Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	1.0	Basic Plane and Coordinate Geometry
Lesson Code	1.3	Distance and Midpoint Formulas
Time Limit		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

By the end of this module, the students will have been able to apply the distance and midpoint formulas for points and segments on the coordinate plane.

HOOK

Time Allocation: 4 minutes
Actual Time Allocation: _____ minutes

Solving problems that involve finding either distances or midpoints are often necessary. Let us recall **line segments** – part of a line which has two endpoints – drawn in a coordinate plane. Given the coordinates of these endpoints, we will be able to determine the *distance* (length of the line segment) and the *midpoint* (halfway between two endpoints). It is essential to fully understand how to find the midpoint and distance of a line segment because this will also enable us to fully understand the concept of absolute value.

Take this problem!

Michael is going on a two-day hike. The map which is shown at the right (see Figure 1) presents the trails he plans to follow. Each unit represents 1 km.

a. Michael hikes from the lodge to point A and choose that he will hike to the midpoint of $\overline{AB}$ before he camps for the night. Determine the point in the plane where Michael plans to camp.

b. How far will Michael hike each day?

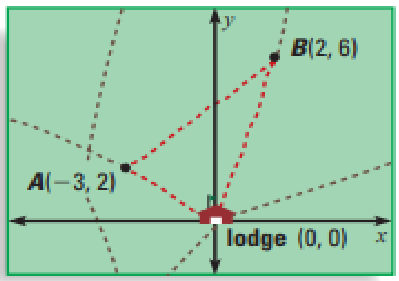


Figure 1

(Retrieved from https://www.classzone.com/eservices/home/pdf/student/LA210AAD.pdf)

In lesson 1.2, we have discussed how to find the midpoint and distance between two points on a number line. This time, to solve this problem, we need to compute for the midpoint and distance between two points on a Cartesian plane.

Try these out! (Recall some concepts learned from Mathematics 2)

Determine whether each statement is **TRUE** or **FALSE.** Justify your answer.

a. Distance is always positive.

b. We can derive a formula that allows us to compute for the distance between two points on coordinate plane from the Pythagorean Theorem.

c. The values computed using the distance formula differs when the first point and the second point are not labeled (x_1, y_1) and (x_2, y_2), respectively.

IGNITE

Time Allocation: 15 minutes
Actual Time Allocation: _____ minutes

Let us start exploring our topic with this example!

Example 1. Determine the distance between points Y and Z which are shown on the coordinate plane at the right (see Figure 2).

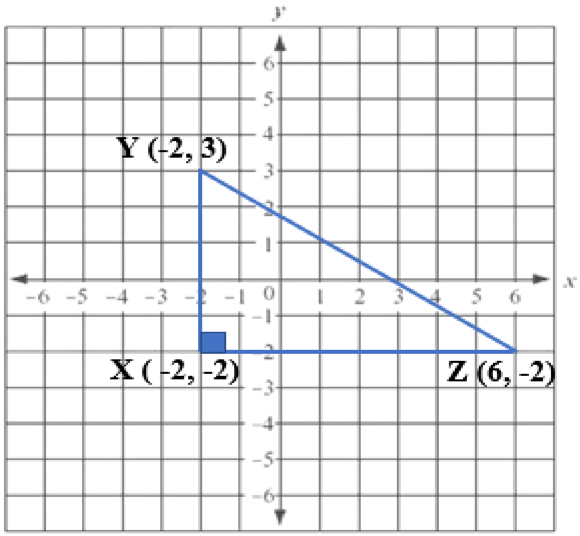


Figure 2

Solution:

To find the distance between points Y and Z, we have added point X with coordinates $(-2,-2)$ to create a right triangle. This will enable us to use the Pythagorean theorem. Isolating right triangle ΔXYZ, we'll have figure 3 shown below.

