DOING PHYSICS WITH PYTHON

THE NEURON MEMBRANE AS A CAPACITOR

Ian Cooper

Any comments, suggestions or corrections, please email me at

matlabvisualphysics@gmail.com

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INTRODUCTION

A neuron's membrane acts as a capacitor because its lipid bilayer separates conductive intracellular and extracellular fluids. The hydrophobic lipid bilayer of the cell membrane, which prevents direct ion flow and acts as an insulator. Ion pumps and channels maintain a higher concentration of positive ions outside (extracellular region) and negative ions inside (intracellular region - cytoplasm), creating a charge imbalance, thus a potential difference across the membrane. This is the membrane potential v_m. Changes in the membrane potential are responsible for the electrical signalling through the central nervous system. The membrane essentially acts as a leaky capacitor, where charged is stored across the nerve membrane. The membrane capacitance C represents the membrane's ability to store this charge. Ion channels through the membrane act as resistors with resistance R and influences how quickly membrane potential changes with time and affecting signal speed and integration, and is fundamental to generating action potentials and transmitting nerve impulses.

The charge separation across the membrane of a neurone means that energy can be stored and allows the neuron to maintain a resting potential and build up charge for firing. Ion channels allow some current to pass, making the membrane act like a leaky capacitor, which is crucial for generating action potentials.

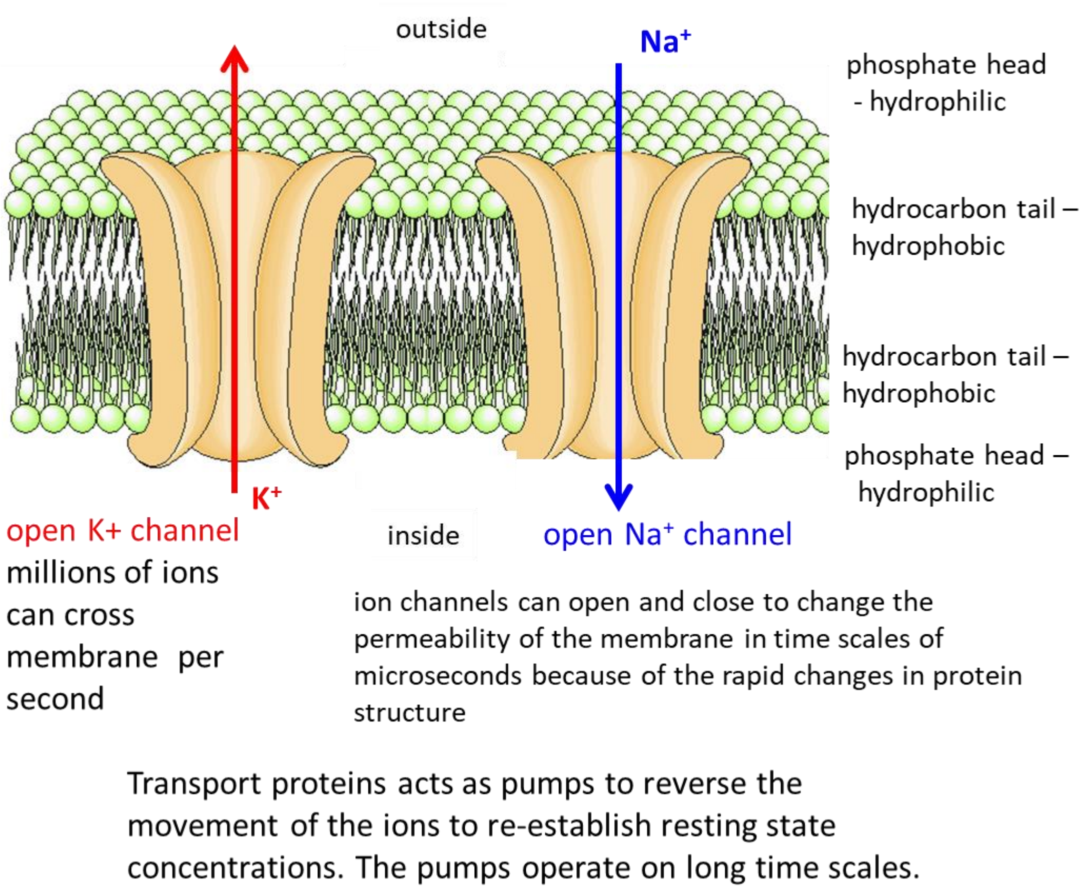


Fig. 1. The neuron membrane

The neurone can be modelled by an equivalent circuit where the membrane components are represented by:

Capacitor - represents the insulating phospholipid bilayer.

