Problem 1

Using the impulse invariance method for analog to digital filter conversion, calculate the Chebyshev lowpass digital filter with parameters: passband frequency 20MHz; stopband frequency = 22MHz; passband ripple 0.5dB; stopband (out-of-band) attenuation 70dB; sampling frequency Fs = 60MHz.

a) Plot the impulse response for both analog and digital systems.

b) Plot the magnitude response for analog and digital systems in the frequency domain.

Provide code.

Generation of a_n and b_n for filter transfer function:

$$H(s) = \frac{B(s)}{A(s)} = \frac{b_1 s^n + b_2 s^{n-1} + \cdots + b_{n+1}}{a_1 s^n + a_2 s^{n-1} + \cdots + a_{n+1}}$$

Calculation of residues r_k and poles pk by partial function expansion of ratio of two polynomials:

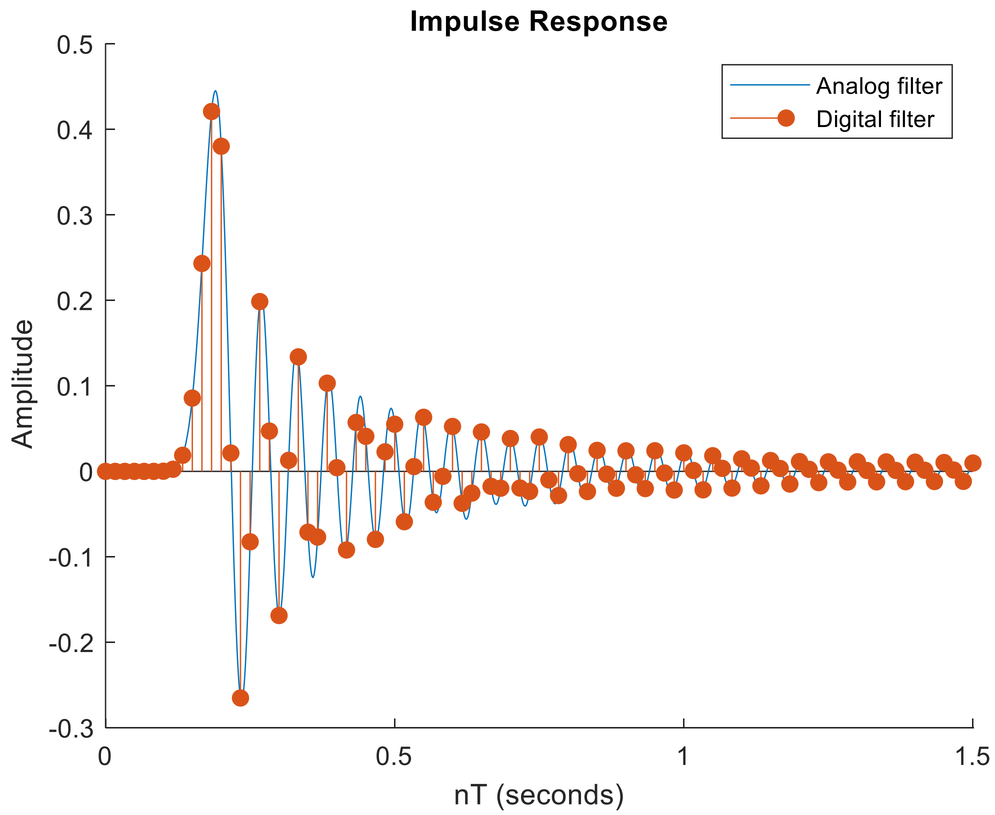
$$H(s) = \sum_k \frac{r_k}{s - p_k}$$

Laplace transform to get impulse response $h(t)$:

