Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	1.0	Basic Plane and Coordinate Geometry
Lesson Code	1.2	Points on a Line and a Plane
Time Limit		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

By the end of this module, the students will have been able to solve problems involving points on the real number line distance of two points, midpoint of a segment)

HOOK

Time Allocation: 4 minutes
Actual Time Allocation: _____ minutes

We can measure the average adult human body in "heads". For instance, the average human is usually measured 7-8 heads tall. In this method, one uses their own head to measure their own body. Other interesting measurements are shown in the picture below.

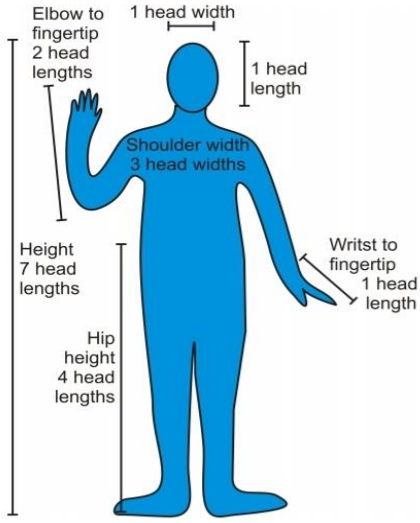

Figure 1. Proportions on the Human body
(Image from: Dillinger, B. (2011). *Chapter 1- Foundation of Geometry.* CK-12 Foundation Inc.)

We can also determine other measurements such as the length from elbow to wrist, the length from the top of the neck to the hip, and each shoulder's width. Well, the length from the wrist to the elbow is one head, the length from the top of the neck is two heads, and the width of each shoulder is one head.

This lesson is about solving problems involving points in the real number line such as the distance between two points and the midpoint of a line segment.

In geometry, we call rules that are recognized even without proof **postulates** or **axioms**. On the other hand, rules that are proven are named **theorems**. In this lesson, we will explore some postulates about the lengths of segments.

Ruler Postulate

The **distance** between points *A* and *B*, written as *AB*, is the absolute value of the difference between the coordinates of *A* and *B*. *AB* is also called the **length** of $\overline{AB}$. We use "absolute value" because distance is always non negative.

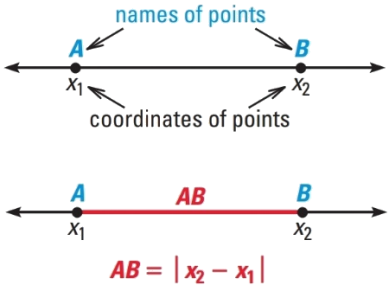

Figure 2. Ruler Postulate
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry.* McDougal Littell, a division of Houghton Mifflin Company.)

Example: Find the distance between the two points A and B.

To do this, we measure the length of the segment to the nearest millimeter.

Figure 3. Line Segment *AB*
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry.* McDougal Littell, a division of Houghton Mifflin Company.)

Solution

Using a ruler to demonstrate the ruler postulate, we place the ruler such that one mark of the ruler is aligned with *A*. After this, we estimate the coordinate of *B*. Looking at the figure below, A is aligned with 3 while B seems to be aligned with 5.5. Hence, the distance between *A* and *B* would be $|5.5 - 3| = 2.5$ mm.

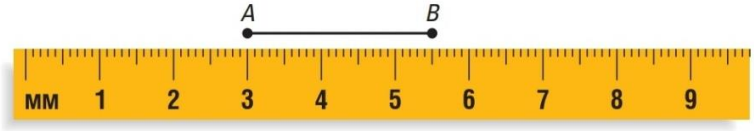

Figure 4. Using a ruler to measure length of line segment *AB*
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry.* McDougal Littell, a division of Houghton Mifflin Company.)

We say that one point is **between** the other two when three points are collinear. As an example, in the figures below, point *B* is between points *A* and *C*, however, point *E* is not between points *D* and *F*.

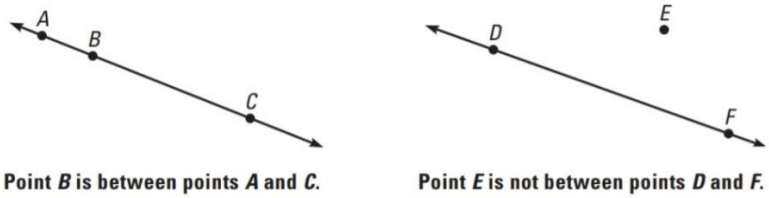

Figure 5. The term "between" means in geometry
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry.* McDougal Littell, a division of Houghton Mifflin Company.)

Segment Addition Postulate

If *B* is between *A* and *C*, then $AB + BC = AC$. If $AB + BC = AC$, then *B* is between *A* and *C*.

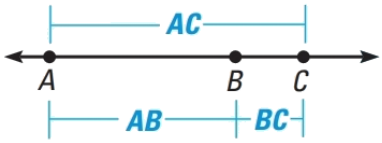

Figure 6. Segment Addition Postulate
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry.* McDougal Littell, a division of Houghton Mifflin Company.)

