# DOING PHYSICS WITH PYTHON

# COMPUTATIONAL OPTICS

# RAY (GEOMETRIC) OPTICS

# FERMAT'S PRINCIPLE

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## DOWNLOAD DIRECTORIES FOR PYTHON CODE

Google drive

GitHub

**S001.py**
Tise Python Code is used to verify that Fermat's principle leads to the laws of reflection and refraction

## FERMAT'S PRINCIPLE: Reflection and Refraction

**Ray optics** (geometrical optics) is the simplest theory of optics where a **ray** is a set of straight lines representing the path of light through an optical system. Ray optics provides a basic understanding of optics before one moves onto the study of **wave (physical) optics**.

The fundamental principle of ray optics is **Fermat's Principle** (**principle of least time**) Fermat's principle, states that light travels between two points along the path that takes the least amount of time. This principle is a fundamental concept in understanding how light behaves in different media and is the link between ray optics and wave optics.

We can define the **optical path length** (**OPL**) as the length of the path weighted by the local refractive index $n$.

$$L = \int_0^d n(r)\, dl$$

Fermat's principle states that the time interval for light to travel the path for light between two points is a minimum. This is why Fermat's principle is often referred to as the **principle of least time**.

We will show how Fermat's principle leads to the laws of reflection and Snell's law for refraction using the Python Code **S001.py**.

## Reflection

Consider an incident ray (AP) and the reflected ray (PB) from a mirror where the ray begins at A$\left(X_0, Y_0\right)$ and ends at the point B$\left(X_1, Y_1\right)$, and P$\left(0, y\right)$ is the set of points the along the Y axis on the mirror from which the incident ray is reflected. The refractive index $n$ of the medium is constant, $n$ = 1.

The OPL for the incident ray from A to P is

$$L_0 = n\sqrt{\left(0 - X_0\right)^2 + \left(y - Y_0\right)^2}$$

The OPL for the reflected ray from P to B is

$$L_1 = n\sqrt{\left(X_1 - 0\right)^2 + \left(Y_1 - y\right)^2}$$

The OPL for the ray from A to B is

$$L_{01} = L_0 + L_1$$

By Fermat's principle, the OPL $L_{01}$ is a minimum. We need to find the value of $y$ so that the OPL $L_{01}$ is a minimum and/or $dL_{01} / dy = 0$. We can do this in a Python Code with doing the algebra. The Python Code **S001.py** defines a square region 5 m x 5m for the reflection and refraction simulation.

Inputs

```
#%% INPUT PARAMETERS:
X = zeros(3); Y =zeros(3); n = zeros(3)
N = 9999      # Grid point
# Refractive indices
n[0] = 1.0; n[1] = 1.0; n[2] = 1.5
# A(X,Y) and B(X,Y) coordinates reflection -5 < X < 0  -5 < Y < 5
X[0] = -2;  Y[0] = 4        # point A
X[1] = -4;  Y[1] = -3       # point B
# C(X<Y) coordinates for refraction  0 < X < -5  -5 < Y < 5
X[2] = 3;   Y[2] = -4       # point C
```

Function to find distance between two points

```
def length(x1,y1,x2,y2,n):
   L = n*sqrt((x2-x1)**2 + (y2-y1)**2)
   return L
```

Function to find angles of incident $\theta_0$, reflection $\theta_1$, and refraction $\theta_2$

```
def Angle(y2,y1,x):
   A = np.arctan((abs((y2-y1)/x))) * (180/pi)
   return A
```

To find the minimum OPL

```
# Minimum OPL: Fermat's Principle
L01min = min(L01); L02min = min(L02)
y01 = y[L01==min(L01)][0]
y02 = y[L02==min(L02)][0]
```

To find the gradient $dL_{01} / dy$

```
m01 = np.gradient(L01,dy); m02 = np.gradient(L02,dy)
```

The results are displayed in the Console Window and in Figure Windows.

```
REFLECTION
L01(min) = 9.220  m
L01(min) --> P(0,y) y = 1.666  m
Incident angle  A0 = 49.403  deg
Reflected angle A1 = 49.397  deg
```

