Ex1 $\neg((\neg p\wedge q)\vee(\neg p\wedge\neg q))\vee(p\wedge q)$

$\cancel{\neg(\neg p\wedge q)}$

$\neg(\neg p\wedge q)\wedge\neg(\neg p\wedge\neg q)\vee(p\wedge q)$

$(p\vee\neg q)\wedge(p\vee q)\vee(p\wedge q)$

$p\vee(p\wedge q)$

$\underline{\underline{p}}$

Ex 2 $((p\wedge q)\vee(p\wedge\neg r)\vee\neg(\neg p\vee q))\vee((r\vee s\vee\neg r)\wedge\neg q)$

$((p\wedge q)\vee(p\wedge\neg r)\vee(p\wedge\neg q))\vee((T_0)\wedge\neg q)$

$((p\wedge q)\vee(p\wedge\neg q)\vee(p\wedge\neg r))\vee\neg q$

$((p\wedge(q\vee\neg q))\vee(p\wedge\neg r))\vee\neg q$

$(p\vee(p\wedge\neg r)\vee\neg q$

$\underline{\underline{p\vee\neg q}}$

Ex 3

i) a) $(\{2,6,4\}\cup\{6,4\})\cap\{4,6,8\}\Rightarrow\{6,4\}$

b) $P(\{7,8,9\})-P(\{7,9\})$

$\Downarrow$

$\{\{\emptyset\},\{7\},\{7,8\},\{7,8,9\},\{8\},\{8,9\},\{9\},\{9,7\}$

$-\{\{\emptyset\},\{7\},\{9\},\{7,9\}$

$\Rightarrow\{\{7,8\},\{7,8,9\},\{8\},\{8,9\}\}$

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Ex 3:

c) $\cancel{P(\emptyset) = \{\{\emptyset\}\}}$ $\{\emptyset\}$

$P(\emptyset) = \{\emptyset\}$

d) $\{1, 3, 5\} \times \{0\} \Rightarrow$

$= \{(1,0), (3,0), (5,0)\}$

e) $\{2, 4, 6\} \times \emptyset = \emptyset$

f) $P(\{0\}) \times P(\{1\})$

$\Rightarrow \{\{0\}, \{\emptyset\}\} \times \{\{1\}, \{\emptyset\}\}$

$\Rightarrow \{(0,1), (0,\emptyset), (\emptyset,1), (\emptyset,\emptyset)\}$

g) $\cancel{p(p(\{2\}))}$

$\Rightarrow \cancel{P(\{\{2\}, \emptyset\})}$

$\Rightarrow \cancel{\{\{2\}, \{\emptyset\}, \{\{2\}, \{\emptyset\}\}, \ldots\}}$

g) $p(p(\{2\}))$

$\Rightarrow P(\{\{2\}, \emptyset\})$

$\Rightarrow \{\{2, \emptyset\}, \emptyset, \{2\}, \{\emptyset\}\}$

iii) ~~his $\emptyset \notin A$ his $\emptyset \in$~~ $A \Rightarrow 0$

$|P(x) - \{\{x\} \mid x \in A\}| = 2^{(n-1)}$ (?)

Ex 4 a) $\emptyset = \{\emptyset\} \Rightarrow$ False $\quad |\emptyset| \neq |\{\emptyset\}|$

b) $\emptyset = \{0\} \Rightarrow$ false $\quad |\emptyset| \neq |\{0\}|$

c) $|\emptyset| = 0 \Rightarrow$ True $\quad |\emptyset| = |\{\}| \Rightarrow \emptyset = \{\}$

d) $|P(\emptyset)| = 0 \Rightarrow$ False $\quad |P(A)| = 2^{|A|} \Rightarrow 2^{|\emptyset|} = 2^0 = 1$

e) $\emptyset \in \{\}$ $\Rightarrow$ True? $\quad \emptyset = \{\}$

f) $\emptyset = \{x \in \mathbb{N} : x \leq 0 \land x > 0\} \Rightarrow$ True $\quad \{x \in \mathbb{N} : x \leq 0 \land x > 0\}$

