

Ex 1  $p \vee (p \wedge q) \Leftrightarrow p$

p	q	$p \wedge q$	$p \vee (p \wedge q)$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

~~Ex 2~~  $\alpha := p, \beta := q, \gamma := r$

~~$p \vee (q \wedge r) \equiv (p \wedge q) \vee (p \wedge r)$~~

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$
0	0	0	0	0
0	0	1	0	0
0	1	0	0	0
0	1	1	1	0
1	0	0	0	1
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

~~b)  $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$~~

~~a)  $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$~~

Ex 2

p	q	r	$q \wedge r$	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	1	0	0	0	0	0	0
0	1	1	1	1	1	0	0
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

Ex 3

$$p \Rightarrow (q \vee r) \equiv (p \wedge \neg q) \Rightarrow r$$

$\Rightarrow$

imp. law

$$\neg p \vee q \vee r \equiv \neg(p \wedge \neg q) \vee r$$

$\Rightarrow$

De.M.L.

$$\neg p \vee q \vee r \equiv \neg p \vee q \vee r$$

4

$$[(q \wedge p) \vee q] \wedge \neg(\neg q \vee p) \equiv q \wedge \neg p$$

$$q \wedge (q \wedge \neg p) \equiv q \wedge \neg p$$

$$q \wedge q \wedge \neg p \equiv q \wedge \neg p$$

$$q \wedge \neg p \equiv q \wedge \neg p$$



Ex

$$5 \ a) \quad \forall x (S(x) \Rightarrow H(x))$$

$$b) \quad \neg (\forall x (S(x) \Rightarrow H(x)))$$

$$c) \quad \cancel{\forall x} \cancel{H(x)} \quad \forall x (S(x) \Rightarrow \neg H(x))$$

$$d) \quad (\neg H(x) \Rightarrow S(x)) \vee (\neg H(x) \Rightarrow \neg S(x)) \quad ?$$

$$6 \ a) \quad \neg ((x < y) \wedge (z < y) \wedge (x < z) \wedge (z < x))$$

$$x < z < y \equiv \text{True}$$

$$b) \quad \cancel{\text{True}} \quad \cancel{\text{False}}$$

$$b) \quad \text{FALSE, } \text{false} \quad \text{P(n)} \quad \text{G(n)}$$

$$c) \quad \text{false } \cancel{z \text{ is not}} \quad z \text{ is after } y$$

$$d) \quad \text{True True}$$

$$7 \quad \forall x [p(x) \wedge q(x)]$$

$$\neg (\forall x [p(x) \wedge q(x)])$$

$$\exists x \neg (p(x) \wedge q(x))$$

$$\underline{\exists x [\neg p(x) \vee \neg q(x)]}$$

$\exists x$

$$\neg \exists x \forall y [p(y) \vee \neg (q(x, y))]$$

$$\forall x \neg (\forall y [p(y) \vee \neg q(x, y)])$$

$$\forall x \exists y \neg [p(y) \vee \neg q(x, y)]$$

$$\underline{\forall x \exists y (\neg p(y) \wedge q(x, y))}$$