Learning Guide Module

Subject Code Math 3		Mathematics 3
Module Code 3.0		Analysis of Graphs: Polynomial and Rational Functions
Lesson Code 3.3		Finding and Estimating Intercepts
Time Frame		30 minutes

TARGET

After completing this module, you should be able to:
1. find the x-intercept and y-intercept of any polynomial and rational function graphically; and
2. identify the x-intercept and y-intercept of any polynomial and rational function algebraically.

HOOK

TRUE or FALSE?

We can identify more than one value of x and y intercepts of any function.

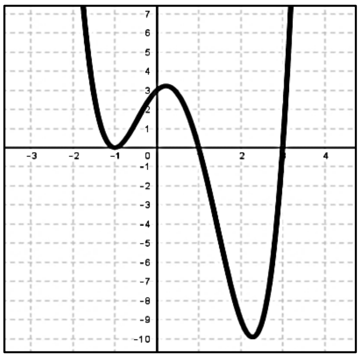
IGNITE

The graphs of polynomial and rational functions are shown below. Study few examples on finding the intercepts of the graphs.

10 MINUTES

EXAMPLE 1

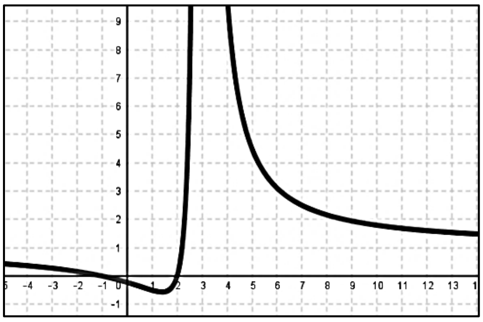
$$f(x) = x^4 - 2x^3 - 4x^2 + 2x + 3$$



$\boldsymbol{x-intercepts}$: $\{-1, 1, 3\}$
$\boldsymbol{y-intercept}$: $\{3\}$

EXAMPLE 2

$$f(x) = \frac{x^2 - x - 2}{x^2 - 6x + 9}$$



$\boldsymbol{x-intercepts}$: $\{0, 2\}$
$\boldsymbol{y-intercept}$: $\left\{-\frac{2}{9}\right\}$

TIP (The Important Point)

GRAPHICALLY

x-intercept is defined as the abscissa of the coordinates where the graph intersects the x-axis.
y-intercept is defined as the ordinate of the coordinates where the graph intersects the y-axis.

Are you confident to find the intercepts without looking at the graph? If not, here's how to do it!

We will use the two examples in the previous page.
Are you ready for the next level? **Let's GO!!!**

EXAMPLE 1

$$f(x) = x^4 - 2x^3 - 4x^2 + 2x + 3$$

A. How to identify the y-intercept:

Let $x = 0$, then solve for $f(x)$

$f(0) = 0^4 - 2(0)^3 - 4(0)^2 + 2(0) + 3$
$f(0) = 3$

Thus, the y-intercept is **3**.

B. How to find the x-intercept:

Get the zero/s of the equation.
Let $f(x) = 0$, then solve for x.

$0 = x^4 - 2x^3 - 4x^2 + 2x + 3$

One of the ways to solve for the zero is factoring:
$0 = (x + 1)^2(x - 1)(x - 3)$
There are 3 values of x that will satisfy the equation.
Thus, the x-intercepts are -1, 1, and 3.

EXAMPLE 2

$$f(x) = \frac{x^2 - x - 2}{x^2 - 6x + 9}$$

A. How to find the y-intercept:

Let $x = 0$, then solve for $f(x)$

$$f(0) = \frac{(0)^2 - (0) - 2}{(0)^2 - 6(0) + 9}$$

Thus, the y-intercept is $-\frac{2}{9}$.

B. How to find the x-intercept:

Get the zero/s of the equation.
Let $f(x) = 0$, then solve for x.

$$0 = \frac{x^2 - x - 2}{x^2 - 6x + 9}$$

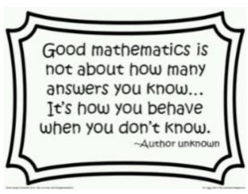
If the numerator is equal to zero, then the rational equation becomes zero.
Hence, we have
$x^2 - x - 2 = 0$
$(x - 2)(x + 1) = 0$

Thus, the x-intercepts are 2, and -1.

TIP (The Important Point)

ALGEBRAICALLY
y-intercept is defined as the value of $f(x)$ when $x = 0$.
x-intercept is defined as zero of the function or the value of x when $f(x) = 0$.

