dOpt Compulsory Assignment 5

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1 Show that JohnsonCut Is An Approximation Algorithm

For an algorithm to be an approximation algorithm, it has to return a feasible solution every time as well as having an approximation ratio (of 2 in this case). The JohnsonCut cannot return an unfeasible solution as it iterates over every edge and always adds a vertex to either S or T but not both at any given point.

Regarding the ratio, the optimal the total weight is given by:

$$\sum_v w(\{v\}, S) + w(\{v\}, T) = W$$

Where W is the total weight of the edges. An optimal solution $|C^*|$ will always be smaller than the total weight:

$$W \geq |C^*|$$

Any one of our proposed solutions $|C|$ will then be at least half of what we found above as we only have the weights of the biggest set, which results in

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our solution also being at least half the size of the optimal solution:

$$|C| \geq \frac{1}{2} \sum_{v} w(\{v\}, S) + w(\{v\}, T) \geq \frac{1}{2}|C^*|$$

Of course our solution cannot exceed the total weight of the graph, which gives it the following property:

$$|C| \leq W$$

Since the cost of the optimal solution is no greater than W and the cost of any given solution is at least $\frac{1}{2}$ the algorithm is bound by the approximation ratio:

$$\frac{W}{\frac{1}{2}W} \leq 2$$

□

2 Running Time

Since JohnsonCut has to iterate all vertices and edges in any given pass it has the running time of $(2|V| + |E|)$ in all cases. The factor 2 is from adding vertices to the sets.