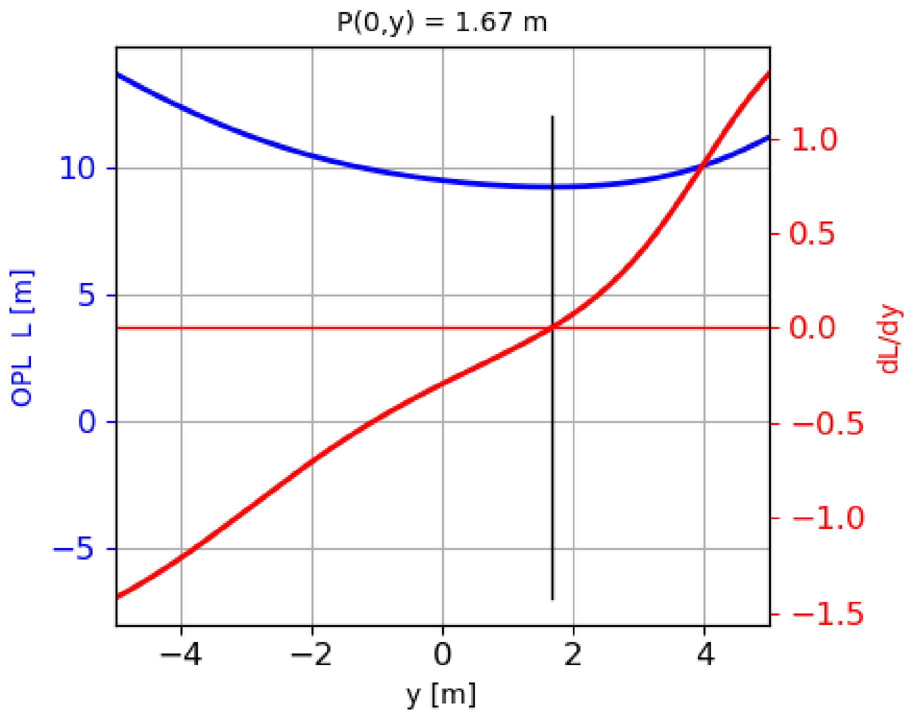


Fig. 1. The OPL $L_{01}$ as a function of $y$ (reflection points from the mirror) and the gradient function $dL_{01}/dy$.

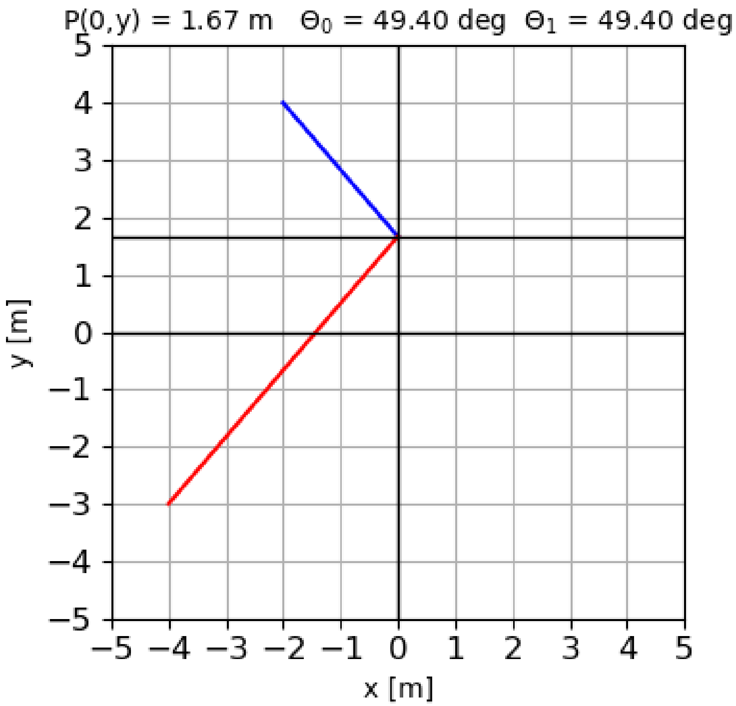


Fig. 2. The incident ray (**AP**) and the reflected ray (**PB**). The angle of incident is equal to the angle of reflection $\theta_0 = \theta_1$.

## Refraction

Consider an incident ray (AP) that begins at A$\left(X_0, Y_0\right)$ and the refracted ray (PC) that ends at the point C$\left(X_2, Y_2\right)$, and P$\left(0, y\right)$ is the set of points the along the Y axis which forms the interface between regions with refractive index $n_0$ and refractive index $n_2$.

The OPL for the incident ray from A to P is

$$L_0 = n\sqrt{\left(0 - X_0\right)^2 + \left(y - Y_0\right)^2}$$

The OPL for the refracted ray from P to C is

$$L_2 = n\sqrt{\left(X_2 - 0\right)^2 + \left(Y_2 - y\right)^2}$$

The OPL for the ray from A to C is

$$L_{02} = L_0 + L_2$$

The results are displayed in the Console Window and in Figure Windows.