Resistors - represent ion channels that allow current to leak through the membrane.

Batteries - represent the equilibrium potentials (Nernst potentials) of specific ions like Na^+ and K^+.

SIMPLEST RC EQUIVALENT CIRCUIT MODEL

In a nerve cell called a **neuron**, currents can pass through the cell membrane from inside to out or from outside to in. Inside and outside the neuron is an electrolytic fluid which is a good conductor and the membrane acts as a dielectric (insulator) separating the two electrolytes. Thus, the simplest model of a segment of the neuron membrane is a capacitor and resistor connecting the outside to the inside of the cell.

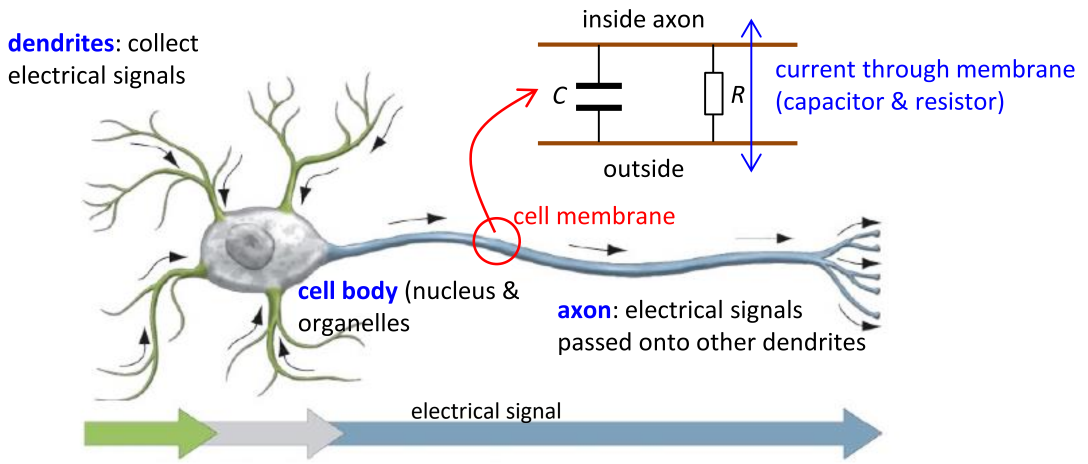
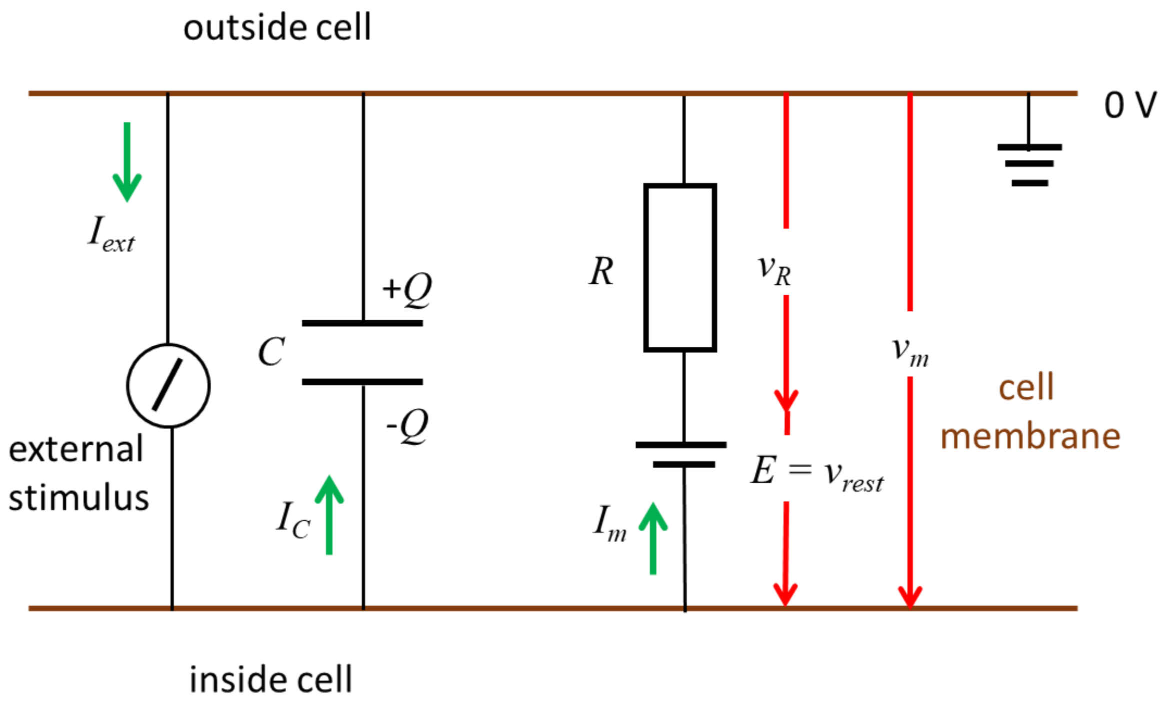


