Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	2.0	Transformations on the Coordinate Plane
Lesson Code	2.1	Translations
Time Limit		30 minutes

In Module 2.0, we will discuss about geometric transformations. The four main types of geometric transformations are translation, reflection, rotation, and dilation. In this module, a number of examples and illustrations to the different types of geometric transformations will be explained in detail.

TARGET

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

By the end of this learning guide module, the students should be able to

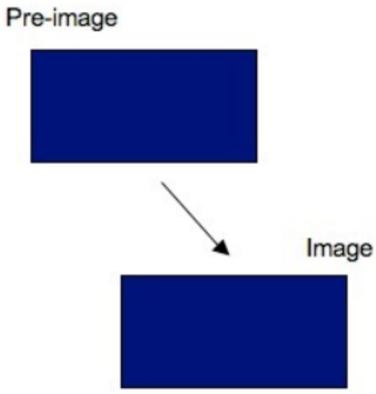
1. Identify and use translations on a plane
2. Identify the use of translations in real-life situations
3. Demonstrate translations using graphing tools and software.

HOOK

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

Geometric transformation is a function, or mapping that results to a change in position, size or orientation of a figure. For this subject, we will discuss five geometric transformations, namely translation, reflection, glide reflection, rotation, and dilation. We often see geometric transformations around us: when we walk around a certain object, we see a rotation. Whenever we see an image of an object, it is a projection. When we are sliding an object, we see a translation. When we are looking at the mirror, we see a reflection. In this module, we will start our discussion with translations.

Suppose we have a rectangle, if we move the rectangle downward then to the right (as illustrated below) then we have already translated the rectangle. In any transformation, the original figure is the *pre-image*, while the resulting figure is called the *image*.



In geometry, a *translation* moves a shape up and down or left and right. Translations only move things from one place to another; they don't change their size or orientation. Now that we've got a basic understanding of what translations are, let's learn how to illustrate them on a coordinate plane.

IGNITE

Time Allocation: 15 minutes
Actual Time Allocation: _____ minutes

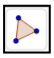
Translations on the Coordinate Plane

Coordinates allow us to be very precise about the translations we perform.
Translations in a coordinate plane can be described by the following coordinate notation:

$$T_{a,b}(x, y) \to (x + a, y + b)$$

where a and b are constants. Each point shifts a units horizontally and b units vertically.

Hands-on Activity Using Geogebra

1. Using the *Polygon Tool* , form a triangle by plotting the coordinates of its vertices $K(2, -1)$, $L(-2, -3)$, and $M(1, -5)$.

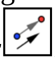
2. Translate triangle KLM 4 units upward. In the input bar, type in the vector u = (a,b) in which direction you wish the figure to be shifted. In this case, u = (0,4). Click the *Transformation Tool*: *Translate by Vector* . Click on the polygon, then the vector. Select a different color shade and label the new figure as triangle $K'L'M'$.

3. Write down the coordinates of the vertices of triangle $A'B'C'$ below.

   $T_{0,4}$: $A(2, -1) \to K'$ (___, ___)

   $T_{0,4}$: $B(-2, -3) \to L'$(___, ___)

   $T_{0,4}$: $C(1, -5) \to M'$(___, ___)

4. Now, translate triangle $K'L'M'$ 2 units downward and 3 units to the right *Transformation Tool*: *Translate by Vector* . Label the figure as triangle $K''L''M''$ then write down the coordinates of the vertices of triangle $K''L''M''$.

   K" (___, ___), L" (___, ___), M" (___, ___)

Example 1.

Figure 1 shows the translation $T_{4,-2}(x, y) \to (x + 4, y - 2)$ which shifts each point of $\overline{PQ}$ 4 units to the right and 2 units down resulting to the position of $\overline{P'Q'}$

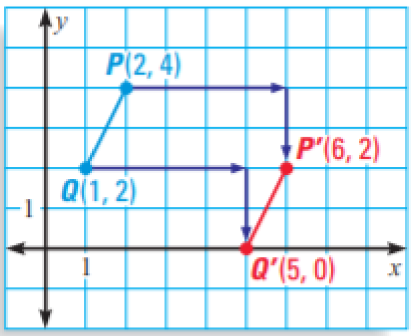


Figure 1: Translation of $\overline{PQ}$ *to* $\overline{P'Q'}$.

Retrieved from: GEOMETRY by Boswell, Laurie, Larson, Ron, Stiff, Lee. McDougal Littell 2004.

Example 2.

Plot the vertices $A\ (-1,-3)$, $B\ (1,-1)$, and $C\ (-1,0)$ then connect the points to form ΔABC . Shift each vertex 3 units to the left and 4 units up. Label these vertices A', B', and C' respectively, then connect each of these points to form $\Delta A'B'C'$.

