DOING PHYSICS WITH PYTHON

COMPUTATIONAL OPTICS

PHYSICIST'S HERMITE POLYNOMIALS

HERMITE-GAUSS FUNCTIONS

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Google drive

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emH01.PY Hermite polynomials and Hermite-Gauss functions

HERMITE POLYNOMIALS

The Hermite polynomials are a set of orthogonal functions widely encounter in many areas of science. The n^{th}-order physicist's Hermite polynomial is a polynomial of degree n, where the leading coefficient (highest power term) is 2^n. A few examples:

$$H_0(x) = 1$$

$$H_1(x) = 2$$

$$H_4(x) = 16x^4 - 48x^2 + 12$$

$$H_5(x) = 32x^4 - 160x^2 + 120$$

The Python function **hermite** can be used to give the polynomial, the coefficients of the polynomial and evaluate the function as shown in the following Python Code **emH01.py**.

```
import numpy as np
from numpy import pi, sin, cos, exp, linspace, zeros, amax, sqrt
import matplotlib.pyplot as plt
import time
from scipy import special
tStart = time.time()

n = 3
H = special.hermite(n, monic=False)
print('Hermite polynomial,coefficients, H(1)')
print(H)
```

```
print(H(1))
x = np.linspace(-3, 3, 400)
y = H(x)

plt.rcParams['font.size'] = 10
plt.rcParams["figure.figsize"] = (5,3)
fig1, ax = plt.subplots(nrows=1, ncols=1)
ax.plot(x, y,'b',lw = 2)
ax.set_title("Hermite polynomial of degree n
= %0.0f" %n,fontsize = 10)
ax.set_xlabel("x")
ax.set_ylabel("H$_n$(x)")
ax.grid()
fig1.tight_layout()
```

print(H) → $8x^3 - 12$

Console Window H → poly1d([8., 0., -12., 0.])

H(1) → -3.9999999999999987

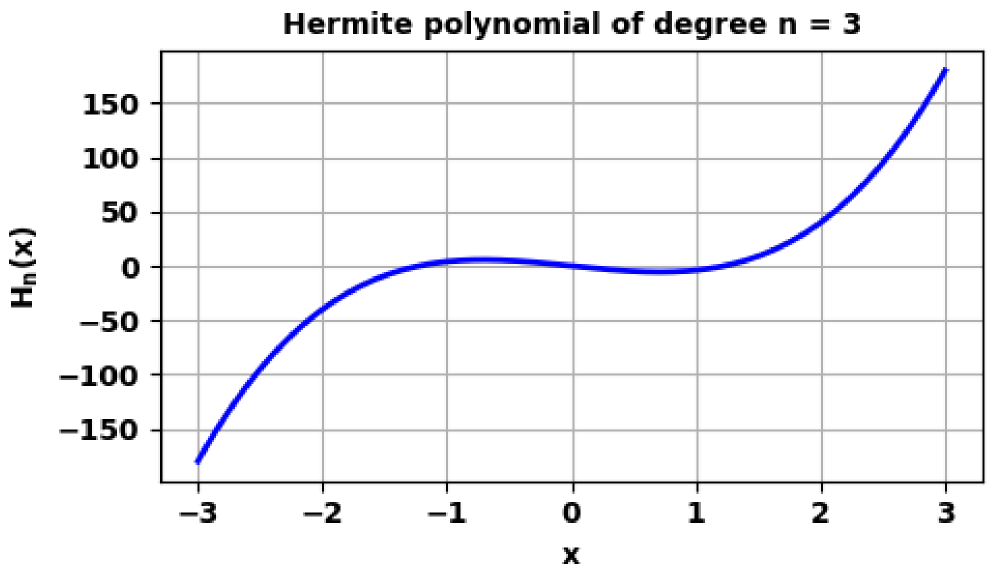

Fig. 1. Hermite polynomial $H_3(x) = 8x^3 - 12$.

HERMITE-GAUSS FUNCTIONS

For example, in spherical mirror resonators, the electric field for stable modes are described by Hermite-Gauss functions and are given by

$$E_{mn}(x,y) = E_0\, H_m\left(\frac{\sqrt{2}\,x}{w_0}\right) H_n\left(\frac{\sqrt{2}\,y}{w_0}\right) \exp\left(-\frac{x^2+y^2}{{w_0}^2}\right)$$

where m and n represent the transverse mode numbers, H are the Hermite-Gauss polynomials and w_0 is a characteristic mode width.

