DOING PHYSICS WITH PYTHON

COMPUTATIONAL OPTICS

PHYSICIST'S HERMITE POLYNOMIALS

HERMITE-GAUSS FUNCTIONS

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emH01.PY Hermite polynomials and Hermite-Gauss functions

HERMITE POLYNOMIALS

The Hermite polynomials are a set of orthogonal functions widely encounter in many areas of science. The n^{th}-order physicist's Hermite polynomial is a polynomial of degree n, where the leading coefficient (highest power term) is 2^n. A few examples:

$$H_0(x) = 1$$

$$H_1(x) = 2$$

$$H_4(x) = 16x^4 - 48x^2 + 12$$

$$H_5(x) = 32x^4 - 160x^2 + 120$$

The Python function **hermite** can be used to give the polynomial, the coefficients of the polynomial and evaluate the function as shown in the following Python Code **emH01.py**.

```
import numpy as np
from numpy import pi, sin, cos, exp, linspace, zeros, amax, sqrt
import matplotlib.pyplot as plt
import time
from scipy import special
tStart = time.time()

n = 3
H = special.hermite(n, monic=False)
print('Hermite polynomial,coefficients, H(1)')
print(H)
```

```
print(H(1))
x = np.linspace(-3, 3, 400)
y = H(x)

plt.rcParams['font.size'] = 10
plt.rcParams["figure.figsize"] = (5,3)
fig1, ax = plt.subplots(nrows=1, ncols=1)
ax.plot(x, y,'b',lw = 2)
ax.set_title("Hermite polynomial of degree n
= %0.0f" %n,fontsize = 10)
ax.set_xlabel("x")
ax.set_ylabel("H$_n$(x)")
ax.grid()
fig1.tight_layout()
```

print(H) → $8x^3 - 12$

Console Window H → poly1d([8., 0., -12., 0.])

H(1) → -3.9999999999999987

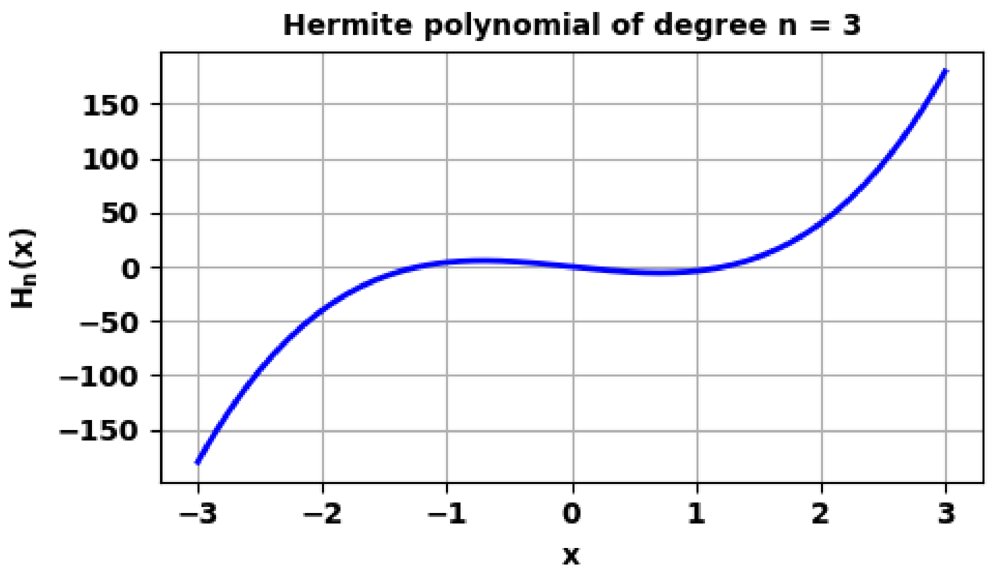

Fig. 1. Hermite polynomial $H_3(x) = 8x^3 - 12$.

HERMITE-GAUSS FUNCTIONS

For example, in spherical mirror resonators, the electric field for stable modes are described by **Hermite-Gauss functions** and are given by

$$E_{mn}(x,y) = E_0\, H_m\left(\frac{\sqrt{2}\,x}{w_0}\right) H_n\left(\frac{\sqrt{2}\,y}{w_0}\right) \exp\left(-\frac{x^2+y^2}{{w_0}^2}\right)$$

where m and n represent the transverse mode numbers, H are the Hermite-Gauss polynomials and w_0 is a characteristic mode width.