Fig. 2. The membrane of a neuron can be modelled as a combination of a resistor R and a capacitor C.

Figure 3 shows the electrical circuit for the simplest model for segment of a neuron.



Fig, 3. Simplest electrical circuit for modelling a segment of the membrane of a neurone. $I_m > 0$ for membrane current direction from inside to outside and $I_{ext} > 0$ for current direction from outside to inside.

Table 1. Model parameters (S.I. units are not used but units that are most commonly found in the literature).

v_m	mV	membrane potential
t	ms	time
C	F	membrane capacitance
c	$F.cm^{-2}$	specific capacitance
Q	C	charge stored on plates of capacitor
R	Ω	membrane resistance to the movement of ions through the membrane

G	S	membrane conductance
g	mS.cm^{-2}	specific membrane conductance
E	mV	reversal potential
v_{rest}	mV	resting membrane potential (fixed point)
v_R	mV	potential across resistor
I_C	μA	capacitor current
J_C	μA.cm^{-2}	capacitor current density
I_m	μA	membrane current (movement of ions through membrane)
J_m	μA.cm^{-2}	membrane ion current densities

Mathematical Relationships

Membrane potential difference measured w.r.t. $v_{out} = 0$

$$v_m = v_{\text{in}} - v_{out}$$

Capacitive current: rate of change of charge Q at the membrane surface

$$I_C = dQ_m / dt$$

Charge stored on surface of membrane

$$Q = v_m C$$

Differentiating Q w.r.t. t at a fixed position x_0

$$I_C = C\, dv_m / dt$$

Membrane current due to movement of ions and

Kirchhoff's current law (conservation of charge)

$$I_{ext} = I_C + I_R$$
$$I_C = I_{ext} - I_R$$

The fundamental differential equation relating the change in membrane potential to the currents through the membrane for a small segment of the membrane

$$C_m \, dv_m / dt = I_{ext} - I_R$$

It is better to use the current density J rather than current I.

$$c_m \, dv_m / dt = J_{ext} - J_m$$

where $J = I / A \quad c_m = C_m / A$

Electrical potential ΔV, current I, resistance R and conductance G and current densities are related by the equations

$$R = \frac{1}{G} \qquad I = \frac{\Delta V}{R} = G \, \Delta V \qquad J = \frac{I}{A} \qquad g = \frac{G}{A} \qquad J = g \, \Delta V$$

$$J_m = g \, (v_m - E)$$

The ODE describing the system is

$$\frac{dv_m}{dt} = \frac{J_{ext} - g\left(v_m - E\right)}{c_m}$$

with initial conditions $t = 0$, $v_m = v_0$.

