Learning Guide Module

Subject Code	Math 3	Mathematics 3
Module Code	3.0	Analysis of Graphs: Polynomial and Rational Functions
Lesson Code	3.1.2	Identifying Graphs of Rational Functions
Time Frame		30 minutes

TARGET

Time Allocation: 1 minute
Actual Time Allocation: _____ minutes

At the end of this lesson, the students should be able to

a. examine key characteristics and properties of rational functions by determining the shape of their graphs.
b. study rational functions with linear or quadratic polynomial expressions in their numerators and/or denominators.
c. establish connections between the algebraic and graphical representations of these functions.

HOOK

Time Allocation: 4 minutes
Actual Time Allocation: _____ minutes

So let us get started by launching your graphing app *(GeoGebra of Desmos)* on your mobile phone or laptop. Sketch the graph of $f(x) = \frac{1}{x}$. What can you say about your graph in terms of characteristics we have previously discussed in lesson 3.1.1?

IGNITE

Time Allocation: 15 minutes
Actual Time Allocation: _____ minutes

Hang on! We will get back to your graph in a while. Let us now consider the definition below.

Rational Function
A rational function is a function of the form $\boldsymbol{f(x) = \frac{g(x)}{h(x)}}$ where $\boldsymbol{g(x)}$ and $\boldsymbol{h(x)}$ are polynomials and $\boldsymbol{h(x) \neq 0}$.

As stated in our definition, rational functions are made up of polynomial functions. Considering this, some characteristics of the graphs of rational functions are similar to other functions. An example of this are possible x-intercepts or y-intercepts. However, it is important to note that some characteristics of rational functions are not the same with other function families.

One of the essential goals for this lesson is to identify important characteristics of rational function graphs. Let us start by looking at the graph of $f(x) = \frac{1}{x}$ which is considered to be the parent function for rational functions. Its graph is a hyperbola which consists of two symmetrical parts called branches. Please refer to the graph on the next page.

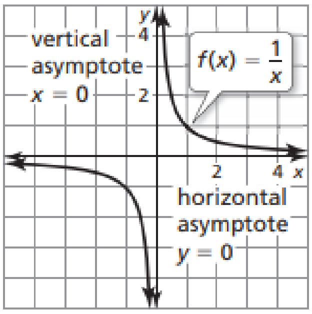


Figure 1. Graph of $f(x) = \frac{1}{x}$
Retrieved from:
https://math.libretexts.org/Bookshelves/Algebra/Map%3A_College_Algebra_(OpenStax)/05%3A_Polynomial_and_Rational_Functions/507%3A_Rational_Functions

Now from the graph, let us consider the following exercises.

1. What value of x would make the denominator zero?
   Let us consider the equation $f(x) = \frac{1}{x}$, you will see that when $x = 0$, $f(x)$ will have a zero denominator. We will see how this affects the graph of the function in the next items.

2. What happens to the y values as the x values approach $x = 0$ from the left side of the graph?
   As we look from left to right, we will see that $y -$ values decrease. In other words, the graph turns downward from the left. In fact, the $y -$ values continue to decrease forever as the graph gets closer and closer to $x = 0$.

3. What happens to the y values as the $x -$values approach $x = 0$ from the right side of the graph?
   As we look from right to left, we will see that $y -$ values increase, that is, the graph turns upward. That is to say that the $y -$values continue to increase forever as the graph gets closer and closer to $x = 0$.

   In other words, numbers 2 and 3 are implying two important things about rational functions.

Short Run Behavior:

If we substitute values of x from the left side, the function values become very largely negative. The function values approach negative infinity. Using mathematical notations, we write: as $x \to 0^-, f(x) \to -\infty$. This is read "as x approaches zero from the left, $f(x)$ approaches negative infinity".

Similarly, if we substitute values of x from the right side the function values become largely positive, approaching positive infinity. Using a similar notation, we write: as $x \to 0^+, f(x) \to \infty$. This is read "as x approaches zero from the right, $f(x)$ approaches positive infinity".

