$\boxed{1}$ $\circ$ $g(x) = b\sqrt{1-\left(\frac{x}{a}\right)^2}$

a) $V_1 = \pi \int g(x)^2 dx \quad , \; 0 \leq x \leq a \quad = \pi \int_0^a g(x)^2 dx$

$= \pi \int_0^a b^2\left(1 - \frac{x^2}{a^2}\right) dx = \pi \int_0^a b^2 - \frac{b^2x^2}{a^2} dx$

$= \pi \left[b^2 x - \frac{b^2 x^3}{3a^2} \right]_0^a =$

$= \pi\left(b^2 a - \frac{b^2 a^3}{3a^2}\right) - \pi(0) = \pi b^2 a\left(1 - \frac{1}{3}\right)$

$= \frac{\pi b^2 a 2}{3}$

b) $V_2 = 2\pi \int_0^a x g(x)\, dx = 2\pi \int_0^a x\, b\sqrt{1-\frac{x^2}{a^2}}\, dx$

$= 2\pi \int_{x=0}^{x=a} x b \sqrt{U} \cdot \frac{dU \cdot a^2}{-2x} \quad , \; U = 1 - \frac{x^2}{a^2}$

$\frac{dU}{dx} = -\frac{2x}{a^2}$

$= -\pi \int_{x=0}^{x=a} b a^2 U^{\frac{1}{2}} dU = -\pi b a^2 \int_{x=0}^{x=a} U^{\frac{1}{2}} = -\pi b a^2 \left[\frac{2}{3} U^{\frac{3}{2}}\right]_{x=0}^{x=a}$

$= -\pi b a^2 \left(\frac{2}{3}\left(1 - \frac{a^2}{a^2}\right) - \left(-\frac{2}{3}(1-0)\right)\right) = \frac{\pi 2 b a^2}{3}$

c) $V_1 = V_2$

$\frac{\pi b^2 a 2}{3} = \frac{\pi 2 b a^2}{3} \nearrow \quad \cancel{b}^2 \cancel{a} = \cancel{b} a^2$

$b = a \Longrightarrow \frac{\pi a^3 2}{3}$

$V_{Halbkugel} = \frac{2}{3} a^3 \pi =$

2 a)

$$\int_1^\infty \frac{1}{x^{5/3} + x^{2/3}}\,dx = \frac{3}{2}\int_0^1 \frac{1}{1+t^{3/2}}\,dt$$

$$\Downarrow$$

$$\int_1^\infty \frac{1}{x^{5/3}(1 - x^{-1})}\,dx$$

$t = x^{-2/3}$

$(x^{-2/3})^y = x^{-1}$

$-\frac{2}{3} \cdot y = -1$

$x^{-1} = (x^{-2/3})^y$

$-1 = -\frac{2}{3} \cdot y$

$y = \frac{3}{2}$

Setter inn for $t = x^{-2/3}$ og får

$$\int_{x=1}^{x=\infty} \frac{1}{x^{5/3}(1 - t^{3/2})}\,dx$$

og: $t = x^{-2/3}$.

$$\frac{dt}{dx} = \frac{-2x^{-5/3}}{3}$$

$$dx = \frac{dt \cdot 3}{-2x^{-5/3}}$$

$$\Downarrow$$

$$\int_{x=1}^{x=\infty} \frac{1}{\cancel{x^{5/3}}(1 - t^{3/2})} \cdot \frac{dt \cdot 3}{-2\cancel{x^{-5/3}}}$$

$$\Downarrow$$

$$\int_{x=1}^{x=\infty} \frac{1}{1 - t^{3/2}} \cdot \frac{-3\,dt}{2}$$

$$\Downarrow$$

$t = x^{-2/3}$

setter in $x = 1$

$t = 1^{-2/3} = 1$

$t = x^{-2/3}$

setter in ~~x = ∞~~ hvis $x \to \infty$

$t = \frac{1}{\sqrt[3]{x^\infty}} = 0$

$$-\frac{3}{2}\int_{t=1}^{t=0} \frac{1}{1-t^{3/2}}\,dt = \underline{\frac{3}{2}\int_0^1 \frac{1}{1-t^{3/2}}\,dt}$$

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$\boxed{2}$ b)

$$f(x) = \int_1^{\infty} \frac{dx}{x^{\frac{5}{3}} + x^{\frac{2}{3}}} = \frac{3}{2}\int_0^1 \frac{dt}{1+t^{3/2}}$$

$$\int_0^1 f(x)\,dx \approx \int_0^1 p(x)\,dx = \frac{h}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right)$$

$$x_i = i \cdot \Delta x = \frac{b-a}{n} = \frac{1}{4} \qquad h = \Delta x = \frac{1}{4}$$

