The Informational Content of High-Frequency Option Prices*

by

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Abstract

We propose the option realized variance as an observable variable to summarize information from high-frequency option data. This variable aggregates intraday option returns from midquote prices to compute the option's total variability for a given day. Using the S&P 500 index time series and options data, this paper documents the incremental information offered by this realized measure in predicting the index realized variance as well as the equity and variance risk premiums. In a parametric study, our results show that the information contained in the option realized variance improves the inference of model variables such as the instantaneous variance and its jumps. Parameter estimates indicate that omitting this information can affect the risk premium breakdown between jump and diffusive risks.

Keywords: High-frequency data, option realized variance, options, jump-diffusions.

1 Introduction

The increasing availability of high-frequency data has paved the way for a better understanding of asset prices and their underlying risks. The fine granularity of intraday prices and recent advances in econometrics have been key in providing new evidence about the importance of stochastic variance and jumps as sources of risk (Todorov and Tauchen, 2011; Andersen et al., 2015a, among others). In addition, mounting nonparametric evidence suggests that these risk factors constitute an important part of the equity and variance risk premiums (see Bollerslev and Todorov, 2011; Andersen et al., 2015b).

In the spirit of the current econometric literature on high-frequency data, this paper aggregates intraday option returns from midquote prices to compute the option's total variability for a given day. As argued in Andersen et al. (2015a), functionals of the variance and jump intensity, such as option prices, inherit the behaviour of these variables at small scales, meaning that the option realized variance (*ORV*) should contain information about the underlying asset dynamics.[1] Accordingly, this paper documents the informational content of this new set of parsimonious variables and provides an economic rationale of the benefits of aggregating high-frequency option prices.

We construct a sample of option realized variances using more than eight years of index option data to explore the informational content of this new observable variable. Throughout a series of heterogeneous autoregressive (HAR) regressions based on the framework proposed by Corsi (2009), we find that option realized variances of out-of-the-money contracts are helpful in predicting future index realized variances. Following Bollerslev et al. (2009) and Andersen et al. (2015b), we also find evidence of the predictive power of these realized variances when studying equity and variance risk premiums. Specifically, our results suggest that lagged values of *ORV*s offer additional information for determining risk premiums, with the greatest gains observed for longer horizons.

To study the informational content of the option realized variance, we employ a continuous-time model of asset returns. This model allows us to characterize which of its components are captured by *ORV* and to

[1] Andersen et al. (2015a) employ a nonparametric framework to infer latent instantaneous volatilities and jump intensities with intraday Black and Scholes (1973) implied volatilities. Using option prices, the authors find that the dynamics of implied volatilities for deep out-of-the-money puts behave as a pure jump process, whereas those of near-the-money are better characterized by a diffusive process.

understand the economic information of this variable, thus complementing the results of HAR regressions. Following Pan (2002), Eraker et al. (2003), Broadie et al. (2007) and Johannes et al. (2009), we study a model that exhibits stochastic variance, jumps in returns with stochastic intensity, and jumps in variance using the general affine framework of Duffie et al. (2000). We show that the option realized variance can incorporate information arising from variance jumps, a feature not available in the realized variance computed from the underlying asset returns (RV). This property comes from a distinctive characteristic of ORV, as depending on the option's moneyness, specific features of underlying processes can be isolated. For instance, deep out-of-the-money (OTM) options have very low deltas and variance vegas, so most of the variability in these option prices comes from discontinuous sample paths generated by return and variance jumps. In-the-money (ITM) options, on the other hand, are highly sensitive to changes in the underling asset variance, giving similar information to that provided by RV.

From this perspective, we investigate the implications of adding option realized variances in the estimation of a parametric model with the aforementioned features. We show that including ORV produces less frequent, larger variance jumps and more frequent, smaller negative return jumps. Also, we find that without ORV, the filtered variance jumps are extremely noisy. These results are consistent with the intuition that this measure contains non-redundant information about the data generating process, specifically about variance jump dynamics. We further explore these differences and find that they have nontrivial implications on the identification of short-term risk premiums. Not only are total average risk premiums affected by the omission of intraday option price information, but the risk premium breakdown between diffusive and jump risks is also altered.[2] Finally, an out-of-sample analysis of the model's performance shows that using ORV to infer parameters and latent states produces lower pricing errors.

To verify the robustness of the option realized variance, we conduct an extensive empirical study. As documented in the literature (e.g., Andersen et al., 2001), the microstructure noise associated with high-

[2]More precisely, we find that the average equity risk premium decreases from 4.51% to 3.39% with the addition of this information. Decomposing this risk between diffusive and jump risk shows that when ORVs are included, the diffusive risk premium is less compensated (decreases from 4.45% to 1.76%). On the other hand, the average jump risk premium increases from 0.06% to 1.63%, but this increment is not sufficient to compensate for the overall average decrease. Regarding compensation for variance risk, we find it increases in absolute value from −0.90 bps to −3.41 bps when ORVs are added. This increase is driven by a higher compensation for variance jump risk (−3.24 bps with ORV, −0.12 bps without) and a lower compensation for variance diffusive risk (−0.17 bps with ORV, −0.78 bps without).

frequency observations can induce biases on realized estimates. Using the Hausman tests proposed in Ait-Sahalia and Xiu (2019) and a sample of daily option realized variances that extends from July 2004 to December 2012, we find that a large proportion of high-frequency option prices contains microstructure noise. High rejection rates of these tests suggest that noise-robust estimates are required when working with these data. In a series of exploratory and statistical analyses, we document the performance of noise-robust estimates from existing methodologies when the object of interest is the option realized variance. We find that methodologies such as the subsampling technique of Zhang et al. (2005), the pre-average method (Hansen and Lunde, 2006; Jacod et al., 2009), and the kernel-based estimator (Barndorff-Nielsen et al., 2008) provide consistent estimates of ORV.

Closer to our study, Audrino and Fengler (2015) employ high-frequency option prices to analyze the consistency of models by comparing the option realized variances with those implied by competing models. Our paper complements theirs in several ways. First, we investigate the presence of microstructure noise in high-frequency option prices and find that noise-robust measures are required when working with these prices. Second, we provide nonparametric evidence about the incremental information provided by these variances: option realized variances predict the future realized variance of the underlying index and contain information about equity and variance risk premiums. Third, we study the economic implications of aggregating intraday option prices for model identification and find that they are especially useful in dealing with models that have jump components in their model dynamics.

The contributions of this paper are fivefold. First, we propose the option realized variance as means to synthesize important information about the dynamics of underlying asset prices. Second, we study microstructure effects in high-frequency option prices and provide evidence that robust estimates commonly used in the literature are also useful when working with these data. Third, we show that ORV contains important economic information as it predicts future index realized variance and risk premiums. Fourth, we identify from a parametrical standpoint that the additional information mentioned above is intimately linked to variance jump dynamics. Finally, we show that omitting option realized variances in the estimation of parametric models leads to the inference of different jump dynamics, generating nontrivial changes in the estimated risk premiums.

The remainder of this paper is organized as follows. Section 2 presents our sample, defines the option realized variance, and examines the robustness of this measure. Section 3 performs a nonparametric study on the informational content of ORV. Section 4 employs an asset pricing model to understand which state variables contribute to option realized variances. Section 5 assesses the benefit of adding ORV to the list of observable variables in the estimation of option pricing models. Section 6 concludes.

2 Data and Computation of Option Realized Variances

This section first presents our dataset and details the method used to compute the option realized variance from high-frequency option prices. It then documents the existence of microstructure noise in option prices and shows how existing robust estimators perform with these data. Finally, the section compares option realized variances based on quote prices to those based on trades.

2.1 Data

The sample period is from July 2004 to December 2012. We employ tick-by-tick Level I quote data provided by Tick Data for European options written on the S&P 500 index. Tick Data prices come from the Options Price Reporting Authority (OPRA), the national market system that provides information about last sale reports and quotation information.

For a quote to be included in the dataset, we require its bid price to be higher than zero and lower than the ask price, to have a timestamp between 9:30 am and 4:00 pm Eastern Time, to not have any condition code, and to not be eligible for automatic execution. We match our Tick Data dataset to that of OptionMetrics, so we only keep options that are available in both datasets. This match is necessary since Tick Data contains other types of options not present in OptionMetrics (i.e. binary options). In addition, we follow Bakshi et al. (1997), Carr and Wu (2011), as well as Christoffersen et al. (2012), and apply the following filters to option contracts in OptionMetrics: (1) we remove options with nil volume or nil open interest; (2) we keep options with a maturity of more than one week and less than one year; (3) we remove options with a price below 0.375; (4) we keep options with a positive bid-ask spread; (5) we remove options that violate arbitrage conditions; (6) we keep options that are at-the-money and out-of-the-money. We use these standard filters to control for option contracts that are thinly traded and could exhibit stale quotes during the trading day. The total sample size is of 423,844 options. Table 1 reports the number of contracts sorted by moneyness and maturity (see 'Number of Contracts' in Table 1).[3]

Table 1: **Description of Option Data (2004–2012).**

		Call-Equivalent Delta, Δ^e					
Maturity	**Variables**	**[0,0.2)**	**[0.2,0.4)**	**[0.4,0.5)**	**[0.5,0.6)**	**[0.6,0.8)**	**[0.8,1]**
[0,30)	Number of Contracts	17,549	10,627	5,082	5,397	14,882	45,326
	Implied Volatility (%)	18.72	19.95	21.38	22.65	24.62	31.54
	ORV^{Quotes} (%)	20.45	11.81	7.91	8.22	11.44	17.93
	ORV^{Trades} (%)	21.93	11.80	7.89	8.46	10.32	12.43
	Volume	2,549.5	3,165.0	3,557.7	4,567.6	3,756.0	2,599.7
	Interval, Quotes	208.1	308.7	323.1	324.0	308.8	200.2
	Number of Trades	47.9	52.3	60.9	69.2	47.7	29.6
[30,90)	Number of Contracts	37,093	23,274	12,099	13,587	32,299	78,781
	Implied Volatility (%)	16.82	18.88	20.38	21.80	23.87	31.09
	ORV^{Quotes} (%)	8.98	3.82	2.40	2.97	3.51	6.74
	ORV^{Trades} (%)	6.39	2.49	1.72	1.93	2.25	3.11
	Volume	1,156.6	1,480.0	2,217.8	3,652.5	2,348.7	1,492.3
	Interval, Quotes	175.0	266.4	289.6	295.0	270.5	172.5
	Number of Trades	15.8	15.7	26.6	41.6	17.7	12.1
[90,180)	Number of Contracts	10,293	8,764	4,504	5,315	12,965	25,244
	Implied Volatility (%)	16.80	18.98	20.63	22.25	24.49	32.20
	ORV^{Quotes} (%)	4.54	1.65	1.05	0.96	1.74	3.26
	ORV^{Trades} (%)	1.43	0.73	0.48	0.54	0.70	0.86
	Volume	802.6	1,055.2	1,327.6	2,512.5	1,762.5	1,009.1
	Interval, Quotes	164.1	244.3	268.4	281.6	254.1	155.3
	Number of Trades	5.2	7.1	11.0	22.1	10.8	5.9
[180,270)	Number of Contracts	4,911	4,721	2,583	2,947	8,206	12,478
	Implied Volatility (%)	16.39	18.60	20.74	22.08	23.86	32.38
	ORV^{Quotes} (%)	3.64	0.98	0.61	0.60	1.07	2.83
	ORV^{Trades} (%)	0.63	0.26	0.17	0.20	0.30	0.37
	Volume	556.1	895.2	809.1	1,220.7	1,112.1	789.0
	Interval, Quotes	158.4	244.2	272.2	283.5	258.0	151.7
	Number of Trades	3.9	4.7	5.3	6.9	6.4	3.6
[270,365]	Number of Contracts	2,961	3,854	2,259	2,325	6,087	7,431
	Implied Volatility (%)	16.45	18.40	20.62	22.28	23.72	31.91
	ORV^{Quotes} (%)	3.42	0.69	0.43	0.37	0.42	2.34
	ORV^{Trades} (%)	0.41	0.15	0.11	0.10	0.14	0.30
	Volume	476.3	779.5	715.4	909.8	889.1	732.8
	Interval, Quotes	150.3	229.2	256.2	274.3	253.1	141.3
	Number of Trades	4.1	3.9	4.7	5.0	4.5	3.2

This table presents average values for different daily variables computed across moneyness (measured in terms of call-equivalent delta) and maturities (defined in days to maturity). We apply the following filters to our option data set: (1) we remove options with nil volume or nil open interest; (2) we keep options with a maturity of more than one week and less than one year; (3) we remove options with a price below 0.375 (Bakshi et al., 1997); (4) we keep options with a positive bid-ask spread; (5) we remove options that violate arbitrage conditions; (6) we keep only at-the-money (ATM) and out-of-the-money (OTM) options. This leaves us with a sample of 423,844 options. The variable ORV^{Quotes} represents the ORV of the option using midquote prices and ORV^{Trades} the ORV of the option using transaction prices. The variable *Volume* is calculated as the number of contract trades reported in Option Metrics, the variable *Interval, Quotes* provides the number of one-minute intervals with at least one midquote price, and the variable *Number of Trades* gives the average number of transaction prices in a trading day. The sample period goes from July 2004 until December 2012.

