dOpt Compulsory Assignment 3

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1 Proof

Since π_1 and π_2 are good representations, they must necessary run in polynomial time and since they are equivalent it is possible to reduce one of them from the other and vice versa. f is a simple function that run in constant time (basically converts '1' to 'yes' and '0' to 'no'). Since:

- L_1 is based upon the functionality of π_1,
- f does the same in both languages,
- $\pi_1 \leq \pi_2$ (per definition of being polynomially equivalent)

L_1 must also be reducible from L_2 and vice versa (i.e. $L_1 \leq L_2$). We know that $L_2 \in P$ as it was restricted by π_2 which is bounded by polynomial time and thus in P as well. We can then finally use the property of downward closure of P on our languages:

$$\mathrm{L}_1 \leq L_2 \wedge L_2 \in P \implies L_1 \in P$$

to make the implication that if L_2 is in P then L_1 must also necessarily be in P and vice versa. Similarly if L_1 or L_2 is not in P (and f is trivial), then the inner π-function is *not* a good representation and both languages must not be

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in P, which follows from the closure. This gives us that $L_1 \in P \Leftrightarrow L_2 \in P$, if π_1 and π_2 are good representations. $\Box$