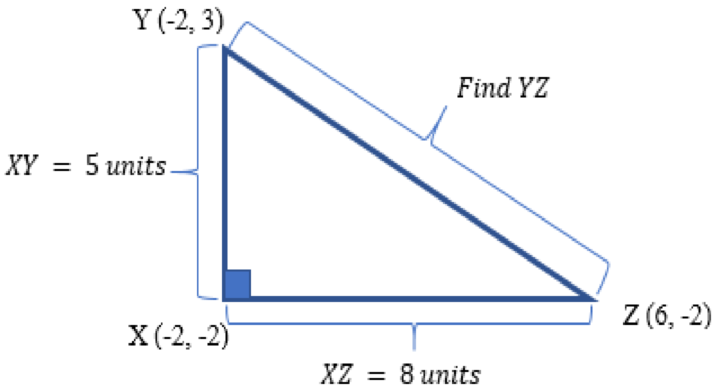


Figure 3

The vertices of the triangle are $X\ (-2,-2)$, $Y\ (-2,3)$ and $Z\ (6,-2)$. Side XY is parallel to the y – axis and points X and Y lie on a vertical line. To compute for the length XY, we simply have $|y_X - y_Y| = |-2-3| = 5$ units. Side XZ is parallel to the x – axis and points X and Z lie on a horizontal line. its length is 8 units. Similarly, we can compute for the length $XZ = |x_X - x_Z| = |-2-6| = 8$ units. Given that the triangle is right, YZ is the longest side which is the hypotenuse and we can use the Pythagorean Theorem to find its length.

Consider the legs of the right triangle XY and XZ as a and b, and the longest side YZ as the hypotenuse c. Now, let us determine the length of side YZ (c) using the Pythagorean theorem.

$$c^2 = a^2 + b^2 \Longrightarrow c^2 = 8^2 + 5^2 \Longrightarrow c^2 = 64 + 25 \Longrightarrow c = \sqrt{\mathbf{89}} \approx \mathbf{9.43}$$

Using the Pythagorean Theorem, we were able to find the length of side YZ which is $\sqrt{\mathbf{89}}$ units and the distance between points Y and Z is $\sqrt{\mathbf{89}}$ or approximately **9.43** units.

DISTANCE FORMULA

From this example, we now have an idea how to derive the distance formula.

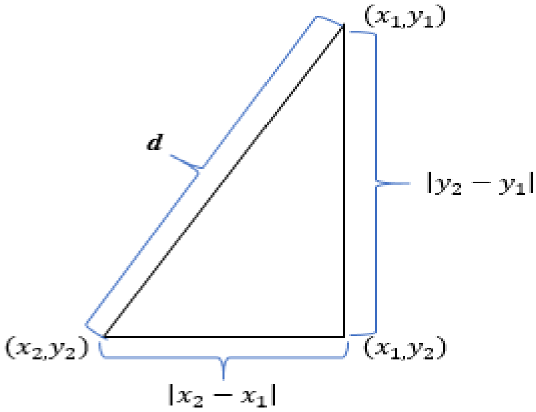


Figure 4

Observe the triangle shown below. Each vertex of the triangle is labeled appropriately. And based from how we have computed the for the distance of the legs of the triangle, we replace side "a" by $|x_2 - x_1|$ and side "b" by $|y_2 - y_1|$, and side "c" by *d*. We determine that

$$\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}\ \mathbf{.}$$

Example 2.

Let us find the distance between points Y $(-2,3)$ and Z $(6,-2)$ from example 1.

Solution:

$$\text{Let } \boldsymbol{Y(x_1, y_1) = (-2, 3)} \text{ and } \boldsymbol{Z(x_2, y_2) = (6, -2)}$$

Therefore,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{[6 - (-2)]^2 + [(-2) - 3]^2}$$
$$d = \sqrt{8^2 + 5^2} = \sqrt{64 + 25} = \sqrt{\mathbf{89}} \approx \mathbf{9.43}$$

We can conclude that we have the same results.

Example 3.

Find the distance between the given points A (-1, 4) and B (-5, -6) in the coordinate plane shown at the right (see Figure 5).

Solution:

Using the Distance Formula, we can determine the distance between points A (-1, 4) and B (-5, -6)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{[-5 - (-1)]^2 + [(-6) - 4]^2}$$
$$d = \sqrt{(-4)^2 + (-10)^2} = \sqrt{16 + 100}$$

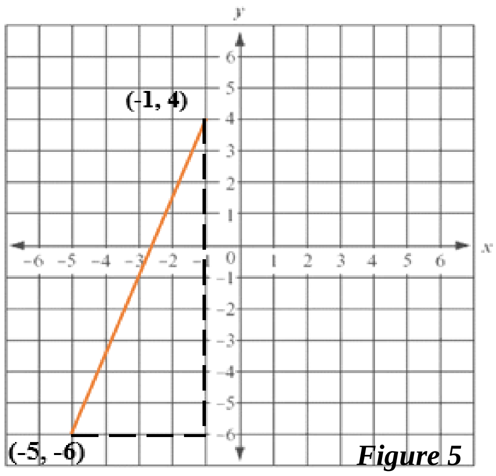


Figure 5

$d = \sqrt{116} = \mathbf{2\sqrt{29}} \approx \mathbf{10.77}$

Therefore, the distance between points A and B is $\mathbf{2\sqrt{29}}$ or approximately **10.77** units.

