Probability Distributions

1.

Actual	Predicted: credit worthy	Predicted: not credit worthy
credit worthy	8000	900
not credit worthy	100	1000

a)

$T_+ = 8000 \qquad F_- = 900$

$T_- = 1000 \qquad F_+ = 100$

$$\text{Recall} = \frac{T_+}{T_+ + F_-} = \frac{8000}{8900} = 89.88\%$$

$$\text{Precision} = \frac{T_+}{T_+ + F_+} = \frac{8000}{8100} = 98.76\%$$

$$\text{Accuracy} = \frac{T_+ + T_-}{T_+ + T_- + F_+ + F_-} = \frac{9000}{10000} = 90\%$$

$$F_1 = 2 \times \frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}} = 2 \times \frac{0.8988 \times 0.9876}{0.8988 + 0.9876} = 0.9411 \approx 94.11\%$$

b) In this case, since the bank is to offer loan, ~~I will be more~~ I will be more interested towards credit customers.

2. $P(H) = 0.6$

$P(T) = 1 - 0.6 = 0.4$.

$$P(X=4) = {}^{10}C_4 \times (0.6)^4 \times (0.4)^{10-4}$$

$$= {}^{10}C_4 \times (0.6)^4 \times (0.4)^6$$

$$= \frac{10!}{4!\,6!} \times 0.1296 \times 0.004$$

$$= \frac{10 \times \cancel{9}^{3} \times \cancel{8} \times 7 \times \cancel{6!}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times \cancel{6!}} \times 0.1296 \times 0.004$$

$$= 10 \times 3 \times 7 \times 0.1296 \times 0.004$$

$$= 0.1088$$

Found different b/w manual and Python solution.

3. Calculating PMF for Geometric Distribution in 2nd & 4th visits

~~PMF~~

$p = 0.15$, $q = 1 - p$

$= 1 - 0.15$

$= 0.85$

$PMF_2 = q^{r-1}p = (0.85)^{2-1} \times 0.15$

$= 0.1275$

$PMF_4 = q^{r-1}p = (0.85)^3 \times 0.15$

$= 0.0921$

4. $P(x=r) = \dfrac{e^{-\lambda}\lambda^r}{r!}$

(a) $\lambda = 3$

$P(x=0) = \dfrac{0.04978 \times 1}{1}$

$= 0.04978$

b) $X \sim Po(\lambda_1)$, $Y \sim Po(\lambda_2)$

$X+Y \sim Po(\lambda_1+\lambda_2)$ $[\lambda_1+\lambda_2 = 6]$

$P(X+Y \geq 2) = 1 - P(X+Y=0) + P(X+Y=1)$

$= 1 - (0.0024 + 0.0144)$

$= 1 - 0.0168$

$= 0.9832$

5. $\mu = 67.2$, $Var = 29.37$

$\therefore \sigma = \sqrt{Var} = 5.42$

$$z = \frac{x-\mu}{\sigma} = \frac{72-67.2}{5.42} = 0.88$$

Referring the table,

$P(z < 0.88) = 0.8106$

$\therefore P(z > 0.88) = 1 - 0.8106$

$= 0.1894$

6. $\sigma^2 = 2000$, $\mu = 700$

$x = 800$ ~~10 Adults~~

$$z_{10} = \frac{x-\mu}{\sqrt{2000}} = 2.236$$

10 Adults

$P(z < 2.236) = 0.987$

12 - Adults

$\sigma^2 = 2400$, $\mu = 840$

$x = 800$

$$z_{12} = \frac{800-840}{\sqrt{2400}} = \frac{-40}{15.49}$$

$= -2.58$

~~$P(z_{12} < -2.58) =$~~

$P(z_{12} < 2.58) = .9951$

$\Rightarrow P(z_{12} > 2.58) = 1 - .9951$

$\therefore P(z_{12} < -2.58) = 0.0049$