Answer to Hook Question.
FALSE. There is at least one x-intercept and at most one y-intercept.



We are confident that you understood the examples. This time we will allow you to challenge yourself and be amazed on what you can achieve.

Do It Yourself

15 MINUTES

Find the intercepts of the graphs of the following polynomial and rational functions. Then show how to find the intercepts analytically.

1. $f(x) = -x^4 - 4x^3 - 2x^2 + 4x + 3$

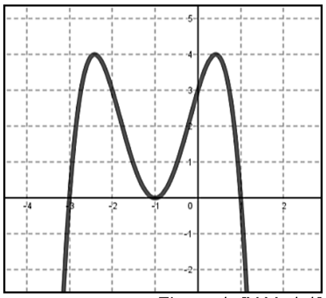


Figure 1. [X:Y=1:1]

***x*-intercepts:** __________________
***y*-intercept:** __________________

2. $f(x) = -x^3 + 2x^2 + 4x - 8$

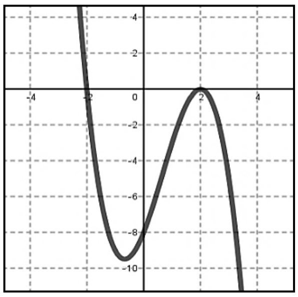


Figure 2. [X:Y=2:2]

***x*-intercepts:** __________________
***y*-intercept:** __________________

SOLUTION:

y-intercept:

x-intercepts:

SOLUTION:

y-intercept:

x-intercepts:

3.

$$f(x) = \frac{x+1}{x^2+2x-3}$$

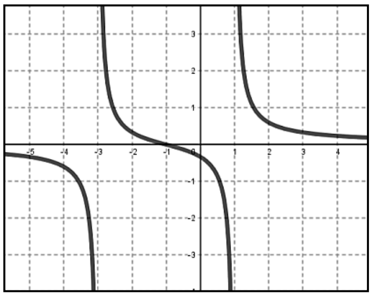


Figure 3. [X:Y=1:1]

***x*-intercepts:** ____________________
***y*-intercept:** ____________________

SOLUTION:

y-intercept:

x-intercepts:

4.

$$f(x) = \frac{x^3+2x^2-4x-8}{x-1}$$

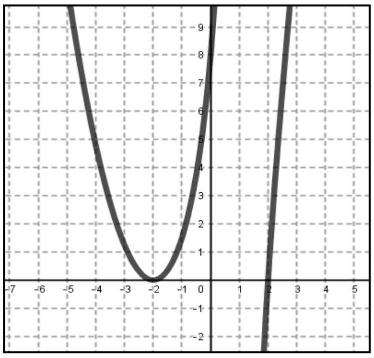


Figure 4. [X:Y=1:1]

***x*-intercepts:** ____________________
***y*-intercept:** ____________________

SOLUTION:

y-intercept:

x-intercepts:

5. $$f(x) = \frac{(x^2 + 2x)(x + 1)}{(x^2 - 3x + 2)(x + 1)}$$

y-intercept:

x-intercepts:

6. $$f(x) = \frac{2x^2 + 10x - 48}{x^2 + 2x - 15}$$

y-intercept:

x-intercepts:

7. Find the third-degree polynomial with y-intercept: 9, x-intercepts: -1, 1.5, &3 and whose graph is shown below.

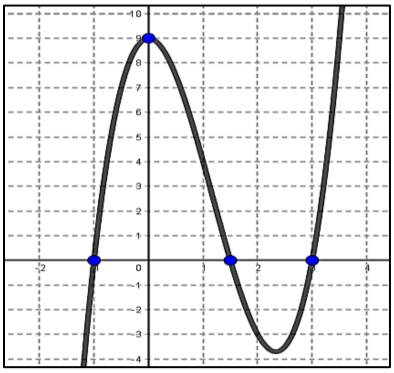


Figure 5. [X:Y=1:1]

SOLUTION:

8. What polynomial has degree 7 for which $f(1) = 3$ and has zeroes -2 (multiplicity 2), 0 (multiplicity 3), and 2 (multiplicity 2)?

SOLUTION:

Before this lesson ends, *K*eep *N*ote of these *O*utstanding *T*houghts:

1. There are two ways in finding the intercepts of polynomial and rational functions: graphical and algebraically.
2. Graphically, intercepts are defined as
   - x-intercept is abscissa of the point where the graph touches the x-axis
   - y-intercept is the ordinate of the point where the graph touches the y-axis
3. Algebraically, intercepts are defined as
   - y-intercept is the value of f(x) when x = 0
   - x-intercept is the zero of the function or the value of x when f(x) = 0

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Reference:

[1] Swokowski, E., & Cole, J. (2008). *Algebra and Trigonometry with Analytic Geometry, 12th Edition.* Thomson Learning, Inc.