[3]Throughout the paper, moneyness is defined in terms of call-equivalent delta, Δ^e. The call-equivalent delta is simply the delta of the option assuming it is a call (regardless of its actual type).

2.2 Option Realized Variances and Microstructure Biases

To measure the option realized variance from midquote prices (option realized variance or *ORV* hereafter) for a given day, we start with following estimator:

$$ORV_{t-\tau,t}^{\text{Naive}}(\Delta_N) = \sum_{j=1}^{N} \left(\log\left(O_{t-\tau+j\Delta_N}\right) - \log\left(O_{t-\tau+(j-1)\Delta_N}\right)\right)^2, \tag{1}$$

where O_t is the prevailing midquote option price at time t, N is the number of elements in an equidistant grid dividing the interval of length τ (e.g., one day), and $\Delta_N = \tau/N$. The definition of the option realized variance in Equation (1) resembles that of the realized variance (*RV*). We label this quantity with the superscript 'Naive' to highlight the fact that it does not consider potential microstructure biases in high-frequency option prices.

As documented in the literature, the microstructure noise associated with high-frequency observations can induce biases in realized measures of variance (e.g., Andersen et al., 2001). In Appendix A.1, we employ a Hausman-type test introduced in Ait-Sahalia and Xiu (2019) to assess the presence of these noises in high-frequency option prices. Irrespective of options' maturity or moneyness level, we find high rejection rates of the null hypothesis supporting the idea that ORV^{Naive} exhibits biases resembling to those reported for the *RV* in the equity market.

The above finding suggests that ORV^{Naive} is not appropriate for empirically measuring the option's intraday variability. Nonetheless, the econometric literature on *RV* provides different approaches for coping with the presence of microstructure noise in the data. From the list of available methodologies, we investigate three approaches: the subsampling approach of Zhang et al. (2005), the kernel-based estimator of Barndorff-Nielsen et al. (2008), and the pre-average method of Hansen and Lunde (2006) and Jacod et al. (2009). In addition, we investigate the impact of the sampling frequency when these estimators are applied to our data. As shown in Appendix A.2, sampling at one-minute and 15-minute frequencies leads to different estimates. However, robust estimators based on five-minute data show similar average values across maturities and moneyness levels, suggesting that at this frequency these estimators are equivalent up to the first order. This last remark is consistent with Christensen et al. (2014), who state that, "when the object to be estimated is quadratic variation, the pre-averaging approach is to first-order equivalent to the realized kernel-based estimator of Barndorff-Nielsen et al. (2008) and the two-scale or multi-scale subsampler of Zhang et al. (2005) or Zhang (2006). This result is in line with Liu et al. (2015) who consider about 400 realized measures of variance over a large cross-section of financial assets, and find very little evidence that the five-minute estimate of RV is outperformed by any of the competing measures.

Based on the previous evidence, we employ the subsampling approach at a five-minute frequency to perform all our empirical analyses. For each trading day, we select option prices at a frequency $\Delta_N =$ 5 minutes and construct $k = 3$ overlapping price grids at an inferior frequency, i.e., $\delta = k\Delta_N$. The subsampling option realized variance estimator is defined as the average of naive ORVs over the k grids defined above, that is:

$$ORV_{t-\tau,t}^{\text{Subsampling}}(\Delta_N, k) = \frac{1}{k}\sum_{i=0}^{k-1} ORV_{t-\tau+i\Delta_N, t+(i+1-k)\Delta_N}^{\text{Naive}}(\delta). \tag{2}$$

With an observation frequency of five minutes, this estimator employs three non-overlapping grids constructed at a 15-minute frequency. For instance, the first grid uses prices at 9:31, 9:46, 10:01, etc., the second grid uses prices at 9:36, 9:51, 10:06 and so on. These three grids are then averaged to get the subsampling estimator of ORV. We select this approach as it has been shown to produce reasonable estimates of volatility with high accuracy in different empirical applications (see Andersen et al., 2011; Liu et al., 2015). For ease of presentation, we drop the superscript from the estimate when referring to this quantity.

2.3 Quote Versus Trade Prices

Since midquote prices are used to evaluate our estimates, we now assess the differences and similarities between option realized variances calculated with trade and midquote prices.[4] Table 1 presents the average values for quote- and trade-based estimates by different maturities and moneyness levels. Regarding quote-base estimates (see 'ORV^{Quotes}' in the table), the highest values of option realized variances are for contracts with short maturities and options that are out-of-the-money. This pattern contrasts with that typically observed for implied volatilities (see 'Implied Volatility' in the table) as out-of-the-money put

[4]See Section C of the Supplementary Material for information about trade data used in these calculations.

options have the highest values and no significant change is observed across maturities. This difference suggests that both measures capture different aspects of the dynamic behaviour of option prices.

The trade-based estimates (see 'ORV^{Trades}' in Table 1) follow a pattern resembling those reported for quote-based ORV, suggesting that both datasets capture similar information in terms of price dynamics. Despite this similarity and the fact that, for short-term maturities, average values are comparable in size, substantial discrepancies emerge as we consider longer maturities.

To understand these differences, Table 1 also presents three additional variables that capture the daily trading activity in these contracts, i.e., volume, number of trades and number of one-minute intervals with at least one quote update. The volume and number of trades are low when the largest discrepancies are observed. For contracts with low-trading activity, a large proportion of consecutive intervals have the same values, producing a large number of zero values, which lowers the ORV estimates. This effect is less likely to have an incidence on quote-based estimates as option market-makers rapidly update quotes as new information is obtained—this is evidenced by the high number of quote updates for all contracts. Hence, the quote-based ORV offers a more robust measure of the option's intraday activity variability, in line with other papers which use option quotes to infer high-frequency properties of asset dynamics (e.g., Andersen et al., 2015a; Audrino and Fengler, 2015).

3 Informational Content of Option Realized Variances

This section provides empirical evidence regarding the informational content of option realized variances. To this end, we construct a balanced panel of these variables and analyze its relationship with measures that typically capture the market activity of the underlying asset. Next, we document the predictive power of ORV in forecasting variables such as the equity index realized variance, the equity risk premium, and the variance risk premium.

3.1 Option Realized Variance Panel and Market Activity

To interpret the large daily cross-section of option data spanning different maturities and moneyness levels, we construct a surface of option realized variances across both dimensions. Specifically, for a given day, we collect ORV measures for all options in our dataset and perform a smoothed interpolation across moneyness levels and maturities.[5] We then construct a daily balanced panel by taking three equally spaced points over the delta dimension (0.2, 0.5, and 0.8) for maturities of 30 and 90 days. We focus our attention on options with these maturities as our previous analyses show that these contracts have the largest trading activity from all maturities available in our sample. In addition, short-dated options are contracts that have been shown to contain unique information about volatility dynamics (Bollerslev and Todorov, 2011; Andersen et al., 2017, among others).[6] In total, the panel contains 12,840 observations (six values for each of the 2,140 days).

Figure 1 reports 30- and 90-day *ORV* measures for three selected levels of moneyness: OTM call options with $\Delta^e = 0.2$ (top panel), ATM options with $\Delta^e = 0.5$ (middle panel) and OTM put options with $\Delta^e = 0.8$ (bottom panel): the time series of these estimates display similar patterns, suggesting that the levels of these variables share common information. As a first step towards understanding the interaction between *ORV* and market activity variables, Figure 2 presents the dynamics of *RV* and the jump variation (*JV*) associated with the S&P 500 index.[7]

Large sporadic spikes in *RV* and *JV* are also present in *ORV* variables, revealing an important degree of commonality during specific periods. This relationship is also observed when we compute sample correlation coefficients between these variables, as reported in Table 2. Typically, there is a high correlation among *ORV* variables and a low correlation between *ORV* and the market activity variables. This result points to the existence of differential information in both sets of variables, which is the subject of the following analyses.

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[5] We perform a locally smoothing quadratic regression using the Matlab `Lowess` procedure on the square root of *ORV*, and then use these values to obtain a result in *ORV* units. The Matlab procedure performs a local regression using weighted linear least squares with a 2nd degree polynomial model. This procedure is robust to other choices of smoothing.

[6] In Section 4.3, we provide a Monte Carlo simulation study that assesses the impact of variance jumps on *ORV* for different option maturities. Overall, option realized variances for short-dated options tend to be more reactive to jumps than their long-dated counterparts.

[7] We use one-minute log-returns of the E-mini S&P 500 futures contract prices, Y_j, to construct realized measures with the subsampling methodology of Zhang et al. (2005). The realized volatility corresponds to the average of $RV^{\text{Naive}}_{t-\tau,t}(\Delta_N) = \sum_{j=1}^{N}\left(Y_{t-\tau+j\Delta_N} - Y_{t-\tau+(j-1)\Delta_N}\right)^2$, which is computed over each subgrid. The realized jump variation *JV* of index returns is defined as $JV_{t-\tau,t} = \max\left[RV_{t-\tau,t} - BV_{t-\tau,t}, 0\right]$, where *BV* is the bipower variation. We compute the *BV* as the average of $BV^{\text{Naive}}_{t-\tau,t}(\Delta_N) = \frac{\pi}{2}\sum_{j=2}^{N}\left|Y_{t-\tau+j\Delta_N} - Y_{t-\tau+(j-1)\Delta_N}\right|\left|Y_{t-\tau+(j-1)\Delta_N} - Y_{t-\tau+(j-2)\Delta_N}\right|$, where $BV^{\text{Naive}}_{t-\tau,t}$ is computed over each grid. We employ five subgrids in our implementation of the subsampling methodology ($\Delta_N = 5$ minutes).