There is a whole family of Hermite–Gaussian modes (**TEM_{mn}**). These are approximate solutions of the wave equation in the paraxial approximation. Their electric field distributions are essentially given by the product of a **Hermite polynomial** and a **Gaussian function** and a phase term.

Figure 2 shows the XY electric field patterns of some low order modes of a stable resonator cavity formed by spherical mirrors. Figure 3 shows the patterns for the electric field and the intensity I, where $I = E^2$ for the transverse mode (3,3).

The indices m and n determine the shape of the profile in the x and y directions. The intensity distribution of a mode is such that there are m nodes in the horizontal direction and n nodes in the vertical direction. For $m = n = 0$, a **Gaussian beam** is obtained. This mode is called the fundamental mode or axial mode.

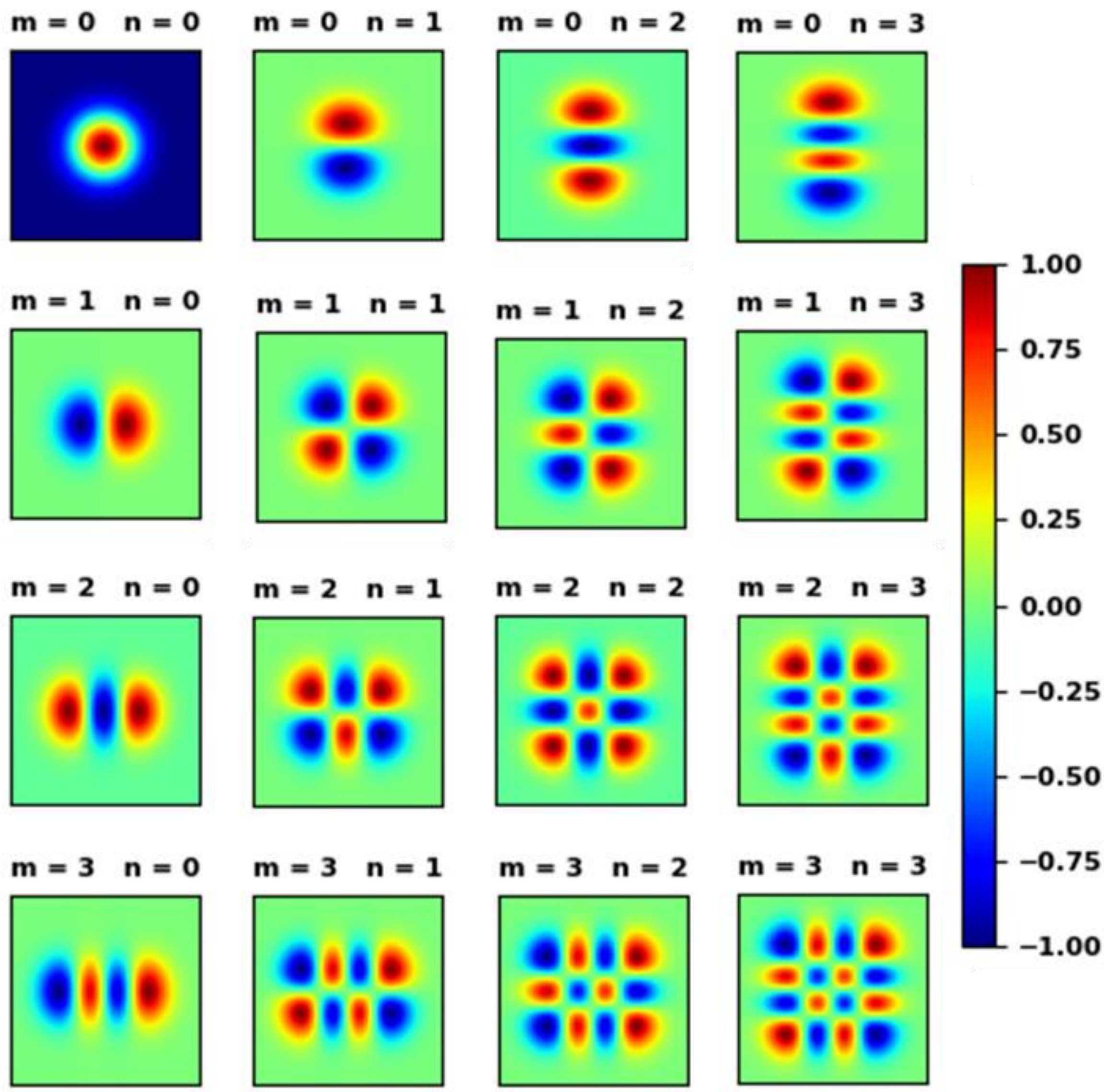

Fig. 2. Electric field lower order transverse resonator modes.

