AI1110 Assignment 1

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I. QUESTION 3 (A)

Question:

Simplify

$$\sin A \begin{pmatrix} \sin A & -\cos A \\ \cos A & \sin A \end{pmatrix} + \cos A \begin{pmatrix} \cos A & \sin A \\ -\sin A & \cos A \end{pmatrix} \tag{1}$$

Solution:

Let,

$$\mathbf{R}_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{2}$$

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3}$$

$$\mathbf{J} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{4}$$

The matrix expression in the question can be written as

$$\left(\mathbf{R}_A^T * \begin{pmatrix} \mathbf{J} & \mathbf{I} \end{pmatrix}\right) \begin{pmatrix} \sin A \\ \cos A \end{pmatrix} \tag{5}$$

Taking dot product of the vectors

$$\begin{pmatrix} \sin^2 A & -\sin A \cos A \\ \sin A \cos A & \sin^2 A \end{pmatrix} + \begin{pmatrix} \cos^2 A & \cos A \sin A \\ -\cos A \sin A & \cos^2 A \end{pmatrix} \tag{6}$$

Adding the matrices

$$\begin{pmatrix} \sin^2 A + \cos^2 A & -\sin A \cos A + \cos A \sin A \\ \sin A \cos A - \cos A \sin A & \sin^2 A + \cos^2 A \end{pmatrix} \tag{7}$$

Simplifying the elements finally gives

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{8}$$