

**DOING PHYSICS WITH PYTHON**

**COMPUTATIONAL OPTICS**

**RS1 DIFFRACTION INTEGRAL:**

**FOCUSED BEAM**

**HALF-CIRCULAR APERTURE**

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**DOWNLOAD DIRECTORIES FOR PYTHON CODE**

[\*\*Google drive\*\*](#)

[\*\*GitHub\*\*](#)

**emRSFBXY2.py**

Calculation of the radiant flux density (irradiance) in a plane perpendicular to the optical axis for the radiant flux of convergent beam emitted from a circular aperture.

## **emRSFBZ2.py**

Calculation of the radiant flux density (irradiance) along the optical axis for the radiant flux of convergent beams emitted from a circular aperture.

**Warning:** The results of the integration may look OK but they may not be accurate if you have used insufficient number of partitions for the aperture space and observation space. It is best to check the convergence of the results as the number partitions is increased. Note: as the number of partitions increases, the calculation time **rapidly** increases.

It is necessary to modify the Python Codes and comment or uncomment lines of code to run the simulations with different input and output parameters.

**Link: essential reference**

[RS1 diffraction integral: Focused beam from a circular aperture](#)

## INTRODUCTION

A full circular aperture produces a diffraction pattern known as the Airy pattern, characterized by a bright central disk (Airy disk) surrounded by concentric rings of decreasing intensity.

When light from a beam passes through a half-circular aperture, the resulting diffraction pattern will exhibit asymmetries compared to a full circular aperture. The diffraction pattern will be a combination of the Airy disk pattern from the circular part of the aperture, with distortions and variations due to the truncation at the edge of the half-circle. The intensity distribution will be shifted and elongated, and the bright central region will be less circular and more stretched. The intensity distribution will no longer be perfectly circular. It will exhibit a shift in the centre, with the bright region being stretched along the truncation edge. The pattern will also show asymmetry in the rings surrounding the Airy disk, with a more pronounced intensity along the truncation edge compared to the opposite side.

The diffraction pattern from a half-circular aperture is computed by integrating the RS1 Diffraction integral using a [2D] form of Simpson's rule.

## SIMULATIONS

The aperture electric field  $E_Q(x_Q, y_Q, 0)$  is calculated over a rectangular grid of  $n_Q \times n_Q$  grid points. The electric field  $E_Q$  is non-zero in only the upper half of the circular aperture. The electric  $E_Q$  in the plane of the aperture field corresponds to a spherical wave converging to the focal point  $(x_S = 0, y_S = 0, z_S = f)$ .

Figure 1 shows a [2D] view of the aperture irradiance for the semi-circle (half-circle) aperture.

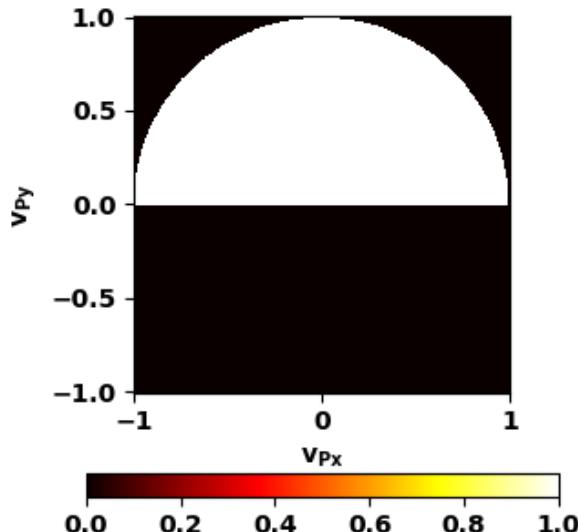


Fig. 1. Aperture irradiance. [emRSFBZ2.py](#)

Two simulations will be given as examples.

1. Small numerical aperture ( $NA$ ) and large Fresnel number ( $NR$ ).
2. Small numerical aperture ( $NA$ ) and small Fresnel number ( $NR$ ).

A summary of the simulation parameters is displayed in the Console Window and the plots in Figure Windows.

## Simulation 1

*Irradiance pattern along the optical axis near the focal point*

emRSFBZ2.py

NQ = 199 NP = 199

nQ = 399 nP = 399

wavelength wL = 500 nm

aperture radius a = 10.000 mm

Source

xS = 0.00 m yS = 0.00 m zS = 0.200 m

Focal length f = 0.200 m

Numerical aperture **NA = 0.0499**

Fresnel number NF = **1000.000**

u1 = -40.20 u2 = 40.20

lmax = 3.11e+28 a.u.