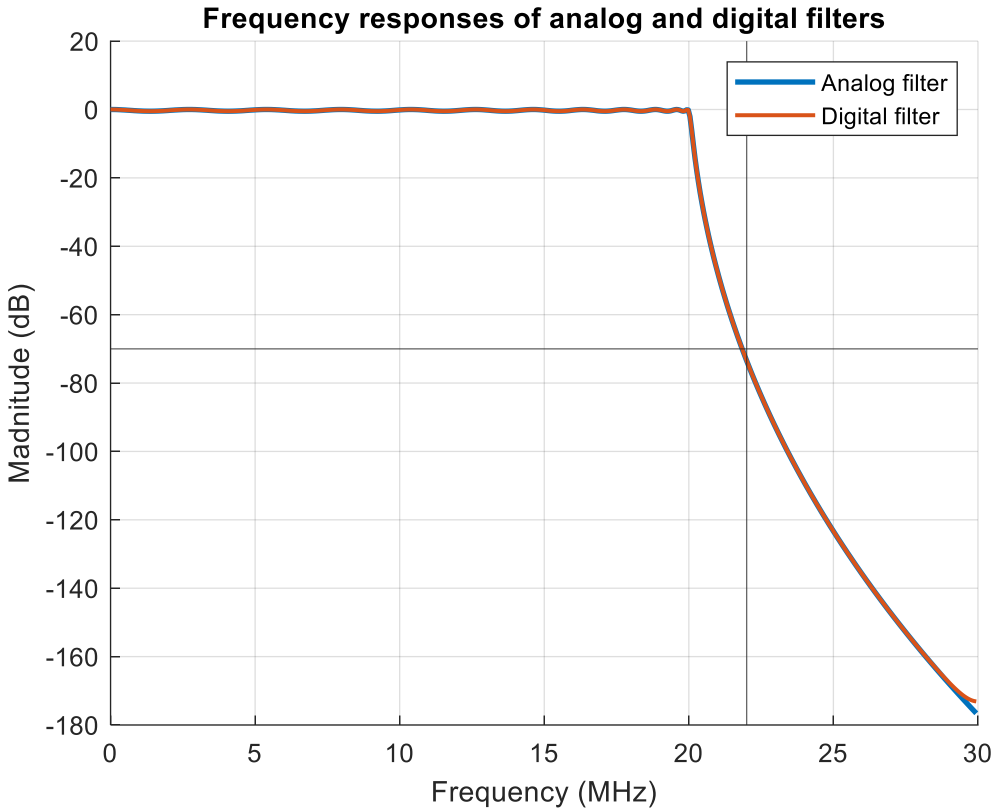
$$h(t) = \sum_k r_k e^{p_k t}$$

Impulse invariant method

$$H(s) \xrightarrow{L^{-1}} h(t) \xrightarrow{t=nT_s} h[t] \xrightarrow{Z} H(z)$$



The impulse response for both analog and digital systems



The magnitude response for analog and digital systems in the frequency domain

```
Fpass = 20; Fstop = 22; Fs = 60; Rp = 0.5; Rs = 70;
n = cheb1ord(2*Fpass/Fs,2*Fstop/Fs,Rp,Rs,'s');
[b,a] = cheby1(n,Rp,2*pi*Fpass,'s');    %analog filter

figure(1);hold on;
[bz,az] = impinvar(b,a,Fs);     %digital prototype of the analog filter
[r,p] = residue(b,a);    %direct term of a Partial Fraction Expansion

t = linspace(0,100/Fs,1000);
h = real(r.'*exp(p.*t)/Fs);     %analog filter impulse response
plot(t,h)

impz(bz,az,[],Fs);  %digital filter impulse invariance
legend('Analog filter','Digital filter'),xlim([0 1.5])

figure(2);hold on;grid on;
[h,w] = freqz(bz,az);
[h_an] = freqs(b,a,w*Fs);

h_db = 20*log10(abs(h));
h_an_db = 20*log10(abs(h_an));

plot(w/pi*Fs/2,h_an_db);
plot(w/pi*Fs/2,h_db);

legend('Analog filter','Digital filter');xline(22);yline(-70);
title('Frequency responses of analog and digital filters');
ylabel('Madnitude (dB)'); xlabel('Frequency (MHz)');
```

Problem 2

Implement a digital prototype of the analog filter with the transfer function

$$H(s) = \frac{s + 2.5}{s^2 + 2.5s + 4}$$

using the Bilinear Transformation. The sample clock frequency is Fs=20Hz.

a) Determine the Linear Difference Equation of the digital filter.

b) Plot impulse and frequency responses for digital and analog filters. Provide code.

