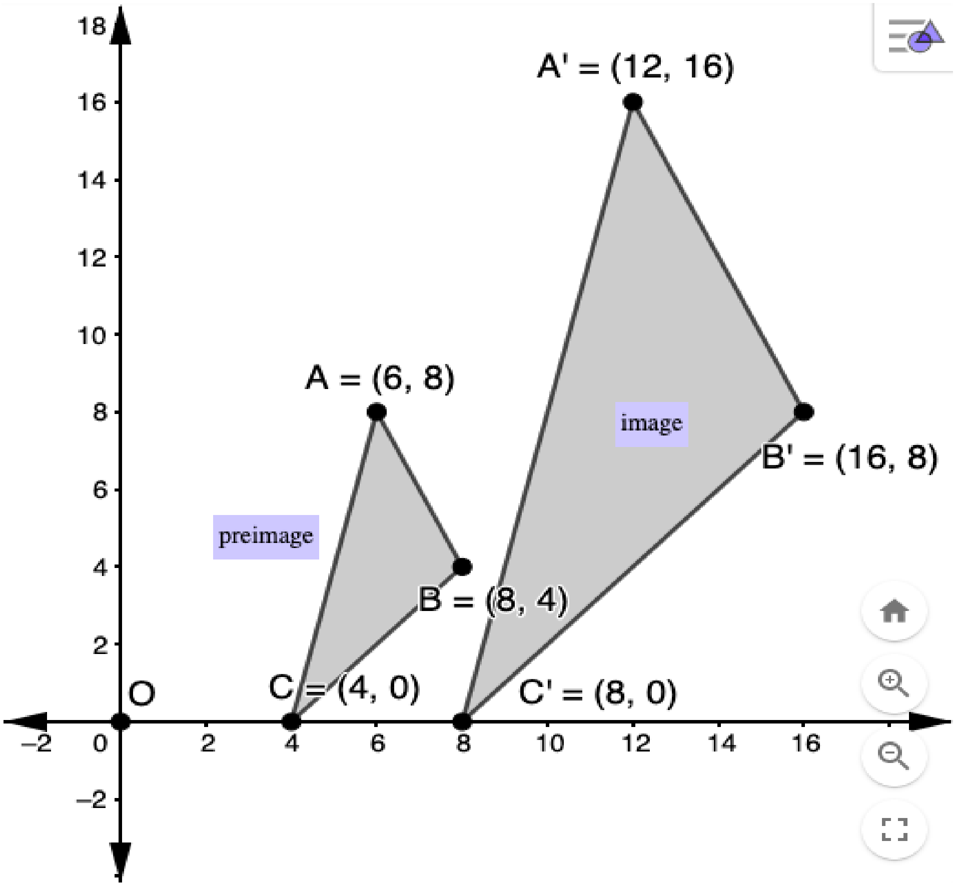


Figure 2. The triangle and its image under dilation with center of dilation at the origin and scale factor 2

Likewise, when $k = \frac{1}{2}$, the vertices of the image are as follows:

$$
\begin{aligned}
D_{1/2}: (6,8) &\rightarrow (3,4) \\
(8,4) &\rightarrow (4,2) \\
(4,0) &\rightarrow (2,0)
\end{aligned}
$$

We can see that each vertex (x, y) is mapped to $(\frac{1}{2}x,\ \frac{1}{2}y)$. In symbols, $D_{1/2}: (x, y) \rightarrow (\frac{1}{2}x,\ \frac{1}{2}y)$.

The image is shown in Figure 3. Notice that the image is reduced in size as a result of the scale transformation.

