AI1110
Assignment 9

Dondapati Chandrahas Reddy
AI21BTECH11010

June 13, 2022

Question(Papoulis Exercise 7.26)

Using the Cauchy criterion, show that a sequence x_n tends to a limit in the MS sense iff the limit of $E(x_n, x_m)$ as $n, m \to \infty$ exists.

Solution

If $E\{x_n, x_m\} \to a$ as $n, m \to \infty$, then, given $\epsilon > 0$, we can find a number θ_n such that, if $n, m > 0$

$$E\{x_n, x_m\} = a + \theta(n, m) \ldots (|\theta| < \epsilon) \tag{1}$$

Hence,

$$
\begin{align}
E\{(x_n - x_m)^2\} =& E\{x_n^2\} + E\{x_m^2\} - 2E\{x_n x_m\} \tag{2} \\
=& a + \theta_1 + a + \theta_2 - 2(a + \theta_3) \tag{3} \\
=& \theta_1 + \theta_2 - 2\theta_3 \tag{4}
\end{align}
$$

and since it $|\theta_1 + \theta_2 - 2\theta_3| < 4\epsilon$ for any ϵ, it follows that $E\{(x_n - x_m)^2\} \to 0$, hence (Cauchy) x_n tends to a limit.

Conversly,

If $x_n \to x$ in the MS sense, then $E\{(x_n - x)^2\} \to 0$.

Furthermore,

$$E\{x_n^2\} \to E\{x^2\} \tag{5}$$

$$E\{xx_n\} \to E\{x^2\} \tag{6}$$

$$E^2\{x_n^2 - x^2\} = E^2\{(x_n - x)(x_n + x)\} \tag{7}$$

$$\leq E\{(x_n - x)^2\}E\{(x_n + x)^2\} \to 0 \tag{8}$$

$$E^2\{x(x_n - x)\} \leq E\{x^2\}E(x_n - x)^2 \to 0 \tag{9}$$

Similarly,

$$E\{(x_n - x)(x_m - x)\} \to 0 \qquad (10)$$

Hence,

$$E\{x_n x_m\} + E\{x^2\} - E x x_n - E x x_m \to 0 \qquad (11)$$

Combining, we conclude that $E\{x_n x_m\} \to E\{x^2\}$.