This behavior creates a vertical asymptote. Recall from your eighth grade that an asymptote is a line that the graph approaches. In this case, the graph of $f(x) = \frac{1}{x}$ is approaching the vertical line $x = 0$ as the values of x gets closer to zero.

Long Run Behavior:

As the values of x approach negative infinity, the function values approach 0. Symbolically, we write as $x \to \pm\infty, f(x) \to 0$.

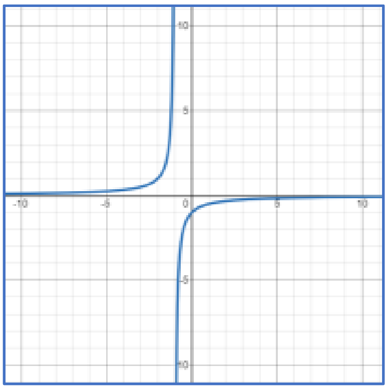
Based on this long run behavior, we can deduce that the function approaches 0 but will not actually reach 0, it simply "levels off" as the inputs become large. This behavior creates a horizontal asymptote. In this case, the graph of $f(x) = \frac{1}{x}$ approaches the horizontal line $y = 0$ as the input becomes very large in both negative and positive directions.

Vertical and Horizontal Asymptotes
A **vertical asymptote** of a graph is a vertical line $\boldsymbol{x = a}$ where the graph approaches positive or negative infinity as the inputs approach a. As $\boldsymbol{x \to a, f(x) \to \pm\infty}$. A **horizontal asymptote** of a graph is a horizontal line $\boldsymbol{y = b}$ where the graph approaches the line as the inputs get large. As $\boldsymbol{x \to \pm\infty, f(x) \to b}$.

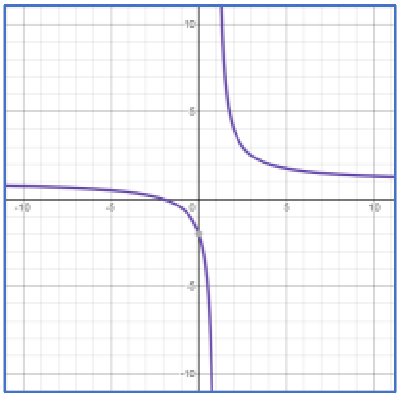
Guided Activity 1

Now explore more about rational functions by considering the four functions below. Each function is a transformation of the parent function $f(x) = \frac{1}{x}$. Match the given function with its corresponding graph. Describe the transformation (with respect to the parent function) in each graph.

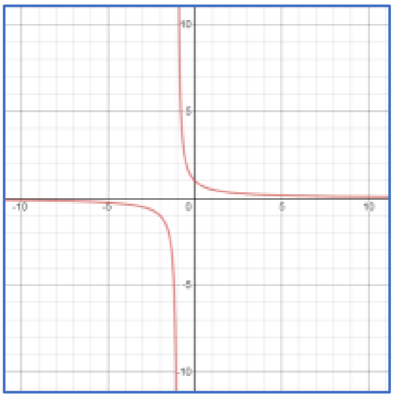
1. $h(x) = \frac{1}{x+1}$
2. $i(x) = \frac{-1}{x+1}$
3. $j(x) = \frac{x+1}{x-1}$
4. $k(x) = \frac{x+2}{x-1}$



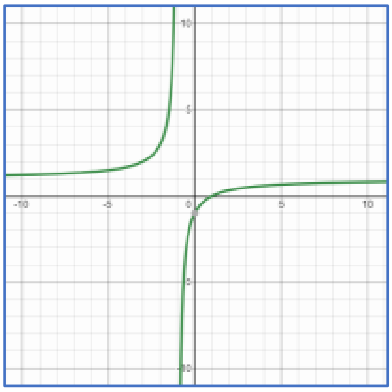
A.



B.



C.