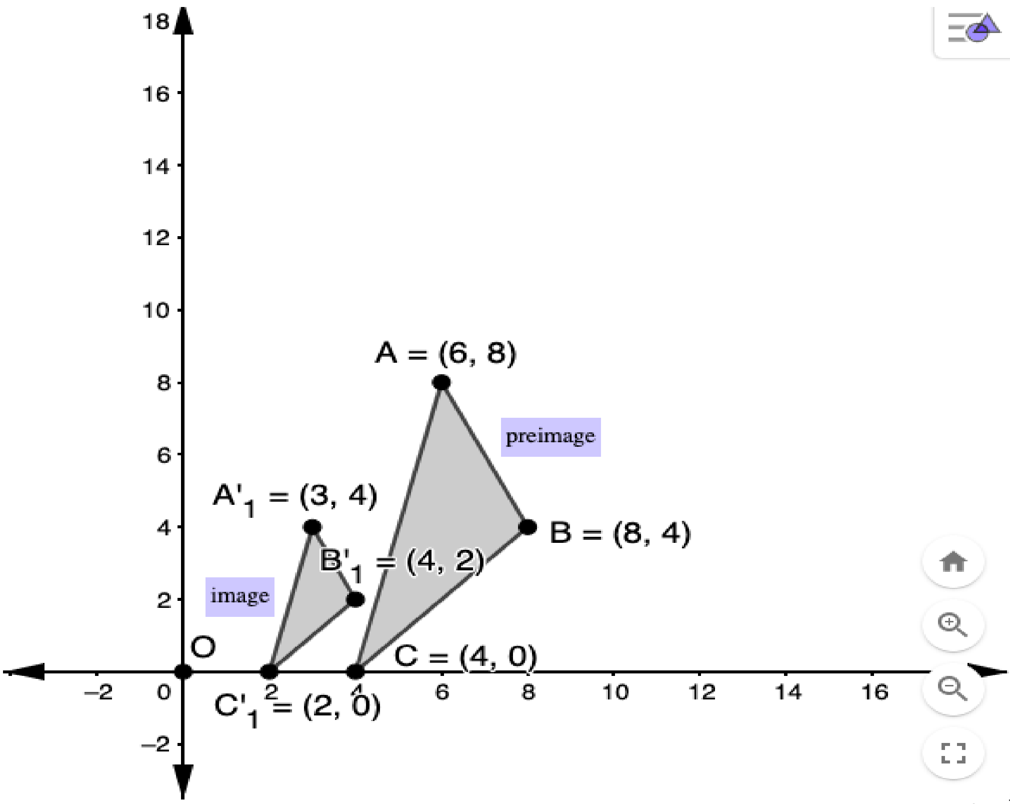


Figure 3. The preimage and its image under dilation with center of dilation at the origin and scale factor $k = \frac{1}{2}$

In general, the coordinates of the vertices of the image are multiplied by the scale factor k. In symbols, we can represent a scale transformation using the following mapping:

$$D_k: (x, y) \rightarrow (kx, ky), \text{ where } k \text{ is the scale factor.}$$

It is important to note that when $k = 1$, the scale transformation maps the figure to itself. That is, $D_1: (x, y) \rightarrow (x, y)$.

Finally, when $k < 0$, say $k = -2$, we have

$$\begin{aligned} D_{-2}: (x, y) &\rightarrow (-2x, -2y) \\ D_{-2}: (6,8) &\rightarrow (-12, -16) \\ (8,4) &\rightarrow (-16, -8) \\ (4,0) &\rightarrow (-8,0) \end{aligned}$$

Figure 4 shows the image when the scale factor is $k = -2$. Note that the image is located at the opposite side of the center of dilation. In addition, we can note that the image is bigger than the preimage. This is so, since $|k| = 2$.We will explain this further in the next module (Module 2.5.2) together with more points/observations from the Hands-on Activity.

