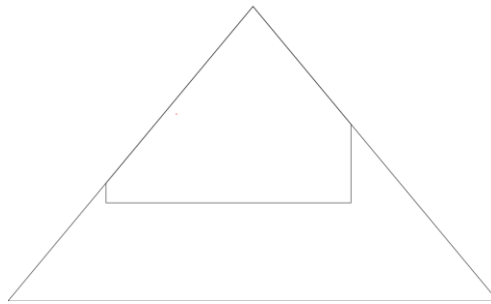


CSE 306 Project 2 report, Dimitrije Zdrle, BX 24

1. Polygon clipping

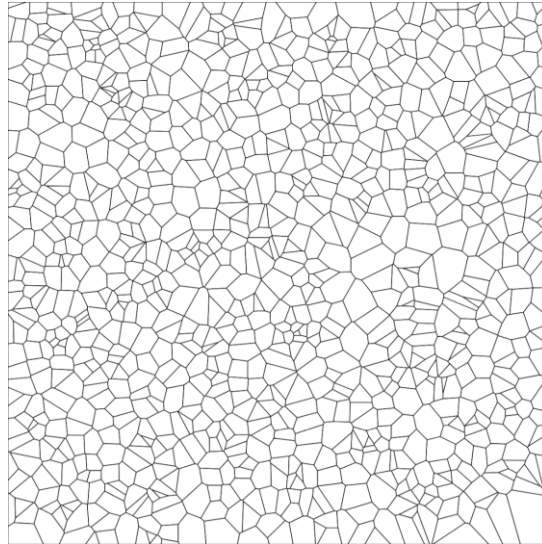
I started the project work by making a simple polygon clipping algorithm, known to us as Sutherland-Hodgman polygon clipping. However, before I started this, I needed to implement basic object classes, such as polygons, vectors, edges, as well as their important functions (containing/inside, intersection, vector operations). After these were done, I could implement the algorithm thanks to the lecture notes. To test it, I made a simple square on the background, and I cut it with a regular triangle, as seen on the image below:



Square clipped by a triangle

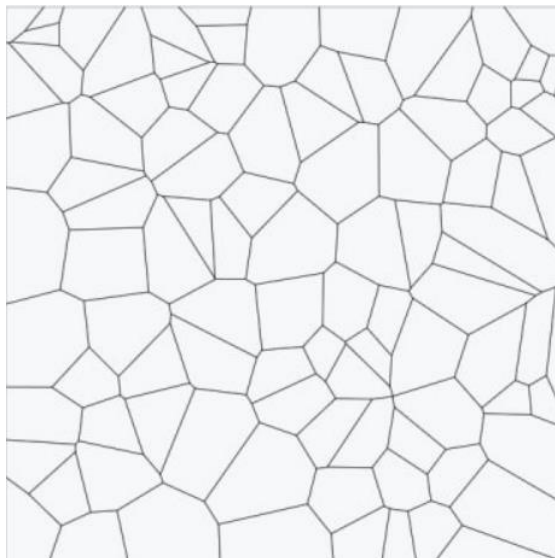
2. Voronoi diagram

With the clipping algorithm in place, I proceeded to do the Voronoi diagram implementation. The first step was to define new intersection and containment functions for my vectors and edges, as the M parameter needed to be modified to M' . This also required a new type of vector, which I named `WeightedVector`, however, I didn't have it as a separate class while I was doing the Voronoi diagram, but rather had a "weight" parameter in the vector class (later on this produced some confusion so I made the `WeightedVector`). After the new functions were done, I had to implement the Voronoi diagram generation. Firstly, I initialized "n" points randomly on the background, and then iterated over them, passing the current iteration point and the `std::vector` of all points to the `clip_voronoi()` function. The `clip_voronoi()` isn't much different than `clip()`, with the exception of using the new containment (`inside_voronoi()`) and new intersection functions (`w_intersect()`), and sometimes using weighted vectors. When this was complete, I first made a Voronoi diagram. I obtained the following image:



1000 points

After playing with weight initialization, like setting them all to a constant value (0.1, 0.5, etc.), with the same number of points, I was getting images like the following one:



1000 points with constant weights

As you can see, some of the polygons got reduced/disappeared.