Solution:

Bilinear transformation equivalent to the substitution

$$s = \left(\frac{2Fs(z-1)}{(z+1)}\right)$$

the transfer function of the analog filter $H(s)$

$$H(z) = \frac{\left(\frac{2Fs(z-1)}{(z+1)}\right) + 2.5}{\left(\frac{2Fs(z-1)}{(z+1)}\right)^2 + 2.5\left(\frac{2Fs(z-1)}{(z+1)}\right) + 4} = \cdots = \frac{85z^2 + 10z - 75}{3408z^2 - 6384z + 3008}$$

I am too lazy to perform all transformations, so I just do this (in mathcad):

$$s := \left[\frac{2\cdot Fs\cdot(z-1)}{z+1}\right] \qquad Hs := \frac{s + 2.5}{s^2 + 2.5s + 4} \qquad Hs \text{ substitute}, Fs = 20 \rightarrow \frac{85.0\cdot z^2 + 10.0\cdot z - 75.0}{3408.0\cdot z^2 - 6384.0\cdot z + 3008.0}$$

$$\frac{\left(85.0\cdot z^2 + 10.0\cdot z - 75.0\right)}{3.408} \text{ coeffs} \rightarrow \begin{pmatrix} -22.007042253521126761 \\ 2.9342723004694835681 \\ 24.941314553990610329 \end{pmatrix}$$

$$\frac{\left(3408.0\cdot z^2 - 6384.0\cdot z + 3008.0\right)}{3408} \text{ coeffs} \rightarrow \begin{pmatrix} 0.88262910798122065728 \\ -1.8732394366197183099 \\ 1.0 \end{pmatrix}$$

$$H(z) = \frac{85z^2 + 10z - 75}{3408z^2 - 6384z + 3008} = \frac{z^2(85 + 10z^{-1} - 75z^{-2})}{z^2(3408 - 6384z^{-1} + 3008z^{-2})} = \frac{Y(z)}{X(z)}$$

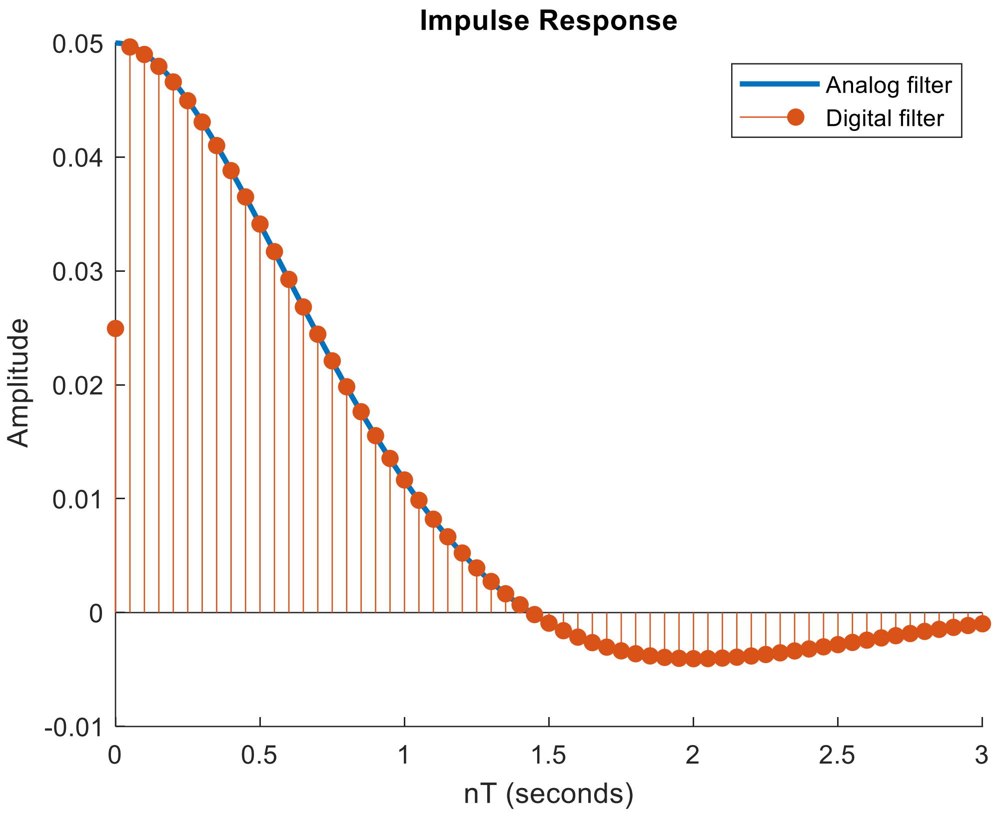
$$Y(z)(3408 + 6384z^{-1} + 3008z^{-2}) = X(z)(85 + 10z^{-1} - 75z^{-2})$$