Max irradiance uP = -0.02

Execution time 10 s

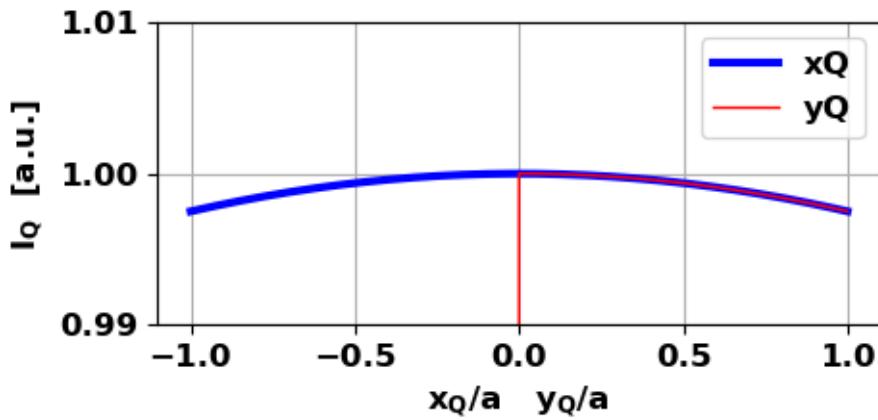


Fig. 1. The normalized aperture irradiance variations in X and Y axis direction where  $E_Q(x_Q, y_Q < 0, 0) = 0$ .

Figure 2 shows the irradiance dependence along the optical axis as a function of the axial radial coordinate  $u_P$ . The irradiance variation along the optical axis is identical to the full circular aperture. For the small numerical aperture and large Fresnel number, the Debye approximation is valid and the irradiance can be described by Fraunhofer diffraction. The **RS1** and the **Fraunhofer** diffraction patterns are both displayed as a linear scale and decibel scale.

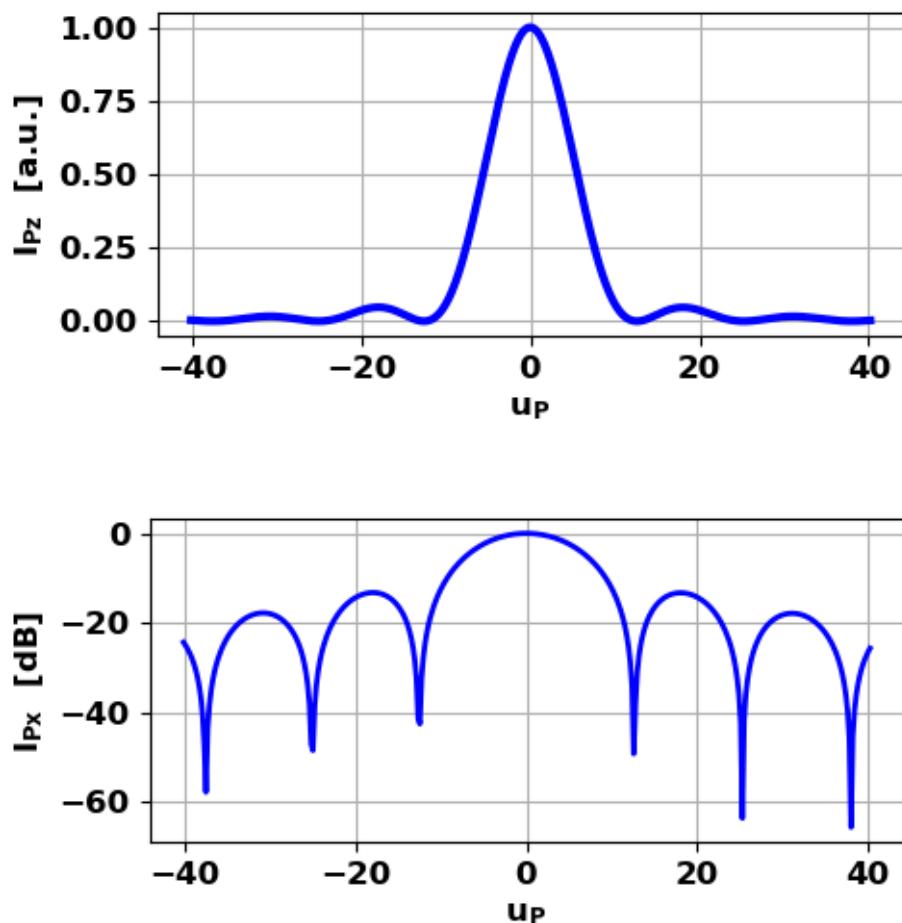


Fig. 2. The irradiance dependence along the optical axis as a function of the axial radial coordinate  $u_P$ . The irradiance pattern corresponds to Fraunhofer diffraction.