The fixed point of the system when J_{ext} = 0 gives the resting potential v_{rest}.

$$\frac{dv_m}{dt} = 0 = \frac{J_{ext} - g\left(v_m - E\right)}{c_m} \quad v_m = v_{rest}$$

$$v_{rest} = E$$

The system ODE is solved using the fourth-order Runge-Kutta method. You can not use the Python ODE solvers since Jext is time dependent.

SIMULATIONS

Simulation 1 DEPOLARIZATION

Model parameters

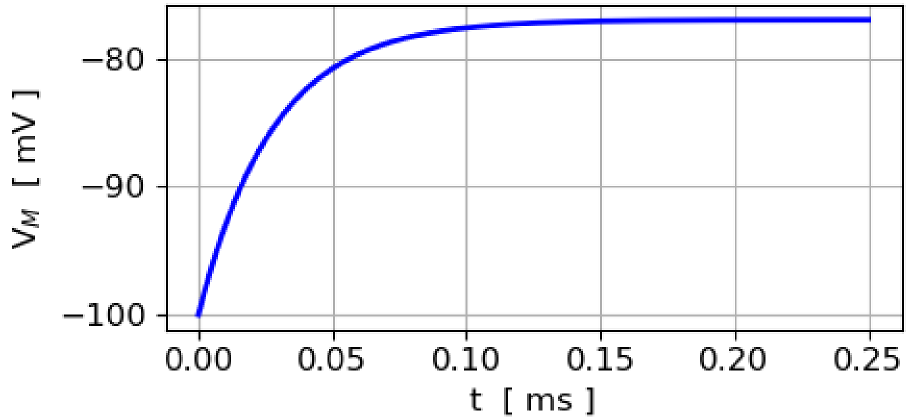
$E = EK$ = -77 mV c = 1.0 μF.cm^{-2} $g = gK$ = 36 mS.cm^{-2}

Initial condition v_0 = -100 mV < v_{rest} $v_{rest} = E$ = -77 mV

Figure 4 $v_0 < v_{rest}$

The membrane potential increases to its resting value v_{rest} = -77 mV by a negative membrane current where positive charges move from the extracellular region (outside) to the intracellular region (inside) as shown in figure 4. The ion current depolarizes the membrane potential as the capacitor charges. The charge Q on the plates of the capacitor C is directly proportional to the membrane potential,

$Q = C\,v_m$, so the t vs v_m plot shows the same time evolution of the charge Q.



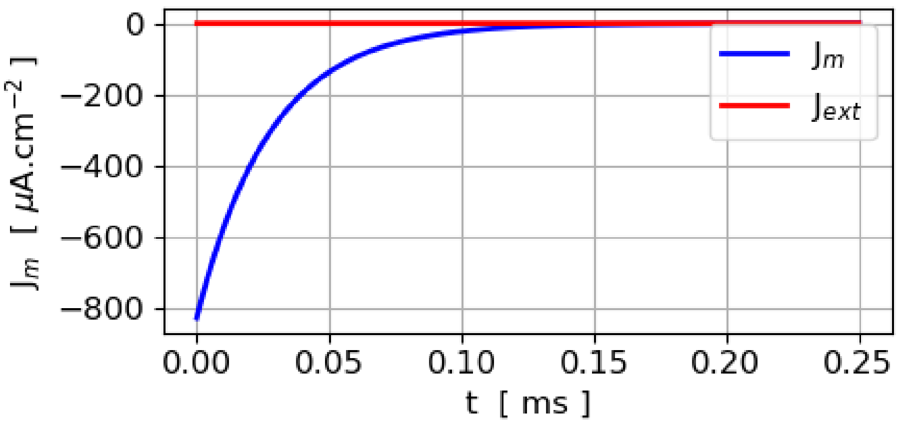


Fig. 4. $v_0 < v_{rest}$ The membrane is depolarized as the capacitor charges by the inward flow of positive ions.

