Matte a ØVING 5

1 a) e^x er kont

$\sqrt{x}$ er kon for alle $x \geq 0$

$\sin(x)$ er kont

$f(x)$ er kontinuerlig på alle punkt

b)

$$g(x)\begin{cases} 2x, & x \geq -3 \\ -x & x < -3 \end{cases}$$

$$\lim_{x \to -3^+} g(x) = 2\cdot(-3) = -6$$

$$\lim_{x \to -3^+} g(x) \neq \lim_{x \to -3^-} g(x)$$

$$\lim_{x \to -3^-} g(x) = -(-3) = 3$$

derav ikke kontinuerlig

2
$$\lim_{x \to \infty} a_n = \frac{\sqrt{n^2+6n+9}}{\sqrt{n^2+1}} = \sqrt{\frac{n^2+6n+9}{n^2+1}} \quad \Big| \cdot \frac{1}{n^2}$$

$(a-b)(a+b)\ a^2-b^2$

$$\sqrt{\frac{\frac{n^2}{n^2}+\frac{6n}{n^2}+\frac{9}{n^2}}{\frac{n^2}{n^2}+1}}$$

$$\lim_{n \to \infty} \sqrt{\frac{1+\frac{6}{n}+\frac{9}{n^2}}{1+\frac{1}{n^2}}} = \sqrt{\frac{1}{1}} = \underline{\underline{1}}$$

$\frac{k}{n} = 0$

3
$$\lim_{n \to \infty} a_n = \sqrt{n^2-n+9} - \sqrt{n^2+9} \quad \Big| \cdot \frac{\sqrt{n^2-n+9}+\sqrt{n^2+9}}{\sqrt{n^2-n+9}+\sqrt{n^2+9}}$$

$$= \frac{n^2-n+\cancel{9}+n^2-\cancel{9}}{\sqrt{n^2-n+9}+\sqrt{n^2+9}} \quad \frac{2n^2-n}{\sqrt{\ }}$$

3

$$a_n = \sqrt{n^2-n+9} - \sqrt{n^2+9} \quad \left| \cdot \frac{\sqrt{n^2-n+9} + \sqrt{n^2+9}}{\sqrt{n^2-n+9} + \sqrt{n^2+9}} \right.$$

$$a_n = \frac{n^2-n+9-n^2+9}{\sqrt{n^2-n+9} + \sqrt{n^2+9}}$$

$$a_n = \frac{-n}{\sqrt{n^2-n+9} + \sqrt{n^2+9}}$$

$$a_n = \frac{\cancel{-n}\ -1}{\cancel{n}\left(\sqrt{1-\frac{1}{n}+\frac{9}{n^2}} + \sqrt{1+\frac{9}{n^2}}\right)}$$

$$\lim_{n\to\infty} a_n = \frac{-1}{\sqrt{1-\frac{1}{n}+\frac{9}{n^2}} + \sqrt{1+\frac{9}{n^2}}} = \frac{-1}{\sqrt{1-0+0} + \sqrt{1+0}} = \frac{-1}{2}$$

4

$$a_{k+1} = 2 - \frac{1}{a_k}$$

$$L = 2 - \frac{1}{L} \quad | \cdot L$$

$$L^2 = 2L - 1$$

$$L^2 - 2L + 1 = 0$$

$$(L-1)(L+2) = 0$$

$$L = 1 \wedge L = 2$$

$$a_k = 1 \wedge a_k = 2$$

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5

$b_0 = 1 \qquad b_1 = 2 \qquad b_{n+1} = (b_n + 2 \cdot b_{n-1})$

$b_2 = 2 + 2 \cdot 1 = b_1 + 2 \cdot b_0$

$b_3 = b_1 + 2 \cdot b_0 + 2 \cdot b_1$

$b_4 = b_3 + 2 \cdot b_2 = b_1 \cdot 2 \cdot b_0 + 2 \cdot b_1 + 2 \cdot b_1 + 2 \cdot b_0$

~~$b_2 = b_1 + 2 \cdot b_0$~~

~~$b_3 = b_1 + 2 \cdot b_0 + 2 \cdot b_1$~~

~~$b_4 = b_1 + 2 \cdot b_0 + 2 \cdot b_1 + 2 \cdot b_1 + 2 \cdot b_0$~~

~~$b_5 = b_1 + 2 \cdot b_0 + 2 \cdot b_1 + 2 \cdot b_1 + 2 \cdot b_0 + 2 \cdot b_1 + 2 \cdot b_0 + 2 \cdot b_1$~~

$2 \cdot 1 \quad 2 \cdot 2 \quad 2 \cdot 2$

$2 \cdot 2 \quad 2 + 2 + 4 \quad 4 \cdot 2$

$2 + 2 + 4 + 4 + 2$

$2 + 2 + 4 + 4 + 2$

$2 + 2 + 4 + 4 + 2 + 2 + 2 + 4$

$4, 8, 14, 22$

$4 \quad 8 \quad 8$

$b_2 = 2 + 2 \cdot 1 = 4 = 2^2$

$b_3 = 4 + 2 \cdot 2 = 8 = 2^3$

$b_4 = 8 + 2 \cdot 4 = 16 = 2^4$

$b_5 = 16 + 2 \cdot 8 = 32 = 2^5$

$b_6 = 32 + 2 \cdot 16 = 64 = 2^6$

$b_7 = 64 + 2 \cdot 32 = 128 = 2^7$

$f(n) = b_n = 2^n$

$f(0) = 1$

$f(1) = 2$

$f(7) = 2^7 = 128$

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