Oppgave 5

$$g(x) = \begin{cases} \frac{1}{x^2} \int_0^{x^2} f(t)\,dt, & x \neq 0 \\ f(0), & x = 0 \end{cases}$$

$$\lim_{x \to 0} g(x) = \lim_{x \to 0} \frac{1}{x^2} \cdot \int_0^{x^2} f(t)\,dt = \text{"}\frac{0}{0}\text{"} \quad \text{l'Hôp}$$

$$= \cancel{\lim_{x \to 0}}$$

$$\left(\int_0^{x^2} f(t)\,dt\right)\frac{d}{dx} = 2x \cdot f(x^2)$$

$$\left(x^2\right)\frac{d}{dx} = 2x$$

$$\lim_{x \to 0} g(x) = \lim_{x \to 0} \frac{2x\, f(x^2)}{2x} = \lim_{x \to 0} f(x^2) = \underline{\underline{f(0)}}$$

Funksjonen er kontinuerlig fordi $\lim_{x \to 0+} g(x) = g(0) = \lim_{x \to 0-} g(x)$

$= f(0)$