DOING PHYSICS WITH PYTHON

COMPUTATIONAL OPTICS

RAY (GEOMETRIC) OPTICS

MATRIX METHODS IN GAUSSIAN OPTICS

THIN LENS

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DOWNLOAD DIRECTORIES FOR PYTHON CODE

Google drive

GitHub

S005.py

The Python Code **S005.py** is used to plot three ray paths from an input point to a set of output points located at a fixed from a thin lens. The ABC matrix method in the paraxial approximation is used as the basis of the computations.

IMAGE FORMATION PROCESS SIMULATIONS

Figure 1 shows the geometry of the single lens optical system, a summary of the simulation parameters, and a summary of the ABCD matrix procedure for the calculation the output vector (y_F, a_F) from the input vector (y_I, a_I) where y is the height a point and a is the elevation of the ray from that point.

In matrix form the transformation is

$$\begin{pmatrix} y_F \\ a_F \end{pmatrix} = \mathbf{M} \begin{pmatrix} y_I \\ a_I \end{pmatrix} \qquad \mathbf{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

The Python Code **S005.py** also calculates the x positions on the optical axis (y = 0) of the cardinal points:

F1, F2 front and rear focal points

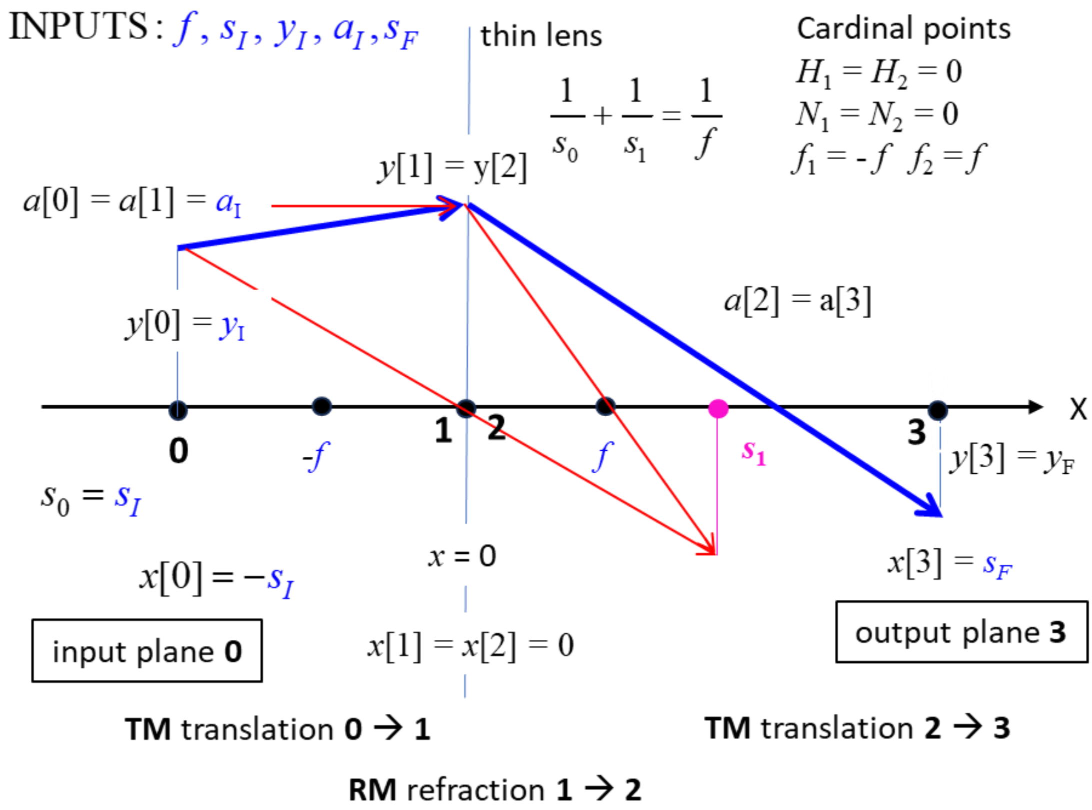
H1, H2 first and second principal points

N1, N2 first and second nodal points

where the Origin (0, 0) is located at the centre of the thin lens. The image point is also calculated from the lens equation

$$\frac{1}{s_0} + \frac{1}{s_1} = \frac{1}{f}$$

A summary of the simulation parameters is displayed in the Console Window.



$$\mathbf{TM} = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \qquad \mathbf{RM} = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} \qquad \mathbf{M} = \mathbf{TM}_{23}\ \mathbf{RM}\ \mathbf{TM}_{01}$$

$$\begin{pmatrix} y[3] \\ a[3] \end{pmatrix} = \mathbf{M} \begin{pmatrix} y[0] \\ a[0] \end{pmatrix} \equiv \begin{pmatrix} y_F \\ a_F \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} y_I \\ a_I \end{pmatrix}$$

Fig. 1. Summary of the ABCD matrix ray tracing procedure.