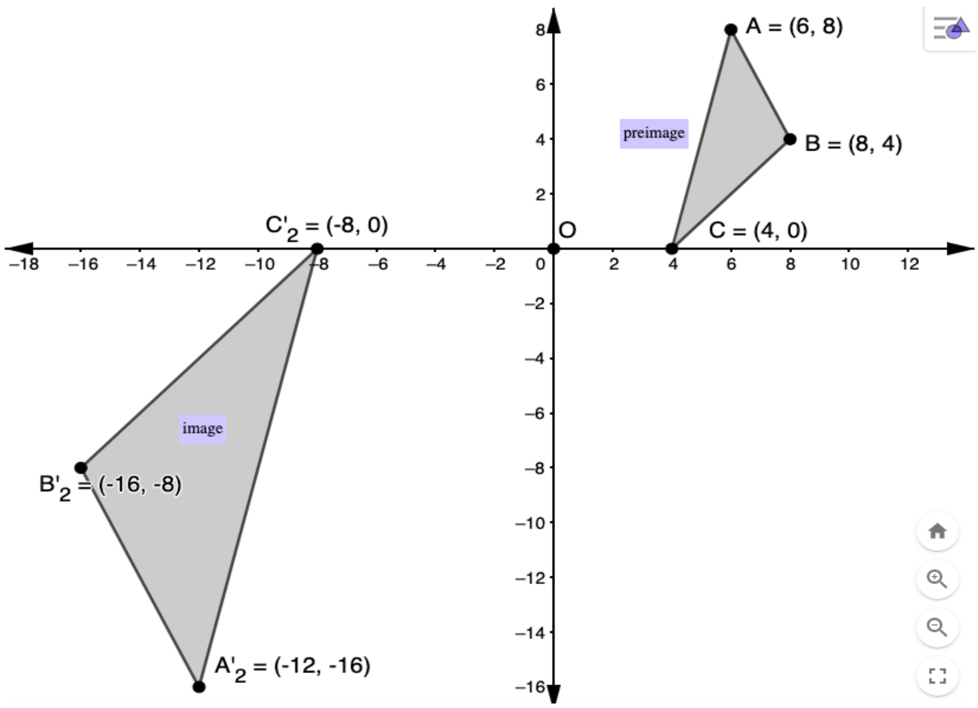
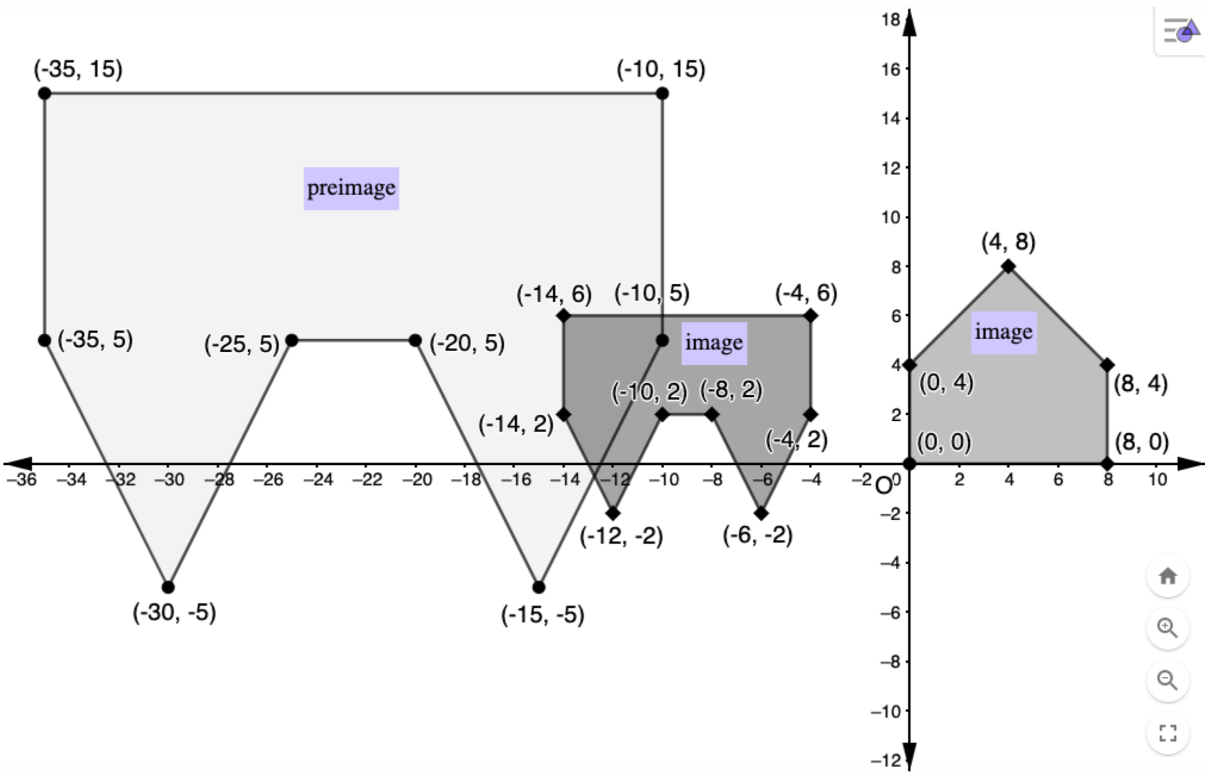


Figure 4. The preimage and its image under dilation with center of dilation at the origin and scale factor $k = -2$

Try this!

A geometric figure and its image under dilation is shown in the Cartesian plane below. (1) Determine the rule of this transformation, then (2) give the coordinates of the preimage of the pentagon under the same rule of transformation.



Solution:

(1) To determine the rule of transformation under dilation, we can pick a few pairs of corresponding vertices (i.e. preimage and image) and obtain the scale factor, k.