Figure 2 shows the XY electric field patterns of some low order modes of a stable resonator cavity formed by spherical mirrors. Figure 3 shows the patterns for the electric field and the intensity I, where $I = E^2$ for the transverse mode (3,3).

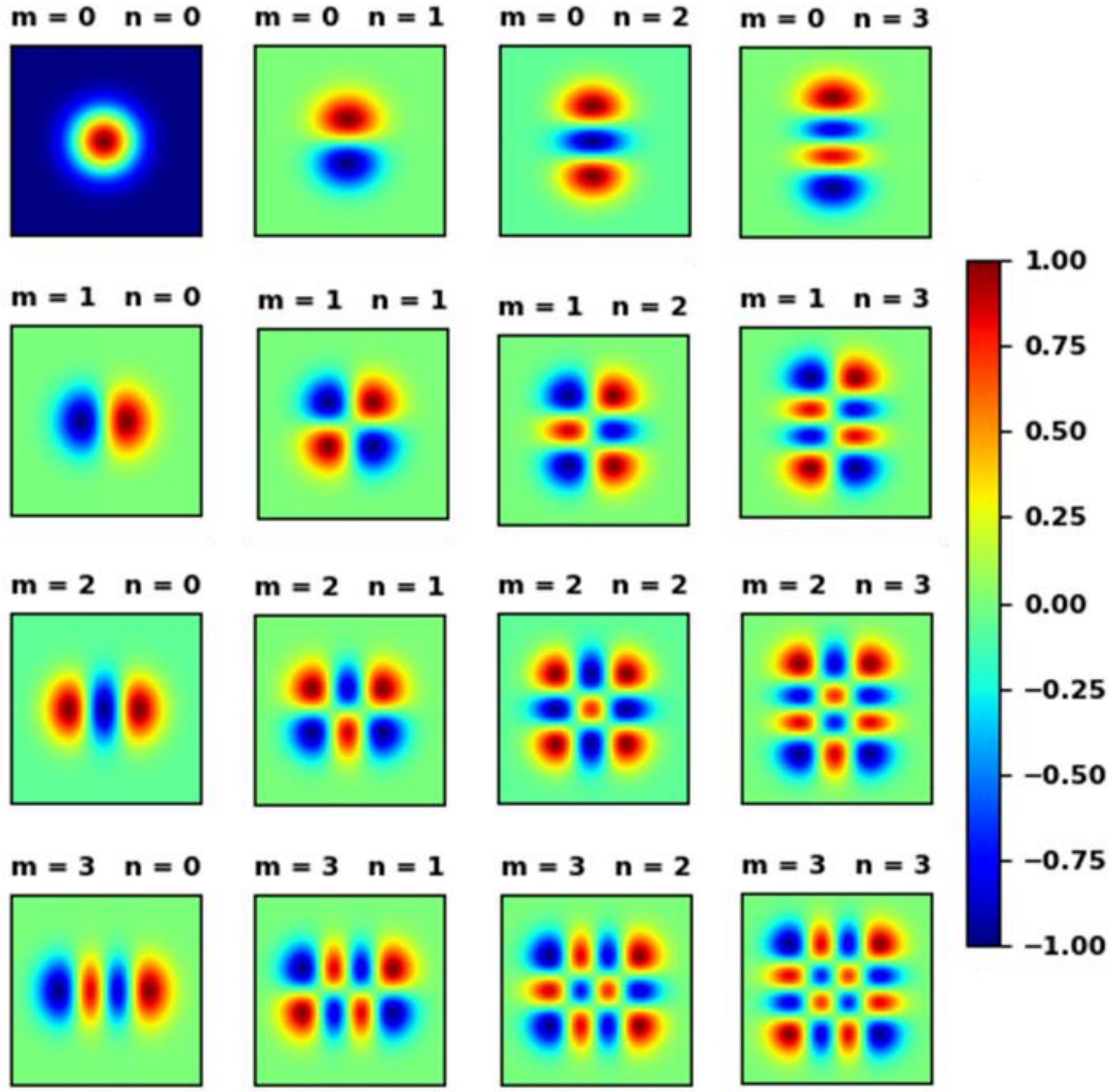

Fig. 2. Electric field lower order transverse resonator modes.

