AI1110
Assignment 11

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Question(Papoulis Exercise 15.3)

Find the stationary distribution $q_0, q_1, \ldots$ for the Markov chain whose only nonzero stationary probabilities are

$$p_{i,1} = \frac{i}{i+1} \qquad p_{i,i+1} = \frac{1}{i+1} \qquad i = 1, 2, \ldots$$

Solution

State transition matrix:

	1	2	3	4	5	
1	0.5	0.5	0	0	0	…
2	0.66667	0	0.33333	0	0	…
3	0.75	0	0	0.25	0	…
4	0.8	0	0	0	0.2	…
5	0.83333	0	0	0	0	…
	⋮	⋮	⋮	⋮	⋮	

Solution(contd.)

$$q_{i+1} = p_{i,i+1}\ q_i = \frac{1}{i+1} q_i \tag{1}$$

$$q_i = p_{i-1,i}\ q_{i-1} = \frac{1}{i} q_{i-1} \tag{2}$$

$$\vdots$$

$$q_1 = p_{0,1}\ q_0 = \frac{1}{1} q_0 \tag{3}$$

$$\implies q_{i+1} = \frac{q_0}{(i+1)!} \tag{4}$$

Solution(Contd.)

Summation of all probabilities equals one.

$$\sum_{k=1}^{\infty} q_k = q_0 \sum_{k=1}^{\infty} \frac{1}{k!} = 1 \tag{5}$$

$$q_0 e = 1 \tag{6}$$

$$\implies q_0 = \frac{1}{e} \tag{7}$$

Therefore, the steady state probabilities are given by,

$$q_k = \frac{1}{e\,k!} \tag{8}$$