$$3408y[n] - 6384y[n-1] + 3008y[n-2] = 85x[n] + 10x[n-1] - 75x[n-2]$$

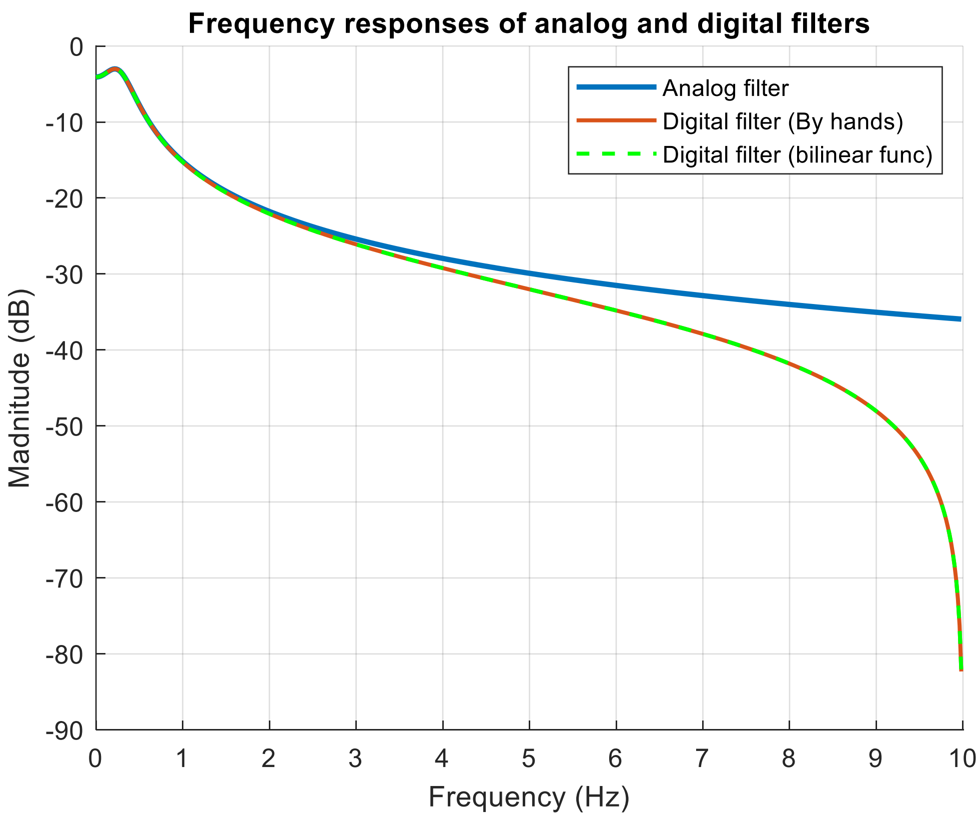
$$y[n] = \frac{6384}{3408}y[n-1] - \frac{3008}{3408}y[n-2] + \frac{85}{3408}x[n] + \frac{10}{3408}x[n-1] - \frac{75}{3408}x[n-2]$$

