dOpt Compulsory Assignment 3 Re-handin

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1 Proof

We want to prove that one of two languages are in $\boldsymbol{P}$ iff the other one is as well. These languages are decided by the same function, but the input to the deciding function is represented differently (both are still strings $x \in \{0,1\}^*$) and the two representations are *good* and *polynomially equivalent*.

Starting off, we can assume that one of the languages, lets say L_1, is in $\boldsymbol{P}$:

$\mathrm{L}_1 \in \boldsymbol{P}$

Under this assumption, the language is decided by a Turing machine T_1 in polynomial time. Now to continue our proof we need to show that L_2 is also in $\boldsymbol{P}$. From our assumption, we know that π_1 is a good representation and we want to map it to π_2. However, we cannot assume that $x \in \pi_2(S)$ just because they are polynomially equivalent.

Since π_2 is also supposedly a good representation we can create a Turing machine T_2 that decides the language it represents in polynomial time. Being

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certain of a good representation AND knowing it is polynomially equivalent with pi_1 from the description, we can perform the mapping

$\pi_1(x) = r_1(\pi_2(x))$.

Since every input to the mapping was a good representation (and thus decidable by a Turing machine) and it can be mapped back to π_1 used by L_1, which was definitely in $\boldsymbol{P}$ from our assumption, which translates back to $L_2 \in \boldsymbol{P}$ iff it is decided by a TM T_2 and L_1 is decided by a TM T_1. $\square$

2 Notes

So the idea is that if $L_1 \in \boldsymbol{P}$ we can utilise a mapping between the polynomially equivalent representations. We know that π_1 is a good representation from our assumption (and our constructed Turing machine). For the mapping to make sense, π_2 must also be a good representation, which is why we construct the second Turing machine that hopefully decides the language L_2. This can be used to confirm that $L_2 \in \boldsymbol{P}$ if and only if our assumptions are true (i.e. that π_2 is a good representation and that $L_1 \in \boldsymbol{P}$).

This was how I interpreted the proof made using Turing machines.