*Irradiance pattern in the XY focal plane surrounding the focal point*

emRSFBXY2.py

$nQ = 179$   $nP = 179$

wavelength  $wL = 500$  nm

aperture radius  $a = 10.000$  mm

Source

$xS = 0.00$  m  $yS = 0.00$  m  $zS = 0.20$  m

Focal length  $f = 0.20$  m

Numerical aperture N.A. = **0.04994**

Fresnel number NF = **1000.00**

$z_P = 0.20000$  m

$I_{max} = 1.25e+27$  a.u.

Execution time 75 s

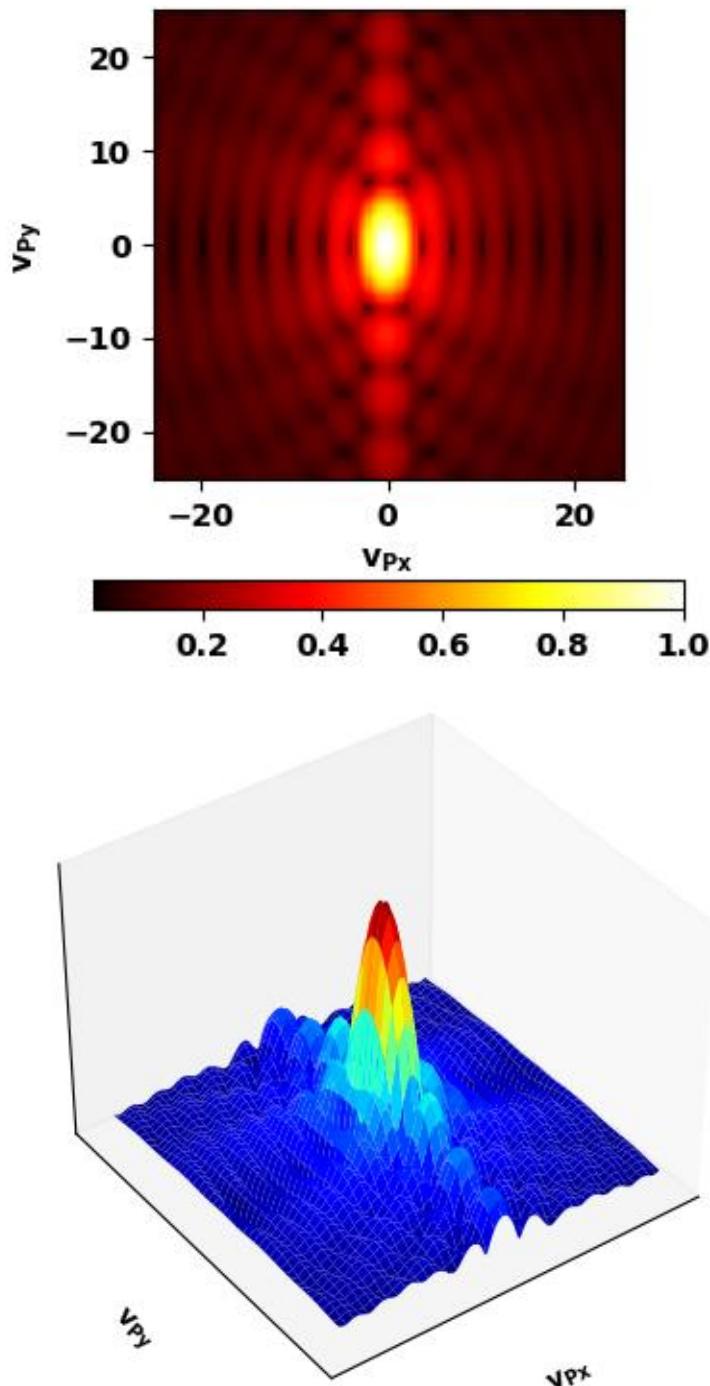


Fig. 3. [2D] and [3D] views of the irradiance in the XY focal plane.