SIMULATIONS

Simulation 1 $s_0 = 2f$

Propagation of a ray through a thin lens optical system

focal length, f = 5.00

Intersection point (image point) xQ = 10.00 yQ = -1.00

Magnification at image point mag = -1.00

position x

[-10. 0. 0. 20.]

heights y

[[1. 1. 1.]

[4.142 1. -2.142]

[4.142 1. -2.142]

[-6.142 -3. 0.142]]

Elevations [deg]

[[18. 0. -18.]

[18. 0. -18.]

[-29.459 -11.459 6.541]

[-29.459 -11.459 6.541]]

System matrix and Cardinal points

M --> A = -3.00 B = -10.00 C = -0.20 D = -1.00

Focal distance F1 = -5.00 F2 = 5.00

Principle plane H1 = 0.00 H2 = 0.00

Nodal plane N1 = 0.00 N2 = 0.00

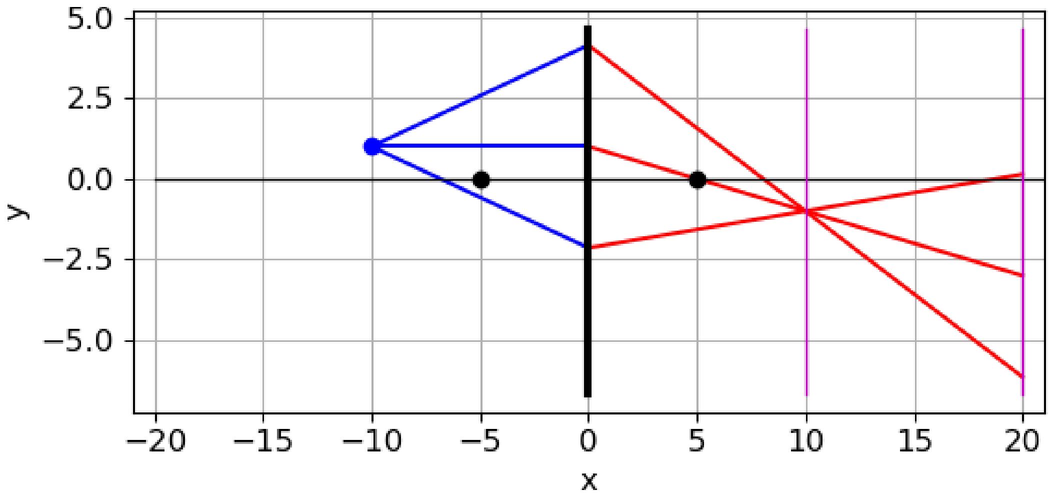


Fig. 2. Ray diagram. Focal points (**black** dots), thick **black** vertical line (lens), input rays 0 → 1 (blue), output rays 2→ 3 (**red**), image plane at s_1 (**magenta**) and output plane at s_F (**magenta**).

The input point (object point) is at

$$x = -s_0 = -s_I = -2f = -10$$

The output point is at

$$x = s_F = 20 \neq 2f$$

A focused image occurs where the three input rays intersect at

$$x = 2f = 10$$

And the magnification would be

$$m = \frac{y_F}{y_I} = -1$$

When $s_F \neq s_1$, the matrix element B of **M** is not zero $(B = -10 \neq 0)$ and the three rays will not intersect at a single point in the output plane.

