Ex 7.) Partial order when: R = Reflexive, antisym, and trans.

a) $R(x,y): x-y \mid 2$, $x,y \in \mathbb{Z}$

Ref. $R(x,x) \Rightarrow x-x = 0 = 2\cdot k$, $k \in \mathbb{Z}$, yes

trans if $R(x,y)$ and $R(y,z)$ then $R(x,z)$

$x-y = 2k_1$, $k \in \mathbb{Z}$

$y-z = 2k_2$, $k \in \mathbb{Z}$

$\Downarrow$

$x-z \Rightarrow x+2k_2-y \Rightarrow x-y = 2k_2$, yes

antisymetric if $R(x,y)$ then not $R(y,x)$

$x-y = 2k_1$, $k_1 \in \mathbb{Z}$

$y-x \Rightarrow -(x-y) = -(2k_1)$, not

not a partial order

b) $R \in (\mathbb{Z}\times\mathbb{Z})\times(\mathbb{Z}\times\mathbb{Z})$

$R([a,b],[c,d])$, $a \leq c$.

$[a<c]$ or $[a=c$ or $b\leq d]$

$a=c, b<d$ $\quad$ $a=c$, $b=d$

Ex 7

b) $R([a,b],[c,d])$; $a,b,c,d \in \mathbb{Z}$: $a \leq c$

$R[(x,y),(x,y)]$

Ref: $x \leq x$, yes

~~Symmetric~~

$R[(x,y),(z,w)]$

antisymmetric

$x \leq z \Rightarrow z \leq x$, if an only if $x = z$

$x \leq z$ and $x \geq z \Leftrightarrow x = z$, yes

transitive:

iff $R[(x,y),(z,w)]$ and $R[(z,w),(u,v)]$, then $R[(x,y),(u,v)]$

$x \leq z$, $z \leq u \Rightarrow \underline{\underline{x \leq u}}$, yes

<u>This Relation is a partial order</u>

Ex 2

$A = \{1, 2, 3, 6, 9, 18\}$ $R(x,y): x | y$, $x, y \in A$

$R(x,y): y = x \cdot k$, $k \in \mathbb{Z}$

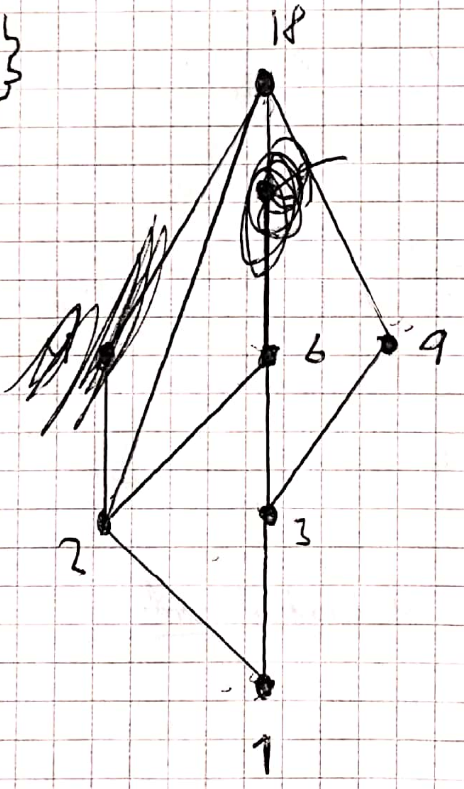
$R(x,y) = \{(1,2), (1,3), (1,6), (1,9), (1,18),$

$(2,6), (2,18),$

$(3,6), (3,9), (3,18)$

$(6,18),$

$(9,18)\}$



Ex) 3, a) $X = \{1, 0\}$ $\quad A = X \times X$ (cartesian p.)

$R \subseteq (A)$ $\quad R[(a,b),(c,d)]$, $a,b,c,d \in (X)$

iff $[(a<c)$ or $((a=c)$ and $(b \le d))]$

Ref. $R[(a,b),(a,b)]$

iii) $a = a$ and $b \le b$, yes

antisymetric:

$R[(a,b),(c,d)] \Rightarrow R[(c,d),(a,b)]$ iff $(a,b) \neq (c,d)$

i) $a < c \Rightarrow c < a$, no

ii) $a = c \Rightarrow c = a$

$b \le d \Rightarrow d \le d$ only if $b = d$, yes, antisym.

Trans.

iff $R[(a,b),(c,d)]$ and $R[(c,d),(e,f)]$ then $R[(a,b),(e,f)]$

i) if $a < c$ and $c < e$, the $a < e$

iii) $= (a=c) \wedge (b \le d) \Leftrightarrow [(a=c)$ and $(b<d)]$ or $(a=c)$ and $(b=d)$

$b < d < f$, yes $\quad$ or $\quad a = c = e$ and $b = d = f$, yes

Ex.