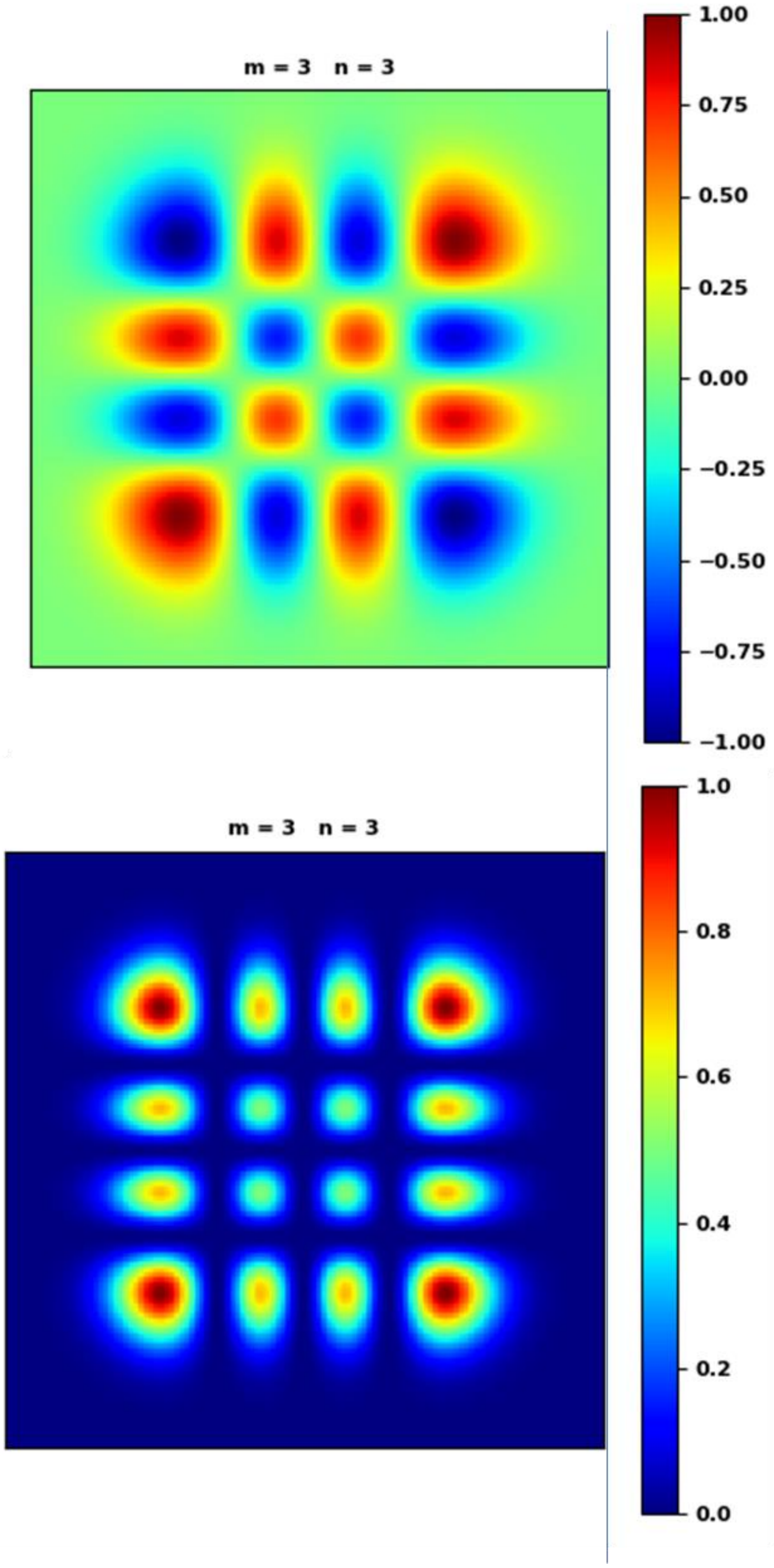

Fig. 3. Transverse mode (3,3) for the electric field (top) and intensity (bottom).