Solution:

As shown in Figure 2, to find the translated vertices, shift each point of ΔABC 3 units to the left and 4 units up.

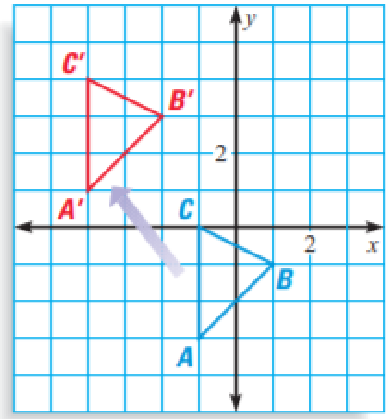


Figure 2. Translating $\Delta ABC\ to\ \Delta A'B'C'$

Retrieved from: GEOMETRY by Boswell, Laurie, Larson, Ron, Stiff, Lee. McDougal Littell 2004.

The coordinates of the vertices of the preimage and image are listed below.

$\boldsymbol{\Delta ABC}$ **(preimage)**	$\boldsymbol{\Delta A'B'C'}$ **(image)**
$A(-1,-3)$	$A'(-4,1)$
$B(1,-1)$	$B'(-2,3)$
$C(-1,0)$	$C'(-4,4$

As you can see, each of the *x*-coordinate of the image is 3 units less than the *x*-coordinate of the preimage and each y -coordinate of the image is 4 units more than the *y*-coordinate of the preimage.

Example 3.

In Figure 3, the quadrilateral QRST maps onto quadrilateral Q'R'S'T' by a translation. Describe the translation made using the coordinate notation.

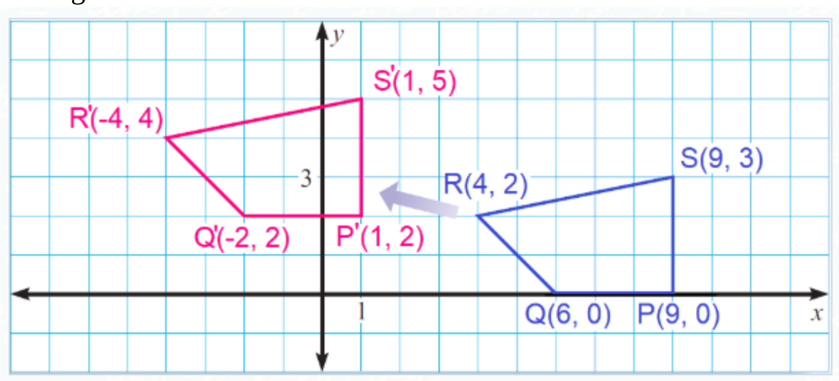


Figure 3: Translating QRST to Q’R’S’T’

Retrieved from: *https://www.onlinemath4all.com/translations-in-a-coordinate-plane.html*

Solution :
Choose any vertex and its image, say S and S'. To move from S to S', we have to move 8 units to the left (from 9 to 1) and 2 units up (from 3 to 5). So, the coordinate form of the translation is given by $T_{-8,2}(x, y) \rightarrow (x - 8, y + 2)$.

In addition, the following are the real-life examples of translations:

Figure 4: Translation in Nature
Retrieved from
https://mathbitsnotebook.com/Geometry/Transformations/TRTransformationTranslations.html

Figure 5: Translation in a slide.
Retrieved from
https://mathbitsnotebook.com/Geometry/Transformations/TRTransformationTranslations.html

Each hexagonal section of a honeycomb can be imagined as a translation of a single honeycomb. Each section is of the same size, the same shape and face in the same direction.

Going down a slide is also an example of a translation. When you slide , you will be moving in a given distance in a given direction. You will not change your size, shape or the direction you will be facing.

NAVIGATE

Time Allocation: 10 minutes
Actual Time Allocation: _____ minutes

Do as indicated.

A. Use the coordinate notation to describe the translation.

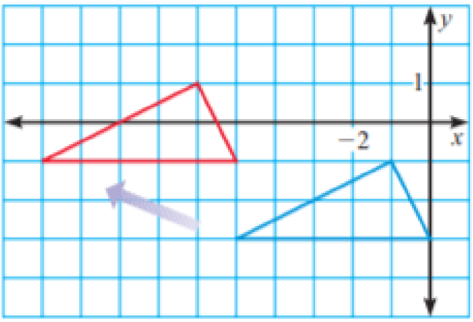
1. 5 units to the right and 2 units down ______________________________
2. 7 units up and 4 units to the right ______________________________
3. 6 units to the left and 1 unit up. ______________________________

