

AI1110 Assignment 11

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Question(Papoulis Exercise 15.3)

Find the stationary distribution q_0, q_1, \dots for the Markov chain whose only nonzero stationary probabilities are

$$p_{i,1} = \frac{i}{i+1} \quad p_{i,i+1} = \frac{1}{i+1} \quad i = 1, 2, \dots$$

Solution

$$q_{i+1} = p_{i,i+1} \quad q_i = \frac{1}{i+1} q_i \quad (1)$$

$$q_i = p_{i-1,i} \quad q_{i-1} = \frac{1}{i} q_{i-1} \quad (2)$$

$$\vdots$$

$$q_1 = p_{0,1} \quad q_0 = \frac{1}{1} q_0 \quad (3)$$

$$\Rightarrow q_{i+1} = \frac{q_0}{(i+1)!} \quad (4)$$

Solution(Contd.)

Summation of all probabilities equals one.

$$\sum_{k=1}^{\infty} q_k = q_0 \sum_{k=1}^{\infty} \frac{1}{k!} = 1 \quad (5)$$

$$q_0 e = 1 \quad (6)$$

$$\implies q_0 = \frac{1}{e} \quad (7)$$

Therefore, the steady state probabilities are given by,

$$q_k = \frac{1}{e k!} \quad (8)$$