Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	3.0	Analysis of Graphs: Polynomial and Rational Functions
Lesson Code	3.1	Exploring Graphs and Properties of Polynomial Functions
Time Frame		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

At the end of this lesson, the students should be able to make use of graphing software to explore the graphs and properties of polynomial functions.

HOOK

Time Allocation: 3 minutes
Actual Time Allocation: _____ minutes

Looking at the night sky, stars are not always where they appear. Because the light from a star travels such a far distance, it is bent by the distribution of objects in between. This process is called "gravitational lensing," and was a prediction of Albert Einstein's General Theory of Relativity.

Assuming we are looking at a particular star, but there is an object in the way. The light travels around the object, and instead of seeing the star at a particular point, we see two images of the star at two different points. To find the locations of lensed images, scientists use polynomial functions, like $y = x^5 + 3x^3 - 4x$. The solutions of these polynomial functions tell us the locations of lensed images.

Now, sketch the graph of $y = x^5 + 3x^3 - 4x$ in your graphing application. Describe the shape of the graph. Does it ever cross the x- axis? If yes, at how many points? Does it cross the $y-$ axis? If yes, at how many points? To which side does the graph fall? To which side does the graph rise?

Now try to sketch the following polynomial functions and try to answer the same set of questions asked of you in $y = x^5 + 3x^3 - 4x$.

Function	Real Life Application of the Function
A. Insertion Sort Algorithm in Computers $y = 0.00339x^2 + 0.00143x - 5.95$	It is used to determine the numbers that a minicomputer can sort in less than 1 second.
B. Legendre Polynomial: $P(x) = \frac{1}{2}(5x^3 - 3x)$	It occurs in the solution of heat transfer problems in physics and engineering.
C. Chebyshev Polynomial: $f(x) = 8x^4 - 8x^2 + 1$	It is used in statistical studies.

Did you notice any similarity or difference with respect to shape or behavior among the four graphs that you have sketched? We will confirm your ideas in the succeeding discussion.

IGNITE

Time Allocation: 12 minutes
Actual Time Allocation: _____ minutes

A polynomial function is a function of the form

$$\boldsymbol{p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0}$$

where n is a nonnegative integer and $a_n \neq 0$. Here n is the degree of the polynomial and $a_n, a_{n-1}, \dots, a_1, a_0$ are the coefficients.

Most references would write that polynomial functions of degree 2 or more have graphs with *smooth* curves. These are graphs without sharp corners. Graphs of this kind also display curves with no jumps or breaks. Such graphs are called *continuous* graphs. Figure 1 shows a graph that represents a polynomial function (left) and a graph that represents a function but is not a polynomial function (right).

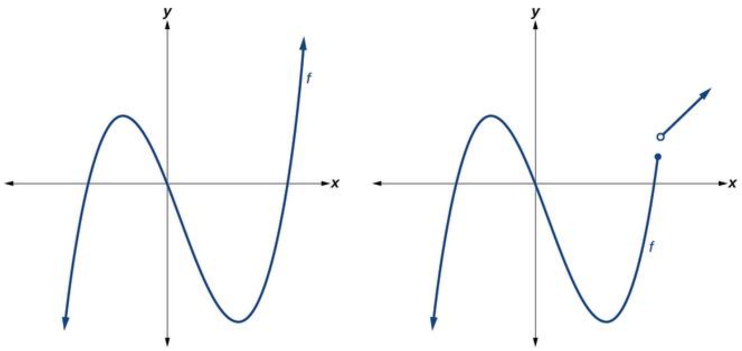

Figure 1: Continuous and Discontinuous Graphs

The graphs of polynomial functions contain a great deal of information. We can find the necessary information to formulate the equation of a function by looking at its graph. In like manner, by analyzing the function, we can be able to determine some characteristics of its graph. And using your app, I bet you sketched the graph without sweat.

Shown in figure 2 below are the general features that will help us understand better the graph of a polynomial function.