B. Consider the translation $T_{5,-8}(x, y) \rightarrow (x + 5, y - 8)$.

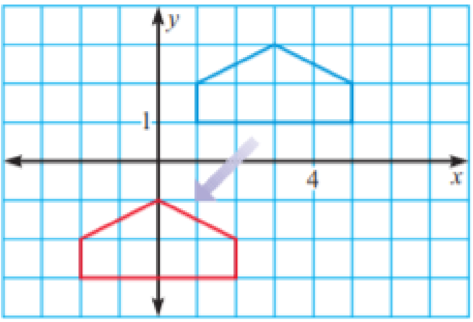
4. What is the image of $(5, 3)$? ______________________________
5. What is the preimage of $(-2, 1)$? ______________________________
6. What is the preimage of $(0, -6)$? ______________________________

C. Describe the translation illustrated in the figures below using coordinate notation.

7.



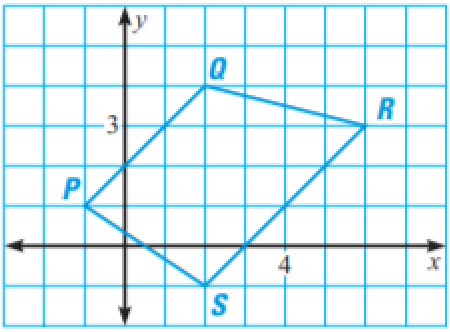
8.



Retrieved from: GEOMETRY by Boswell, Laurie, Larson, Ron, Stiff, Lee. McDougal Littell 2004.

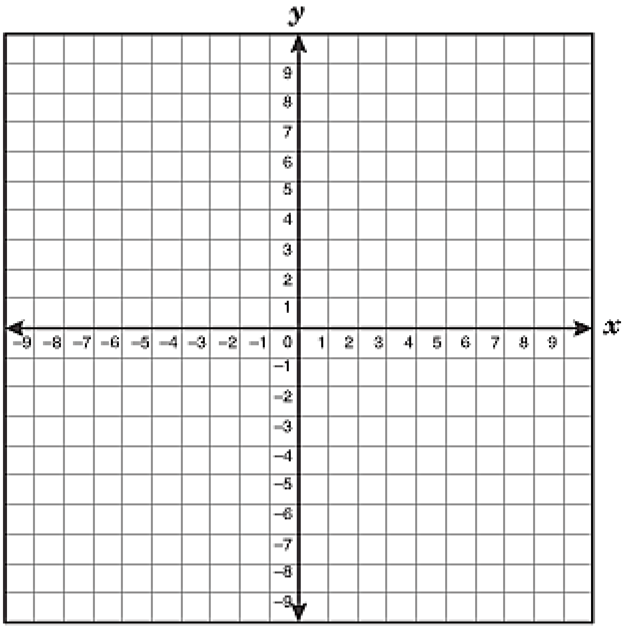
______________________________ ______________________________

D. Copy figure PQRS and draw its image after translation.



9. $T_{1,-4}(x, y) \rightarrow (\ x + 1, y - 4)$

10. $T_{-6,4}(x, y) \rightarrow (\ x - 6, y + 4)$



KNOT

Time Allocation: 3 minutes

Actual Time Allocation: _____ minutes

In summary,

- Translation moves the shape either up or down, sideways or diagonally but it does not change the size and orientation of the shape of the pre-image.
- In a coordinate plane, translations can be described by $T_{a,b}(x, y) \rightarrow (\ x + a, y + b)$ where a and b are constants and each point shifts a units horizontally and b units vertically.

Determine whether each statement is **true or false**.

1. If line m is the image of a different line n under translation, then m is parallel to n.
2. It is possible for a translation to map a line l onto a perpendicular line v.
3. Consider the translation defined by $T_{10,-5}(x, y) \rightarrow (x + 10, y - 5)$.
   a. What is the image of $(6, -10)$?
   b. What is the preimage of $(-8, 12)$?
   c. What is the image of $(1 - a, -2a)$?
   d. What is the preimage of $(-5, 3\sqrt{2})$?

References:

Albarico, J.M. (2013). THINK Framework. Based on Science LINKS by E.G. Ramos and N. Apolinario. Quezon City: Rex Bookstore Inc.

Boswell, Laurie, Larson, Ron, Stiff, Lee. GEOMETRY. McDougal Littell 2004. Houghton Mifflin Company.