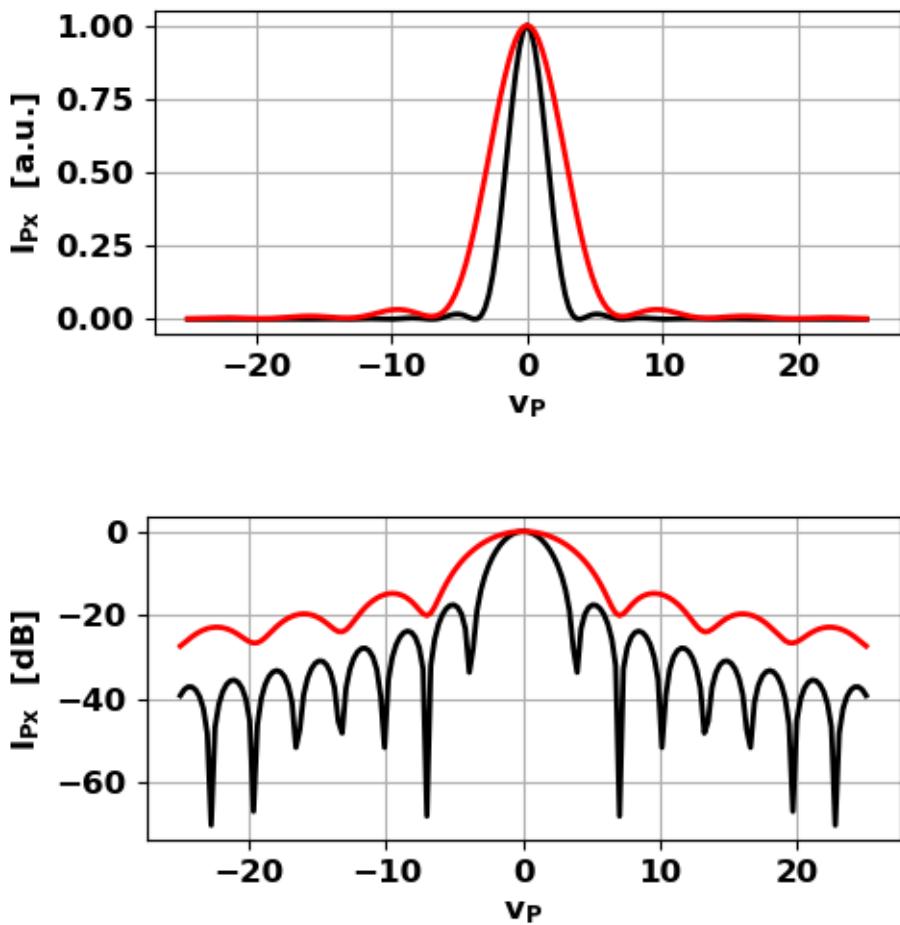


Fig. 4. Irradiance in the XY focal plane. **Black** curve pattern along the **X** axis and **red** curve pattern along the **Y** axis.

## Simulation 2

### *Irradiance pattern along the optical axis near the focal point*

emRSFBZ2.py

NQ = 199 NP = 199

nQ = 399 nP = 399

wavelength wL = 500 nm

aperture radius a = 0.700 mm

Source

xS = 0.00 m yS = 0.00 m zS = 0.200 m

Focal length f = 0.200 m

Numerical aperture **NA = 0.0035**

Fresnel number NF = **4.900**

u1 = -25.10 u2 = 60.20

lmax = 3.27e+28 a.u.