Simulation 2 $s_F = s_1 = 2f = 10$

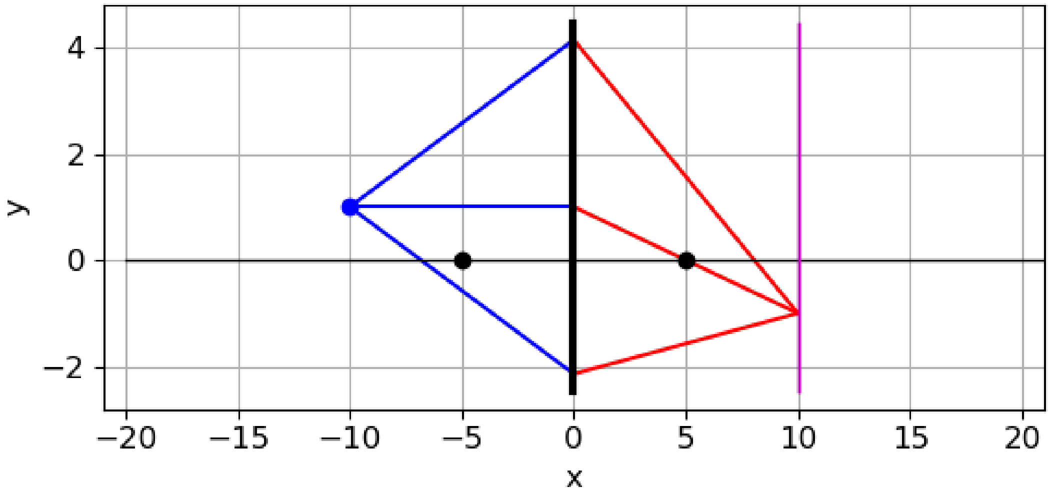


Fig. 3. $s_F = s_1 = 2f = 10$ The image plane and output plane coincide and the three rays form the input point intersect at the image point. The image point is real, and the same height as the object point, but inverted.

Simulation 3 $s_0 > 2f$

focal length, f = 5.00

Intersection point (image point) xQ = 6.67 yQ = -0.33

Magnification at image point **mag = -0.33**

position x

[-20. 0. 0. 10.]

heights y

[[1.   1.   1.   ]

 [7.283 1.   -5.283]

 [7.283 1.   -5.283]

 [-4.142 -1.   2.142]]

Elevations [deg]

[[18.   0.   -18.   ]

 [18.   0.   -18.   ]

 [-65.459 -11.459 42.541]

 [-65.459 -11.459 42.541]]

System matrix and Cardinal points

M --> A = -1.00 **B = -10.00** C = -0.20 D = -3.00

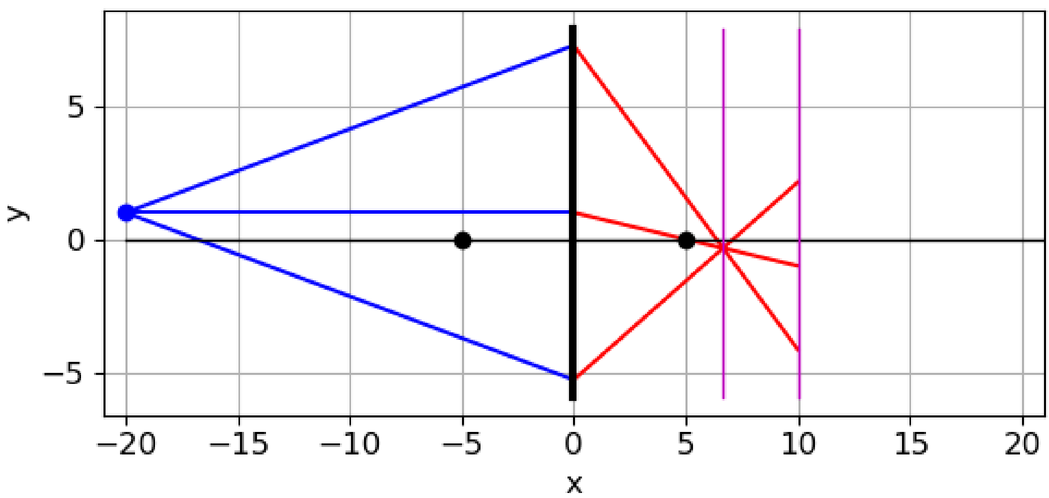


Fig. 4. $s_0 > 2f$ → the image is real, inverted and diminished $\left(|m| = 0.33 < 1\right)$.

Simulation 4 $f < s_0 < 2f$

focal length, f = 5.00

Intersection point (image point) **xQ = 17.50 yQ = -2.50**

Magnification at image point **mag = -2.50**

position x

[-7. 0. 0. 20.]

heights y

[[1. 1. 1.]

[3.199 1. -1.199]

[3.199 1. -1.199]

[-3.314 -3. -2.686]]

Elevations [deg]

[[18. 0. -18.]

[18. 0. -18.]