$D_k: (x, y) \to (kx, ky) \quad \Rightarrow k = \frac{kx}{x} = \frac{ky}{y}$

$D_k: (-25,5) \to (-10,2) \Rightarrow k = \frac{-10}{-25} = \frac{2}{5} \quad \Rightarrow k = 2/5$

$D_k: (-10,15) \to (-4,6) \Rightarrow k = \frac{-4}{-10} = \frac{6}{15} \quad \Rightarrow k = 2/5$

$D_k: (-15,-5) \to (-6,-2) \quad \Rightarrow k = \frac{-6}{-15} = \frac{-2}{-5} \quad \Rightarrow k = 2/5$

Therefore, $k = ⅖$ and the rule of transformation is $D_{2/5}$. This also makes sense because the image is smaller than the preimage, hence, k must be between 0 and 1.

(2) To get the preimage of the pentagon, we can work backwards as follows:

$D_k: (x, y) \to (kx, ky) \quad \Rightarrow x = \frac{kx}{k}, y = \frac{ky}{k}$

$D_{2/5}: (x_1, y_1) \to (0,0) \quad \Rightarrow x_1 = 0, y_1 = 0 \quad \Rightarrow (0,0)$

$D_{2/5}: (x_2, y_2) \to (0,4) \quad \Rightarrow x = \frac{0}{2/5}, y = \frac{4}{2/5} \quad \Rightarrow (0,10)$

$D_{2/5}: (x_3, y_3) \to (4,8) \quad \Rightarrow x = \frac{4}{2/5}, y = \frac{8}{2/5} \quad \Rightarrow (10,20)$

$D_{2/5}: (x_4, y_4) \to (8,4) \quad \Rightarrow x = \frac{8}{2/5}, y = \frac{4}{2/5} \quad \Rightarrow (20,10)$

$D_{2/5}: (x_5, y_5) \to (8,0) \quad \Rightarrow x = \frac{8}{2/5}, y = \frac{0}{2/5} \quad \Rightarrow (20,0)$

As expected, the preimage must be bigger than the image because of the value of the scale factor that we got in (1).

NAVIGATE

Time Allocation: 5 minutes
Actual Time Allocation: _____ minutes

Check your understanding by answering the items below.

1. Given the scale factors below, tell whether the image formed under dilation is *enlarged* or *reduced*.
   (a) $k = 5$
   (b) $k = 0.5$
   (c) $k = 0.1$
   (d) $k = -0.2$
   (e) $k = 2\sqrt{3}$
   (f) $k = \frac{1}{5}a$, where a is a real number

2. Determine the coordinates of the image of a rectangle with vertices (2,7), (8,10), (9,8), and (3,5) under dilation given each scale factor below. (The center of dilation is at the origin.)
   (a) $k = 3$
   (b) $k = 0.25$
   (c) $k = \frac{2}{3}$
   (d) $k = -2$
   (e) $k = \sqrt{3}$
   (f) $k = a - b$, where a and b are real numbers

3. A pentagon is dilated by a scale factor of $k = -3$. If the origin is the center of dilation and the vertices of its image are $(0, -6)$, $(3, -15)$, $(12, -9)$, $(6, -3)$, and $(0,0)$, determine the coordinates of the vertices of the pre-image. Verify your answer using a graphing software.

4. Fill in the table below with the unknown preimage, image, or scale factor.

	Preimage	Image	Scale Factor, k
(a)	$(3, -1)$		$k = 8$
(b)		$(2, 2)$	$k = 1/6$
(c)	$(a + b, a - b)$	$(-2a - 2b, -2a + 2b)$	
(d)		$(1/5, -2/5)$	$k = a/b\ (a, b \neq 0)$

KNOT

Time Allocation: 2 minutes
Actual Time Allocation: _____ minutes

Key terms to remember:

1. dilation
2. center of dilation
3. scale factor
4. enlargement
5. reduction

Some points to remember:

1. Under scale transformation where the origin is the center of dilation and scale factor k, the point (x, y) is mapped to (kx, ky).
2. A dilation of a geometric figure produces an *enlarged* image when the scale factor k is greater than one and a *reduced* image when the scale factor k is between *0* and *1*.
3. When $k = 1$, the image has the same size as its preimage. The preimage is mapped to itself.
4. When $k < 0$, the image will be located at the opposite side of the center of dilation.
5. Scale transformation produces similar figures.

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Answer Key:

Ignite: Hands-On Activity (Answers may vary.)

Navigate:

1. (a) enlarged (b) reduced (c) reduced (d) reduced (e) enlarged (f) enlarged when $|a| > 5$, reduced when $0 < |a| < 5$

3. The vertices of the pre-image are: $(0,2)$, $(-1,5)$, $(-4,3)$,$(-2,1)$, and $(0,0)$.
The pre-image (smaller pentagon) and the image are shown in the Cartesian plane below.