The time constant τ for the charging of the capacitor is

$$\tau = R\,C = C\,/\,G = c\,/\,g = 0.028 \text{ ms}$$

and the potential across the capacitor which is equal to the membrane potential is given by the equation

$$v_C = v_m = v_0 + \left(v_{rest} - v_0\right)\left(1 - e^{-t/\tau}\right)$$

The charging of the capacitor and its time constant is displayed in figure 5.

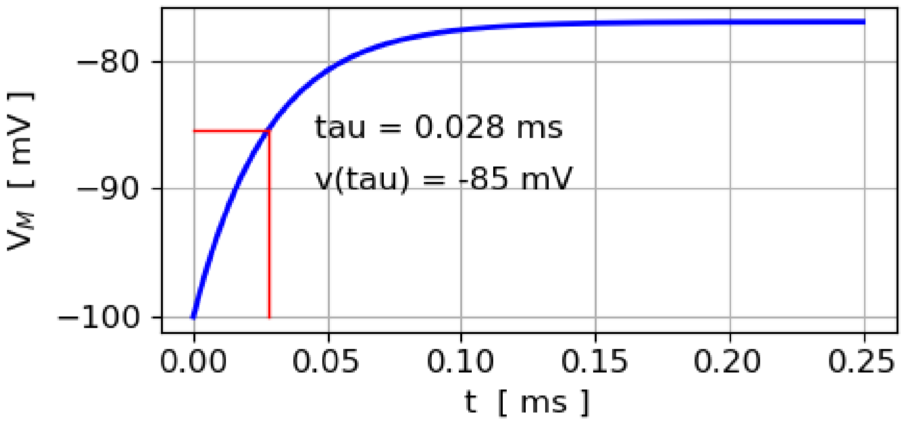


Fig. 5. Depolarization of the membrane and the time constant τ .

Simulation 2 HYPERPOLARIZATION

Model parameters

$E = EK = \text{-77 mV}$ $c = 1.0\ \mu\text{F.cm}^{-2}$ $g = gK = 36\ \text{mS.cm}^{-2}$

Initial condition v_0 = -100 mV < v_{rest} v_{rest} = E = -77 mV

Figure 6 $v_0 > v_{rest}$

The membrane potential decreases to its resting value v_{rest} = -77 mV by a positive membrane current where positive charges move from the intracellular region (inside) to the extracellular region (outside) as shown in figure 6. The ion current hyperpolarizes the membrane potential as the capacitor discharges. The charge Q on the plates of the capacitor C is directly proportional to the membrane potential,

$Q = C\, v_m$, so the t vs v_m plot shows the same time evolution of the charge Q.

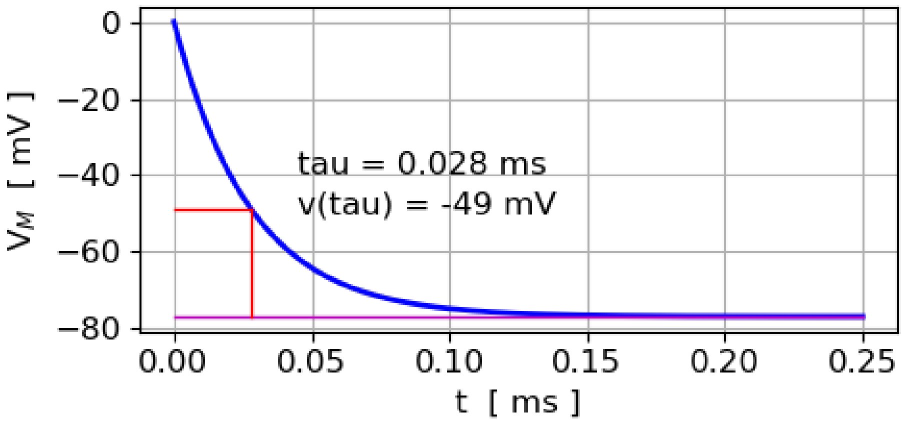
The time constant τ for the charging of the capacitor is

$$\tau = RC = C / G = c / g = 0.028 \text{ ms}$$

and the potential across the capacitor which is equal to the membrane potential is given by the equation

$$v_C = v_m = v_{rest} - \left(v_{rest} - v_0\right) e^{-t/\tau}$$

The discharging of the capacitor and its time constant is displayed in figure 6.



