Skritlig innlevering 1 [illegible]

1 $$f(x) = \begin{cases} \cos(x)\sin(\tan(x)) & , x \neq \frac{\pi}{2} \\ 0 & , x = \frac{\pi}{2} \end{cases}$$

Vi vet at dersom $\lim_{x \to \frac{\pi}{2}} \cancel{f} = 0$ er ~~funksjonen~~ grafen kont. i $x = \frac{\pi}{2}$

$$\left|\sin(\tan(x))\right| \Rightarrow \left|\sin\left(\frac{\sin(x)}{\cos(x)}\right)\right| \leq 1$$

Vi skviser $f(x)$ mellom $-|\cos(x)|$ og $|\cos x|$ og får

$$-|\cos(x)| \leq \cos(x)\sin(\tan(x)) \leq |\cos(x)|$$

setter inn $x = \frac{\pi}{2}$

$$0 \leq \cos(x)\sin(\tan(x)) \leq |\cos(x)|$$

$$\lim_{x \to \frac{\pi}{2}} f(x) = 0$$

og siden $\tan(x)$ er kont. i ~~I~~ $[0, \frac{\pi}{2}), (\frac{\pi}{2}, 0]$ og $\sin(x)$ og $\cos(x)$ er kont. får vi at grafen er kontinuerlig i ~~0~~ $[0, \pi]$

~~$y(0)$~~

2 $$\ln(1+y) + \sin(xy) = \ln(5)$$

$y(0)$

$$\ln(1+y) + \sin(0) = \ln(5)$$

$$e^{\ln(1+y)+0} = e^{\ln 5}$$

$$1+y = 5$$

$$y = 4$$

$$y(0) = 4$$

$\frac{dy}{dx}\left(\ln(1+y) + \sin(xy) = \ln(5)\right)$

$\ln(1+y)\frac{dy}{dx} + \sin(xy)\frac{dy}{dx} = \ln(5)\frac{dy}{dx}$ $\qquad \ln(u)' = \frac{u'}{u}$

$\left(\frac{y}{1+y}\right)\frac{dy}{dx}$

$\sin(u)' = \cos u \cdot u'$

$(u \cdot v)' = u'v + uv'$

$$\frac{y'}{1+y} + \cos(xy)\cdot(y + xy') = 0$$

$y'(0)$

$$\frac{y'(0)}{1+4} + \cos(0)(4+0) = 0$$

$$\frac{y'(0)}{5} = -4 \qquad \frac{y'}{1+y} + \cos(xy)(y + xy') = 0$$

$$y'(0) = -20 \qquad \frac{dy}{dx}\left(\frac{y'}{1+y} + \cos(xy)\cdot y + \cos(xy)\,xy' = 0\right)$$

$(uv)' = u'v + uv'$

$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ $\qquad \frac{y''(1+y) - y'\cdot y'}{(1+y)^2}$ $\qquad -\sin(xy)(y+xy')\cdot y +$

$\cos(xy)\cdot y'$

$u'v + uv'$ $\quad (xy')' = y' + xy''$

$-\sin(xy)(y+xy')\cdot xy' + \cos(xy)(y' + xy'')$

$$\frac{y''(1+y) - (y')^2}{(1+y)^2} - \sin(xy)(y+xy')y + \cos(xy)\,y' - \sin(xy)(y+xy')xy' + \cos(xy)(y'+xy'')$$

$y''(0)$

$\sin(0) = 0$

$$\frac{y''(0)\cdot(1+4) - (20)^2}{(1+4)^2} - 0 + \cos(0)\,20' - 0 + \cos(0)(-20+0) = 0$$

$$\frac{y''(0)}{5} - \frac{(4\cdot 5)^2}{5^2} - 20 + -(20) = 0$$

$$\frac{y''(0)}{5} = 16 +$$

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$$\frac{y''(0)\cdot(1+4) - (4\cdot 5)^2}{(1+4)^2} - 20 - 20 = 0$$

