Skriftlig innlevering 2

1 $f(x) = \cosh(x) = \frac{e^x + e^{-x}}{2}$

$$f'(x) = \frac{e^x}{2} + \frac{e^{-x}\cdot(-1)}{2}$$

$$f'(x) = \frac{e^x}{2} - \frac{e^{-x}}{2} = \frac{e^x - e^{-x}}{2}$$

$$e^x \geq e^{-x}, \quad x \in [0, \infty)$$

$$f'(x) \leq 0, \quad x \in [0, \infty)$$

$f(x)$ er strengt voksende på $[0, \infty)$

$$f(0) = \frac{e^0 + e^{-0}}{2} = \frac{1+1}{2} = 1$$

$$\lim_{x\to\infty} f(x) = \frac{e^x + e^{-x}}{2} = \frac{e^x + \frac{1}{e^x}}{2} = \frac{\infty + 0}{2} = \infty$$

$V_f = [1, \infty)$

$$\left(\cancel{f(f^{-1}(x)) = x}\right)\frac{d}{dx} \qquad \left(\cancel{f^{-1}(f(x)) = x}\right)\frac{d}{dx}$$

$$\cancel{(f^{-1})'(f(x))}$$

$$\cancel{(f)'(f^{-1}(x))} \qquad \cancel{f'}(f(x))$$

$$\left(f(f^{-1}(x)) = x\right)\frac{d}{dx}$$

$$f'(f^{-1}(x)) \cdot (f^{-1})'(x) = 1$$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$f(x) = \sqrt{170}$$

$$\frac{e^x + e^{-x}}{2} = \sqrt{170}$$

$$e^x + e^{-x} = 2\sqrt{170} \quad \Big| \cdot e^x$$

$$z = e^x$$

$$e^{x2} - 2e^x\sqrt{170} + 1 = 0$$

$$z^2 - 2\sqrt{170}\,z + 1 = 0$$

$$z = \frac{2\sqrt{170} \pm \sqrt{4 \cdot 170 - 4}}{2}$$

$$z = \sqrt{170} \pm 13$$

$$e^x = \sqrt{170} + 13$$

$$x = \ln(\sqrt{170} + 13)$$

$$(f^{-1})'(x) = \frac{1}{f'(\ln(13 + \sqrt{170}))}$$

$$f'(x) = \frac{e^x - e^{-x}}{2}$$

$$f'(\ln(13 + \sqrt{170})) = \frac{13 + \sqrt{170} - \frac{1}{13+\sqrt{170}}}{2}$$

$$= \frac{13 + \sqrt{170}}{2} - \frac{1}{2(13+\sqrt{170})} = 13$$

$$(f^{-1})'(x) = \frac{1}{f'(\ln(13 + \sqrt{170}))}$$

$$(f^{-1})'(x) = \frac{1}{13}$$

2

$f(x)$, $-f(x) = f(-x)$, $f(x+2\pi) = f(x)$

$f(x) \geq 0 \leq x \leq \pi$, $f(\frac{\pi}{2}) = 2$ $\int_0^{\pi} f(x)\,dx = 2$

a) $\int_{17\pi}^{19\pi} |f(x)|\,dx = \int_{-\pi}^{\pi} |f(x)|\,dx = \int_{-\pi}^{0} |f(x)|\,dx + \int_0^{\pi} |f(x)|\,dx$

$= \int_0^{\pi} |f(-x)|\,dx + \int_0^{\pi} |f(x)|\,dx$, $|f(-x)| = |f(x)|$

$2\int_0^{\pi} |f(x)|\,dx$

since $f(x) \geq 0$ $|f(x)| = f(x)$

$2\int_0^{\pi} f(x)\,dx = 2\cdot 2 = 4$ //

b) $\int_0^{\pi/4} e^{f(2x)} f'(2x)\,dx$

$f(2x) = u$

$f'(2x)$ $u = 2x$

$f(2x)\frac{d}{dx} = f'(2x)\cdot u' = 2f'(2x)$

$\int_{x=0}^{x=\frac{\pi}{4}} e^{u}\cdot f'(2x)\,du$

~~$u = 2x$~~ $\cancel{\int_{x=0}^{x=\frac{\pi}{4}} e^{u} f'(u)\,dx}$

$U = f(2x)$ $\int_{x=0}^{x=\frac{\pi}{4}} e^{u}\cdot u'\,dx$ $\frac{du}{dx} = 2f'(2x)$

$dx = \frac{du}{2f'(2x)}$

$\int_{x=0}^{x=\frac{\pi}{4}} \left[e^{f(2x)} \frac{f'(2x)\,du}{2f'(2x)}\right]_0^{\frac{\pi}{4}} = \frac{1}{2}\left[e^{f(2x)} + C\right]_0^{\frac{\pi}{4}}$

$$\frac{1}{2}\left(e^{f(\frac{\pi}{2})} - e^{0}\right) = \underline{\underline{\frac{1}{2}\left(e^{f(\pi/2)} - 1\right)}} \quad \left(f(\tfrac{\pi}{2}) = 1\right) = \underline{\underline{\frac{1}{2}(e-1)}}$$

c) $\int_{-\pi}^{\pi} e^{(f(x))^2} \sin(f(x))$

$g(x) = e^{(f(x))^2}$

$h(x) = \sin(f(x))$

siden funksjonen er odde [illegible] $f(-x) = f(x)$

[illegible]

$$\left[\int e^{(f(x))^2} \sin(f(x))\right]_{x=-\pi}^{x=\pi} = \underline{\underline{0}}$$

