Enzyme kinetics

1. In the first step of the reaction, an equilibrium between E and ES is rapidly reached and maintained throughout. The second step is the rate-limiting step and is irreversible. So, apparently the rate of change of P is:

$$V_P = k_3[ES]$$

And the rate of change of ES is:

$$V_{ES} = k_1[E][S] - k_2[ES] - \ k_3[ES] = \frac{d[ES]}{dt}$$

However, when the reaction is stable, although S and P are constantly changing, and the ES is constantly being formed and decomposed, the formation rate and decomposition rate are roughly equal, so ES remains basically unchanged.

The rate of change of S is:

$$V_S = -\ k_1[E][S]$$

The rate of change of E is:

$$V_E = k_3[ES] - k_1[E][S]$$

2. Runge-Kutta methods are an important class of implicit or explicit iterative methods for the solution of nonlinear ordinary differential equations.

As the title says:

$$k_1 = 100\mu M/min = \frac{5}{3}\mu M/s$$

$$k_2 = 600\mu M/min = 10\mu M/s$$

$$k_3 = 150\mu M/min = 2.5\mu M/s$$

$$E = 1\ \mu M$$

$$S = 10\mu M$$

Then, I choose 0.04 as the step size as it is moderate after testing 0.01, 0.02, 0.03, 0.04, 0.05, and 0.1. And for each of these variables, there is:

$$v_1 = f(t_n, y_n)$$

$$v_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}v_1)$$

$$v_3 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}v_2\right)$$

$$v_4 = f(t_n + h, y_n + hv_3)$$

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Thus, the next value (y_{n+1}) is determined by the current value (y_n) plus the product of the time interval (h) and an estimated slope. The slope is a weighted average of the following slopes (In order to avoid duplication with the above rate constant, v is used here to present the slope):

v_1 is the slope at the beginning of the time period;

v_2 is the slope of the midpoint of the time period, and the slope v_1 is used to determine the value of y at point $t_n + \frac{h}{2}$ through the Euler method;

v_3 is also the slope of the midpoint, but this time the slope v_2 is used to determine the y value;

v_4 is the slope at the end of the time period, and its y value is determined by v_3.

When the four slopes are averaged, the slope at the midpoint has more weight:

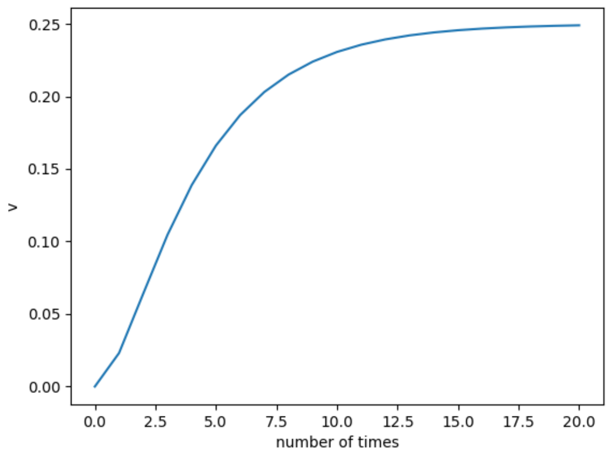
$$slope = \frac{v_1 + 2v_2 + 2v_3 + v_4}{6}$$

Next, I solve the problem by writing a code in Python, which uses the same algorithm as I mentioned above, and 20 iterations were performed. And the file is named **RK4.py**.

The results of ES, E, S, P in 20 iterations is:

------ ES ------	------ E ------	------ S ------	------ P ------
0	1	10	0
0.36470462658762326	0.5428304747688772	9.519714250108002	0.02311622466087487
0.42157008146505465	0.32191421548250454	9.257785289719394	0.06412892576311022
0.3767045704750665	0.20500966525128644	9.100438224182874	0.10457144106841178
0.3068578037232647	0.1377623894169166	8.99891743770196	0.13884495171495467
0.23945037510424227	0.09612277687649315	8.930016064871676	0.16610671200481614
0.1827487390368165	0.06875578559707908	8.881631916755563	0.1871238688415148
0.13780206199345865	0.04995867462219991	8.846898858776113	0.20305981584608535
0.10321649065812119	0.03665147871101044	8.821618471053291	0.21503300765771707
0.07702325989613153	0.02704425517518448	8.803061133943011	0.22398312123217098
0.057358900474642496	0.020022993600400666	8.789368467119159	0.23065452648123919
0.04266765954044123	0.014853554148874645	8.779233857571201	0.235619696577671
0.03172120120350568	0.011030852195074646	8.771718865544717	0.2393119866503549
0.023576733744894888	0.00819687697220953	8.766140279651482	0.24205659732072388
0.017521519040986245	0.006092894371988997	8.76199649772523	0.24409639664675617
0.013021195089956809	0.004529646717198371	8.758917357168984	0.24561228954821118
0.009676953415579753	0.0033676878225797208	8.756628848132117	0.24673883969046012
0.007191879844298442	0.0025038357490835676	8.754927764647427	0.2475760711016545
0.0053451993813253005	0.001861557432539233	8.753663246636004	0.24819831079653387
0.003972849959038166	0.0013840106723291066	8.752723225830168	0.2486607848421582
0.0029529432430033817	0.001028948562598165	8.752024421513996	0.24900452704859963

And The rate of production of product P as a function of time is plotted as:



3. When the concentrations of S are small, the velocity V increases approximately linearly, which is approximately in the dashed box and is the first-order reaction. However, at large concentrations of S, the velocity V saturates to approaches the asymptote of V_{max} or V_m (circled part). At this time, the active site of the enzyme is almost completely occupied by the substrate molecule and the rate is independent of the substrate concentration, which is the zero-order reaction. Between first-order and zero-order reaction is the mixed-order reaction.