$\Downarrow$

$\{\}$

Ex 5 i)

$$A \cap (A \cup B) \equiv A \equiv \{x : x \in A \cap (A \cup B)\}$$

$$\equiv \{x : x \in A \wedge x \in A \cup B\}$$

$$\equiv \{x : x \in A \wedge (x \in A \vee x \in B)\}$$

$$\equiv \{x : x \in A\}$$

$$\equiv A.$$

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5 1)

$$A-(B\cap C) \overset{?}{=} (A-B)\cup(A-C)$$

$$\Downarrow$$

$$A\cap\overline{(B\cap C)} \overset{?}{=} (A\cap\bar{B})\cup(A\cap\bar{C})$$

$$\Downarrow$$

$$\{x,\ x\in A\cap\overline{(B\cap C)}\} \overset{?}{=} \{y,\ y\in(A\cap\bar{B})\cup(A\cap\bar{C})\}$$

$$\Downarrow$$

$$\{x,\ x\in A\wedge x\in\overline{(B\cap C)} = \{y,\ y\in(A\cap\bar{B})\vee y\in(A\cap\bar{C})\}$$

$$\Downarrow$$

$$\{x,\ x\in A\wedge x\notin B\cap C\} = \{y,\ [y\in A\wedge y\notin B]\vee y\in A\wedge y\notin C\}$$

$$\Downarrow$$

$$\{x,\ x\in A\wedge x\notin(B\cap C)\} = \{y,\ y\in A\wedge(y\in(\bar{B}\vee y\in\bar{C})\}$$

$$\{x,\ x\in A\wedge x\notin(B\cap C) = \{y,\ y\in A\wedge y\notin(B\cap C)\}$$

$$\underline{x=y}$$

Ex 6

i) $A \Delta B = (A \cap \bar{B}) \cup (B \cap \bar{A})$

$\Downarrow$

$((A \cap \bar{B}) \cup B) \cap ((A \cap \bar{B}) \cup \bar{A})$

$\Downarrow$

$(B \cup (A \cap \bar{B})) \cap (\bar{A} \cup (A \cap \bar{B}))$

$\Downarrow$

$(B \cup A) \cap (B \cup \bar{B}) \cap (\bar{A} \cup A) \cap (\bar{A} \cup \bar{B})$

$\Downarrow$

$(B \cup A) \cap \mathcal{U} \cap \mathcal{U} \cap (\bar{A} \cup \bar{B})$ Identity laws

$\Downarrow$

$(B \cup A) \cap (\bar{A} \cup \bar{B}) = (B \cup A) \setminus (\overline{\bar{A} \cup \bar{B}})$

$\Downarrow$

$\underline{(B \cup A) \setminus (A \cap B)}$ □

$(N - M) = N \cap \bar{M}$

Distributive Law

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Distributive law

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

ii) $U := \mathbb{N}$, $A := \{1, 2, 3, 4, 5, 6, 7, 9, 10\}$

$B := \{1, 3, 5, 8, 9, 10\}$

$A \Delta B = (A \setminus B) \cup (B \setminus A)$

$\Downarrow$

$\{2, 4, 6, 7\} \cup \{8\}$

$\Downarrow$

$\{2, 4, 6, 7, 8\}$

Ex 2

7) $X := \{\{1,2,3\},\{2,3\},\{e,f\}\} \cup \{\{e\}\}$

$= \{\{1,2,3\},\{2,3\},\{e,f\},\{e\}\}$

$P(X) = \{\emptyset, \{1,2,3\}, \{2,3\}, \{e,f\}, \{e\}, \{\{1,2,3\},\{2,3\}\}, \{\{1,2,3\},\{e,f\}\}, \{\{1,2,3\},\{e\}\}, \{\{1,2,3\},\{2,3\},\{e,f\}\}, \{\{1,2,3\},\{2,3\},\{e\}\}, \{\{1,2,3\},\{e\},\{e,f\}\}, \{\{1,2,3\},\{2,3\},\{e\},\{e,f\}\}, \{\{2,3\},\{e,f\}\}, \{\{2,3\},\{e\}\}, \{\{2,3\},\{e,f\},\{e\}\}, \{\{e,f\},\{e\}\}\}$.