I decided to stop with the $O(N^2)$ implementation and proceed with semi-discrete optimal transport.

3. Semi-discrete optimal transport and L-BFGS

This part was by far the most difficult for me, as I've spent the most time debugging and running it.

I began by obtaining the L-BFGS files from the provided github repository and linking them with my Makefile with the rest of the code. After this was done, I decided to change the sample.cpp provided in the repository to my needs.

Firstly, I had to decide on how the function will obtain all the necessary parameters. Since the points and the lambdas don't change during the gradient ascent, I decided

to pass them to the `objective_function` class instance. The lambdas are initialized as in the formula provided in the lecture notes, and I normalized them, as they should be a probability distribution. Points are generated in the same manner as in the Voronoi diagram. These two lists are then stored within the class and used later.

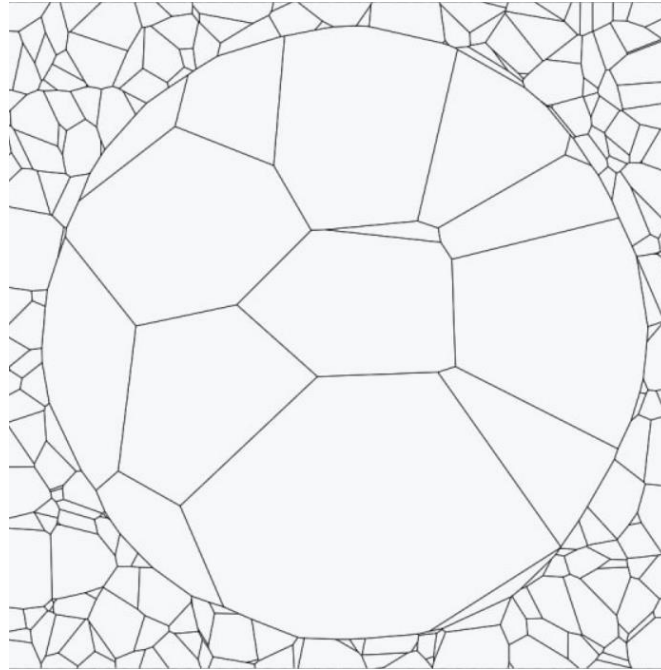
I then moved on to the evaluation function, the one where we implement the formulas.

First, I make a new set of points with updated weights, the $x[i]$ from the library. These points were then used to generate another Voronoi diagram. Later on, for convenience (and after being told that the diagram generation should technically be in a separate class), I moved the diagram generation into a `power_diag()` function. After obtaining the power diagram of the current step, I proceeded to perform integral computation. Since we were given that $f(x) = 1$, after looking at the formulas, I realized that I could split the computation in the following manner: return $F_x = \text{main_integral} - \text{weights} * \text{polygon_areas} + \text{lambdas} * \text{weights}$.

The main integral is done with triangulation. I will now explain every step of iteration over the points in the diagram.

- 1) We compute the area of the current polygon using the provided formula (Shoelace/Gauss's formula) from the lecture notes.
- 2) We update the gradient with $g[i] = \text{area_of_polygon} - \text{lambdas}[i]$. Sign is flipped because we perform gradient ascent.
- 3) Next, I perform the triangulation operation. It is a function of the polygon class. Initially, I connected vertices to the "main point" of the polygon, the `points[i]`. However, after remembering the discussions on Slack, I decided to connect all vertices to the first one, `vertex[0]`. This function returns a `std::vector` of Triangle classes, which is, essentially, as simpler polygon class.
- 4) With the obtained triangles, I iterate over them and perform the area computation as specified in the lecture notes. I obtain the $|T|$ by making a polygon out of the triangle vertices and computing its area with the same function as from step 1).
- 5) This gives me the `main_integral` value for the certain point "i", so now, I increment the `fx` with (obtained value (`integral_approx`) - $x[i] * \text{area of the polygon} + \text{lambdas}[i] * x[i]$). As you can see, the exact split as explained above.
- 6) After we iterate over all the points "i", we return $-fx$ as we want the gradient to ascend.