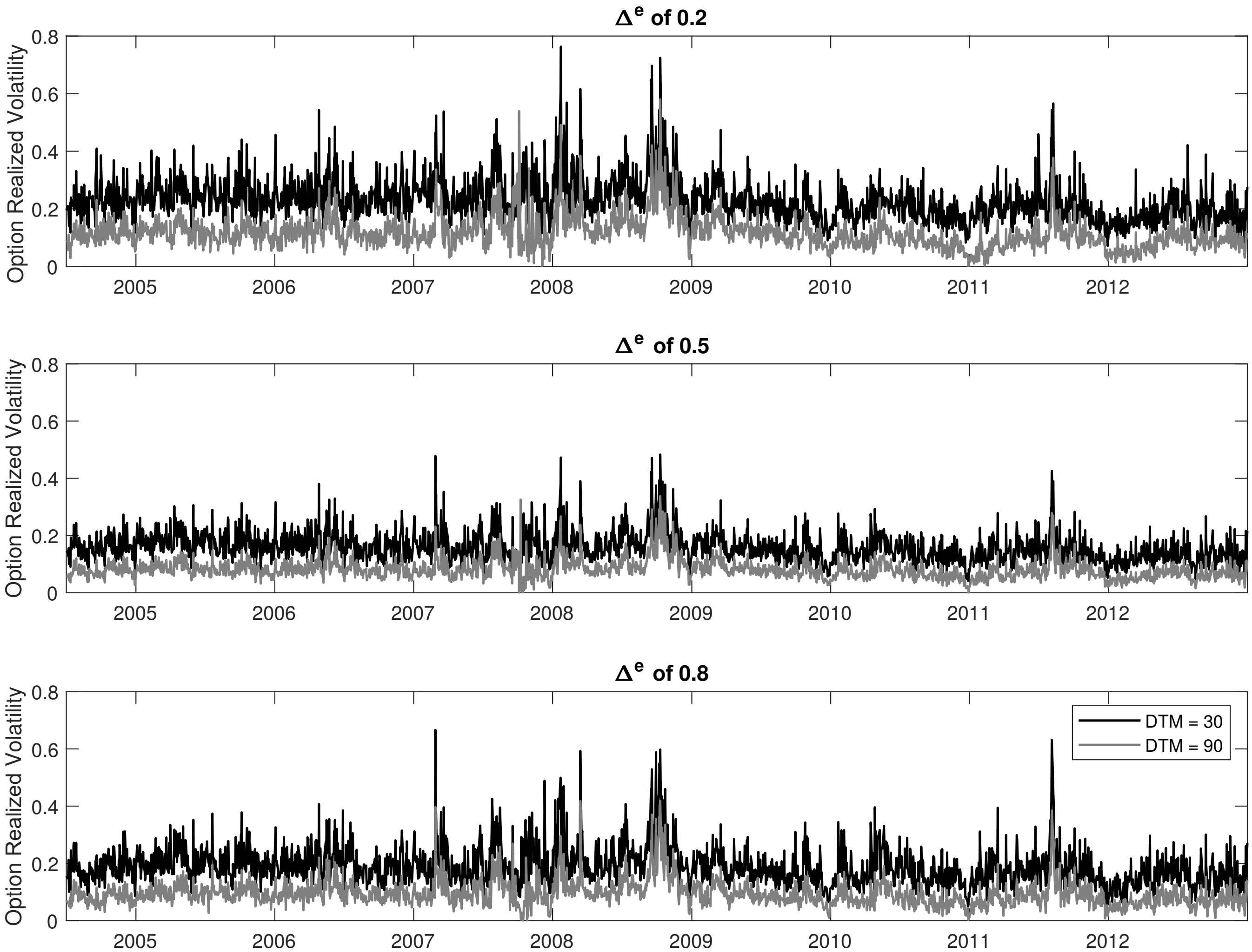

Figure 1: **Option Realized Volatility for Different Moneyness and Maturities.**
This figure presents daily values of *ORV* for selected moneyness and maturities. We perform a daily smoothed interpolation across moneyness and maturity of all *ORV*s in our sample to obtain a balanced panel. The top panel presents values for OTM call options ($\Delta^e = 0.2$), the middle panel for ATM options ($\Delta^e = 0.5$), and the bottom panel for OTM put options ($\Delta^e = 0.8$). The option realized volatility presented in the figure corresponds to the square root of the option realized variance. DTM stands for days to maturity. The sample period is July 2004 to December 2012.

3.2 Predictive Regressions

We use a series of predictive regressions to document the informational content of option realized variances. Throughout these analyses, we employ two extensions of Corsi's (2009) HAR model as it offers a unifying framework to include the *ORV* measures along with other market activity variables.

In what follows, we compute the variables of interest over the period $(t, t + h]$ and regress them on a selected option realized variance measured at time t, in addition to other predictors. In all of the reported

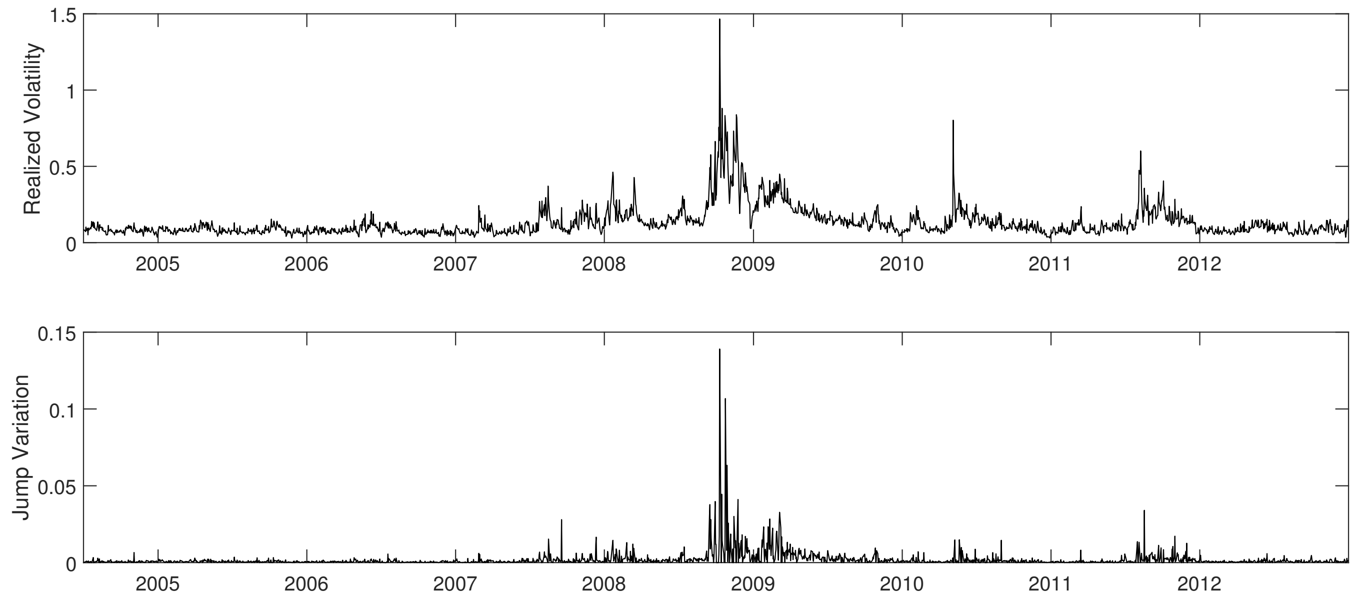

Figure 2: **Realized Volatility and Jump Variation of the S&P 500 Index.**
From July 2004 to December 2012, we use one-minute returns from the E-mini S&P 500 futures to compute two types of realized measures. The top panel shows the annualized realized volatility (i.e., the square root of *RV*) of index returns. The bottom panel reports the realized annualized jump variation (*JV*). This variation is defined as the positive part of the difference between *RV* and the bipower variation (*BV*), where the latter variable measures the variability of the diffusive component governing the return process (Barndorff-Nielsen and Shephard, 2004). To mitigate for the presence of microstructure errors, these two variables are constructed with the subsampling method of Zhang et al. (2005) using five subgrids.

Table 2: **Correlation Matrix for Option Realized Variances and Realized Measures.**

	$\Delta^e = 0.2$ DTM = 30	$\Delta^e = 0.5$ DTM = 30	$\Delta^e = 0.8$ DTM = 30	$\Delta^e = 0.2$ DTM = 90	$\Delta^e = 0.5$ DTM = 90	$\Delta^e = 0.8$ DTM = 90	*RV*
$\Delta^e = 0.5$, DTM = 30	0.9200						
$\Delta^e = 0.8$, DTM = 30	0.7972	0.9187					
$\Delta^e = 0.2$, DTM = 90	0.7987	0.7737	0.7136				
$\Delta^e = 0.5$, DTM = 90	0.8137	0.8561	0.8220	0.8468			
$\Delta^e = 0.8$, DTM = 90	0.7739	0.8493	0.9005	0.8068	0.9325		
RV	0.4995	0.5217	0.5061	0.6995	0.7410	0.6756	
JV	0.3812	0.3752	0.3628	0.5137	0.5095	0.4689	0.7175

This table presents sample correlations between selected *ORV* variables, the realized variance (*RV*), and the jump variation (*JV*). The option realized variances are obtained from a smoothed interpolation across moneyness and maturities of all available *ORV*s in a given day. Selected maturities correspond to 30 and 90 days (*DTM*). Selected moneyness correspond to OTM call options ($\Delta^e = 0.2$), ATM options ($\Delta^e = 0.5$), and OTM put options ($\Delta^e = 0.8$). The realized variance and the realized jump variation are computed from E-mini S&P 500 futures using Zhang et al.'s (2005) microstructure-noise robust estimates. The sample period is July 2004 to December 2012.

analyses, we use 30-day *ORV* values with call-equivalent deltas of 0.2 and 0.8. We include only one *ORV* series at a time in our regression to avoid multicollinearity issues, as evidenced from correlations reported in Table 2. We select moneyness of 0.2 and 0.8 for this exercise since they correspond to deep out-of-the-money options, which are contracts that have been used to study information about volatility dynamics (e.g., Bollerslev and Todorov, 2011).[8]

3.2.1 Index Price Variations

We start by studying the predictability of the future index price variations, as measured by RV, over forecasting horizons of one day, one week, one month and one year. Table 3 presents estimates, standard errors and adjusted R^2s for the two different specifications.

The first specification includes lagged realized variances measured over the previous day ($RV_t^{(\mathrm{d})}$), week ($RV_t^{(\mathrm{w})}$) and month ($RV_t^{(\mathrm{m})}$), as well as the lagged jump variation over the previous day ($JV_t^{(\mathrm{d})}$) and the implied variance (the square of the implied volatility) of an ATM 30-day option (IV_t).[9] This model combines the standard HAR-RV model of Corsi (2009), Andersen et al.'s (2007) jump-induced variation model, and Busch et al.'s (2011) forward-looking variation model. Panel A of Table 3 reports the first specification's results. The coefficients associated with the option realized variance are all positive and significant, meaning that an increase in the current option realized variance raises future index price variations. As evidenced by larger R^2s, this positive relationship is stronger for OTM put options with short-term forecasting horizons and for OTM call options with longer horizons. Interestingly, the ORV provides the largest gains in terms of R^2s for the longer horizon as their values increase from 11.04% to 17.31% for calls and to 15.07% for puts.

We reassess this evidence in the HAR-RV framework by controlling for the leverage effect captured in lagged values of negative returns as proposed in Corsi and Renò (2012). Similar to the first specification, we add the ATM 30-day option implied variance to control for forward-looking variability information. Panel B of Table 3 reports similar patterns to those reported in Panel A for coefficients of ORV variables.[10] There are no significant gains in terms of adjusted R^2s with the second specification for short horizons,

[8] As a robustness test, we use different call-equivalent delta values and obtain similar results. Section E.2 of the Supplementary Material reports regressions as those of Panel A of Table 3 using ORV from call-equivalent delta values that range between 0.1 and 0.9. In all of the 36 reported regressions, the parameter associated with ORV is positive. Moreover, for 34 out of the 36 specifications it is significantly different from zero.

[9] The past lagged realized variances are measured over three different horizons: (1) over the previous day, $RV_t^{(\mathrm{d})} \equiv RV_{t-1/252,t}$; (2) over the previous week, $RV_t^{(\mathrm{w})} \equiv RV_{t-5/252,t}$; and (3) over the previous month, $RV_t^{(\mathrm{m})} \equiv RV_{t-21/252,t}$. We also consider the jump variation over the previous day in the regressions, i.e., $JV_t^{(\mathrm{d})} \equiv \max[0, RV_{t-1/252,t} - BV_{t-1/252,t}]$.

[10] Three additional specifications using various conventional regressors are presented in Section E.1 of the Supplementary Material. The conclusions are similar to those presented in the core of the paper.

suggesting that the leverage effect captured by negative returns is also present in ORV. Given the empirical evidence of the relationship between variance jumps and the leverage effect (see Todorov and Tauchen, 2011), one could postulate that ORV's incremental information is related to this type of jump.[11] We elaborate further on this conjecture in Section 4 when we study ORV in the context of a parametric model.

Overall, the positive relationship between the current ORV and the future RV points to the existence of predictive information in option realized variances beyond that found in other variables associated with price variability.[12]

3.2.2 Equity Risk Premium and Variance Risk Premium

We now examine the predicting power of ORV with respect to *ex-post* compensations of risks. We follow Bollerslev et al. (2009) and Andersen et al. (2015b), and study the equity and variance risk premiums in a model-free fashion.