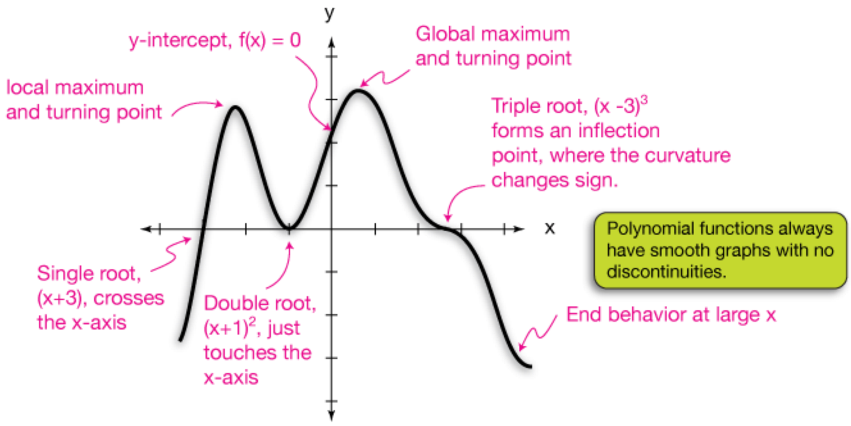

Figure 2: General Features of Polynomial Functions

Retrieved from https://openstax.org/books/precalculus/pages/3-4-graphs-of-polynomial-functions

By looking at the graph, we can determine the end behavior, x- and y-intercepts, domain, range and the number of turning points of a polynomial function. It also reflects whether the multiplicity of a root is single, double or triple.

What happens to the y-values as the x-values get bigger or smaller, meaning as x approaches positive or negative infinity, determines the *end behavior* of a graph. Such end behavior can be summarized in the following table.

Table 1: End Behavior of Polynomial Functions Using the Leading Coefficient Test

Degree	Even		Odd	
Leading Coefficient	Positive, $a_n > 0$	Negative, $a_n < 0$	Positive, $a_n > 0$	Negative, $a_n < 0$
End Behavior	$x \to \infty$, $f(x) \to \infty$ $x \to -\infty$, $f(x) \to \infty$	$x \to \infty$, $f(x) \to -\infty$ $x \to -\infty$, $f(x) \to -\infty$	$x \to \infty$, $f(x) \to \infty$ $x \to -\infty$, $f(x) \to -\infty$	$x \to \infty$, $f(x) \to -\infty$ $x \to -\infty$, $f(x) \to \infty$
Arrow Notation	↑↑	↓↓	↑↓	↓↑

The *intercepts* are the points where the graph touches or intersects the x or y-axis. Graphs behave differently at various x-intercepts. The zeros of a function f correspond to the x-intercepts of its graph. The *multiplicity* of a zero refers to the number of times an associated factor of a polynomial appears. If f has a zero of odd multiplicity, its graph will cross the x-axis at that x-value. If f has a zero of even multiplicity, its graph will only touch the x-axis at that point.

The *domain* of polynomial functions is the set of real numbers, $\mathbb{R}$, and the *range* highly depends on the type of polynomial we are considering. For instance, in constant functions with form $y = c$, the range is $\{c\}$. For linear and cubic functions, it is $\mathbb{R}$. The range of quadratic functions depend on whether the parabola opens upward or downward.

We determine the maximum number of *turning points* of a polynomial function with the formula $(n - 1)$. Hence, the maximum number of turning points is one less than the degree of the function. The turning point of the graph is any point where the graph of a function changes from increasing to decreasing or decreasing to increasing. This point is either classified as a local maximum/minimum or global maximum/minimum. When a graph moves up to down or down to up, a maximum or a minimum value is formed. The highest or lowest points on the graph are called the global maximum or minimum. Otherwise, it is simply referred to as a local maximum/ minimum. It is also important to emphasize that a global maximum or global minimum may not exist. For instance, the parabola $f(x) = x^2$ has a global minimum at $x = 0$ but has no global maximum since the graph shows that it increases without bound. The parabola $f(x) = -x^2$ which opens downward, has a global maximum at $x = 0$ but has no global minimum.

Going back to the functions you were asked to sketch earlier with a graphing app, let us try to analyze them in the light of the terms discussed earlier.

Using a graphing app, you may have sketched the graph of $y = x^5 + 3x^3 - 4x$ similar to the one below.