$n = 4$

$$\int_0^1 p(x)\,dx = \frac{h}{3}\left(f(0) + 4f(\tfrac{1}{4}) + 2f(\tfrac{1}{2}) + 4f(\tfrac{3}{4}) + f(1)\right)$$

$$= \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{3}{2}\left(\frac{1}{1+\left(\frac{0}{4}\right)^{\frac{3}{2}}} + 4\,\frac{1}{1+\left(\frac{1}{4}\right)^{\frac{3}{2}}} + 2\,\frac{1}{1+\left(\frac{1}{2}\right)^{\frac{3}{2}}} + 4\,\frac{1}{1+\left(\frac{3}{4}\right)^{\frac{3}{2}}} + \frac{1}{1+1}\right)$$

$$\approx \underline{\underline{1{,}119762}}$$

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3

$f(x) = \cos(x)\ , \ 0 \leq x \leq \pi$

$c_1 \in (1, \pi/2)$ $\quad q : (1, 0]$ $\quad \vec{pq}:$

$|(c, f(c)) - q| \leq |\vec{pq}|$

$\cancel{\vec{pq} = \vec{op} \ \vec{oq} = \vec{po} - \vec{qo} = [}$

$\vec{pq} = \cancel{\vec{qo} + \vec{op}} \ \cancel{\vec{qp}} + \vec{po} + \vec{oq} = \vec{oq} - \vec{op} = [1-x, -\cos(x)]$

$|\vec{pq}|^2 = (1-x)^2 + (-\cos(x))^2 = (x-1)^2 + (\cos^2 x)$

$((1-x)^2 = (-(x-1))^2 = (x-1)^2$

$g(x) = |\vec{pq}|^2 = (x-1)^2 + \cos^2 x$

$g'(x) = 2(x-1) + (-2\cos(x)\sin(x)) = 2(x-1) - \sin(2x)$

$g'(x) = 2x - \sin(2x) - 2$

$\cancel{g'(x) = 0}$

$\cancel{2x - \sin 2x - 2 = 0}$

$\cancel{g(1) = \cos^2(1) = 0{,}29}$

$g'(1) = 2 - \sin(2) - 2 = -\sin(2) \simeq -0{,}91$

$g'(\pi/2) = \pi - \sin(\pi) - 2 = \pi - 2 \simeq 1{,}14$

~~Også vet at~~ vi vet med sekantsetningen siden grafen g' er kontinuerlig at det finnes en c slik at $g'(c) = 0$

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1 a) som vil være minimumspunktet i grafen $g(x)$

3 b)

$$g'(x) = 2x - 2\sin(2x) - 2$$

$$g'(x) = 0$$

$$g''(x) = 2 - 2\cos(2x)$$

$$2x - 2\sin(2x) - 2 = 0$$

$$x_{n+1} = x_n + \frac{g'(x)}{g''(x)}$$

$$x_0 = \pi/3$$

$$x_1 = \frac{\pi}{3} - \frac{2\pi/3 - \sin\left(\frac{\pi}{3}\right) - 2}{2 - 2\cos\left(\frac{2\pi}{3}\right)} = 1{,}3044076517$$

$$x_2 = 1{,}2773677232$$

$$g'(x) = 0$$

$$x = 1{,}2770979964$$

$$x_3 = 1{,}2770979897$$

$$\cancel{x_3 = 1{,}277}$$

$$x_3 = 1{,}27700$$

$$x_3 = 1{,}277098 = 1{,}2771$$

$\boxed{4}$

$$\lim_{x \to 1} \frac{\ln(x) - (x-1) + \frac{(x-1)^2}{2}}{(x-1)^3}$$

$f(x) = \ln(x)$

$\cancel{P_n(x) =}$

$\cancel{\ln'(x) =}$

$f'(x) = \frac{1}{x}$

$f''(x) = -\frac{1}{x^2}$

$f'''(x) = -\frac{2}{x^3}$

$f^n(x) = \frac{(-1)^{n+1}(n-1)}{x^n}$

$\frac{f^n(x)}{n!}(x-1)^n$

$a = 1$

$$P_n(x) = 0 + \frac{1}{1 \cdot 1!}(x-1) - \frac{(x-1)^2}{2!} + \frac{2(x-1)^3}{3!} + O(\cancel{x^4}(x-1)^4)$$

$$P_n(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} + O((x-1)^4)$$

$$\lim_{x \to 1} \frac{(x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} + O((x-1)^4) - (x-1) + \frac{(x-1)^2}{2}}{(x-1)^3}$$

$$\lim_{x \to 1} \frac{\frac{(x-1)^3}{3} + O((x-1)^4)}{(x-1)^3} = \frac{\frac{\cancel{(x-1)^3}}{3}}{\cancel{(x-1)^3}} + \frac{O((x-1)^{\cancel{4}})}{\cancel{(x-1)^3}} = \frac{1}{3} + 0$$

$$= \underline{\frac{1}{3}}$$