To proxy for the unobserved equity risk premium at time t, we compute log-excess-cum-dividend returns of the index over the interval $(t, t+h]$ and denote it by $ERP_{t,t+h}$. The variance risk premium is defined as the difference between the *ex-post* realized price variation over the interval $(t, t+h]$ and its *ex-ante* risk-neutral expectation,

$$VRP_{t,t+h} = \frac{1}{h}\left(RV_{t,t+h} - \mathrm{E}_t^{\mathbb{Q}}\left[RV_{t,t+h}\right]\right), \tag{3}$$

where the risk-neutral expectation is computed in a model-free way as done in Bollerslev et al. (2009) and in the computation of the VIX Index (Chicago Board Options Exchange, 2009).

Panels A and B of Table 4 present the results of HAR regressions in which the dependent variable is $ERP_{t,t+h}$, measured over one-day, one-week, one-month and one-year horizons.[13] As it was the case

[11] We conducted similar regressions with the future (return) jump variation instead of the realized variance as the dependent variable. In this case, ORV is statistically significant at a 95% confidence level for the one-month and one-year time horizons. The sign of the coefficients associated with the ORV measure in these regressions is positive, meaning that an increase in ORV has a positive impact on the jump variation.

[12] Section E.3 of the Supplementary Material assesses the impact of ORV from an out-of-sample perspective. Results reported in that section show that the in-sample gains documented above are also observed out of sample.

[13] In this table, we show only coefficients related to ORV variables, as our main focus is to provide evidence about the incremental information offered by these variables. We refer the interested reader to Section E.1 of the Supplementary Material for the complete set of estimates and additional robustness tests.

Table 3: **Predictive Heterogeneous Autoregressive Regressions on the Realized Variance.**

Panel A: Regression HAR-*RV*-*JV*-*IV*-*ORV*.

$RV_{t,t+h} = \beta_0 + \beta_1 RV_t^{(d)} + \beta_2 RV_t^{(w)} + \beta_3 RV_t^{(m)} + \beta_4 JV_t^{(d)} + \beta_5 IV_t + \beta_6 ORV_t^{\Delta^e=0.2} + \beta_7 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_0	**-0.0106**	**-0.0167**	**-0.0252**	**-0.0042**	**-0.0102**	**-0.0148**	**0.0039**	-0.0051	-0.0072	**0.0201**	**0.0063**	**0.0109**
	(0.0048)	(0.0056)	(0.0097)	(0.0020)	(0.0032)	(0.0051)	(0.0019)	(0.0045)	(0.0046)	(0.0019)	(0.0023)	(0.0019)
β_1	**0.2918**	0.2452	0.1401	**0.3094**	0.2623	0.1989	0.1427	0.0730	0.0275	0.0047	**-0.1024**	**-0.0907**
	(0.1293)	(0.1373)	(0.1286)	(0.1428)	(0.1457)	(0.1256)	(0.0780)	(0.0830)	(0.0727)	(0.0216)	(0.0308)	(0.0258)
β_2	**0.4374**	**0.4236**	**0.4139**	**0.2829**	**0.2689**	**0.2657**	**0.2598**	**0.2392**	**0.2419**	0.0699	0.0382	0.0551
	(0.1686)	(0.1674)	(0.1564)	(0.0989)	(0.0980)	(0.0950)	(0.1225)	(0.1149)	(0.1164)	(0.0367)	(0.0310)	(0.0345)
β_3	-0.4029	-0.4057	-0.3576	-0.0837	-0.0864	-0.0507	0.1902	0.1861	0.2246	0.0176	0.0113	0.0462
	(0.2757)	(0.2772)	(0.2546)	(0.1650)	(0.1648)	(0.1541)	(0.1803)	(0.1794)	(0.1653)	(0.0494)	(0.0453)	(0.0450)
β_4	-2.1064	-2.0010	**-1.7597**	**-1.6024**	-1.4960	**-1.3499**	-0.5027	-0.3451	-0.2396	0.0399	**0.2822**	**0.2581**
	(1.1028)	(1.0852)	(0.8751)	(0.8024)	(0.7785)	(0.6701)	(0.5580)	(0.5130)	(0.4291)	(0.1686)	(0.1302)	(0.1152)
β_5	**0.7482**	**0.7885**	**0.8010**	**0.4688**	**0.5094**	**0.5072**	0.1851	**0.2452**	**0.2251**	**0.1020**	**0.1945**	**0.1352**
	(0.2556)	(0.2561)	(0.2568)	(0.1017)	(0.1069)	(0.1029)	(0.0988)	(0.1072)	(0.0958)	(0.0446)	(0.0483)	(0.0425)
β_6 / β_7	–	**0.0973**	**0.3712**	–	**0.0983**	**0.2704**	–	**0.1456**	**0.2817**	–	**0.2238**	**0.2336**
	–	(0.0349)	(0.1362)	–	(0.0320)	(0.0965)	–	(0.0712)	(0.1109)	–	(0.0400)	(0.0470)
Adjusted R^2	0.5867	0.5887	0.6054	0.6897	0.6929	0.7044	0.5800	0.5891	0.6004	0.1104	0.1731	0.1507

Panel B: Regression LHAR-*RV*-*IV*-*ORV*.

$RV_{t,t+h} = \beta_0 + \beta_1 RV_t^{(d)} + \beta_2 RV_t^{(w)} + \beta_3 RV_t^{(m)} + \beta_4 r_t^{(d)} + \beta_5 r_t^{(w)} + \beta_6 r_t^{(m)} + \beta_7 IV_t + \beta_8 ORV_t^{\Delta^e=0.2} + \beta_9 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_0	**-0.0131**	**-0.0181**	**-0.0222**	**-0.0061**	**-0.0120**	**-0.0143**	0.0035	-0.0042	-0.0045	**0.0201**	**0.0077**	**0.0121**
	(0.0055)	(0.0067)	(0.0094)	(0.0026)	(0.0045)	(0.0057)	(0.0021)	(0.0045)	(0.0045)	(0.0019)	(0.0022)	(0.0020)
β_1	-0.0253	-0.0524	-0.0872	0.0604	0.0289	0.0045	0.0478	0.0066	-0.0070	0.0006	**-0.0656**	**-0.0544**
	(0.1908)	(0.1870)	(0.1961)	(0.0764)	(0.0713)	(0.0726)	(0.0528)	(0.0440)	(0.0459)	(0.0150)	(0.0283)	(0.0222)
β_2	**0.4204**	**0.4114**	**0.4108**	**0.2827**	**0.2721**	**0.2740**	**0.2611**	**0.2474**	**0.2527**	0.0337	0.0116	0.0252
	(0.1746)	(0.1712)	(0.1686)	(0.0829)	(0.0809)	(0.0812)	(0.1166)	(0.1096)	(0.1130)	(0.0404)	(0.0362)	(0.0384)
β_3	-0.1617	-0.1694	-0.1693	0.0778	0.0690	0.0710	**0.3221**	**0.3105**	**0.3154**	0.0533	0.0346	0.0466
	(0.1448)	(0.1469)	(0.1499)	(0.1454)	(0.1425)	(0.1437)	(0.1486)	(0.1494)	(0.1496)	(0.0500)	(0.0437)	(0.0462)
β_4	**-1.1932**	**-1.1811**	**-0.9890**	**-0.4428**	**-0.4287**	-0.2585	**-0.5575**	**-0.5390**	**-0.3765**	-0.1633	-0.1337	0.0182
	(0.4386)	(0.4388)	(0.3778)	(0.1386)	(0.1385)	(0.1391)	(0.2194)	(0.2060)	(0.1621)	(0.0979)	(0.0865)	(0.0819)
β_5	-2.3594	-2.3203	-2.0599	-2.2990	-2.2535	-2.0287	**-1.6970**	**-1.6373**	-1.4316	-0.0475	0.0482	0.2188
	(1.8967)	(1.8891)	(1.7447)	(1.3341)	(1.3199)	(1.2265)	(0.7868)	(0.7861)	(0.7751)	(0.3952)	(0.3856)	(0.4038)
β_6	**-5.7548**	**-5.3875**	**-4.8144**	-3.2521	-2.8244	-2.4031	-1.4019	-0.8414	-0.5683	-2.7308	-1.8319	-1.8946
	(2.3086)	(2.2688)	(2.1760)	(2.5053)	(2.4445)	(2.2979)	(2.0507)	(2.0107)	(1.9411)	(1.8670)	(1.6872)	(1.7639)
β_7	**0.4678**	**0.5056**	**0.5503**	**0.3126**	**0.3566**	**0.3871**	0.0054	0.0631	0.0786	0.0424	**0.1350**	**0.1158**
	(0.1365)	(0.1410)	(0.1588)	(0.1025)	(0.1075)	(0.1118)	(0.0875)	(0.0924)	(0.0944)	(0.0491)	(0.0496)	(0.0481)
β_8 / β_9	–	**0.0840**	**0.2370**	–	**0.0978**	**0.2140**	–	**0.1281**	**0.2101**	–	**0.2055**	**0.2107**
	–	(0.0368)	(0.1215)	–	(0.0387)	(0.0933)	–	(0.0706)	(0.1060)	–	(0.0375)	(0.0457)
Adjusted R^2	0.6123	0.6137	0.6188	0.6971	0.7003	0.7051	0.6006	0.6076	0.6104	0.1230	0.1758	0.1515

This table presents two HAR regressions in the spirit of Corsi (2009) where the dependent variable is the realized variance over future horizons. The variables $ORV_t^{\Delta^e=0.2}$ and $ORV_t^{\Delta^e=0.8}$ are the 30-day option realized variance for OTM calls with call-equivalent delta of 0.2 and the option realized variance for OTM calls with call-equivalent delta of 0.8. Lagged realized variance are daily $RV_t^{(d)} = RV_{t-1/252,t}$, weekly $RV_t^{(w)} = RV_{t-5/252,t}$ and monthly $RV_t^{(m)} = RV_{t-21/252,t}$. The implied variance, IV_t, comes from ATM options with a maturity of 30 days. The last panel includes the daily, weekly, and monthly lagged negative returns defined as $r_t^{(d)} = \min(0, r_t)$, $r_t^{(w)} = \min\left(0, \sum_{j=1}^{5} r_{t-j-1}\right)$, $r_t^{(m)} = \min\left(0, \sum_{j=1}^{21} r_{t-j-1}\right)$, respectively. Here r_t is the equity return during day t. Coefficients for OTM call ORVs in Columns 2, 5, 8 and 11 are given by β_6 (Panel A) and β_8 (Panel B). Coefficients for OTM put ORVs in Columns 3, 6, 9 and 12 are given by β_7 (Panel A) and β_9 (Panel B). Newey-West standard errors are presented in parentheses. Values in bold are statistically significant at a confidence level of 95%.

Table 4: **Predictive Regression on the Risk Premium.**

Panel A: Regression HAR-*RV-JV-IV-ORV*.

$ERP_{t,t+h} = \beta_0 + \beta_1 RV_t^{(d)} + \beta_2 RV_t^{(w)} + \beta_3 RV_t^{(m)} + \beta_4 JV_t^{(d)} + \beta_5 IV_t + \beta_6 ORV_t^{\Delta^e=0.2} + \beta_7 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_6 / β_7	–	**-0.0309**	-0.0225	–	**-0.0477**	-0.0450	–	**-0.1592**	-0.1937	–	**-1.4558**	**-1.4430**
	–	(0.0109)	(0.0168)	–	(0.0222)	(0.0340)	–	(0.0788)	(0.1008)	–	(0.2587)	(0.2988)
Adjusted R^2	0.0401	0.0458	0.0416	0.0424	0.0457	0.0439	0.0674	0.0789	0.0774	0.0478	0.1105	0.0841

Panel B: Regression LHAR-*RV-IV-ORV*.

