Ex 1

a) $\overline{xy} + x\bar{y}$

$\overline{0\cdot 1} + \bar{1}\cdot\bar{0}$

$\bar{0} + 0\cdot 1$

$1 + 0 = 1$

$x := 1$, $y := 0$

$w := 1$

$z = 0$

b) $w + \bar{x}\cdot y$

$1 + \bar{x}\cdot y = 1$

c) $wx + \bar{y} + y\bar{z}$

$1\cdot 1 + \bar{y} + y\bar{z} = 1$

d) $(wx + y\bar{z}) + w\bar{y} + \overline{(w+y)(\bar{x}+y)}$

$(wx + y\bar{z}) + 1\cdot\bar{0} + \overline{(w+y)(\bar{x}+y)}$

$\Downarrow$

1

Ex 2)

a) $xy + (x+y)\bar{z} + y$

$xy + y + (x+y)\bar{z}$

$y + (x+y)\bar{z}$

$\cancel{y + (\bar{z}+x)(\bar{z}+y)}$

$y + (x\bar{z}) + (\bar{z}y)$

$y + (x\bar{z})$

b) $x + y + \overline{(\bar{x} + y + z)}$

$x + y + (x\bar{y}\bar{z})$

$y + x + x(\bar{y}\bar{z})$

$y + x$

c) $y\bar{z} + wx + \bar{z} + [w\bar{z}(xy + w\bar{z})]$

$a(b+a) \Rightarrow a$

$\bar{z} + wx + w\bar{z}$

$wx + \bar{z}$

3) $S_n = \sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

$a_n = n^2$

$S_k = \frac{k(k+1)(2k+1)}{6}$

$S_{k+1} \Leftrightarrow S_k + a_{k+1}$

$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$= \frac{(k^2+k)(2k+1) + 6(k+1)^2}{6}$

$= \frac{2k^3 + 3k^2 + k + 6k^2 + 12k + 6}{6}$

$= \frac{2k^3 + 9k^2 + 13k + 6}{6}$

$= \frac{(k+1)(k+2)(2(k+1)+1)}{6}$

$= \frac{(k^2+3k+2)(2k+3)}{6}$

$= \frac{2k^3 + 3k^2 + 6k^2 + 9k + 4k + 6}{6}$

$= \frac{2k^3 + 9k^2 + 13k + 6}{6}$

$\Leftrightarrow$

qed

proof by induction

Ex 4

$$S(n) = \sum_{i=0}^{n} 2^{-i}$$

a) $S(0) = 1 \quad S(1) = \frac{1}{2} \quad S(2) = \frac{1}{4} \quad S(3) = \frac{1}{8}$

b) $S(n) = 2 - \frac{1}{2^n}$

c) $S(0) = 2 - 1 = 1$ (Base case)

$$S(n+1) = 2 - \frac{1}{2^{n+1}} = S(n) + \frac{1}{2^{n+1}}$$

$$\Downarrow$$

$$2 - \frac{1}{2^{n+1}} = \sum_{i=0}^{n} 2^{-i} + \frac{1}{2^{n+1}}$$

$$\Downarrow$$

$$\sum_{n=0}^{n+1} 2^{-i} = \sum_{n=0}^{n} 2^{-i} + \frac{1}{2^{n+1}}$$

$$\sum_{i=0}^{n+1} 2^{-i} - \sum_{i=0}^{n} 2^{-i} = \frac{1}{2^{n+1}}$$

$$\Downarrow$$

$$\sum_{i=n}^{n+1} 2^{-i} = \frac{1}{2^{n+1}}$$

$$2 - 2^{-(n+1)} - (2 - 2^{-n}) = \frac{1}{2^{n+1}}$$

$$-\frac{1}{2^{n+1}} + \frac{1}{2^n}$$

$$\Downarrow$$

$$2 - 2^{-(n+1)} = 2 - 2^{-n} + 2^{-(n+1)}$$

$$\Downarrow$$

$$2^{-n} = 2^{-(n+1)} + 2^{-(n+1)}$$

$$\Downarrow$$

$$2^{-n} = 2(2^{-(n+1)}$$

$$\Downarrow$$

$$2^{-n} = 2^{-n-1+1}$$

$$\Downarrow$$

$$\underline{2^{-n} = 2^{-n}}$$

Ex 4

d)

$\cancel{S(n) \geq |2-\epsilon|}$

$2 - 2^{-n}$

$\cancel{S(n) < |2-\epsilon|}$

$S(n) \geq |2-\epsilon|$

$2 - 2^{-n} \geq |2-\epsilon|$

$-2^{-n} \geq -\epsilon$

$2^{-n} \leq \epsilon \quad | \ln$

$\ln(2^{-n}) \leq \ln(\epsilon)$

$-n\ln(2) \leq \ln(\epsilon)$

$-n \leq \frac{\ln(\epsilon)}{\ln(2)}$

$n \geq \frac{\ln(\epsilon)}{\ln(2)}$

Ex 5)

1) $S_n = \sum_{i=1}^{n} 2^{i-1} \cdot i = 2^n \cdot (n-1) + 1$

$S_1 = 2^0 \cdot 1 = 1$ (Base case)

$a_n = 2^{n-1} \cdot n$

beviser at det gjelder for alle $n \in \mathbb{N}$

$S(n+1) = S(n) + a_{n+1}$ (antar at det gjelder for alle n)

$2^{n+1}(n+1-1) + 1 = 2^n(n-1) + 1 + 2^{n+1-1} \cdot (n+1)$

$2^{n+1}(n) + 1 \stackrel{?}{=} 2^n(n-1) + 1 + 2^n(n+1)$

~~$2 \cdot 2^n n + 1 = 2^n(n-1)(n+1) + 1$~~

~~$= 2^n(n^2-1) + 1$~~

$n2^{n+1} + 1 = 2^n(n-1+n+1) + 1$

$\underline{\underline{2n \cdot 2^n + 1 = 2^n \cdot 2n + 1}}$