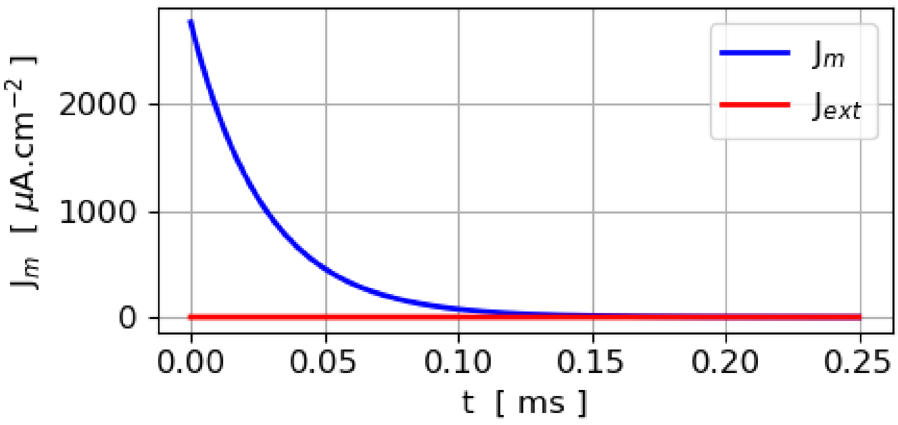


Fig. 6. Hyperpolarization of the membrane and the time constant τ .

SIMULATION 3 ACTION POTENTIAL

We have seen in simulations 1 and 2 that when the membrane potential is perturbed from its resting value it will either increase or decrease exponentially back to its resting value with a time constant τ .

How is an action potential generated?

An external current may trigger an action potential if the membrane potential increases to a threshold level v_{TH} . The external current is given by a pulse of height J_{max} and duration dt and the threshold level is v_{TH} = -50 mV as shown in figures 7 and 8. When the external current pulse is strong enough a spike or action potential is fired and then the membrane potential is reset to a value less than the resting potential.