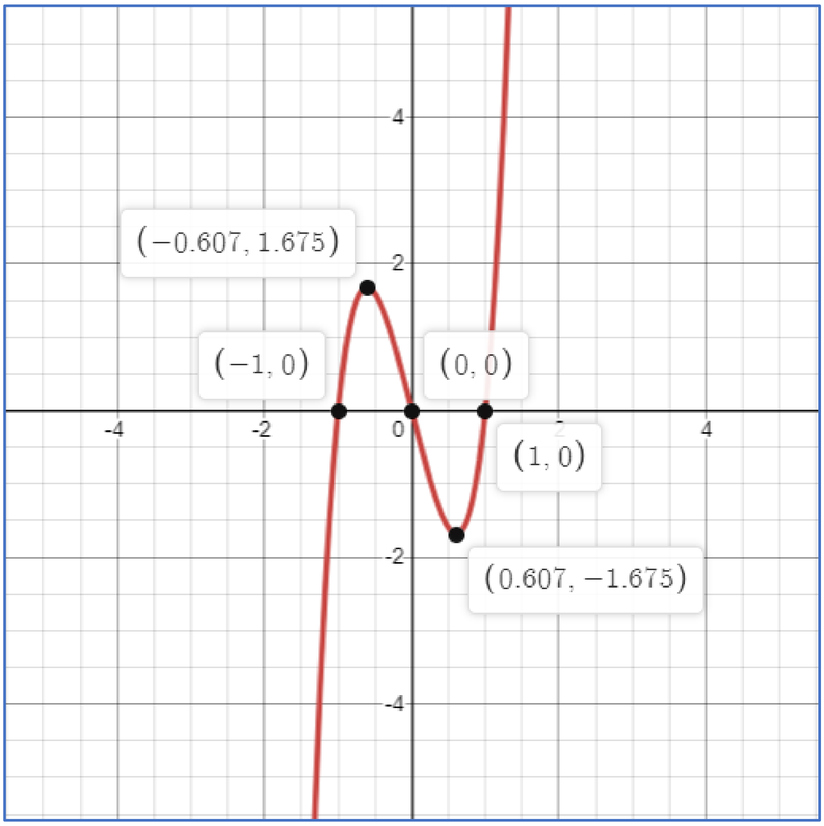

Figure 3: $\boldsymbol{y = x^5 + 3x^3 - 4x}$

In the sketch, we see that the graph touches the x-axis at three points. At which points do the graph intersect the x-axis? Click those locations on your graphing app. It will help you identify the point more accurately. So yes, the points are $(-1,0)$, $(0,0)$ and $(1,0)$! At what point does it intersect the y-axis? Yes, it is at $(0,0)$. The app also shows that the graph rises to the right and falls to the left.

End Behavior:	With the graph on hand, it is easy to determine that as $x \to \infty, f(x) \to \infty$ and as $x \to -\infty,\ f(x) \to -\infty$. The leading coefficient test as indicated in Table 1 may be used as a tool to verify your answer.
x-intercepts:	$(-1,0), (0,0), (1,0)$
y-intercept:	$(0,0)$
Domain:	$\mathbb{R}$
Range:	$\mathbb{R}$
Turning points:	2 $(-0.607,1.675)$ is a local maximum point since the graph extends upwards or that it increases without bound. $(0.607,-1.675)$ is a local maximum point since the graph extends downwards or that it decreases without bound. There is no global maximum/minimum.

NAVIGATE

Time Allocation: 8 minutes
Actual Time Allocation: _____ minutes

Complete the table below guided by the previous example given.

Function/ Graph	Intercepts	Domain and Range	Number of Turning Points/ Classification
1. $y = 0.00339x^2 + 0.00143x - 5.95$			
2. $P(x) = \frac{1}{2}(5x^3 - 3x)$			
3. $f(x) = 8x^4 - 8x^2 + 1$			

KNOT

Time Allocation: 6 minutes
Actual Time Allocation: _____ minutes

As a final assessment, search for another polynomial function used in other learning areas (biology, chemistry, computer science, physics, engineering, economics, geology etc.). Graph this function with your graphing app. Then, identify the end behavior, intercepts, domain, range, and turning points of the graph. Also, elaborate how such a function is used in that learning area. Use the space below for your answer.

SYNTHESIS JOURNAL (PSHS System, 2020)

What are the things I have learned about exploring polynomial functions?

What were the difficulties that I encountered throughout the lessons on exploring polynomial functions?

How did I overcome these difficulties?