[-18.659 -11.459 -4.259]

[-18.659 -11.459 -4.259]]

M --> A = -3.00 **B = -1.00** C = -0.20 D = -0.40

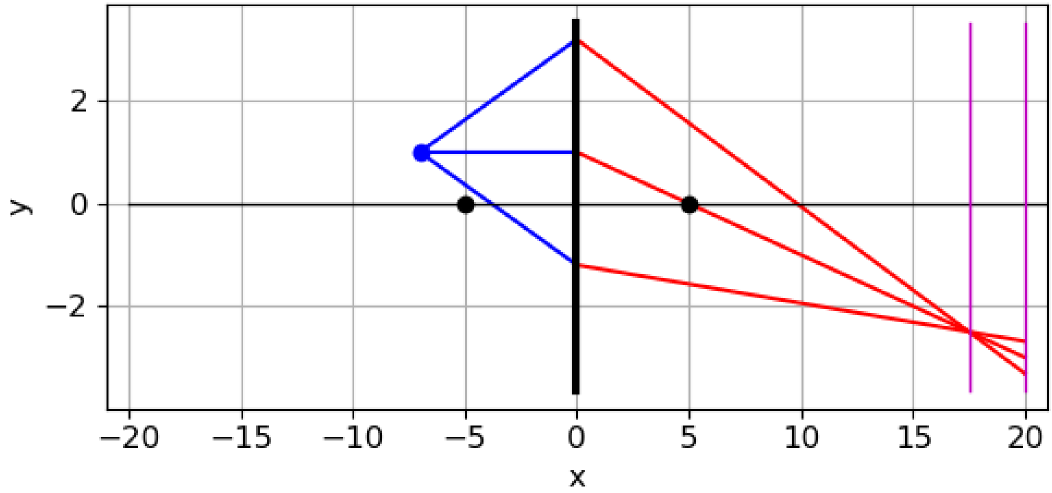


Fig. 5. $f < s_0 < 2f$ The image is real, inverted and diminished in height.

Simulation 5 $s_0 < f$

focal length, f = 5.00

Intersection point (image point) xQ = -7.50 yQ = 5.00

Magnification at image point **mag = 2.50**

position x

[-3. 0. 0. 20.]

heights y

[[2. 2. 2.]

[2.942 2. 1.058]

[2.942 2. 1.058]

[5. 5. 5.]]

Elevations [deg]

[[18. 0. -18.]

[18. 0. -18.]

[-15.718 -22.918 -30.118]

[-15.718 -22.918 -30.118]]

System matrix and Cardinal points

M --> A = 2.50 **B = 0.00** C = -0.20 D = 0.40

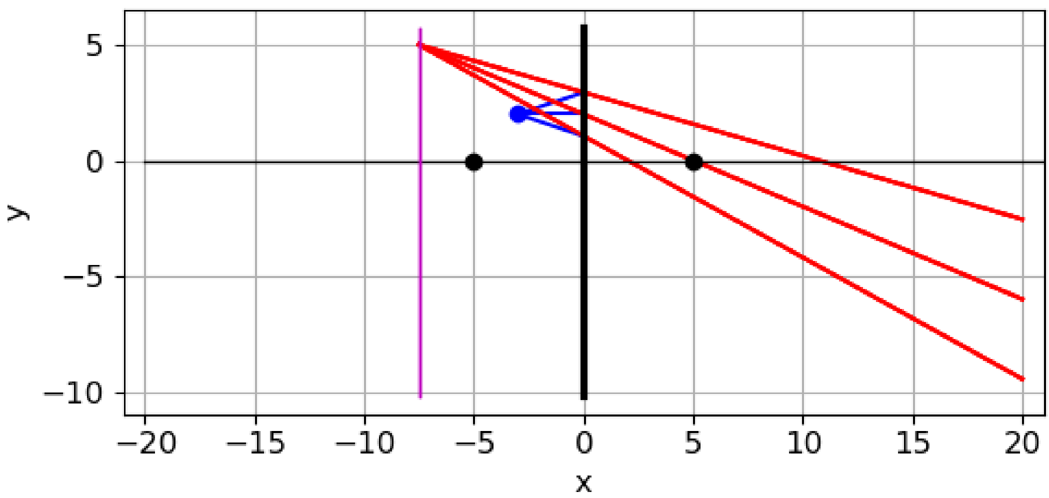


Fig. 6. $s_0 < f$ The image is virtual, upright and enlarged.

The light appears to come from the image point where the rays intersect.