$ERP_{t,t+h} = \beta_0 + \beta_1 RV_t^{(d)} + \beta_2 RV_t^{(w)} + \beta_3 RV_t^{(m)} + \beta_4 r_t^{(d)} + \beta_5 r_t^{(w)} + \beta_6 r_t^{(m)} + \beta_7 IV_t + \beta_8 ORV_t^{\Delta^e=0.2} + \beta_9 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_8 / β_9	–	**-0.0436**	**-0.0581**	–	**-0.0607**	**-0.0794**	–	**-0.1661**	**-0.2091**	–	**-1.3728**	**-1.3204**
	–	(0.0147)	(0.0253)	–	(0.0223)	(0.0363)	–	(0.0749)	(0.1002)	–	(0.2477)	(0.2941)
Adjusted R^2	0.0291	0.0409	0.0399	0.0520	0.0577	0.0570	0.0771	0.0897	0.0874	0.0568	0.1126	0.0832

Panel C: Regression HAR-*RV-JV-IV-ORV*.

$VRP_{t,t+h} = \beta_0 + \beta_1 RV_t^{(d)} + \beta_2 RV_t^{(w)} + \beta_3 RV_t^{(m)} + \beta_4 JV_t^{(d)} + \beta_5 IV_t + \beta_6 ORV_t^{\Delta^e=0.2} + \beta_7 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_6 / β_7	–	**0.1205**	**0.3242**	–	**0.1211**	**0.2238**	–	**0.1616**	**0.2713**	–	**0.3125**	**0.3730**
	–	(0.0353)	(0.1206)	–	(0.0322)	(0.0824)	–	(0.0713)	(0.1109)	–	(0.0434)	(0.0556)
Adjusted R^2	0.2379	0.2439	0.2647	0.3959	0.4052	0.4149	0.4631	0.4768	0.4862	0.3444	0.3962	0.3880

Panel D: Regression LHAR-*RV-IV-ORV*.

$VRP_{t,t+h} = \beta_0 + \beta_1 RV_t^{(d)} + \beta_2 RV_t^{(w)} + \beta_3 RV_t^{(m)} + \beta_4 r_t^{(d)} + \beta_5 r_t^{(w)} + \beta_6 r_t^{(m)} + \beta_7 IV_t + \beta_8 ORV_t^{\Delta^e=0.2} + \beta_9 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_8 / β_9	–	**0.1257**	**0.2622**	–	**0.1390**	**0.2388**	–	**0.1540**	**0.2389**	–	**0.2789**	**0.2611**
	–	(0.0364)	(0.1100)	–	(0.0417)	(0.0853)	–	(0.0696)	(0.1045)	–	(0.0403)	(0.0529)
Adjusted R^2	0.2442	0.2508	0.2594	0.3780	0.3903	0.3969	0.4748	0.4873	0.4904	0.3952	0.4366	0.4138

This table presents HAR regressions in the spirit of Corsi (2009) where the dependent variable is the equity risk premium $ERP_{t,t+h}$ (Panels A and B) or the variance risk premium $VRP_{t,t+h}$ (Panels C and D) realized over a future period of length h. Independent variables are the same as those employed in regressions of Table 3. We only present coefficients associated with ORV variables. For OTM call options ($ORV_t^{\Delta^e=0.2}$), Panels A and C present the coefficient β_6 in Columns 2, 5, 8 and 11, while Panels B and D present the coefficient β_8 in the same columns. For OTM put options ($ORV_t^{\Delta^e=0.8}$), Panels A and C present the coefficient β_7 in Columns 3, 6, 9 and 12, while Panels B and D present the coefficient β_9 in the same columns. Newey-West standard errors are presented in parentheses. Values in bold are statistically significant at a confidence level of 95%.

when predicting future returns, the degree of predictability is quite low for short horizons. It increases for longer horizons, yet the fact that we are using multi-period return regressions hinders the interpretation of these R^2s as they tend to increase in proportion to the return horizon and the length of the overlap period (see Bollerslev et al., 2009). Nonetheless, we can conclude that, over a long horizon, the *ORV* of the OTM call brings the largest gains compared to the specification that does not include these variables. Finally, the negative sign of the coefficients associated with *ORV* suggests that the effect of option realized variance is dominated by different episodes in our sample during which large variance spikes are followed by decreases in the index value.

We now turn to the relationship between *ORV* and the variance risk premium. Panels C and D of Table 4 report the results with $VRP_{t,t+h}$ on the left-hand side of the two HAR specifications.[14] All coefficients associated with $ORV_t^{\Delta^e=0.2}$ and to $ORV_t^{\Delta^e=0.8}$ are positive and statistically significant at a 95% level of confidence. In this case, positive coefficients are in line with the results obtained when forecasting *RV* in Table 3. Regarding adjusted R^2s, a high degree of predictability is observed, which can be attributed to the satisfactory empirical performance of HAR regressions in the case of variance forecasting. Furthermore, as observed when predicting *RV*, the R^2s are larger when we include *ORV* measures.

In summary, the option realized variance offers additional information on risk premiums. Specifically, OTM calls seem to produce substantial gains for long-term horizons, while OTM puts do so for shorter terms.

4 Properties of Option Realized Variances

In the previous two sections, we introduced a robust measure of intraday option price variability that predicts index realized variances as well as the equity and variance risk premiums. To analyze the economic mechanism behind these relationships, this section first presents an asset pricing model that captures different properties of asset prices: diffusive stochastic volatility, return jumps, and variance jumps. These three risk factors impound different characteristics in asset prices, so we analyze in the second part of this section how intraday option price variability relates to these components. We conclude with a Monte Carlo experiment that provides further intuition about the results of this section.

[14]Other regressions are presented in Section E.1 of the Supplementary Material.

4.1 Model

The model belongs to the family of stochastic volatility models with jumps (SVJ). This affine jump-diffusion model yields semi-closed-form solutions for option pricing as shown in Duffie et al. (2000). Consider a security with a price of S. Under the objective measure $\mathbb{P}$, the process governing the dynamics of the log-equity price, $Y = \log(S)$, and its instantaneous stochastic variance, V, are

$$dY_t = \left(r - q + \left(\eta_Y - \frac{1}{2}\right) V_{t^-} + \left(\gamma_Y - \left(\varphi^{\mathbb{P}}_{Z_Y}(1) - 1\right)\right) \lambda_{Y,t^-}\right) dt + \sqrt{V_{t^-}}\, dW_{Y,t} + dJ_{Y,t}, \tag{4}$$

$$dV_t = \kappa (\theta - V_{t^-})\, dt + \sigma \sqrt{V_{t^-}}\, dW_{V,t} + dJ_{V,t}, \tag{5}$$

$$W_{Y,t} = \rho W_{V,t} + \sqrt{1-\rho^2} W_{\perp,t},$$

$$J_{Y,t} = \sum_{n=1}^{N_{Y,t}} Z_{Y,n}, \qquad Z_{Y,n} \sim \mathcal{N}(\mu_Y; \sigma_Y^2),$$

$$J_{V,t} = \sum_{n=1}^{N_{V,t}} Z_{V,n}, \qquad Z_{V,n} \sim \mathrm{Exp}(\mu_V),$$

where $\{W_{V,t}\}_{t\geq 0}$ and $\{W_{\perp,t}\}_{t\geq 0}$ are two independent standard $\mathbb{P}$-Brownian motions. The risk-free interest rate is given by r and the instantaneous dividend yield by q. The parameters η_Y and γ_Y capture the diffusive and jump risk premiums, respectively.[15] The function $\varphi^{\mathbb{P}}_{Z_Y}(1)$ is the cumulant generating function of Z_Y evaluated at 1. The return jumps are generated by a Cox process $\{N_{Y,t}\}_{t\geq 0}$ with a stochastic intensity that depends on the instantaneous variance: $\lambda_{Y,t^-} = \lambda_{Y,0} + \lambda_{Y,1} V_{t^-}$. The size of these jumps are given by Gaussian random variables with mean μ_Y and standard deviation σ_Y.[16,17] Regarding variance jumps, these are governed by a Poisson process $\{N_{V,t}\}_{t\geq 0}$ that has a constant intensity $\lambda_{V,t^-} = \lambda_{V,0}$. The variance jump

[15] Our Radon-Nikodym derivative explained in Appendix B.1 includes four equivalent martingale measure processes: Λ_{Y,u^-}, Λ_{V,u^-}, Γ_Y and Γ_V. The process Λ_{Y,u^-} is $\eta_Y \sqrt{V_{u^-}}$, as in Heston (1993) among others. Λ_{V,u^-} is defined analogously. Moreover, γ_Y is a nontrivial function of Γ_Y. Even though η_V and Γ_V are not involved directly in our $\mathbb{P}$-measure modelling, these two parameters deal with the change of measure of the variance diffusive and jump components.

[16] This kind of jump process is used by Bates (1996), Bakshi et al. (1997), Duffie et al. (2000), Pan (2002), Eraker et al. (2003), Johannes et al. (2009) to name a few.

[17] For this model, the jump convexity correction is given by $\varphi^{\mathbb{P}}_{Z_Y}(1) - 1 = \exp(\mu_Y + \sigma_Y^2/2) - 1$.

sizes $\{Z_{V,n}\}_{n=1}^{\infty}$ are given by independent exponentially distributed random variables with mean μ_V.[18,19]

The drift process is a by-product of the Radon-Nikodym derivative used in this study. The change of measure is specified so that model dynamics under both measures keep the same structure. This implies that each risk factor is priced, from equity and variance Brownian motions to the jump processes $J_{Y,t}$ and $J_{V,t}$. A similar methodology is used in Bates (1991, 2006), Liu et al. (2005), Eraker (2008), Christoffersen et al. (2012), and Ornthanalai (2014), among others.

4.2 Option Quadratic Variation

The option realized variance examined in Sections 2 and 3 corresponds to an estimate of the quadratic variation associated with the logarithm of the option price. Under usual conditions, the price of a European option is given by

$$O_t(Y_t,V_t) \equiv \mathrm{E}^{\mathbb{Q}}\left[e^{-r(T-t)}F(Y_T,K)\,\Big|\,Y_t,V_t\right],$$

where $F(Y_T,K)$ is the time-T payoff function, K the strike price of the option, and $\mathbb{Q}$ a risk-neutral probability measure.

In the proposed framework, the option quadratic variation over an interval of time $(t-\tau,t]$ can be written as[20]

$$\begin{aligned}\Delta OQV_{t-\tau,t} = &\int_{t-\tau}^{t}\left(S_{u^-}^2\,\Delta_{u^-}^2+2\sigma\rho S_{u^-}\,\Delta_{u^-}\mathcal{V}_{u^-}+\sigma^2\,\mathcal{V}_{u^-}^2\right)\frac{V_{u^-}}{O_{u^-}^2}\,du\\ &+\sum_{t-\tau<u\leq t}\left(\log(O_u(Y_u,V_u))-\log(O_{u^-}(Y_{u^-},V_{u^-}))\right)^2,\end{aligned} \tag{6}$$

where $\Delta_{u^-} = \frac{\partial O_{u^-}}{\partial S}(Y_{u^-},V_{u^-})$ is the option delta and $\mathcal{V}_{u^-} = \frac{\partial O_{u^-}}{\partial V}(Y_{u^-},V_{u^-})$ represents the option variance

[18] As argued in Bandi and Renò (2016), among others, jumps in log-equity and variance processes tend to happen at the same time, as in the so-called stochastic volatility with correlated jumps (SVCJ) specification. Although we do not include this feature directly in the proposed framework, both Poisson processes could jump during a given interval, which reconciles with the co-jump empirical evidence. Moreover, our specification makes the inference of latent states more difficult as the noise-to-signal ratio is higher.

[19] Among others, Duffie et al. (2000), Eraker et al. (2003), Johannes et al. (2009) and Andersen et al. (2015b) use exponentially distributed variance jumps. Note that Bates (2000), Duffie et al. (2000), Pan (2002), Eraker et al. (2003) and Todorov and Tauchen (2011) provide evidence for the presence of positive jumps in volatility.