$$a = [3408 \quad -6384 \quad 3008]/3408; \; b = [85 \quad 10 \quad -75]/3408$$

$$a = [1 \quad -1.87 \quad 0.88]; \; b = [24.94 \quad 2.93 \quad -22]\cdot 10^{-3}$$



The impulse response for both analog and digital systems



The magnitude response for analog and digital systems in the frequency domain

```
Fs = 20;
as = [1 2.5 4];      az = [3408  -6384   3008]/3408;
bs = [0 1 2.5];      bz = [85    10      -75  ]/3408;

figure(1);hold on;
[r,p] = residue(bs,as);   %direct term of a Partial Fraction Expansion

t = linspace(0,100/Fs,1000);
h = real(r.'*exp(p.*t)/Fs);     %analog filter impulse response
plot(t,h,'LineWidth',2)
impz(bz,az,[],Fs);  %digital filter impulse invariance

legend('Analog filter','Digital filter'),xlim([0 3])

figure(2);hold on;grid on;
[h,w] = freqz(bz,az);
[h_an] = freqs(bs,as,w*Fs);

h_db = 20*log10(abs(h));
h_an_db = 20*log10(abs(h_an));

plot(w/pi*Fs/2,h_an_db,'LineWidth',2);
plot(w/pi*Fs/2,h_db,'LineWidth',1.5);

%%compare to MATLAB
[bz,az] = bilinear(bs,as,Fs);     %digital prototype of the analog filter
[h,w] = freqz(bz,az);
h_db = 20*log10(abs(h));
plot(w/pi*Fs/2,h_db,'g--','LineWidth',1.5);

legend('Analog filter','Digital filter (By hands)','Digital filter (bilinear func)');
title('Frequency responses of analog and digital filters');
ylabel('Madnitude (dB)'); xlabel('Frequency (Hz)');
```

Problem 3

A filter has the transfer function

$$H(z) = 3z^{-0} + 4z^{-1} + 6z^{-2} + 8z^{-3}$$

Determine the impulse response of the filter with the modified frequency response

$$F(\omega) = H\left(\omega - \frac{3\pi}{4}\right)$$

Solution:

First of all, let's move from Z-domain to frequency domain. For this substitute $z = re^{j\omega}$ to the expressions for the $H(z)$. Note: for simplicity I defined $r = 1$ so $z = e^{j\omega}$. I can do it, because r is just a constant (like a scaling coefficient for each summand).

$$H(\omega) = 3e^{-0j\omega} + 4e^{-1j\omega} + 6e^{-2j\omega} + 8e^{-3j\omega}$$

Then for our task substitute modified frequency: $\omega' = \omega - \frac{3\pi}{4}$

$$H(\omega) = 3e^{-0j\left(\omega-\frac{3\pi}{4}\right)} + 4e^{-1j\left(\omega-\frac{3\pi}{4}\right)} + 6e^{-2j\left(\omega-\frac{3\pi}{4}\right)} + 8e^{-3j\left(\omega-\frac{3\pi}{4}\right)} =$$

$$H(\omega) = 3e^{-0j\omega}e^{0j\frac{3\pi}{4}} + 4e^{-1j\omega}e^{1j\frac{3\pi}{4}} + 6e^{-2j\omega}e^{2j\frac{3\pi}{4}} + 8e^{-j3\omega}e^{3j\frac{3\pi}{4}}$$

So, then I need to calculate inverse DTFT:

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(\omega)e^{j\omega n}d\omega$$

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(3e^{-0j\omega}e^{0j\frac{3\pi}{4}} + 4e^{-1j\omega}e^{j\frac{3\pi}{4}} + 6e^{-2j\omega}e^{2j\frac{3\pi}{4}} + 8e^{-j3\omega}e^{3j\frac{3\pi}{4}}\right)e^{j\omega n}d\omega =$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(3e^{(n-0)j\omega}e^{0j\frac{3\pi}{4}} + 4e^{(n-1)j\omega}e^{j\frac{3\pi}{4}} + 6e^{(n-2)j\omega}e^{2j\frac{3\pi}{4}} + 8e^{(n-3)j\omega}e^{3j\frac{3\pi}{4}}\right)d\omega$$

$$= \frac{1}{2\pi}\left(3e^{0j\frac{3\pi}{4}}\frac{e^{j\omega(n-0)}}{j(n-0)}\bigg|_{-\pi}^{\pi} + 4e^{1j\frac{3\pi}{4}}\frac{e^{j\omega(n-1)}}{j(n-1)}\bigg|_{-\pi}^{\pi} + 6e^{2j\frac{3\pi}{4}}\frac{e^{j\omega(n-2)}}{j(n-2)}\bigg|_{-\pi}^{\pi} + 8e^{3j\frac{3\pi}{4}}\frac{e^{j\omega(n-3)}}{j(n-3)}\bigg|_{-\pi}^{\pi}\right)$$

$$= \frac{1}{2\pi}\left(3e^{0j\frac{3\pi}{4}}\left(\frac{e^{j\pi(n-0)} - e^{-j\pi(n-0)}}{j(n-0)}\right) + 4e^{1j\frac{3\pi}{4}}\left(\frac{e^{j\pi(n-1)} - e^{-j\pi(n-1)}}{j(n-1)}\right) + \cdots\right.$$

$$\left. + 6e^{2j\frac{3\pi}{4}}\left(\frac{e^{j\pi(n-2)} - e^{-j\pi(n-2)}}{j(n-2)}\right) + 8e^{3j\frac{3\pi}{4}}\left(\frac{e^{j\pi(n-3)} - e^{-j\pi(n-3)}}{j(n-3)}\right)\right)$$

$$= \frac{2}{2\pi}\left(\left(\frac{3e^{0j\frac{3\pi}{4}}}{(n-0)}\right)\left(\frac{e^{j\pi(n-0)} - e^{-j\pi(n-0)}}{2j}\right) + \left(\frac{4e^{1j\frac{3\pi}{4}}}{(n-1)}\right)\left(\frac{e^{j\pi(n-1)} - e^{-j\pi(n-1)}}{2j}\right) + \cdots\right.$$

$$\left. + \left(\frac{6e^{2j\frac{3\pi}{4}}}{(n-2)}\right)\left(\frac{e^{j\pi(n-2)} - e^{-j\pi(n-2)}}{2j}\right) + \left(\frac{8e^{3j\frac{3\pi}{4}}}{(n-3)}\right)\left(\frac{e^{j\pi(n-3)} - e^{-j\pi(n-3)}}{2j}\right)\right) =$$

$$= \frac{1}{\pi}\left(3e^{0j\frac{3\pi}{4}}\left(\frac{\sin(\pi(n-0))}{(n-0)}\right) + 4e^{j\frac{3\pi}{4}}\left(\frac{\sin(\pi(n-1))}{(n-1)}\right) + 6e^{2j\frac{3\pi}{4}}\left(\frac{\sin(\pi(n-2))}{(n-2)}\right) + 8e^{3j\frac{3\pi}{4}}\left(\frac{\sin(\pi(n-3))}{(n-3)}\right)\right) =$$

