Basic Probability Problems

Ex.1 What is the chance that a leap year selected at random will contain 53 Wednesdays?

Ans leap year = 366 days.

So. $52 \times 7 = 364 + 2$ days.

↓ full week

$\therefore P(X = \text{Wednesday}) = \frac{2}{7}$

Ex.2 $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$,

$P(AB) = \frac{1}{4}$ [AB means $A \cap B$]

a) find the following:

$P(\bar{A})$, $P(A \cup B)$, $P(A|B)$; $P(\bar{A}B)$.

$P(\bar{A} \cup B)$.

~~b) state how the events are:~~

Ans. $P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$

$P(A \cup B) = P(A) + P(B) - P(AB)$

$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{7}{12}$

$$P(A|B) = \frac{P(AB)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4}$$

$$P(\bar{A}B) = P\{(S-A)B\} = P(SB - AB)$$

$$= P(SB) - P(AB)$$

$$= P(B) - P(AB)$$

$$= \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

$$P(\bar{A}\bar{B}) = 1 - P(A \cup B) = 1 - \frac{7}{12} = \frac{5}{12}$$

$$P(\bar{A} \cup B) = P(\bar{A}) + P(B) - P(\bar{A} \cap B)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{12}$$

$$= \frac{3}{4}$$

Q. An urn contains 4 white & 6 black balls. Two balls are successively drawn from the urn without replacement of the first ball. If the ~~fool~~ first ball is seen to be white, what is probability that the 2nd ball is also white.

Ans Probability of both balls as white.

$$P(A_1 \cap A_2) = \frac{4}{10} \times \frac{3}{9} = \frac{2}{15}$$

Now Probability of ~~2nd~~ 2nd ball as white when 1st ball is white

$$P(A_2 | A_1) = \frac{P(A_1 \cap A_2)}{P(A_1)}$$

$$= \frac{2/15}{4/10} = \frac{1}{3}.$$

Ex There are two identical urn containing respectively ~~urns containing respecting~~ 4 white, 3 red balls and 3 white, 7 red balls. As urn is chosen at random and a ball is drawn from it. Find the probability that the ball is white. if the ball drawn is white, what is the prob- that it is from the first urn?

$P(\text{First Urn}) = P(\text{Second Urn}) = \frac{1}{2}$

$\cancel{P(\text{First Urn} \mid \text{White Ball}) = \frac{4}{7}}$

$\cancel{P(\text{Second Urn} \mid \text{White Ball}) = \frac{3}{10}}$

$\cancel{P(\text{White Ball} \mid \text{First Urn}) = P(}$

$P(\text{White Ball} \mid \text{First Urn}) = \frac{4}{7}$

$P(\text{White Ball} \mid \text{Second Urn}) = \frac{3}{10}$

Now, $P(\text{White Ball}) = \cancel{P(\text{White Ball} \mid \text{First Urn}}$

$= P(WB \mid FU) \times P(FU) + P(WB \mid SU) \times P(SU)$

$= \frac{4}{7} \times \frac{1}{2} + \frac{3}{10} \times \frac{1}{2}$

$= \frac{61}{140}$

~~Prob~~ $\cancel{P(FU \mid WB) = \frac{P(FU) \times P(WB \mid FU)}{P(WB)}}$

$\cancel{= \frac{61/140 \times \frac{4}{7}}{\frac{1}{2}}}$

$$P(FU \mid WB) = \frac{P(FU) \times P(WB \mid FU)}{P(WB)}$$

$$= \frac{\frac{1}{2} \times \frac{4}{7}}{\frac{61}{140}}$$

$$= \frac{40}{61}$$

Ex Two urns contain respectively 5 white 7 black balls and 4 white, 2 black balls. One of the urns is selected by the toss of a fair coin and then 2 balls are drawn witho replacement from the selected urn. If both balls drawn are white, what the proba that the first urn is selected?

Ans $P(FU) = \frac{1}{2}$, $P(SU) = \frac{1}{2}$

$$\cancel{P(WB) = \frac{{}^5C_2}{{}^{12}C_2} = \frac{5}{33}}$$

Ex $P(WB \mid FU) = \frac{{}^5C_2}{{}^{12}C_2} = \frac{5}{33}$

Ans $P(FU|WB) = \dfrac{P(FU) \times P(WB|FU)}{P(WB)}$

$$= \frac{\frac{1}{2} \times \frac{5}{33}}{\frac{91}{330}} = \frac{25}{91}$$

$$P(WB) = P(WB|FU) \times P(FU) + P(WB|SU) \times P(SU)$$

$$= \frac{5}{33} \times \frac{1}{2} + \frac{2}{5} \times \frac{1}{2}$$

$$= \frac{91}{330}$$

Ex A speaks the truth 3 out of 4 times and B 7 times out of 10. They agree in their statements that from a bag containing 6 balls of different colours a white ball has been drawn. Find the probability that the statement is true.

Ans $P(A_T) = \frac{3}{4}$, $P(B_T) = \frac{7}{10}$

$P(T) = \frac{1}{6}$, $P(NT) = \frac{5}{6}$

$P(X|T) = \frac{3}{4} \times \frac{7}{10} = \frac{21}{40}$

$P(T|C)$

$\cancel{P(T) = P(T|W) \times P(W) + P(T|NW) \times P(NW)}$

$$P(X|NT) = \left(\frac{1}{4} \times \frac{1}{5}\right) \times \left(\frac{3}{10} \times \frac{1}{5}\right)$$

$$= \frac{3}{1000}$$

$$P(X) = P(X|T) \times P(T) + P(NT) P(X|NT)$$

$$= \frac{21}{40} \times \frac{1}{6} + \frac{3}{1000} \times \frac{5}{6}$$

$$= \frac{9}{100}$$

$$\cancel{P(T|X) = \frac{P(T) \times P(X|T)}{P(X)}}$$

$$\cancel{= \frac{9/100 \times 21/40}{\frac{1}{6}}}$$

$$P(T|X) = \frac{P(T)\, P(X|T)}{P(X)}$$

$$= \frac{1/6 \times 21/40}{9/100} = \frac{35}{36}$$

Ex: In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total of their output. 5%, 4% and 2% are defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probability that it was manufactured by machines A, B and C?

Ans $P(A) = \frac{1}{4}$, $P(B) = \frac{7}{20}$, $P(C) = \frac{2}{5}$

$$P(D|A) = \frac{5}{100} = \frac{1}{20}$$

$$P(D|B) = \frac{4}{100} = \frac{1}{25}$$

$$P(D|C) = \frac{2}{100} = \frac{1}{50}$$

$$P(D) = P(A) \times P(D|A) + P(B) \times P(D|B) + P(C) \times P(D|C)$$

$$= \frac{1}{4} \times \frac{1}{20} + \frac{7}{20} \times \frac{1}{25} + \frac{2}{5} \times \frac{1}{50}$$

$$= \frac{69}{2000}$$

$$P(A \mid D) = \frac{P(A) \times P(D \mid A)}{P(D)}$$

$$= \cancel{\frac{1}{4} \times \frac{69}{1000}}$$

$$= \frac{1/4 \times 1/20}{69/2000} = \frac{25}{69}$$