Transformations and Isometries.Retrieved from https://www.xceleratemaths.com/geometry/transformations

Transformations: Translations. Retrieved from https://mathbitsnotebook.com/Geometry/Transformations/TRTransformationTranslations.html

Translations in a Coordinate Plane. Retrieved from: https://www.onlinemath4all.com/translations-in- a-coordinate-plane.html

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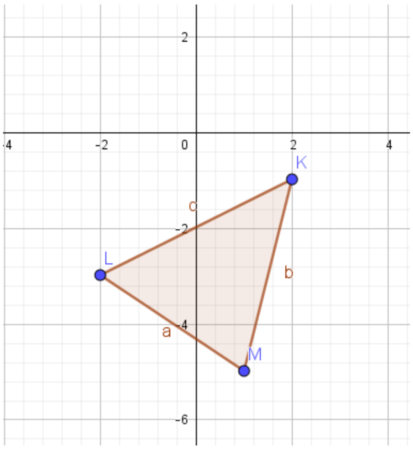
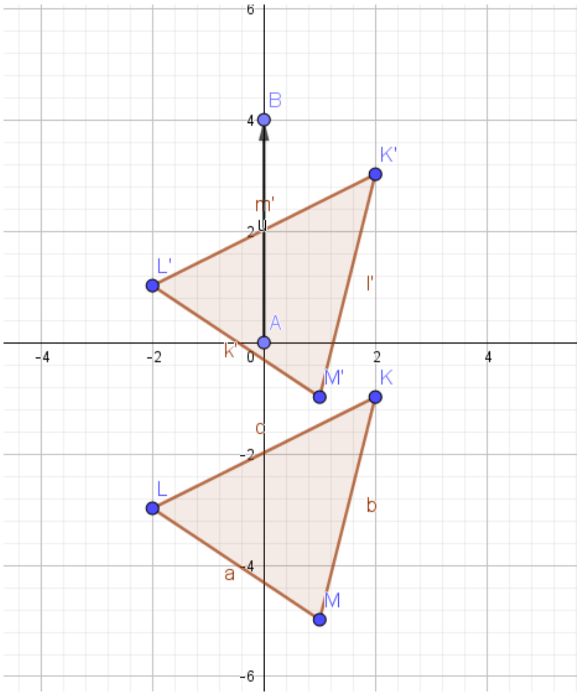
Answer Key

<u>Hands-on Activity:</u>

1.



2.



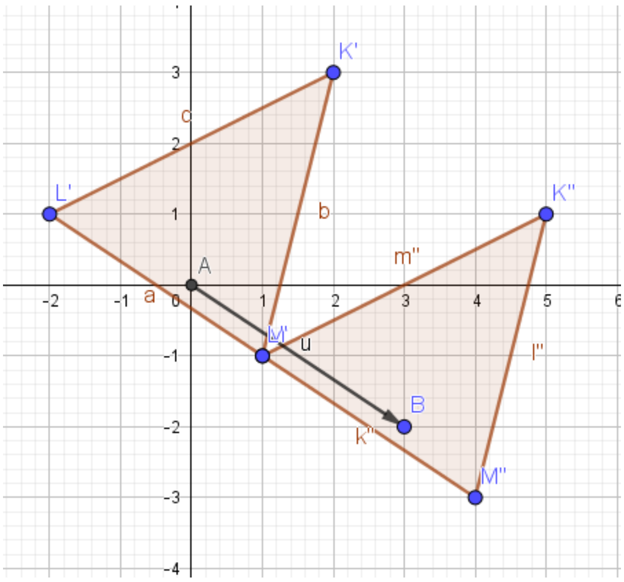
3. Coordinates of the Translated Triangle:

$T_{0,4}$: $K(2,-1) \rightarrow K'$ (2, 3)

$T_{0,4}$: $L(-2,-3) \rightarrow L'$(-2, 1)

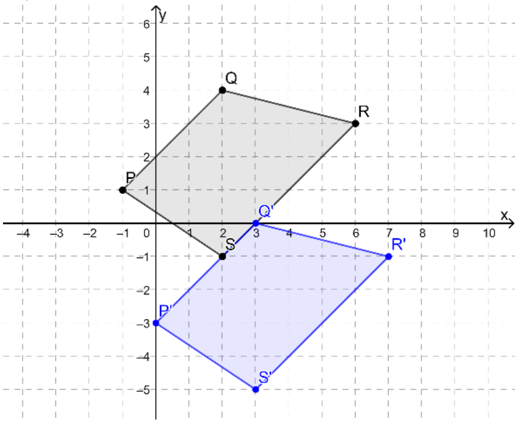
$T_{0,4}$: $M(1,-5) \rightarrow M'$(1, -1)

4.



Navigate:

1. $T_{5,-2}(x, y) \to (\, x + 5, y - 2)$
3. $T_{-6,1}(x, y) \to (\, x - 6, y + 1)$
5. $(-7, 9)$
7. $T_{-3,-4}(x, y) \to (\, x - 3, y - 4)$
9.



Knot:

1. True
3. $(16, -15)$
5. $(11 - a, -2a - 5)$