$$= 3e^{0j\frac{3\pi}{4}}\left(\frac{\sin\big(\pi(n-0)\big)}{\pi(n-0)}\right) + 4e^{j\frac{3\pi}{4}}\left(\frac{\sin\big(\pi(n-1)\big)}{\pi(n-1)}\right) + 6e^{2j\frac{3\pi}{4}}\left(\frac{\sin\big(\pi(n-2)\big)}{\pi(n-2)}\right) + 8e^{3j\frac{3\pi}{4}}\left(\frac{\sin\big(\pi(n-3)\big)}{\pi(n-3)}\right)$$

$$= 3e^{0j\frac{3\pi}{4}}\operatorname{sinc}\big(\pi(n-0)\big) + 4e^{j\frac{3\pi}{4}}\operatorname{sinc}\big(\pi(n-1)\big) + 6e^{j\frac{6\pi}{4}}\operatorname{sinc}\big(\pi(n-2)\big) + 8e^{j\frac{9\pi}{4}}\operatorname{sinc}\big(\pi(n-3)\big) =$$

$$3\delta(n) + 4\left(\frac{i-1}{\sqrt{2}}\right)\delta(n-1) + 6\left(\frac{i}{\sqrt{2}}\right)\delta(n-2) + 8\left(\frac{i+1}{\sqrt{2}}\right)\delta(n-3)$$

$$h[n] = \begin{cases} 3, & n = 0 \\ 4\left(\frac{i-1}{\sqrt{2}}\right), & n = 1 \\ 6\left(\frac{i}{\sqrt{2}}\right), & n = 2 \\ 8\left(\frac{i+1}{\sqrt{2}}\right), & n = 3 \\ 0, & otherwise \end{cases}$$

Problem 4

For a linear system with the transfer function

$$H(z) = \frac{z+1}{z^3 + z^2 + 2z^1 + 2}$$

a) Calculate the difference equation relating the input x[n] to the output y[n]

b) Design block diagram realizations (Direct-Form 1 and Direct-Form 2)

c) Plot impulse and frequency responses Provide code.

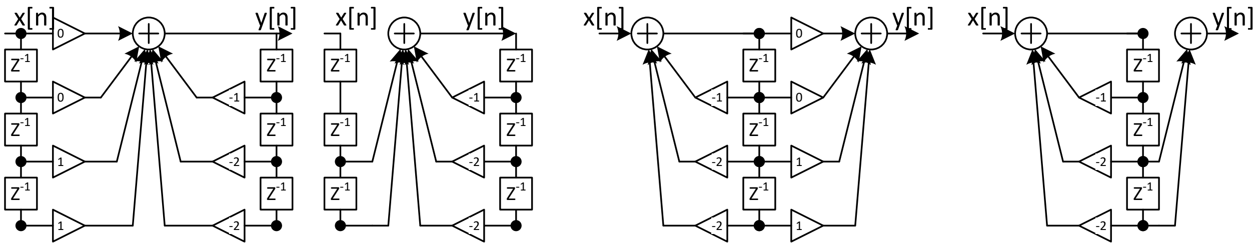
$$H(z) = \frac{z+1}{z^3 + z^2 + 2z^1 + 2} = \frac{z^3(z^{-2} + z^{-3})}{z^3(1 + z^{-1} + 2z^{-2} + 2z^{-3})} = \frac{Y(z)}{X(z)}$$

$$Y(z)(1 + z^{-1} + 2z^{-2} + 2z^{-3}) = X(z)(z^{-2} + z^{-3})$$

$$y[n] + y[n-1] + 2y[n-2] + 2y[n-3] = x[n-2] + x[n-3]$$

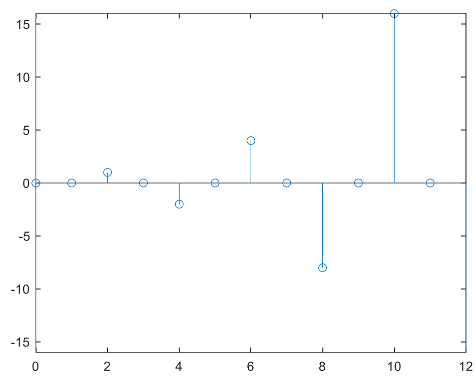
$$y[n] = x[n-2] + x[n-3] - y[n-1] - 2y[n-2] - 2y[n-3]$$