$$\frac{y''(0)\cdot 5}{5^2} - \frac{4^2\cdot 5^2}{5^2} - 2\cdot 20 = 0$$

$$\frac{y''(0)}{5} = 40 + 16$$

$$\underline{\underline{y''(0) = 56\cdot 5 = 280}}$$

3

$$\lim_{x\to -2} \frac{3x^3 + x^2 - 12x - 4}{x^2 - 4} = -5$$

$$\frac{3x\cancel{(x^2-4)} + \cancel{x^2-4}}{\cancel{x^2-4}} = -5$$

$$3x + 1 = -5 \qquad (x \neq 2;\ x \neq -2$$

$$\boxed{\Delta = 8}$$

Definisjonen at for enhver $\varepsilon > 0$ skal det finnes en $\Delta > 0$ ~~Som~~ - avhengig av ε slik att $0 < |x-a| < \Delta$ slik at $|f(x) - L| < \varepsilon$

$$0 < |x+2| < \Delta$$

$$|3x + 1 - (-5)| < \varepsilon$$

$$|3x + 6| < \varepsilon$$

$$3\,|x+2| < \varepsilon$$

$$|x+2| < \frac{\varepsilon}{3}$$

siden $f(x) = L$ og $\frac{\varepsilon}{3}$ er et positivt nummer, finnes det en $\Delta > 0$ som att

$$0 < |x+2| < \Delta \implies |x+2| < \frac{\varepsilon}{3}$$

4

a) $\lim_{x \to -1} \frac{(1+x)^2}{\sqrt{1+(x+1)^2} - 1} \quad \Big| \cdot \frac{\sqrt{1+(x+1)^2} + 1}{\sqrt{1+(x+1)^2} + 1}$

$$\frac{(\cancel{1+x})^2 (\sqrt{1+(x+1)^2} + 1)}{\cancel{1+(x+1)^2 - 1}} \quad = \lim_{x \to -1} \sqrt{1+(x+1)^2} + 1 \; = \sqrt{1+(2)^2} + 1 = \sqrt{5} + 1$$

$\sqrt{1}$

$\sqrt{1+(-1+1)^2} + 1$

$\sqrt{1}$

$\sqrt{1} + 1 = \underline{\underline{2}}$

b)

$\lim_{x \to 0} \frac{19\pi x \sin(7x)}{14(1-\cos(19x))}$

bruker l'Hôpitals regel

$g(x) = 19\pi x \sin(7x)$

$g'(x) = 19\pi \cdot \sin(7x) + 19\pi x \cdot \cos(7x) \cdot 7$

$h(x) = 14(1-\cos(19x)) = 14 - 14\cos(19x)$

$h'(x) = -14 \cdot (-\sin(19x) \cdot 19) = 14 \cdot 19 \sin(19x)$

$\frac{g'(x)}{h'(x)} = \frac{19\pi(\sin(7x) + x\cos(7x) \cdot 7)}{14 \cdot 19 \sin(19x)}$

$g''(x) = 19\pi \cos(7x) \cdot 7 + 19\pi \cdot \cos(7x) \, 7 \cdot \cancel{1} + 19\pi x(-\sin(7x)) \cdot 7 \cdot 7$

$h''(x) = 14 \cdot 19 \cos(19x) \cdot 19$

$\frac{g''(x)}{h''(x)} = \frac{\cancel{19}\pi \cdot \cancel{7}(\cos(7x) + \cos(7x) \cdot \cancel{1} + x \sin(7x) \cdot 7}{\cancel{14} \; 2 \cdot \cancel{7} \cdot \cancel{19} \cdot 19 \cdot \cos(19x)}$

$\lim_{x \to 0} \frac{g''(x)}{h''(x)} \quad \frac{\pi(1 + \cancel{1} - 0)}{2 \cdot 19 \cdot 1} = \frac{4\pi}{2 \cdot 19} \quad \frac{4\pi}{19} = \frac{2\pi}{2 \cdot 19} = \underline{\underline{\frac{\pi}{19}}}$