$Y = \{\{1,2,3,e,f\}\}$

$X \cap Y = \emptyset$

$P(\emptyset) = \{\emptyset\}$

nothing to compare really

Ex8 1) $A_1 \cap A_2 \cap A_3$, $\bar{A}_1 \cap A_2 \cap A_3$, $A_1 \cap \bar{A}_2 \cap A_3$, $A_1 \cap A_2 \cap \bar{A}_3$

$\bar{A}_1 \cap \bar{A}_2 \cap A_3$, $\bar{A}_1 \cap A_2 \cap \bar{A}_3$, $A_1 \cap \bar{A}_2 \cap \bar{A}_3$,

$\bar{A}_1 \cap \bar{A}_2 \cap \bar{A}_3$

2) ~~p~~ independent probability, 2 combinations exist $= 2^n$
~~$\cap A$~~ of the combination of $A_1, A_2, \ldots, A_n$.

Ex 9 $(A\setminus(B\setminus C))\setminus((A\setminus B)\setminus C) = A\cap C$ $A\setminus B \Rightarrow A\cap\bar{B}$

$(A\cap\overline{(B\cap\bar{C})})\cap\overline{((A\cap\bar{B})\cap\bar{C})}$ D.M.L

$(A\cap(\bar{B}\cup C))\cap((\bar{A}\cup B\cup C))$ A.L.

$(\bar{B}\cup C)\cap A\cap((\bar{A}\cup B)\cup C)$ D.L

$(\bar{B}\cup C)\cap(A\cap(\bar{A}\cup B))\cup(A\cap C))$ D.L

$(\bar{B}\cup C)\cap(A\cap\bar{A})\cup(A\cap B)\cup(A\cap C)$ A.L

$(\bar{B}\cup C)\cap(\emptyset\cup(A\cap B)\cup(A\cap C)$ Inverse Law

$(\bar{B}\cup C)\cap(A\cap(B\cup C))$ ASS. Law

$A\cap(\bar{B}\cup C)\cap(B\cup C)$ D.L

$A\cap C\cup(B\cap\bar{B})$ Inverse Law

$\underline{\underline{A\cap C}}$ $\square$

Ex 10

$(A \cup B) \triangle (A \cup C) = (B \triangle C) \setminus A \Rightarrow ((B \setminus C) \cup (C \setminus B)) \setminus A$

$A \triangle B = (A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$

$\cancel{((A \cup B) \setminus (A \cup C)) \cup ((A \cup B) \setminus A \cup C}$

$((A \cup B) \setminus (A \cup C)) \cup ((A \cup C) \setminus (A \cup B))$

$((A \cup B) \cap \overline{(A \cup C)}) \cup ((A \cup C) \cap \overline{(A \cup B)})$ — B | D.M.L.

$((A \cup B) \cap (\bar{A} \cap \bar{C})) \cup ((A \cup C) \cap (\bar{A} \cap \bar{B}))$

$(\bar{A} \cap (A \cup B) \cap \bar{C}) \cup (\bar{A} \cap (A \cup C) \cap \bar{B})$ — ASS. LAW

$((\bar{A} \cap A) \cup (\bar{A} \cap B) \cap \bar{C}) \cup ((\bar{A} \cap A) \cup (\bar{A} \cap C) \cap \bar{B})$ — D.L.

Identity LAW

$(A \cap B) \cap \bar{C}) \cup (A \cap C \cap \bar{B})$ — ASS. LAW

$A \cap B \cap \bar{C} \cup A \cap C \cap \bar{B}$ — D.L

~~A∩(A∩B(~~

$A \cap ((B \cap \bar{C}) \cup (C \cap \bar{B}))$ — ~~D~~L.

$\cancel{A \cap}$ $\Downarrow$ — ~~Ass.~~ Com. LAW

$((B \cap \bar{C}) \cup (C \cap \bar{B})) \cap A$

$\Downarrow$

$((B \setminus C) \cup (C \setminus B)) \setminus A$

$\Downarrow$

$\underline{(B \triangle C) \setminus A}$ $\square$

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