$$a = [1 \quad 1 \quad 2 \quad 2]; b = [0 \quad 0 \quad 1 \quad 1]$$

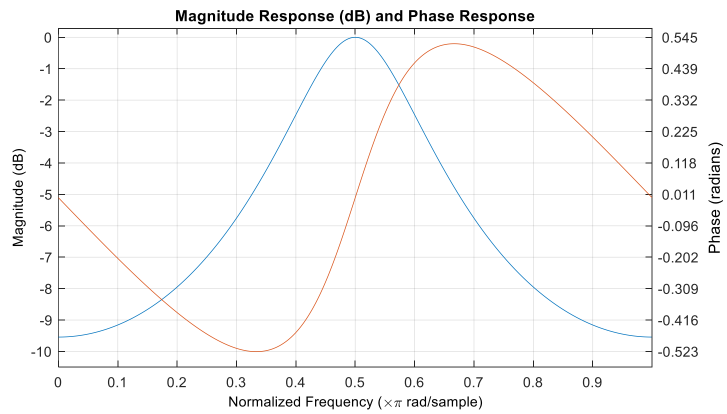


Direct-Form 1 Direct-Form 2

3.9661 3.9661



impulse response



Frequency&phase response

```
x = zeros(1,16); x(4) = 1;
y = zeros(1,16);
for n = 4:16
    y(n) = x(n-2)+x(n-3)-y(n-1)-2*y(n-2)-2*y(n-3);
end
figure(1);stem(0:12,y(4:end))
xlim([0 12]);ylim([-16 16]);

b = [0 0 1 1] ;
a = [1 1 2 2] ;
[h,t] = impz(b,a);
figure(1);stem(t,h)
xlim([0 12]);ylim([-16 16]);
[h,w] = freqz(b,a);
figure(2); plot(w,20*log10(abs(h)))
title('Frequency response of the linear system')
ylabel('Madnitude (dB)');xlabel('Normalized frequency(x\pi rad)');
```

Problem 5

Using 10-steps CORDIC algorithm, calculate

Justify the approach. Compare with the actual value. Provide code.

a) arctan (1.5)	b) abs (2.2+3.3*j)
Minimization was performed by Y; Start Y0 and X0 value was set to 3.3 and 2.2. correspondingly. Then final Z will consist result of the atan(Y0/X0);	*Minimization was performed by Y; Start Y0 and X0 value was set to 3.3 and 2.2. Then final X will consist result of the abs(3.3 + j2.2)*K, where K is constant equal to 0.60725235*

clear all
j = 0:9; tn = 2.^(-j);
a = [45 26.6 14 7.1 3.6 1.8 0.9 0.4 0.2 0.1];
k = 0.607;
x(1) = 2.2; y(1) = 3.3; z(1) = 0;
for i = 1:10
    d = -sign(y(i));
    x(i+1) = x(i) - d * tn(i) *y(i);
    y(i+1) = y(i) + d * tn(i) *x(i);
    z(i+1) = z(i) - d * a(i);
end
[z(11) atand(y(1)/x(1))]
	clear all
	j = 0:9; tn = 2.^(-j);
	a = [45 26.6 14 7.1 3.6 1.8 0.9 0.4 0.2 0.1];
	k = 0.60725235;
	x(1) = 2.2; y(1) = 3.3; z(1) = 0;
	for i = 1:10
	    d = -sign(y(i));
	    x(i+1) = x(i) - d * tn(i) *y(i);
	    y(i+1) = y(i) + d * tn(i) *x(i);
	    z(i+1) = z(i) - d * a(i);
	end
	[x(11)*k abs([x(1)+y(1)*1i])]

56.5000 56.3099	3.9661 3.9661