3 b) ~~$\{(a,b),(c,d)\} = \{(0,0),$~~

~~minimal order~~

minimal el $= \{(0,0)\}$

maximal el $= \{(1,1)\}$

c) no, because it is reflexive (iii) $a \le c$ and $b \le d$

4 a) $\{(x,y) \mid x, y \in \mathbb{Z}, y = x^2 + 7\}$ $R: \mathbb{Z} \to \mathbb{Z}$

since $f(x) = x^2 + 7$

it is a function

~~$x = \pm\sqrt{f(x) - 7}$, x maps to 2 values, which makes it a~~

~~function Relation, and not function~~

~~the range: $\pm\sqrt{x-7}$, $x \in \mathbb{Z}$ $x \in \mathbb{N} = \{0,1,2,3,4,5,6\}$~~

the range: $7 + a^2$, $a \in \mathbb{N}$

b) $\{(x,y) \mid x, y \in \mathbb{R}, y^2 = x\}$ $R: \mathbb{R} \to \mathbb{R}$

$f(x)^2 = x$

$f(x) = \pm\sqrt{x}$, $\mathbb{R}^+ \to \mathbb{R}$

x maps to 2 values, not a function.

The range is $\mathbb{R}$

Ex 4

i) $\{(x,y) \mid x,y \in \mathbb{R},\ y = 3x+1\}$, $R: \mathbb{R} \to \mathbb{R}$

$y = f(x) = 3x+1$

$f'(x) = 3$, injektive

$x = \frac{f(x)}{3} - 1$, ~~maps~~ $f(x)$ maps only to one y value wich makes it a function

Range : $\mathbb{R}$

ii) $\{(x,y) \mid x,y \in Q \quad x^2+y^2 = 1\}$ $R: Q \to Q$

$y^2 = f(x)^2 = 1 - x^2$

$f(x) = \pm\sqrt{1-x^2}$ which maps x to two values of y

since $1-x^2 \geqslant 0 \Rightarrow x^2 \leqslant 1 \Rightarrow x \leqslant \pm 1$

The range : $x \in R$, $-1 \leqslant x \leqslant 1$, Ex

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$f: \mathbb{Z} \to \mathbb{Z}$

a) $f(x) = 2x - 3 = 2(x-2) - 1$ which is ODD $(2k+1)$

$f'(x) = 2$, which makes the function monotone and injective

$x = \frac{f(x)+3}{2}$ since $f(x)$ is odd x will be an integer

range $= 2k+1, k \in \mathbb{Z}$ (odd numbers)

b)

$f(x) = x^2$ $f: \mathbb{Z} \to \mathbb{Z}$

$f(x) = n^2, n \in \mathbb{N}$ because n^2 ~~always~~ $\geq 0, \forall n$

$x = \pm\sqrt{y}$ which is all integers since $y = n^2, n \in \mathbb{N}$

$f(x)$ is not injective because $f(x) = f(-x)$ $\pm\sqrt{n^2} = \pm n$

$f: \mathbb{Z} \to \mathbb{Z}_+$

c) $y = f(x) = x^3 + x = x(x^2+1)$

~~$x^3 + x = y$~~ $f'(x) = 3x^2 + 1$, here is $f'(x) \geq 0$ for all $x \in \mathbb{Z}$

~~$x(x^2+1) = y$~~ which means it is injective

$\cancel{z = x^3 + x}$ 0

$\sqrt[3]{y+x} = x$, wont map to any integer i.e. $f(x) = 7$

Ex

6) a) $f: \mathbb{R} \to \mathbb{R}$

$y = f(x) = 2x - 3$ $\quad$ $f'(x) = 2$, injective and one-to-one

$f(\mathbb{R}) \to \mathbb{R}$

b) $f: \mathbb{R} \to \mathbb{R}$

$f(x) = x^2$ $\quad$ $x = \pm\sqrt{f(x)}$ $\quad$ $f(x) \geq 0$ because $\sqrt{-f(x)}$ is not defined in $\mathbb{R}$

$f'(x) = 2x$ not injective because $f(x) = f(-x)$

$f(\mathbb{R}) \to \mathbb{R}^{+} \cup \{0\}$, because $x^2 \geq 0$ for all $x \in \mathbb{R}$

c) $f: \mathbb{R} \to \mathbb{R}$

$f(x) = x^3 + x$

$f'(x) = 3x^2 + 1$ $\quad$ ~~which~~ is $f'(x) > 0 \ \forall x \in \mathbb{R}$

which means $f(x)$ is injective

$f(\mathbb{R}) \to \mathbb{R}$