Δx $x_{i+1} - x_i = \frac{1}{n}$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{3n+4i} \qquad \frac{1}{3n+4i} \qquad \frac{1}{n}\frac{4n}{3n+4i}$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{4n}{3n+4i}\right)$$

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$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{3n+4i} \qquad \Delta x = \frac{1}{n} \quad ; \quad x_i = \frac{i}{n}$$

$$\Updownarrow$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\cdot\frac{4}{3+\frac{4i}{n}} = \lim_{n\to\infty} \sum^{n} \Delta x \frac{4}{3+x_i}$$

$$\lim_{n\to\infty} \Delta x \sum_{i=1}^{n} \frac{4}{3+x_i} = \text{Riemann} = \int \frac{4}{3+4x}\,dx$$

$$= \int \frac{4}{3+4x}\,dx \qquad U = 3+4x \quad U' = 4 \quad \frac{du}{dx} = U' \quad dx = \frac{du}{U'}$$

nedre grense ved $i = 0 \Rightarrow \frac{0}{n} = 0$

øvre ved $i = n \Rightarrow \frac{n}{n} = 1$

$$\int \frac{4}{u}\,\frac{du}{4}$$

$$\int \frac{1}{u}\,du = \ln|U| = \ln|3+4x|$$

$$\int_0^1 \frac{4}{3+4x}\,dx = \Big[\ln|3+4x|\Big]_0^1 = \ln(3+4) - \ln(3)$$

$$\Updownarrow$$

$$\ln\left(\frac{7}{3}\right)$$

$$f(x) = a^x$$

$$a^y = x \qquad (x \ln a)'$$

$$f(f^{-1}(x)) = x$$

$$y = \frac{\ln x}{\ln a} \qquad a^x \ln a$$

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}$$

$$y = \ln$$

$$a^{\frac{\ln x}{\ln a}}$$

4 $a > 0$

$$\int_0^\infty e^{-ax} \cos x \, dx$$

$$dU = \cos x$$
$$U = \sin x$$
$$V = e^{-ax}$$
$$dV = e^{-ax} \cdot (-a)$$

$$\int dUV = UV - \int U dV$$

$$\int e^{-ax} \cos x dx = \sin x e^{-ax} - \int \sin x \cdot (-a e^{-ax})$$

$$\int \sin x \cdot (-a e^{-ax}) \qquad dU = \sin x \quad V = -a e^{-ax}$$
$$\Downarrow \qquad U = -\cos x \quad dV = a^2 e^{-ax}$$

$$-\cos x (-a e^{-ax}) - \int -\cos x \, a^2 e^{-ax} dx)$$
$$-\cos x (-a e^{-ax}) + a^2 \int \cos x e^{-ax} dx$$

$$\int e^{-ax} \cos x dx = \sin x e^{-ax} - (\cos x a e^{-xa} + a^2 \int \cos x e^{-ax} dx)$$

$$(a^2 + 1) \int e^{-ax} \cos x dx = \sin x e^{-ax} - \cos x a e^{-ax}$$

$$\int_0^\infty e^{-ax} \cos x dx = \frac{\sin(x) e^{-ax} - \cos(x) a e^{-ax}}{a^2 + 1}$$

$$\lim_{m \to \infty} \int_0^m e^{-ax} \cos x dx = \left[\frac{\sin(x) e^{-ax} - \cos(x) a e^{-ax}}{a^2+1} \right]_0^m$$

$$\Updownarrow$$

$$\lim_{b \to \infty} \frac{\sin b e^{-ab} - \cos(b) a e^{-ab}}{a^2 + 1} - \frac{-a e^0}{a^2 + 1}$$

feneste side ↓

$$\lim_{b\to\infty} \frac{e^{-ab}\sin(b) - a e^{-ab}\cos b}{a^2+1} + \frac{a}{a^2+1}$$

nar $b \to \infty$ ga e^{-b} mot 0 ...

$$\int_0^\infty e^{-ax}\cos(x)\,dx = \frac{a}{a^2+1}$$

$\Downarrow$

$$F(x) = \frac{a}{a^2+1}$$

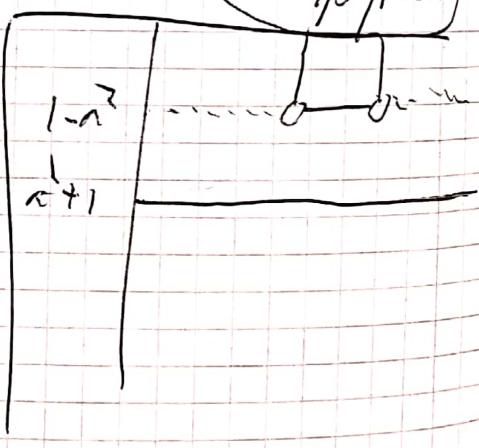
$\cancel{f'(x)=0}$ $\quad g(a) = \frac{a}{a^2+1}$ $\qquad u'=1 \quad v'=2a$

$\cancel{f(x)}$ $\quad g'(a) = \frac{a^2+1-2a^2}{(a^2+1)^2}$ $\qquad \frac{u'v - uv'}{v^2}$

$$g'(a) = 0$$

$$\frac{a^2-2a^2+1}{(a^2+1)^2} = 0 = \frac{1-a^2}{(a^2+1)}$$

$a=-1 \cup 1=a$



$a^3-2a^2+1 = 0$

$\cancel{(a+1)(a}$ $\quad (a-1)^2 = 0$

$(a-1)^2 \cancel{(a+1)^2}$

$a>0 \qquad a=1$

$$F(1) = \frac{1}{1^2+1} = \frac{1}{2}$$

$$a = \frac{1}{2}$$