[20] Since the option price is a smooth function of Y and V, Itō's lemma can be applied to determine its quadratic variation. Details are available in Section J of the Supplementary Material.

vega. Two components determine this quadratic variation. On the one hand, the first term captures diffusive information, which relates to the smooth incorporation of changes in the information set. On the other hand, the last term incorporates sudden and abrupt changes arising from jumps. This last component can be further divided into two parts:

$$
\begin{aligned}
&\sum_{t-\tau<u\leq t} \left(\log\left(O_u\left(Y_u, V_u\right)\right) - \log\left(O_u\left(Y_{u^-}, V_{u^-}\right)\right)\right)^2 \\
&= \underbrace{\sum_{t-\tau<u\leq t} \left(\log\left(O_u\left(Y_u, V_u\right)\right) - \log\left(O_u\left(Y_{u^-}, V_u\right)\right)\right)^2}_{\text{Related to Return Jumps}} + \underbrace{\sum_{t-\tau<u\leq t} \left(\log\left(O_u\left(Y_u, V_u\right)\right) - \log\left(O_u\left(Y_u, V_{u^-}\right)\right)\right)^2}_{\text{Related to Variance Jumps}}, \qquad (7)
\end{aligned}
$$

which come from the two types of jumps in Equations (4) and (5). However, the explicit presence of variance jumps in Equation (7) makes ΔOQV an appealing variable, especially when contrasted with the type of jumps captured in the quadratic variation measures of the underlying asset price process. Indeed, the quadratic variation of the log-price process Y_t, measured empirically by RV, can be written as

$$\Delta QV_{t-\tau,t} = \int_{t-\tau}^{t} V_{u^-}\, du + \sum_{n=N_{Y,t-\tau}+1}^{N_{Y,t}} \left(Z_{Y,n}\right)^2. \qquad (8)$$

Notice that the second term in Equation (8), the jump component only incorporates return jumps whereas all the information related to the variance goes in the first term—the integrated variance. Even though it is possible to disentangle these two terms using the realized variance and bipower variation estimators, it is not possible to identify variance jumps with these estimators from ΔQV alone.[21] Therefore, the option quadratic variation—or an estimator of the latter such as the ORV—emerges as an alternative source to identify model features related to variance jumps.[22]

An additional aspect of interest in Equation (6) is that only partial derivatives appear in the diffusive component of ORV, suggesting that this quantity can be characterized with respect to the option money-

[21]The bipower variation is an estimate of the integrated variance–the first term in Equation (8)–and could be used in concert with the realized variance to capture the return jumps (e.g., Barndorff-Nielsen and Shephard, 2006). However, this is not possible for variance jumps since they are not involved in this decomposition.

[22]To the best of our knowledge, nonparametric tests to capture variance jumps from option prices do not exist and would be extremely hard to construct in a model-free manner. The main limitation is that asset pricing models are required to link option prices with the unobserved volatility.

ness level. We know from Equation (6) that the option realized variance depends on the option's delta and variance vega. This means that the option moneyness determines which model components drive the option realized variance. Specifically, for deep OTM options, the delta and variance vega should be close to zero, so most of the option quadratic variation comes from jump-induced variations:

$$\Delta OQV_{t-\tau,t}^{\mathrm{OTM}} \cong \sum_{t-\tau<u\leq t} \left(\log(O_u(Y_u,V_u)) - \log(O_{u^-}(Y_{u^-},V_{u^-}))\right)^2 . \tag{9}$$

This is consistent with the nonparametric evidence presented in Andersen et al. (2015a): OTM options behave like a pure-jump process. It is precisely this last property that makes ΔOQV an interesting quantity from a model estimation perspective. Moreover, this property also implies that ORV for ITM options would not contain additional information about return and variance jumps to that already impounded in OTM options and realized variances of the underlying asset.

4.3 Monte Carlo Experiment

To illustrate the results of Section 4.2, we run a Monte Carlo experiment to assess the impact of variance jumps on option returns—the input used to calculate ORV. To do so, we generate one-year paths of the data generating process at a sampling frequency of $1/1{,}560$ (every 15 seconds for each 6.5-hour trading day). Then for each period in a given path, we compute prices of OTM options with call-equivalent deltas of 0.2, 0.5 and 0.8, and maturities of 30, 90, and 150 days (a total of nine option contracts). We use parameters estimated from market data and provided in Table 6 (see Section 5).

We start our analysis with a simple case study that focuses on one specific day during which a variance jump occurred. Figure 3 reports intraday values of equity returns, the instantaneous variance and option returns for nine different options. From these figures, we observe that the variance jump is more pronounced in intraday option returns than in price returns, so looking only at the latter type of returns could hide key information about the presence of these jumps. It is precisely this type of information that the option's quadratic variation in Equation (9) captures, as variance jumps form a separate component in this expression. Regarding the type of options that better capture this jump activity, we observe that the variance jump has a larger impact on OTM options than ATM options, especially for short-term maturities.[23]

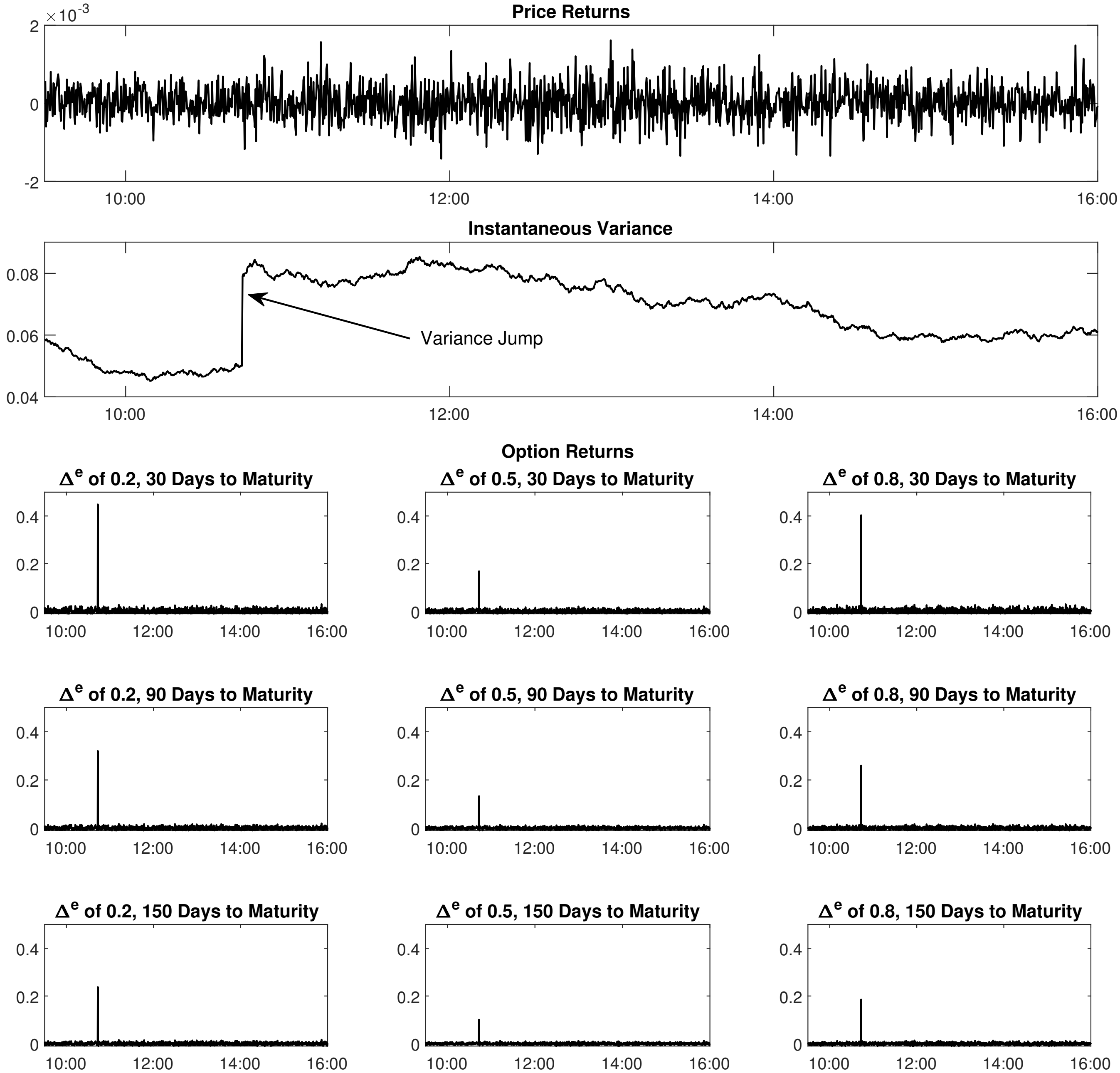

Figure 3: **Example of Option Returns for Different Maturities and Levels of Moneyness.**
This figure reports the intraday equity returns, the instantaneous variance, and the intraday returns associated with nine different option contracts. We generate a one-day path of the data generating process at a sampling frequency of 1/1,560 (every 15 seconds for each 6.5-hour trading day) during which a variance jump occurred. We then compute model option prices for different contracts. We consider OTM call, ATM, and OTM put options with call-equivalent deltas 0.2, 0.5 and 0.8, respectively, as well as maturities of 30, 90, and 150 days. Model parameters are presented in Table 6.

[23]In Section G of the Supplementary Material, we show that returns of ITM options exhibit much more smaller jumps, so these contracts are less informative about the arrival of variance jumps. This result further justifies our choice of using OTM and ATM options in this study.

Table 5: **Mean of the Average Option Jump Size Relative to the Variance Jump.**

	$\Delta^e = 0.2$	$\Delta^e = 0.5$	$\Delta^e = 0.8$
DTM = 30	13.340	7.298	11.340
DTM = 90	6.361	3.243	5.106
DTM = 150	4.220	2.046	3.166

This table reports the mean of the average option jump size relative to the variance jump across 100 paths of the data generating process using Equation (10). Specifically, we generate 100 one-day paths of the data generating process at sampling frequency of 1/1,560 (every 15 seconds during a 6.5-hour trading day). We then compute model option prices for multiple different contracts. We consider OTM call, ATM, and OTM put options with call-equivalent deltas 0.2, 0.5 and 0.8, respectively, as well as maturities of 30, 90, and 150 days. Model parameters are presented in Table 6.

We now use all simulated paths to assess the relative contribution of options in the identification of variance jumps. To quantify the sensitivity of a given option with respect to a variance jump, we average across a sample path the ratio between the change in option price and the size of the variance jump originating this change, that is:

$$\frac{1}{N_{V,1}} \sum_{n=1}^{N_{V,1}} \frac{\log\left(O_{\tau_n}(Y_{\tau_n}, V_{\tau_n})\right) - \log\left(O_{\tau_n}(Y_{\tau_n}, V_{\tau_n^-})\right)}{Z_{V,n}}, \tag{10}$$

where τ_n is the arrival time of the n^{th} variance jump. Table 5 reports the mean of this quantity across the 100 simulated paths. The largest sensitivities are observed for short-maturity options, with that of OTM calls being the highest at 13.34, six times greater than that observed for the ATM option with the longest maturity. Overall, these results support those presented in Section 3: OTM and short-dated maturity options contain valuable information regarding variance jumps.

In summary, the additional information contained in *ORV* can be linked to variance jumps since this information is not explicitly incorporated in variables that employ intraday index returns like the *RV* or *JV*. In addition, daily information from option prices, such as the implied volatility, does not include intraday activity in option prices as *ORV* does. The results of this section provide an economic interpretation of the evidence presented in the predictive regressions of Section 3, pointing to variance jumps as a plausible source driving these results.

5 Empirical Implications

This section provides empirical results using the model introduced in Section 4. We start by estimating the parameters of the model with different sets of information to assess the impact of adding *ORV* to the information set. We then use these parameters to disentangle latent variables and analyze the impact of using the option realized variances. We also compute the model-implied (integrated) risk premiums and study model-implied latent factors for forecasting realized risk premiums. Finally, we look at the fitting performance using out-of-sample analyses.