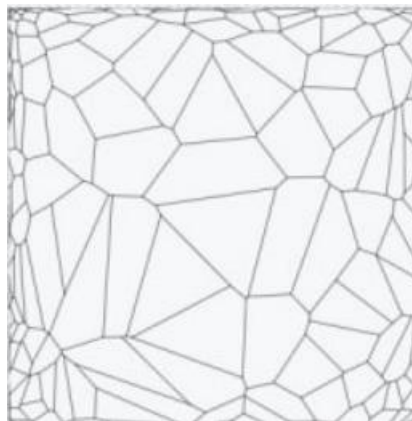
This is where the difficult part started for me. The `lbfgs` library failed after 2-5 iterations, and the results were bad, like the following:



So I started re-checking everything, as I really didn't understand why this was happening.

After a lot of debugging, code restructuring and re-writing many functions, the lbfgs started iterating, however, for a very long time.

I decided to try and run in with less points and see what happens. With 100 points, and after 1000-2500 iterations, depending on my initialization of $m_x[i]$ (usually a constant), I would get images like the following:

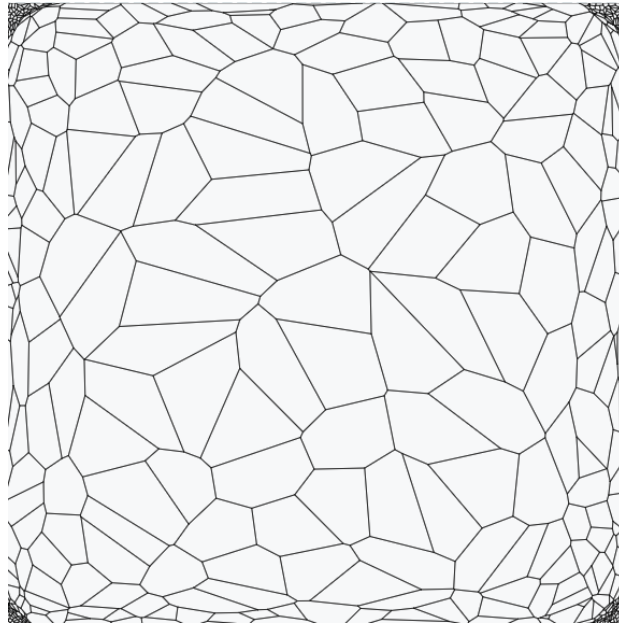


100 points

I could now see that it begins to look like the image in lecture notes, but I needed more points. However, when running it on 1000 points, each iteration would take a long time (even though I tried parallelizing), and the algorithm wouldn't stop even after 2500 iterations.

I did notice that fx didn't change much, so I decided to set a hard upper-bound on the number of iterations. After some experimentation, I settled with 200.

With 200 iterations, the execution time was okay, and I got the following:



1000 points, 200 iterations

After all of the debugging I did, I was more than happy with this result. However, I didn't find out why my program needed so many iterations. It would terminate for 100 points with successful convergence, but for 1000 points it was too long.

4. Fluid dynamics

Firstly, I linked all the required files for saving frames with my project.

For the fluid animation, I made it as a class similar to the `objective_function`, with a `step` and `run` method.

Within the `step`, I perform all the necessary computations and updates to compute and apply the spring and gravitational forces, as well as update the position and velocity of the points and polygons. The `objective_function` is called with `lambdas` being the total volume divided by the number of points. The `run` function simply simulates the steps and prints our debugging statements. The points are also instantiated randomly.

I proceeded to play with many parameters, and then I merged all the collected frame images into gifs. Here are some of the results: youtu.be/e9emZEF8Sws

(Contact me if the link doesn't work).

5. Structure and compilation

Due to Visual Studio issues with the last project, I decided to do this one on the lab machines via ssh, so the structure is different. All the `.cpp` and `.h` files are in their respective relevant folders (classes, lbfgs, functions), and everything is compiled with a Makefile. To run the project, you can go into the project folder on a Linux machine, type "make" in the terminal, and then `./fluid`. There are simple tests for all the major methods in the `main.cpp` file, they just need to be uncommented.