The Segment Addition Postulate is illustrated with the figure above. If $AB = 11$ cm and $BC = 3$ cm, then *AC* must be equal to 14 cm. We may also consider this as the "sum of the partial lengths will the equal to the whole length."

We can extend the concept of the Segment Addition Postulate to three or more segments, provided that the segments lie on a line. For instance, if *P*, *Q*, *R*, and *S* lie on a line as shown in the figure below, then $PS = PQ + QR + RS$.

Figure 8. Segment Addition Postulate on three or more segments
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry.* McDougal Littell, a division of Houghton Mifflin Company.)

Example:

For $\overline{AB}$, suppose that *C* is between *A* and *B*. If $AB = x + 14, AC = 3x + 2$ and $CB = 8$, find the lengths of *AB* and *AC*.

Solution:
Using the Segment Addition Postulate, we have

$$AC + CB = AC$$
$$(3x + 2) + 8 = x + 14$$
$$2x + 10 = 14$$
$$2x = 4$$
$$x = 2$$

So, if $x = 2$, then $AB = 16$ and $AC = 8$.

Midpoint

The point that divides, or bisects, the segment into two congruent segments is called the **midpoint** of the segment.

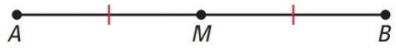

Figure 9. Midpoint of a segment
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry.* McDougal Littell, a division of Houghton Mifflin Company.)

In the line segment shown above, the midpoint of $\overline{AB}$ is *M* if *M* is located on $\overline{AB}$ and $AM = MB$.

Midpoint Postulate: There is exactly <u>one</u> midpoint for any line segment.

<u>Example:</u> Given the figure below, is *M* a midpoint of $\overline{AB}$?

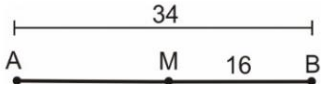

Figure 10. Determining if *M* is a midpoint or not
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry.* McDougal Littell, a division of Houghton Mifflin Company.)

No, because $MB = 16$ and $AM = 34 - 16 = 18$.

<u>Example:</u> Algebra Connection

Suppose that *B* is the midpoint of line segment *AC*. $AB = 5x - 6$ and $BC = 2x + 12$. What is the length of the line segment *AC*?

<u>Solution</u>

Using the definition of the midpoint, we have

$$AB = BC$$
$$5x - 6 = 2x + 12$$
$$3x = 18$$
$$x = 6$$

So, if $x = 6$, then $AB = 24 = BC$. Hence, $AC = 2(24) = 48$.

NAVIGATE

Time Allocation: 5 minutes
Actual Time Allocation: _____ minutes

It's your time to work on these questions. Please answer the following questions.

A. Refer to the figure below

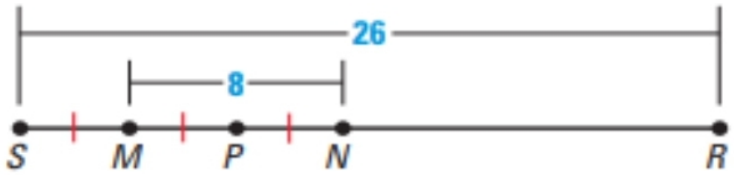

Figure 11. Line Segment *SR* divided into segments
(Image from: Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry.* McDougal Littell, a division of Houghton Mifflin Company.)

Find the length of the following segments.

1. $\overline{MP}$
2. $\overline{SM}$
3. $\overline{NR}$
4. $\overline{MR}$
5. $\overline{PR}$

B. Suppose that *B* is the midpoint of line segment *AC*. $AB = 3x + 1$ and $AC = 8x - 6$. Find the length of the line segment *AC*.

KNOT

Time Allocation: 5 minutes
Actual Time Allocation: _____ minutes

The following are postulates about the lengths of segments.

Ruler postulate: We compute for the distance between two points by taking the absolute value of the difference between coordinates of each point. We use "absolute value" because distance is always non negative.

Segment Addition Postulate: If *B* is between *A* and *C*, then $AB + BC = AC$. If $AB + BC = AC$, then *B* is between *A* and *C*. We also consider this as the "sum of the partial lengths will the equal to the whole length."

Midpoint Postulate: For any line segment, there is exactly one midpoint that bisects the segment into two congruent segments.

References:

Albarico, J.M. (2013). THINK Framework. Based on Science LINKS by E.G. Ramos and N. Apolinario. Quezon City: Rex Bookstore Inc.

Larson, R. Bowell, L. & Stiff, L. (2004). *Geometry.* McDougal Littell, a division of Houghton Mifflin Company.

Dillinger, B. (2011). *Chapter 1- Foundation of Geometry.* CK-12 Foundation Inc.

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Answer Key

A. **Length of the segments.**
   1. $MP = 4$
   3. $NR = 14$
   5. $PR = 18$