5.1 Parameter Estimates

We obtain model parameters by combining several observable variables into a likelihood function using a Monte Carlo-based filtering approximation, as described in Appendix C. This procedure combines several observable variables into a likelihood function computed using a Monte Carlo-based filtering approximation. The first set of observable variables are daily S&P 500 index log-returns and their realized variance and bipower variation, which were defined in Section 3. The second set is composed of daily option prices and their realized variances. [24] Option prices come from OptionMetrics and correspond to European S&P 500 index option contracts. As argued in Bates (2000), the large number of options in a given day becomes a hurdle for estimation routines, so we use a representative sample of options by restricting our attention to OTM and ATM options with maturities closest to 30 and 90 days, and call-equivalent deltas closest to 0.2, 0.35, 0.5, 0.65 and 0.8.[25]

Options with positive volume and bid price are included in the sample, as well as those satisfying the no-arbitrage conditions of Bakshi et al. (1997).[26] Thus, the final sample of options consists of a panel of 21,400 contracts.

To assess the information contained in *ORV*, we first estimate the model excluding these variables from

[24] As explained in Appendix C, we use price changes to compute *ORV* in the estimation procedure, which provides some numerical advantages such as a more stable estimation scheme and a decrease in the computational time. This price-based version preserves the variance jump term and contains the same information as the return-based *ORV* used in the previous sections. More details are provided in Section H of the Supplementary Material.

[25] These maturities and moneyness levels are consistent with the contracts employed in Section 3. All of our results are robust to other choices of moneyness levels and maturities.

[26] Option realized variances are computed following the procedure discussed in Section 2.2 from the Tick Data dataset.

Table 6: **S&P 500 Parameter Estimates.**

Panel A: With *ORV*.								
Log-Price Process			Variance Process			Standard Deviations of Error Terms		
η_Y	0.7635	(0.0230)	η_V	−0.2199	(0.0013)	η_1	0.2160	(0.0023)
γ_Y	0.0058	(0.0010)	Γ_V	0.5811	(0.0019)	η_2	0.2296	(0.0023)
ρ	−0.4336	(0.0011)	κ	5.7808	(0.0061)	η_3	0.2471	(0.0018)
$\lambda_{Y,0}$	2.1530	(0.0498)	θ	0.0085	(0.0177)	η_4	0.4600	(0.0028)
$\lambda_{Y,1}$	29.0744	(0.0682)	σ	0.8935	(0.0007)			
μ_Y	−0.0050	(0.0018)	$\lambda_{V,0}$	7.6498	(0.0090)			
σ_Y	0.0163	(0.0129)	μ_V	0.0214	(0.0101)			
			V_0	0.0118	(0.0174)			
Panel B: Without *ORV*.								
Log-Price Process			Variance Process			Standard Deviations of Error Terms		
η_Y	1.8734	(0.0311)	η_V	−1.5870	(0.0033)	η_1	0.2407	(0.0034)
γ_Y	0.0009	(0.0004)	Γ_V	0.7061	(0.0016)	η_2	0.2288	(0.0022)
ρ	−0.3912	(0.0015)	κ	4.7614	(0.0107)	η_3	0.2453	(0.0018)
$\lambda_{Y,0}$	0.0045	(2.5987)	θ	0.0030	(0.0825)	η_4	-	-
$\lambda_{Y,1}$	30.0106	(0.0992)	σ	0.9121	(0.0010)			
μ_Y	−0.0062	(0.0049)	$\lambda_{V,0}$	11.4438	(0.0065)			
σ_Y	0.0189	(0.0464)	μ_V	0.0174	(0.0041)			
			V_0	0.0114	(0.0242)			

Model parameters are estimated using daily index returns, realized variances, bipower variations, daily option prices, and option realized variances, from July 2004 to December 2012. We use a representative sample of options and option realized variances by restricting our attention to OTM or ATM options with maturities closest to 30 and 90 days, and call-equivalent deltas closest to 0.2, 0.35, 0.5, 0.65 and 0.8. We obtain model parameters using the estimation procedure described in Section C. The final sample of options consists of a panel of 21,400 contracts. Parameters are estimated using multiple simplex search method optimizations (fminsearch in Matlab). Robust standard errors are computed from the outer product of the gradient at optimal parameter values and are multiplied by 100 in the table. Panel A presents estimation results using the full sample of observable covariates, while Panel B presents parameter estimates when *ORV*s are excluded from the information set.

the set of observables and then re-estimate it using the complete set. Table 6 reports parameter estimates and standard errors (multiplied by 100, in brackets) for both cases. We first consider the parameters governing the jump process intensities. When *ORV*s are introduced, we observe a decrease in the intensity of the jump process governing variance jumps, $\lambda_{V,0}$, and an increase in the parameter governing the size of the jumps, μ_V. These parameters suggest that *ORV* is detecting variance jumps that arrive less frequently and have higher magnitudes. Regarding jumps in the log-equity price process, we observe an increase in the base intensity, $\lambda_{Y,0}$, as well as in the average size of jumps, μ_Y, in absolute value. These values show that *ORV* is favouring more frequent negative jumps with lower magnitudes, which is also observed from the average intensity—0.0112 for the estimation with *ORV* and 0.0028 without.[27]

Figure 4 displays filtered return jumps and variance jumps including *ORV* (first row) and excluding

[27]For the estimation results including *ORV*s in the information set, 51.7% of variance jumps happen on days where a return jump is filtered. This percentage of common jumps at a daily frequency shows that our specification is consistent with previous models such as the SVCJ, in which this feature is directly incorporated into the data process.

ORV (second row). Whereas the dynamics of return jumps are similar in both cases, we identify different dynamics for variance jumps. In particular, without option realized variances, the variance jumps are more frequent and tend to be smaller on average. However, there is a striking decrease in the activity of variance jumps when *ORV* is included in the estimation set. The posterior standard deviation of filtered variance jumps is about fivefold smaller in this case, meaning that variance jumps are harder to extract when option realized variances are not included in the model estimation. Moreover, variance jumps filtered with *ORV* are associated with specific periods, arrive less frequently, and display larger sizes—this is very much in line with the notion of burst in volatility as documented in Todorov and Tauchen (2011) and Christensen et al. (2014).

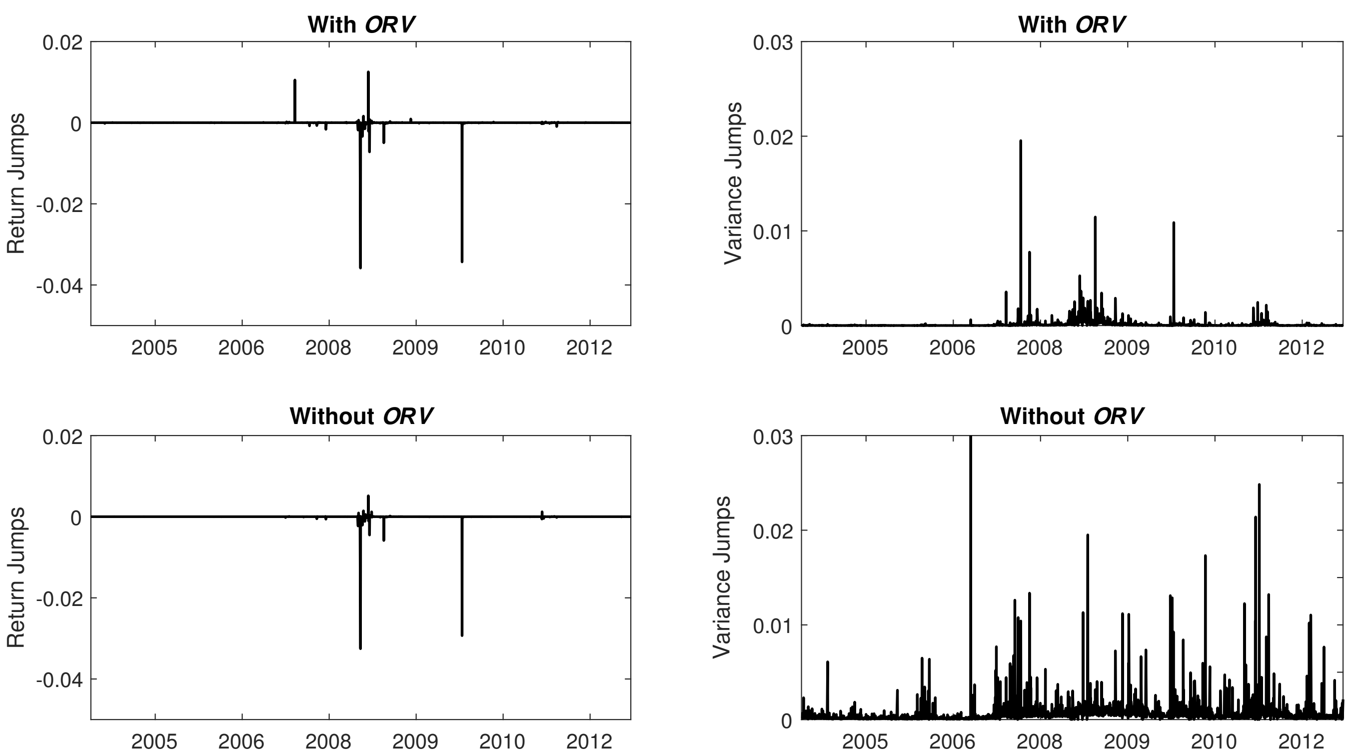

Figure 4: **Filtered Return Jumps and Variance Jumps.**
The index parameters are estimated using daily index returns, realized variances, bipower variations, daily option prices, and option realized variances, from July 2004 to December 2012. We use a representative sample of options and option realized variances by restricting our attention to OTM or ATM options with maturities closest to 30 and 90 days, and call-equivalent deltas closest to 0.2, 0.35, 0.5, 0.65 and 0.8. We obtain model parameters using the estimation procedure described in Section C. The final sample of options consists of a panel of 21,400 contracts. Model parameters correspond to those of Table 6. Using the estimated parameters, the (average) return jumps (left panels) and variance jumps (right panels) are extracted from the filtering procedure. The top panels show the filtered jumps based on returns, option implied volatilities, realized variances, bipower variations and option realized variances. The bottom panels exclude our new source of information, the option realized variances (i.e., without *ORV*).

In summary, the above results complement those of Sections 3 and 4 as they explicitly link the additional information in ORV to the dynamics of variance jumps.

5.2 Model-Implied Risk Premiums

We turn next to analyzing the equity and variance risk premiums obtained from the two parameter sets under consideration. Since jumps play a decisive role in the short-term of risk-premium term structures (see, e.g., Bardgett et al., 2019), we focus our analysis on this part of the term structure. Following Aït-Sahalia et al. (2015) and Bardgett et al. (2019), we define the integrated equity risk premium as:[28]

$$IERP_{t,t+h} = \frac{1}{h}\left(\mathrm{E}_t^{\mathbb{P}}\left[Y_{t+h} - Y_t\right] - \mathrm{E}_t^{\mathbb{Q}}\left[Y_{t+h} - Y_t\right]\right),$$

which can be decomposed into two parts: $IERP_{t,t+h}^{\mathrm{Diffusion}}$ and $IERP_{t,t+h}^{\mathrm{Jump}}$. Similarly, the integrated variance risk premium is defined as

$$IVRP_{t,t+h} = \frac{1}{h}\left(\mathrm{E}_t^{\mathbb{P}}\left[\Delta QV_{t,t+h}\right] - \mathrm{E}_t^{\mathbb{Q}}\left[\Delta QV_{t,t+h}\right]\right),$$

and, again, can be decomposed into diffusive and jump components. These two integrated risk premiums are computed in closed-form solution for the particular model used in our study and are functions of the time-t instantaneous variance. We provide derivations of these quantities in Section I of the Supplementary Material.

Figure 5 exhibits the evolution of one-day integrated equity ($IERP$) and variance risk premiums ($IVRP$) for the two information sets under consideration. In line with previous studies, we observe that both premiums vary significantly across time and have sporadic spikes during turbulent times. Regarding the $IERP$, we find that the average level of compensation decreases with the addition of ORV to the information set—it goes from 4.51% to 3.39%. This decrease can be explained by the fact that ORV is favouring more frequent negative price jumps, so compensation for bearing this type of risk increases (i.e.,

[28]As discussed in Aït-Sahalia et al. (2015), the $IERP$ represents the familiar equity risk premium. Following these authors, we use term "integrated" to differentiate the ERP from the instantaneous risk premium that is commonly studied in these models.

