DSP Homework 3 *Mitchenkov Dmitriy*

# **Problem 1**

Using the impulse invariance method for analog to digital filter conversion, calculate the Chebyshev lowpass digital filter with parameters: passband frequency 20MHz; stopband frequency = 22MHz; passband ripple 0.5dB; stopband (out-of-band) attenuation 70dB; sampling frequency Fs = 60MHz.

a) Plot the impulse response for both analog and digital systems.

b) Plot the magnitude response for analog and digital systems in the frequency domain. Provide code.

Generation of and for filter transfer function:

Calculation of residues and poles pk by partial function expansion of ratio of two polynomials:

Laplace transform to get impulse response :

Impulse invariant method



The impulse response for both analog and digital systems



The magnitude response for analog and digital systems in the frequency domain

Fpass = 20; Fstop = 22; Fs = 60; Rp = 0.5; Rs = 70;

n = cheb1ord(2\*Fpass/Fs,2\*Fstop/Fs,Rp,Rs,'s');

[b,a] = cheby1(n,Rp,2\*pi\*Fpass,'s'); %analog filter

figure(1);hold on;

[bz,az] = impinvar(b,a,Fs); %digital prototype of the analog filter

[r,p] = residue(b,a); %direct term of a Partial Fraction Expansion

t = linspace(0,100/Fs,1000);

h = real(r.'\*exp(p.\*t)/Fs); %analog filter impulse response

plot(t,h)

impz(bz,az,[],Fs); %digital filter impulse invariance

legend('Analog filter','Digital filter'),xlim([0 1.5])

figure(2);hold on;grid on;

[h,w] = freqz(bz,az);

[h\_an] = freqs(b,a,w\*Fs);

h\_db = 20\*log10(abs(h));

h\_an\_db = 20\*log10(abs(h\_an));

plot(w/pi\*Fs/2,h\_an\_db);

plot(w/pi\*Fs/2,h\_db);

legend('Analog filter','Digital filter');xline(22);yline(-70);

title('Frequency responses of analog and digital filters');

ylabel('Madnitude (dB)'); xlabel('Frequency (MHz)');

# **Problem 2**

Implement a digital prototype of the analog filter with the transfer function

using the Bilinear Transformation. The sample clock frequency is Fs=20Hz.

a) Determine the Linear Difference Equation of the digital filter.

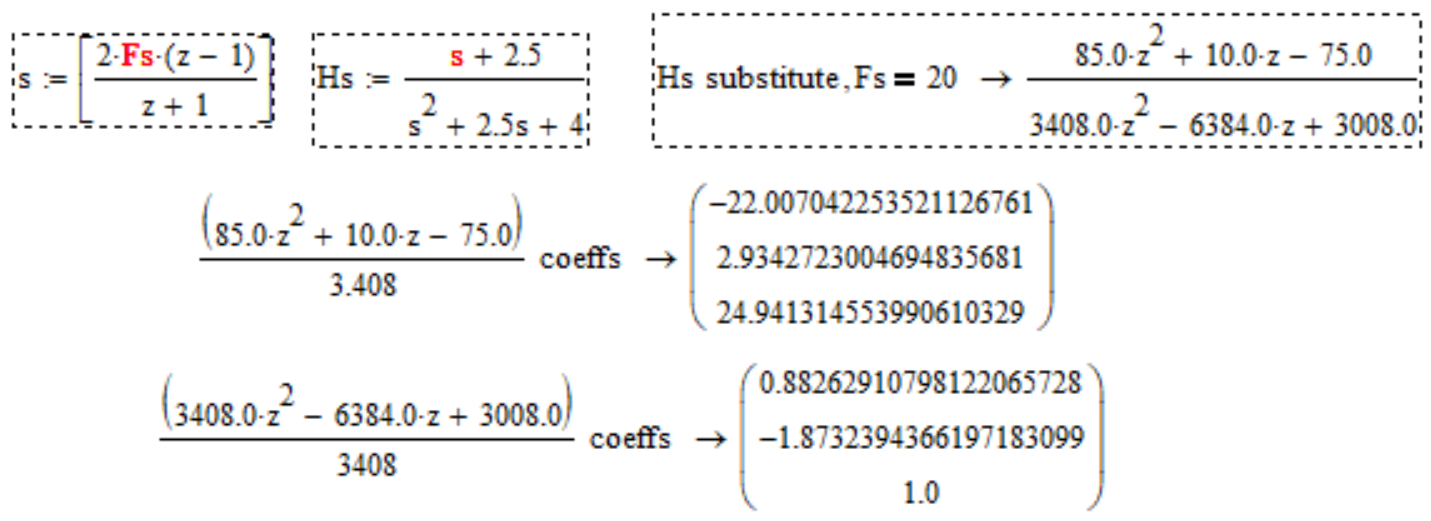
b) Plot impulse and frequency responses for digital and analog filters. Provide code.