Max irradiance uP = -1.31

Execution time 10 s

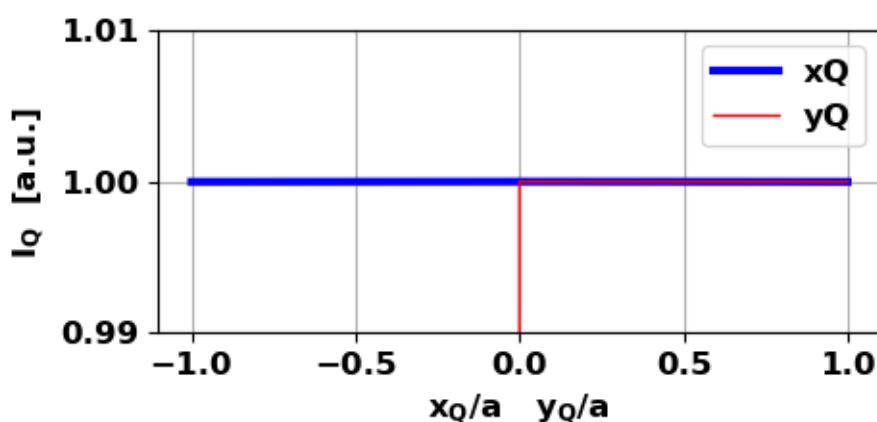


Fig. 5. The normalized aperture irradiance variations in X and Y axis direction where  $E_Q(x_Q, y_Q < 0, 0) = 0$ .

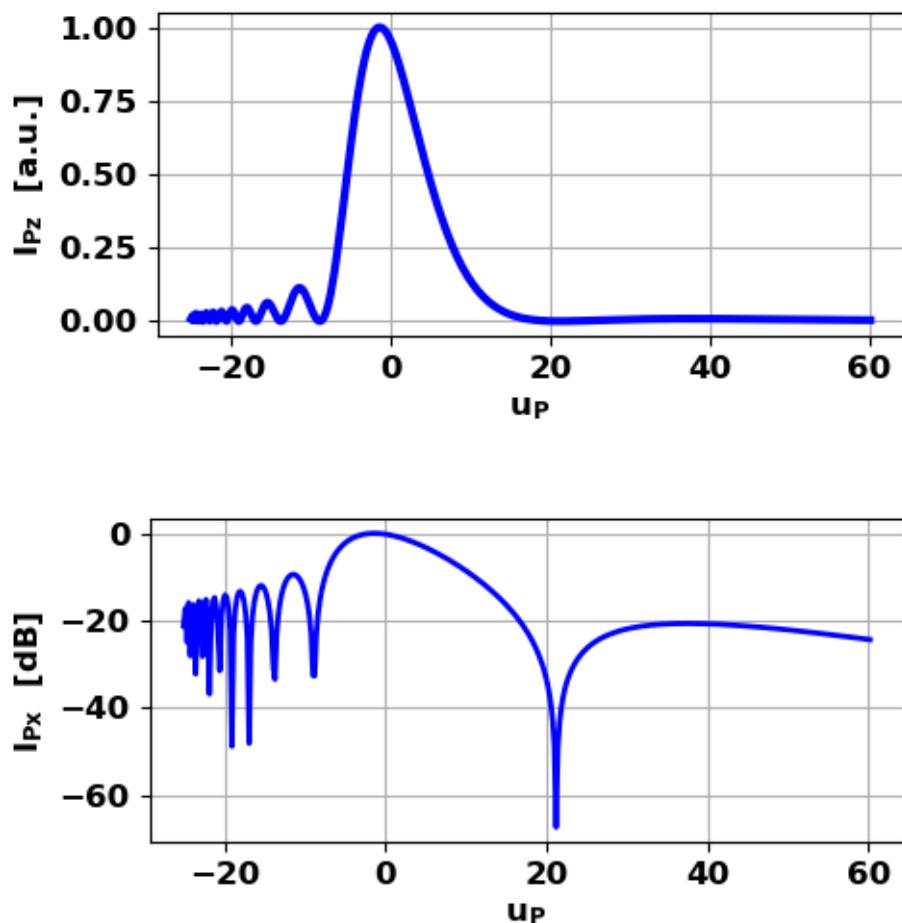


Fig. 6. Observation space: Irradiance pattern along the optical axis near the focal point.

The irradiance pattern is not symmetrical and the peak irradiance is shifted towards the aperture  $u_P \left( I_{Ppeak} \right) = -1.31$