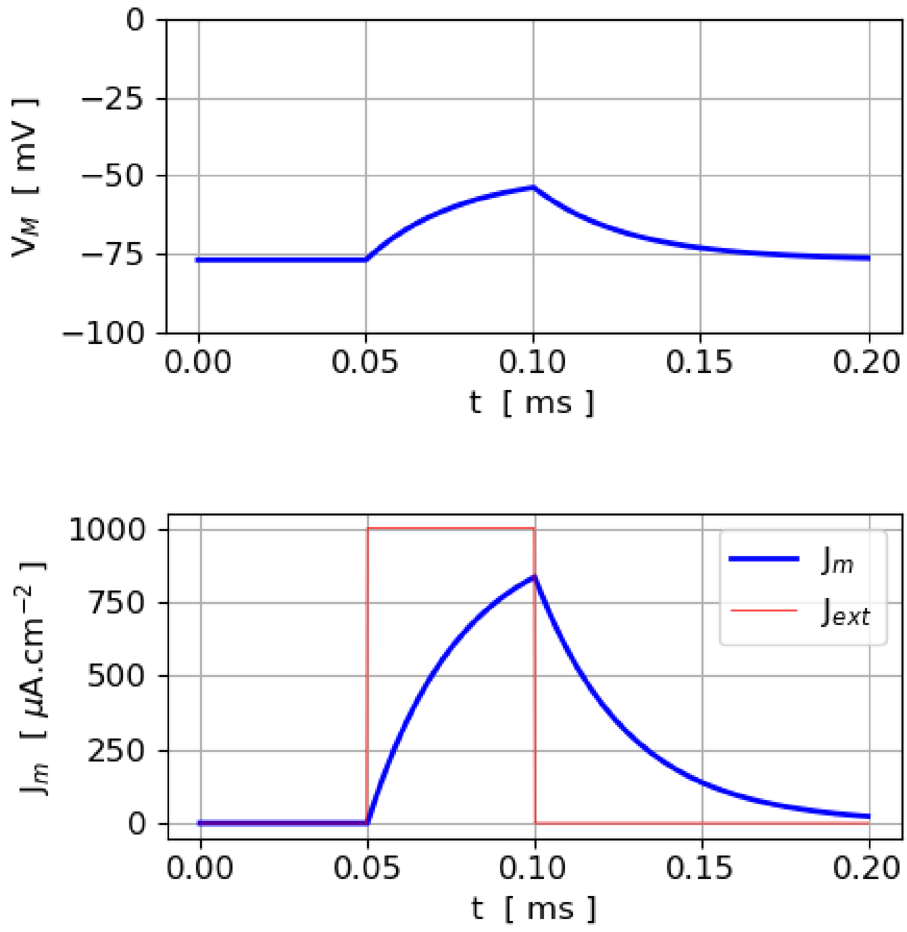


Fig. 7. No spike is produced by the external current stimulus.

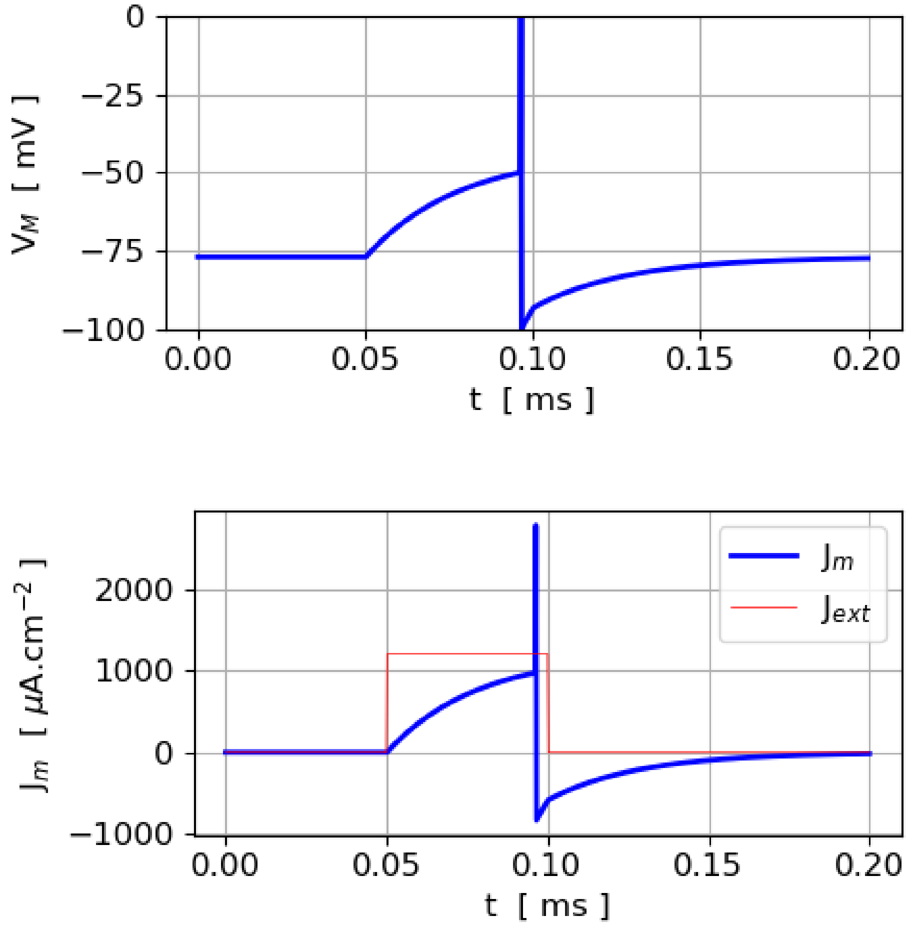


Fig. 8. A spike is produced by the external current stimulus.

This simple RC circuit model used to produce the plots in figures 7 and 8 is known as the **Leaky-Integrate-and-fire model** (LIF model).

https://www.youtube.com/watch?v=LdumhvDBpzQ&t=75s