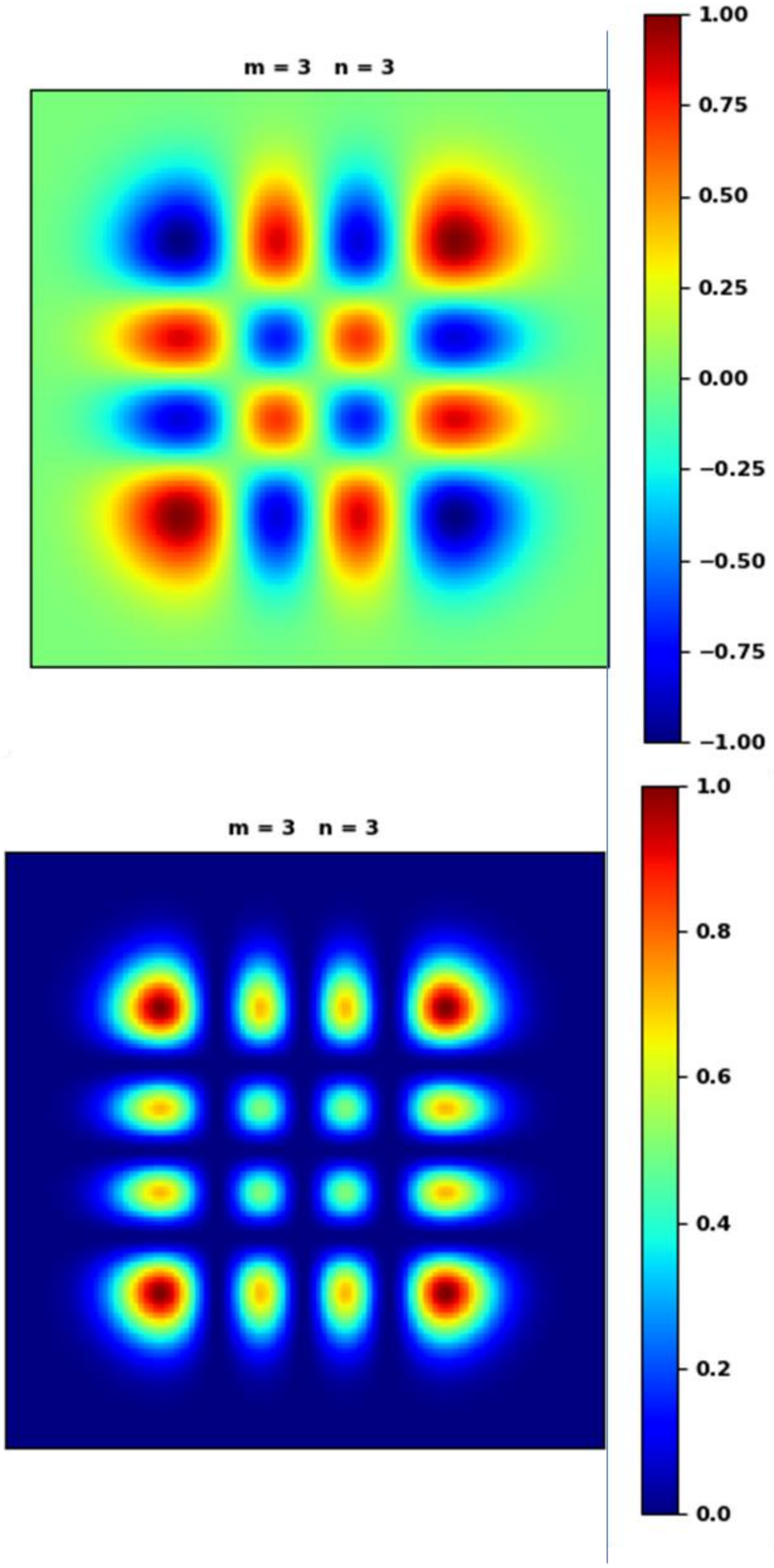

Fig. 3. Transverse mode (3,3) for the electric field (top) and intensity (bottom).