Remember this!

It does not matter which point is assigned as (x_1, y_1) and which point is assigned as (x_2, y_2), the result will be exactly the same.

MIDPOINT FORMULA

Let us proceed to the discussion of the midpoint formula.
Kindly click the link below to see how midpoint formula was derived.

https://www.youtube.com/watch?v=zXpOIcCLRio

Let us consider two points A (x_1, y_1) and B (x_2, y_2), the point which is halfway between A and B on a line is called ***midpoint***. We can calculate the midpoint (M) using the midpoint formula.

$$\boldsymbol{M_x = \frac{x_1 + x_2}{2}} \quad \textbf{and} \quad \boldsymbol{M_y = \frac{y_1 + y_2}{2}}$$

Then, if we combine $\boldsymbol{M_x}$ **and** $\boldsymbol{M_y}$

$$\boldsymbol{M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)}$$

Example 4.

Determine the coordinates of the midpoint between the points $S(-1, 6)$ and $T(4, -6)$.

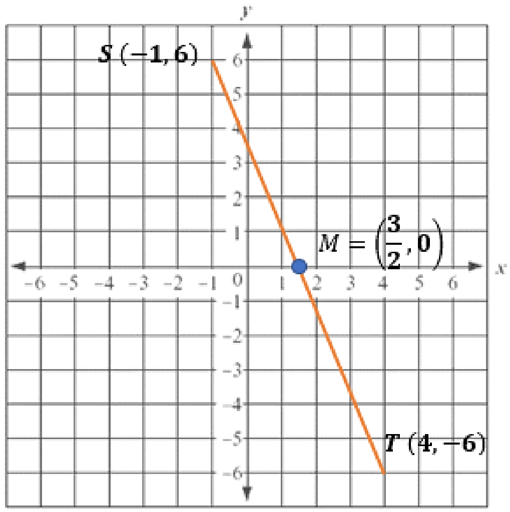
Solution:

Consider $(x_1, y_1) = (-1,6)$ and $(x_2, y_2) = (4, -6)$.
We will use the midpoint formula,

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$M = \left(\frac{-1 + 4}{2}, \frac{6 + (-6)}{2}\right)$$

$$M = \left(\frac{3}{2}, \frac{0}{2}\right) = \left(\frac{\mathbf{3}}{\mathbf{2}}, \mathbf{0}\right)$$



Thus, the midpoint of ST is at $\left(\frac{\mathbf{3}}{\mathbf{2}}, \mathbf{0}\right)$.

Example 5.

If one of the endpoints of a line segment drawn in a coordinate plane is at $(3, -4)$ and the midpoint of the segment is at $(-2, 1)$. Determine the coordinates of the other endpoint.

Solution:

Using $(x_1, y_1) = (3, -4)$ and $\left(M_x, M_y\right) = (-2,1)$. We will use the midpoint formula to determine the coordinates of the other endpoint, (x_2, y_2).

$$\boldsymbol{M_x = \frac{x_1 + x_2}{2}} \quad \textbf{and} \quad \boldsymbol{M_y = \frac{y_1 + y_2}{2}}$$

To find $\boldsymbol{x_2}$, we are going to use $\boldsymbol{M_x = \frac{x_1 + x_2}{2}}$

$$-2 = \frac{3 + x_2}{2} \Longrightarrow -4 = 3 + x_2 \Longrightarrow x_2 = -4 - 3 = \mathbf{-7}$$

To find $\boldsymbol{y_2}$, we are going to use $\boldsymbol{M_y = \frac{y_1 + y_2}{2}}$

$$1 = \frac{-4 + y_2}{2} \Longrightarrow 2 = -4 + y_2 \Longrightarrow y_2 = 2 + 4 = \mathbf{6}$$

Thus, the coordinates of the other endpoint are $\mathbf{(-7, 6)}$**.**

NAVIGATE

Time Allocation: 5 minutes
Actual Time Allocation: _____ minutes

Directions: Answer the following problems.