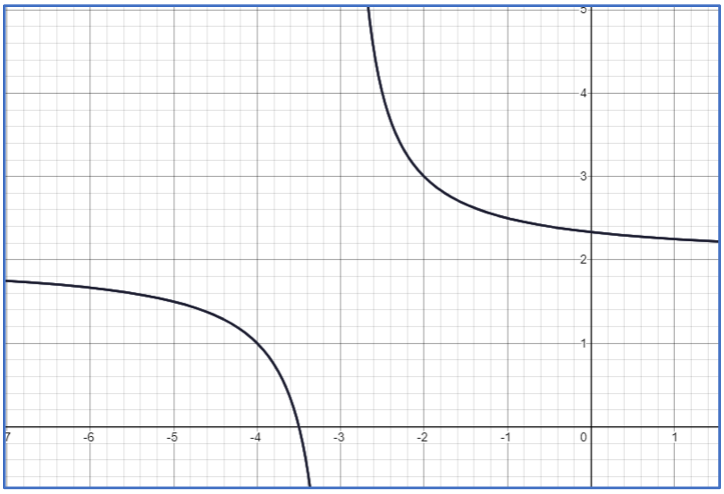
D.

Guided Activity 2

Sketch a graph of the parent rational function shifted three units to the left and two units upward. Identify the horizontal and vertical asymptotes of the graph, whichever is applicable.

Transforming the graph three units to the left and two units upward would result in the function $f(x) = \frac{1}{x+3} + 2$ which is the same as $f(x) = \frac{2x+7}{x+3}$.

Using your graphing app will give you a sketch such as the one below.



Notice that this equation is undefined at $x = -3$, and the graph is also showing a vertical asymptote at $x = -3$. Thus, as $x \to -3^-, f(x) \to -\infty$ and as $x \to -3^+, f(x) \to \infty$.

As the inputs grow large, the graph appears to be leveling off at output values of 2, indicating a horizontal asymptote at $y = 2$. As $x \to \pm\infty, f(x) \to 2$.

Using the parent function as our reference, it is also noticeable that the horizontal and vertical asymptotes are shifted 3 units to the left and 2 units upward, respectively.

NAVIGATE

Time Allocation: 4 minutes
Actual Time Allocation: _____ minutes

Now, it is your turn to work!

1. Using your graphing app, sketch the graph of the function $f(x) = \frac{1}{x^2}$. Use symbolic notation to describe its long run behavior and short run behavior.
2. Sketch the graph and find the horizontal and vertical asymptotes of $f(x) = \frac{1}{x^2}$ that has been shifted 3 units to the right and 4 units down.

KNOT

Time Allocation: 5 minutes
Actual Time Allocation: _____ minutes

As a final assessment, think of possible applications of rational function in the real world. Then illustrate how it works.

SYNTHESIS JOURNAL (PSHS System, 2020)

What are the things I have learned about identifying graphs of rational functions?

What were the difficulties that I encountered throughout the lessons on identifying graphs of rational functions?

How did I overcome these difficulties?

References:

1. Albarico, J.M. (2013). THINK Framework. Based on Ramos, E.G. and N. Apolinario. (n.d.) *Science LINKS*. Quezon City: Rex Bookstore Inc
2. Larson, R., Hostetler, R., and Edwards, B(2005). *College Algebra: A Graphing Approach* 4th Edition. Boston, New York: Houghton Mifflin Company.
3. Swokowski, E., and Cole, J. (2010). *Algebra and Trigonometry with Analytic Geometry*. Classic 12th Edition. Belmont, CA: Cengage Learning
4. PSHS System. (2020). *Math 1 Chapter 1 Module Version 2* [PDF]. Philippines: PSHS System.
5. PSHS CBZRC. (2020). *Template-Editable-1* [DOC]. Batangas: PSHS CBZRC.

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Answer Key:

NAVIGATE:

1. Long run Behavior, as $x \to \pm\infty, f(x) \to 0$
   Short Run Behavior, as$x \to 0, f(x) \to \pm\infty$

KNOT

Answers may vary.