*Irradiance pattern in the XY focal plane surrounding the focal point*

emRSFBXY.py

nQ = 179 nP = 179

wavelength wL = 500 nm

aperture radius a = 0.700 mm

Source

xS = 0.00 m yS = 0.00 m zS = 0.20 m

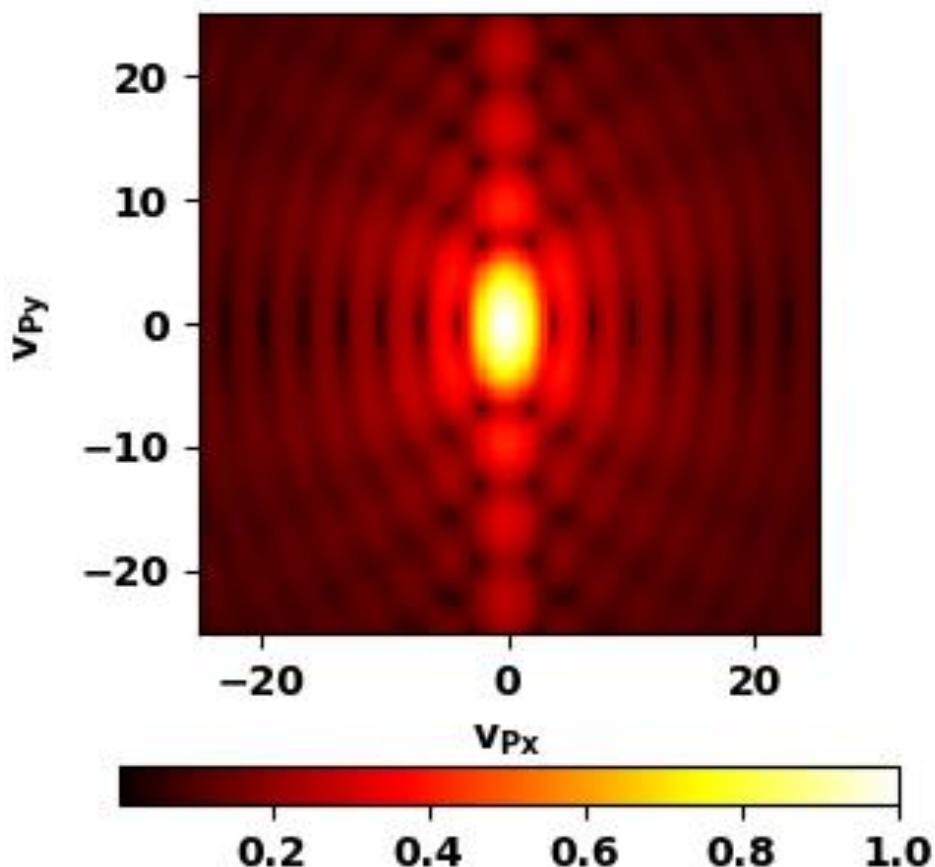
Focal length f = 0.20 m

Numerical aperture N.A. = 0.00350

Fresnel number NF = 4.90