1. Determine the distance between the points A$(4, -3)$ and F$(0, 6)$ in a coordinate plane
2. Given a point $(-2, -4)$ and the midpoint of the line segment joining (2, 4) and (5, 7), find the distance between these points.
3. If (3, 6) is the midpoint of points (x, y) and $(-6, 7)$, find the values of x and y.
4. Given the vertices of a triangle, $A(0, 1)$, $B(2, 3)$ and $C(2, -1)$. Show that ΔABC is a right isosceles.
5. Determine the value of $\boldsymbol{y}$ in the point (0, y) which is equidistant from $(5, -10)$ and $(1, -3)$.

KNOT

Time Allocation: 5 minutes
Actual Time Allocation: _____ minutes

Let us summarize!

The ***distance between two points*** in a coordinate plane can be solved using the **distance formula**. **Distance formula** is derived from the Pythagorean Theorem.

Pythagorean Theorem: $c^2 = a^2 + b^2$ where *a* and *b* are the legs of a right triangle and c is the hypotenuse (side opposite the right angle)

Distance Formula: $\boldsymbol{d} = \sqrt{(\boldsymbol{x_2} - \boldsymbol{x_1})^2 + (\boldsymbol{y_2} - \boldsymbol{y_1})^2}$

Distance is always positive.

Midpoint Formula

If a line segment has endpoints $(\boldsymbol{x_1}, \boldsymbol{y_1})$ and $(\boldsymbol{x_2}, \boldsymbol{y_2})$. We can find the coordinates of the segment's midpoint using

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

We determine the midpoint by taking the average of the two x – coordinates and the average of the two y-coordinates.

How much have you learned? Take the quiz below!

SUMMATIVE TEST

Directions: Read and analyze each statement/problem carefully. Circle the letter of your answer.

1. Determine the midpoint of a line segment whose endpoints are $(-3, -8)$ and $(9,3)$.

a. $(21, 14)$
b. $(3, -2.5)$
c. $(-5{,}5, 6)$
d. $(-6, -5.5)$

2. Calculate the distance between the points $S(-4, 3)$ and $T(6,1)$. Round your answers to the nearest tenth.

a. 3.5
b. 4.5
c. 9.8
d. 10.2

3. The coordinates of X are $(5,2)$. If Y is the midpoint of $\overline{XZ}$ and its coordinates are $(-1,0)$, then the coordinates of Z is

a. $(2, 5)$
b. $(-7, -2)$
c. $(-5, -2)$
d. $(-2, -7)$

4. If a line segment is drawn in a coordinate plane whose endpoints are $R(8, 2)$ and $S(6, 10)$, then the coordinates of the midpoint of $\overline{RS}$ is

a. $(1, 4)$
b. $(14, 12)$
c. $(7, 6)$
d. $(2, 8)$

5. Determine the perimeter of ΔFAZ with vertices $F(0,-6)$, $A(4,-6)$, and $Z(0,-3)$.

a. 7 units | c. 14 units
b. 12 units | d. 32 units

6. The coordinates of T are $(10,10)$ and the coordinates of H which is the midpoint of $\overline{TE}$ are $(9,8)$. What are the coordinates of E?

a. $(9.5,9)$ | c. $(11,12)$
b. $(18,16)$ | d. $(8,6)$

7. Given the coordinates of the vertices of a triangle below, can these be vertices of a right triangle?

$$X(-4,7),\ Y(2,9),\ Z(1,4)$$

a. Yes | c. Cannot be determined
b. No

8. Determine the coordinates of the other endpoint if one endpoint of a line segment is at $(2,-5)$ and its midpoint is at $(4,-2)$,

a. $(6,1)$ | c. $(6,-8)$
b. $(-2,-11)$ | d. $(8,-1)$

9. Given the vertices of a $rt.\Delta STU$, $S(-2,1)$, $T(1,1)$ and $U(1,2)$. Determine the length of the hypotenuse of the right triangle.

a. $\sqrt{10}$ | c. 4
b. $2\sqrt{3}$ | d. $2\sqrt{5}$

10. If $Z(0,2)$ is equidistant from $X(3,a)$ and $Y(a,5)$, determine the value of $\boldsymbol{a}$.

a. -3 | c. 1
b. 0 | d. 3

References:

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ANSWER KEY
Try these Out!
1. a. *TRUE*

   b. *TRUE*

   c. *FALSE. It does not matter which point is designated as* (x_1, y_1) *and which point is designated by* (x_2, y_2)*, the result will be exactly the same*.

Check Your Understanding!
1. $\sqrt{97} \approx 9.85$
3. $x = 12$ and $y = 5$
5. $y = -\frac{115}{14} \approx 8.21$

Summative Test

1. b
3. b
5. b
7. b
9. a