REFRACTION

L02(min) = 11.709  m   L02(min) --> P(0,y)  y = -1.581  m

Incident angle  B0 = 70.285  deg

Reflected angle B1 = 38.877  deg

Snells Law:    n0*sin(B0) = 0.941      n2*sin(B2) = 0.941

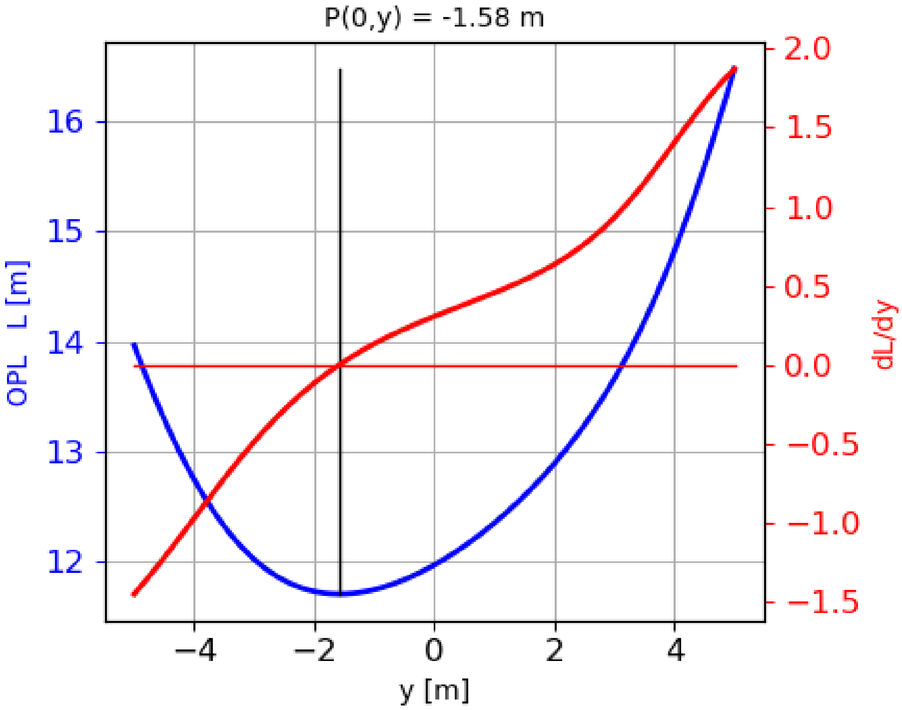


Fig. 3. The OPL $L_{02}$ as a function of $y$ (reflection points from the mirror) and the gradient function $dL_{02} / dy$.

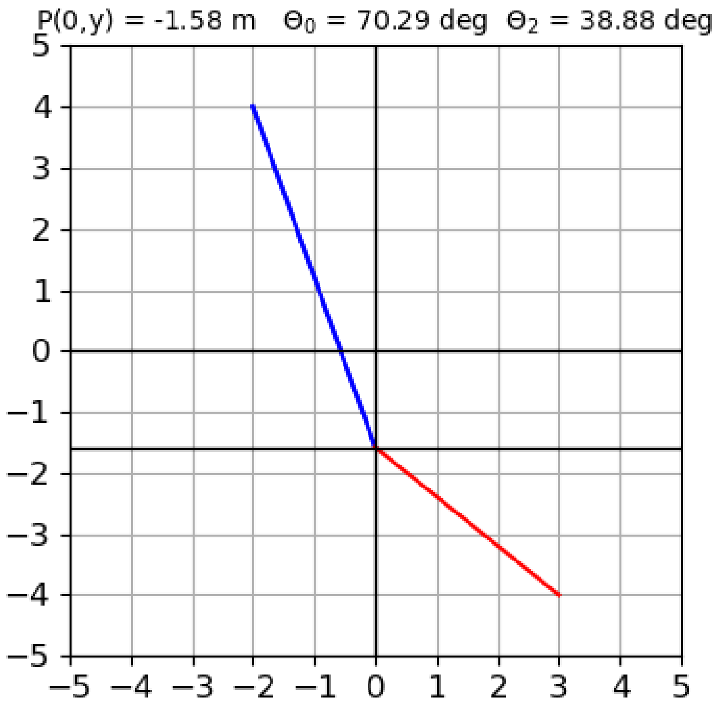


Fig. 2. The incident ray (**AP**) and the refracted ray (**PC**). Snell's law is satisfied: $n_0 \sin\theta_0 = n_2 \sin\theta_2$.