

AI1110

Assignment 7

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Question (Papoulis Problem 4.1)

Suppose that x_u is the u percentile of the random variable x , that is, $F(x_u) = u$. Show that if $f(-x) = f(x)$, then $x_{1-u} = -x_u$.

Solution

Given,

$$f(x) = f(-x) \quad (1)$$

$$\Rightarrow \int_{-\infty}^{-x} f(x) = \int_{-\infty}^{-x} f(-x) \quad (2)$$

$$\Rightarrow \int_{-\infty}^{-x} f(x) = - \int_{\infty}^x f(x) \quad (3)$$

$$\Rightarrow \int_{-\infty}^{-x} f(x) = \int_x^{\infty} f(x) \quad (4)$$

$$\Rightarrow F(-x) - F(-\infty) = F(\infty) - F(x) \quad (5)$$

$$\Rightarrow F(-x) = 1 - F(x) \quad (6)$$

Solution(Contd.)

Given,

$$F(x_u) = u \quad (7)$$

$$\implies 1 - F(x_u) = 1 - u \quad (8)$$

from eqn. (6),

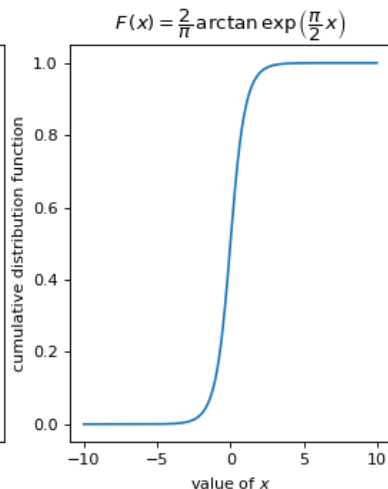
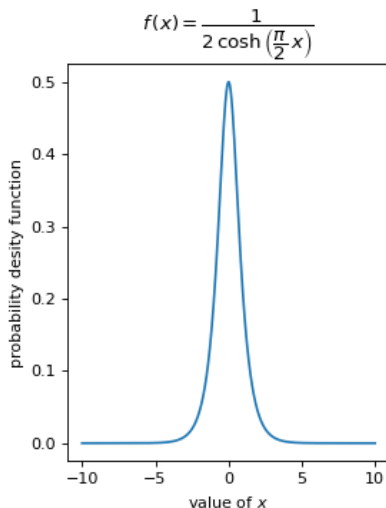
$$1 - F(x_u) = F(-x_u) \quad (9)$$

$$\implies F(-x_u) = 1 - u \quad (10)$$

$$\implies F(-x_u) = F(x_{1-u}) \quad (11)$$

$$\implies -x_u = x_{1-u} \quad (12)$$

Example



Example: Let $u = 0.8$, From above equations: $x_{0.8} = 0.716$, $x_{0.2} = -0.716$
Hence Verified.