References:

1. Albarico, J.M. (2013). THINK Framework. Based on Ramos, E.G. and N. Apolinario. (n.d.) Science LINKS. Quezon City: Rex Bookstore Inc
2. Larson,Ron. Hostetler,Robert. Edwards, Bruce (2005). College Algebra: A graphing Approach4thEdition. Boston, New York: Houghton Mifflin Company.
3. Swokowski, Earl. Cole, Jeffrey (2010). Algebra and Trigonometry with Analytic Geometry. Classic 12th Edition. Belmont, CA: Cengage Learning
4. PSHS System. (2020). *Math 1 Chapter 1 Module Version 2* [PDF]. Philippines: PSHS System.
5. PSHS CBZRC. (2020). *Template-Editable-1* [DOC]. Batangas: PSHS CBZRC.

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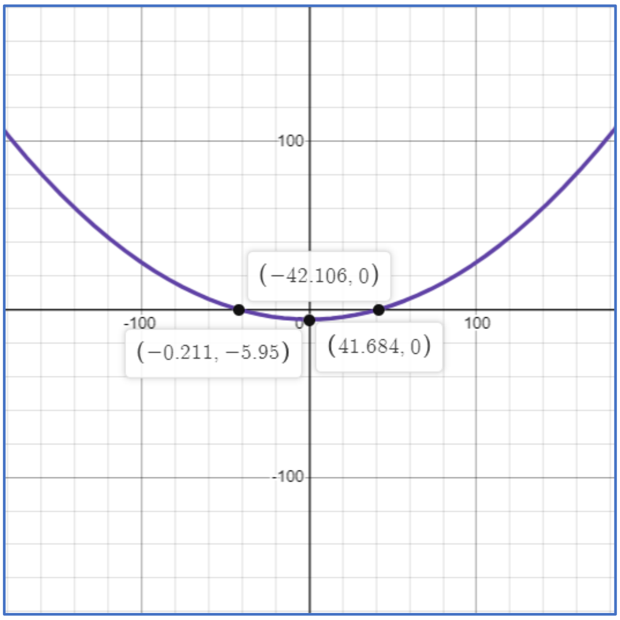
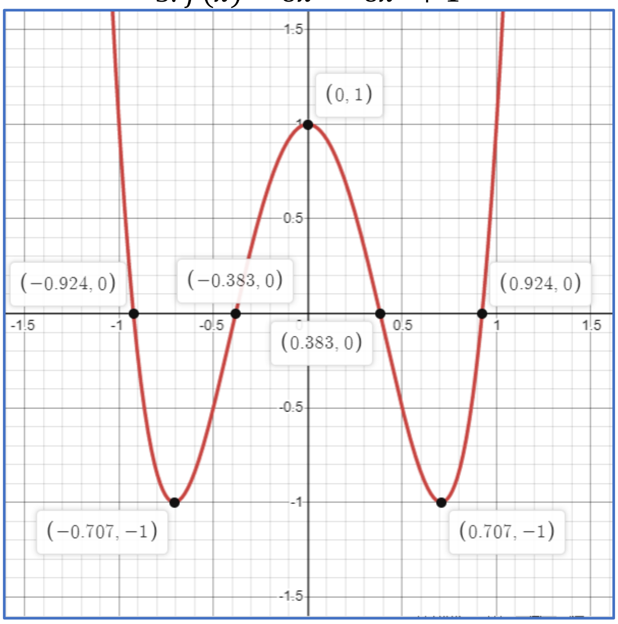
Answer Key:

NAVIGATE:

Function/ Graph	Intercepts	Domain and Range	Number of Turning Points/ Classification
1. $y = 0.00339x^2 + 0.00143x - 5.95$ 	x-intercepts: $(-42.106{,}0)$ $(41.684{,}0)$ y-intercept: $(0.211, -5.95)$	Domain: $(-\infty, \infty)$ Range: $(-5.95, \infty)$	1 turning point global minimum at $(-0.211{,}5.95)$
3. $f(x) = 8x^4 - 8x^2 + 1$ 	x-intercepts: $(-0.924{,}0)$ $(-0.383{,}0)$ $(0.383{,}0)$ $(0.924{,}0)$ y-intercept: $(0{,}1)$	Domain: $(-\infty, \infty)$ Range: $(-1, \infty)$	3 turning points global minimum at $(-0.707, -1)$ $(-0.707, -1)$ Local maximum at $(0{,}1)$

KNOT:
Answers may vary.