[2] International Geogebra Institute. (2020). *GeoGebra.* https://www.geogebra.org/

[3] Albarico, J.M. (2013). THINK Framework. (Based on Ramos, E.G. and N. Apolinario. (n.d.) *Science LINKS*. Rex Bookstore, Inc.)

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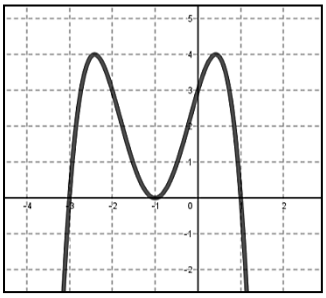
ANSWER KEY:

1. $f(x) = -x^4 - 4x^3 - 2x^2 + 4x + 3$



$\boldsymbol{x - intercepts}$: $\mathbf{-3, -1}, \& \ 1$

$\boldsymbol{y - intercept}$: $\mathbf{3}$

SOLUTION:

y-intercept: Let x = 0

$f(0) = -(0)^4 - 4(0)^3 - 2(0)^2 + 4(0) + 3$
$f(0) = 3$

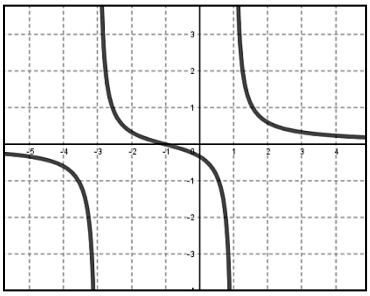
Thus, the y-intercept is 3.

x-intercepts: Let f(x) = 0

$$f(x) = -x^4 - 4x^3 - 2x^2 + 4x + 3$$
$$0 = -x^4 - 4x^3 - 2x^2 + 4x + 3$$
$$0 = -(x^4 + 4x^3 + 2x^2 - 4x - 3)$$
$$0 = -(x+3)(x+1)(x-1)$$

Thus, the x-intercepts are -3,-1, and 1.

3. $f(x) = \dfrac{x+1}{x^2 + 2x - 3}$



$\boldsymbol{x - intercepts}$: $\mathbf{-1}$

$\boldsymbol{y - intercept}$: $\mathbf{-\frac{1}{3}}$

SOLUTION:

y-intercept: Let x = 0

$$f(0) = \frac{(0)+1}{(0)^2 + 2(0) - 3} = -\frac{1}{3}$$

Thus, the y-intercept is $-\frac{1}{3}$.

x-intercepts: Let f(x) = 0

$$f(x) = \frac{x+1}{x^2 + 2x - 3}$$
$$0 = \frac{x+1}{x^2 + 2x - 3}$$
$$0 = x + 1$$
$$x = -1$$

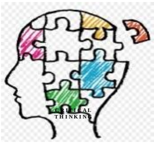
Thus, the x-intercept is -1.

5.

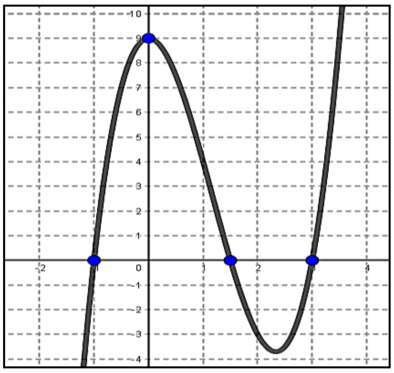
$$f(x) = \frac{(x^2 + 2x)(x + 1)}{(x^2 - 3x + 2)(x + 1)}$$

y-intercept: 0
x-intercepts:

First, simplify the expression,
then equate numerator to 0.
$0 = (x^2 + 2x)$
$0 = x(x + 2)$
$x = -2$ & 0



7. Find the third-degree polynomial with y-intercept: 9,
x-intercepts: {-1, 1.5, 3} and whose graph is shown below.



SOLUTION:
Using the x-intercepts, the equation becomes
$f(x) = a(x + 1)(x - 1.5)(x - 3)$

Using the y-intercept,
$9 = a(0 + 1)(0 - 1.5)(0 - 3)$
$a = 2$

Thus, the polynomial is
$f(x) = 2(x + 1)(x - 1.5)(x - 3)$