zP = 0.20000 m

Execution time 73 s



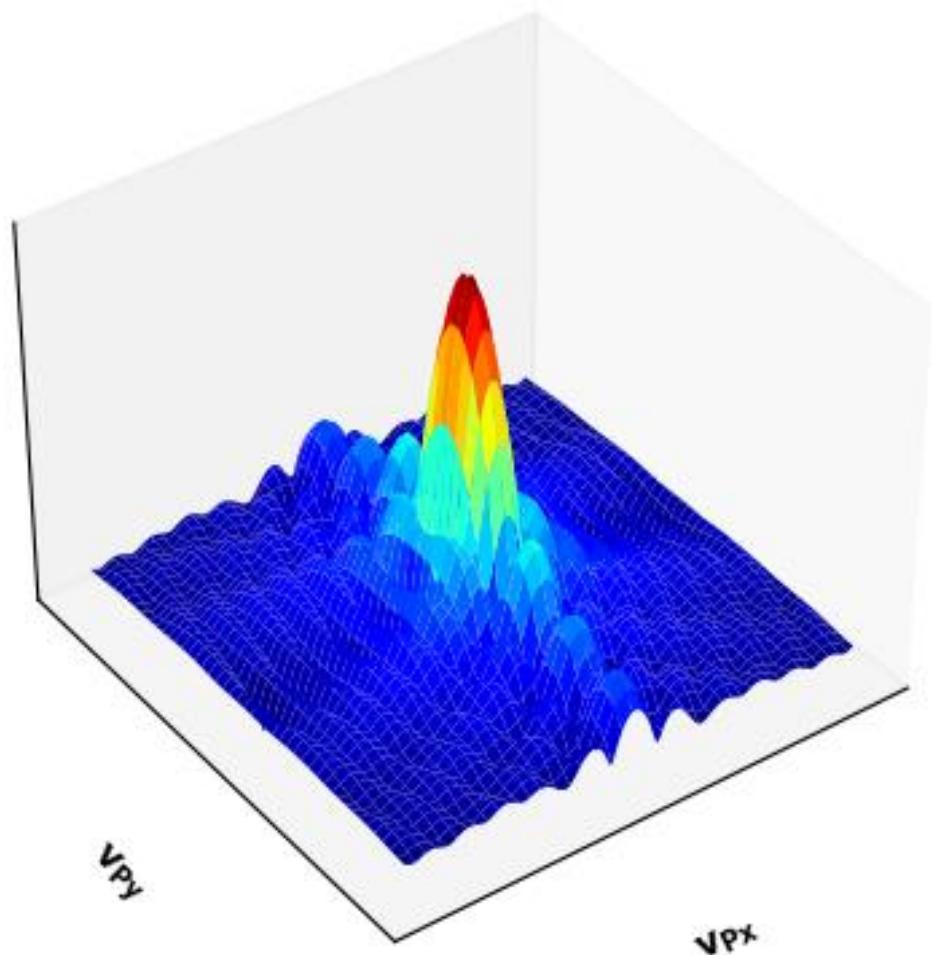


Fig. 7. [2D] and [3D] views of the irradiance in the XY focal plane.

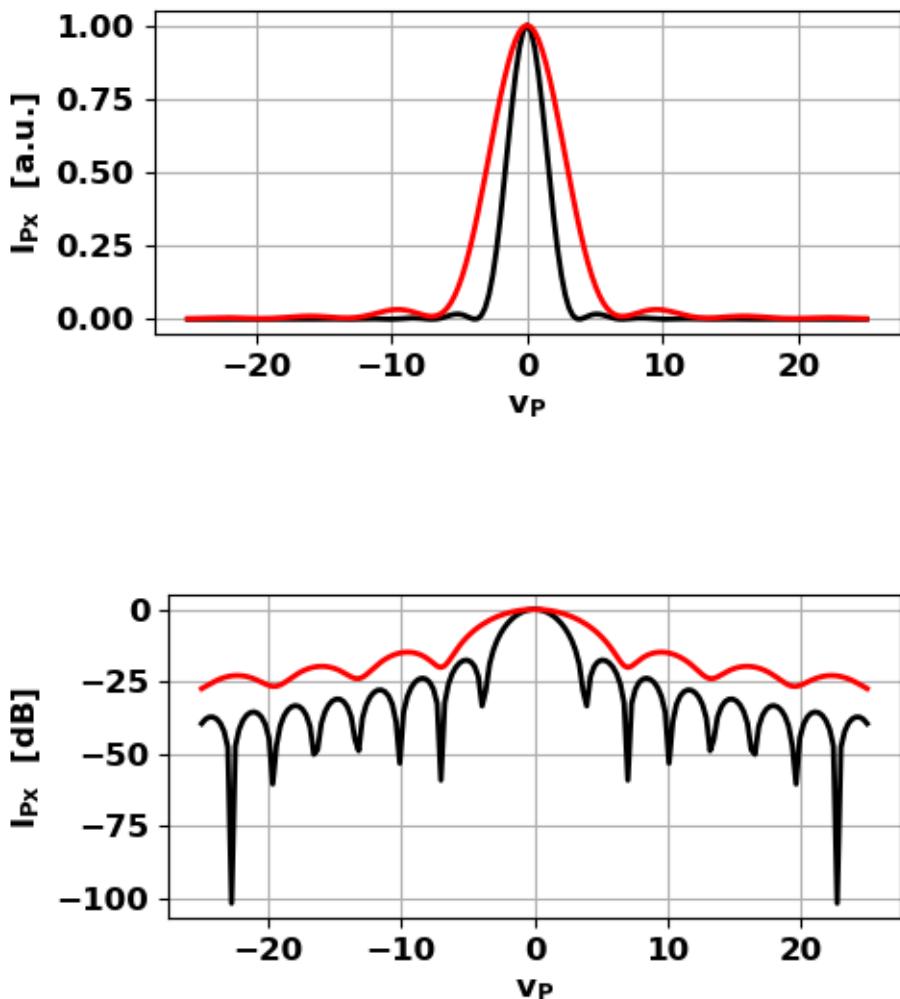


Fig. 8. Irradiance in the XY focal plane. **Black** curve pattern along the **X** axis and **red** curve pattern along the **Y** axis.