**Solution:**

Bilinear transformation equivalent to the substitution

the transfer function of the analog filter

I am too lazy to perform all transformations, so I just do this (in mathcad):





The impulse response for both analog and digital systems



The magnitude response for analog and digital systems in the frequency domain

Fs = 20;

as = [1 2.5 4]; az = [3408 -6384 3008]/3408;

bs = [0 1 2.5]; bz = [85 10 -75 ]/3408;

figure(1);hold on;

[r,p] = residue(bs,as); %direct term of a Partial Fraction Expansion

t = linspace(0,100/Fs,1000);

h = real(r.'\*exp(p.\*t)/Fs); %analog filter impulse response

plot(t,h,'LineWidth',2)

impz(bz,az,[],Fs); %digital filter impulse invariance

legend('Analog filter','Digital filter'),xlim([0 3])

figure(2);hold on;grid on;

[h,w] = freqz(bz,az);

[h\_an] = freqs(bs,as,w\*Fs);

h\_db = 20\*log10(abs(h));

h\_an\_db = 20\*log10(abs(h\_an));

plot(w/pi\*Fs/2,h\_an\_db,'LineWidth',2);

plot(w/pi\*Fs/2,h\_db,'LineWidth',1.5);

%%compare to MATLAB

[bz,az] = bilinear(bs,as,Fs); %digital prototype of the analog filter

[h,w] = freqz(bz,az);

h\_db = 20\*log10(abs(h));

plot(w/pi\*Fs/2,h\_db,'g--','LineWidth',1.5);

legend('Analog filter','Digital filter (By hands)','Digital filter (bilinear func)');

title('Frequency responses of analog and digital filters');

ylabel('Madnitude (dB)'); xlabel('Frequency (Hz)');

# **Problem 3**

A filter has the transfer function

Determine the impulse response of the filter with the modified frequency response

**Solution:**

First of all, let’s move from Z-domain to frequency domain. For this substitute to the expressions for the . Note: for simplicity I defined so . I can do it, because is just a constant (like a scaling coefficient for each summand).

Then for our task substitute modified frequency:

So, then I need to calculate inverse DTFT:

# **Problem 4**

For a linear system with the transfer function

a) Calculate the difference equation relating the input x[n] to the output y[n]

b) Design block diagram realizations (Direct-Form 1 and Direct-Form 2)

c) Plot impulse and frequency responses Provide code.

|  |  |
| --- | --- |
|  |  |
| Direct-Form 1 | Direct-Form 2 |

3.9661 3.9661

|  |  |
| --- | --- |
|  |  |
| impulse response | Frequency&phase response |

x = zeros(1,16); x(4) = 1;

y = zeros(1,16);

for n = 4:16

y(n) = x(n-2)+x(n-3)-y(n-1)-2\*y(n-2)-2\*y(n-3);

end

figure(1);stem(0:12,y(4:end))

xlim([0 12]);ylim([-16 16]);

b = [0 0 1 1] ;

a = [1 1 2 2] ;

[h,t] = impz(b,a);

figure(1);stem(t,h)

xlim([0 12]);ylim([-16 16]);

[h,w] = freqz(b,a);

figure(2); plot(w,20\*log10(abs(h)))

title('Frequency response of the linear system')

ylabel('Madnitude (dB)');xlabel('Normalized frequency(x\pi rad)');

# **Problem 5**

Using 10-steps CORDIC algorithm, calculate

Justify the approach. Compare with the actual value. Provide code.

|  |  |
| --- | --- |
| a) arctan (1.5) | b) abs (2.2+3.3\*j) |
| *Minimization was performed by Y; Start Y0 and X0 value was set to 3.3 and 2.2. correspondingly. Then final Z will consist result of the atan(Y0/X0);* | *Minimization was performed by Y; Start Y0 and X0 value was set to 3.3 and 2.2. Then final X will consist result of the abs(3.3 + j2.2)\*K, where K is constant equal to 0.60725235* |
| clear all  j = 0:9; tn = 2.^(-j);  a = [45 26.6 14 7.1 3.6 1.8 0.9 0.4 0.2 0.1];  k = 0.607;  x(1) = 2.2; y(1) = 3.3; z(1) = 0;  for i = 1:10  d = -sign(y(i));  x(i+1) = x(i) - d \* tn(i) \*y(i);  y(i+1) = y(i) + d \* tn(i) \*x(i);  z(i+1) = z(i) - d \* a(i);  end  [z(11) atand(y(1)/x(1))] | clear all  j = 0:9; tn = 2.^(-j);  a = [45 26.6 14 7.1 3.6 1.8 0.9 0.4 0.2 0.1];  k = 0.60725235;  x(1) = 2.2; y(1) = 3.3; z(1) = 0;  for i = 1:10  d = -sign(y(i));  x(i+1) = x(i) - d \* tn(i) \*y(i);  y(i+1) = y(i) + d \* tn(i) \*x(i);  z(i+1) = z(i) - d \* a(i);  end  [x(11)\*k abs([x(1)+y(1)\*1i])] |
| 56.5000 56.3099 | 3.9661 3.9661 |