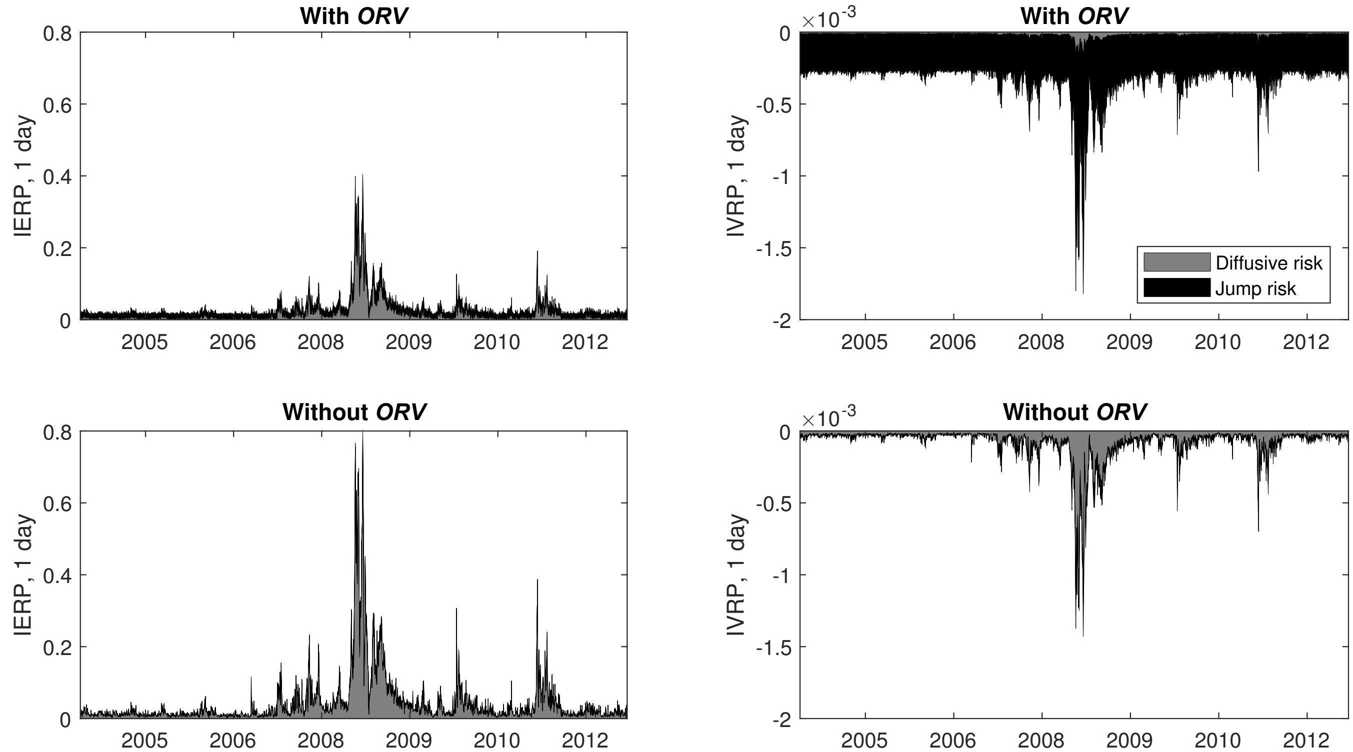

Figure 5: **Integrated Equity and Variance Risk Premiums over a Time Horizon of One Day.**
The index parameters are estimated using daily index returns, realized variances, bipower variations, daily option prices, and option realized variances, from July 2004 to December 2012. We use a representative sample of options and option realized variances by restricting our attention to OTM or ATM options with maturities closest to 30 and 90 days, and call-equivalent deltas closest to 0.2, 0.35, 0.5, 0.65 and 0.8. We obtain model parameters using the estimation procedure described in Section C. The final sample of options consists of a panel of 21,400 contracts. Model parameters are given in Table 6. Using the estimated parameters, the (average) instantaneous variance is extracted from the filtering procedure and is used to calculate the integrated risk premiums over a one-day time horizon (see Section I of the Supplementary Material for more details on the computation of these integrated risk premiums). The figure shows both the integrated diffusive and jump risk premiums (on an annual basis) as given by the model ($IERP$ and $IVRP$, respectively). Both areas are stacked. The top panels show the filtered premiums based on returns, option implied volatilities, realized variances, bipower variations and option realized variances. The bottom panels exclude option realized variances (i.e., without ORV).

0.06% without ORV against 1.63% with ORV, on average) at the expense of a decrease in the compensation of equity diffusive risk (4.45% when ORV is excluded against 1.76% when ORV is used). However, given the infrequent character of these jumps, the combined compensation decreases on average with the addition of this new information.[29]

When we look at the effect of ORV on the compensation for variance risk (rightmost panels of Figure 5), a different pattern emerges: the inclusion of ORV increases the average premium for bearing this risk.

[29]Estimates for equity jump risk premiums are close but lower than those reported in Broadie et al. (2007), Christoffersen et al. (2012), and Ornthanalai (2014). Variations in the sampling period, datasets and methodologies could also explain these small differences to some extent.

Adding *ORV* produces less frequent variance jumps but increases their sizes, which impacts positively the average compensation for variance jump risk (-3.24 bps with *ORV* against -0.12 bps without). In addition, variance diffusive risk premium decreases from -0.78 bps to -0.17 bps. In contrast to the *IERP* case, the *IVRP* increases with the incorporation of *ORV* as jumps in the variance are more frequent than those in the returns.

Overall, the previous results suggest that the premiums of discontinuous risks are underestimated when information about intraday option prices is excluded from the estimation step. The effect of this misspecification on the total compensation for equity and variance risk is not trivial as the contribution of discontinuous risk plays an important role.

The previous analyses were *ex-ante* in nature, helping to provide an understanding of the contribution of *ORV* in disentangling diffusive and discontinuous risk premiums. We now look at the *ex-post* contribution of *ORV* in determining risk premiums. To this end, we follow the work of Andersen et al. (2015b) and Santa-Clara and Yan (2010) and explore the relationship between the model-implied risk factor V_t and the future realized risk premiums.

Given that an estimator of V_t—the filtered value in this case—is already available for the two information sets, we use these values and conduct the following predictive regressions:

$$RP_{t,t+h} = \beta_0 + \beta_1 \widetilde{V}_t + \varepsilon_{t,t+h}, \tag{11}$$

where $\widetilde{V}_t$ is the filtered value of the time-t instantaneous variance. In these regressions, the left-hand side term, $RP_{t,t+h}$, is either the equity risk premium ($ERP_{t,t+h}$) or the variance risk premium ($VRP_{t,t+h}$) for the time horizon h.[30] The risk premiums $ERP_{t,t+h}$ and $VRP_{t,t+h}$ are computed as in Section 3.2.2. Table 7 presents the coefficients, the standard errors, and the adjusted R^2s for the regressions of Equation (11) over time horizons of one week, one month, three months, and one year (similar to the horizons considered in Santa-Clara and Yan, 2010).

[30]Since the model-implied risk premiums $IERP_{t,t+h}$ and $IVRP_{t,t+h}$ are linear functions of V_t, regressing on these model-implied risk premiums or on the instantaneous variance directly leads to the same results when we perform the regression.

Table 7: **Predictive Regression on the Risk Premium using the Model-Implied Factor.**

Panel A: Regression, Equity Risk Premium.
$ERP_{t,t+h} = \beta_0 + \beta_1 \widetilde{V}_t + \varepsilon_{t,t+h}$

h	One Week		One Month		Three Months		One Year	
	With *ORV*	Without *ORV*	With *ORV*	Without *ORV*	With *ORV*	Without *ORV*	With *ORV*	Without *ORV*
β_0	0.0016	0.0016	**0.0057**	**0.0061**	**0.0172**	**0.0170**	**0.0289**	**0.0286**
	(0.0012)	(0.0012)	(0.0026)	(0.0026)	(0.0051)	(0.0051)	(0.0116)	(0.0116)
β_1	-0.0379	-0.0394	-0.1266	-0.1393	**-0.3253**	**-0.3109**	**0.5239**	**0.5240**
	(0.0547)	(0.0551)	(0.1007)	0.0998)	(0.1247)	(0.1218)	(0.1899)	0.1889)
Adjusted R^2	0.0032	0.0035	0.0113	0.0138	0.0225	0.0205	0.0122	0.0122

Panel B: Regression, Variance Risk Premium.
$VRP_{t,t+h} = \beta_0 + \beta_1 \widetilde{V}_t + \varepsilon_{t,t+h}$

h	One Week		One Month		Three Months		One Year	
	With *ORV*	Without *ORV*	With *ORV*	Without *ORV*	With *ORV*	Without *ORV*	With *ORV*	Without *ORV*
β_0	**-0.0143**	**-0.0140**	**-0.0121**	**-0.0120**	**-0.0148**	**-0.0147**	**-0.0234**	**-0.0233**
	(0.0022)	(0.0023)	(0.0020)	(0.0020)	(0.0028)	(0.0029)	(0.0028)	(0.0028)
β_1	**-0.4795**	**-0.4792**	**-0.5863**	**-0.5747**	**-0.5796**	**-0.5699**	**-0.2978**	**-0.2940**
	(0.1409)	(0.1419)	(0.1362)	(0.1348)	(0.1428)	(0.1427)	(0.1054)	(0.1053)
Adjusted R^2	0.2301	0.2301	0.2856	0.2747	0.2224	0.2153	0.0730	0.0713

The index parameters are estimated using daily index returns, realized variances, bipower variations, daily option prices, and option realized variances, from July 2004 to December 2012. We use a representative sample of options and option realized variances by restricting our attention to OTM or ATM options with maturities closest to 30 and 90 days, and call-equivalent deltas closest to 0.2, 0.35, 0.5, 0.65 and 0.8. The final sample of options consists of a panel of 21,400 contracts. Using each set of parameter estimates in Table 6, the (average) instantaneous variance is extracted from the filtering procedure and is used to calculate the filtered instantaneous variance on each day. We rely on the following predictive regressions to assess the incremental information contained in *ORV* and whether it is helpful to predict the risk premiums:

$$RP_{t,t+h} = \beta_0 + \beta_1 \widetilde{V}_t + \varepsilon_{t,t+h},$$

where the left-hand side term $RP_{t,t+h}$ is either the equity risk premium or the variance risk premiums, i.e., $RP_t \in \{ERP_{t,t+h}, VRP_{t,t+h}\}$. The realized integrated risk premiums are computed as in Section 3.2.2. This table presents the coefficients, the Newey-West standard errors and the adjusted R^2s for the regressions over different time horizons h, i.e., one week, one month, three months and one year.

For the equity risk premium, the adjusted R^2s are small, which is consistent with Andersen et al. (2015b) and Santa-Clara and Yan (2010). We find similar values for coefficients and adjusted R^2s, suggesting that both parameter sets convey the same information about the model-implied risk factor. Regarding the variance risk premium, we find that adding ORV to the information set improves adjusted R^2s for time horizons of one month, three months and one year. The difference in the two adjusted R^2 is about 1% in these three cases. For a time horizon of one week, there is virtually no difference between the two adjusted R^2s.

When compared to the gains reported in the predictive regressions of Section 3, we find that the parametric one-factor model used in this study constrains the shape of the volatility process and, ultimately, the risk premiums. Nonetheless, the results of this section are consistent with those of Section 3: having ORV in the estimation seems to be beneficial in forecasting risk premiums, especially for the variance risk premiums.

5.3 Out-of-Sample Assessment

Finally, to investigate whether the previous documented differences have significant implications on the model's performance, we assess the goodness of fit of the two parameter sets reported in Section 5.1. We employ the parameters obtained over the sample period between July 2004 and December 2012, and use daily information between January 2013 and December 2013 to compute one-day-ahead forecast errors.[31] We first assess the ability of both parameter sets to fit historical implied volatilities. To perform this exercise, we use the relative implied volatility root mean square error (RIVRMSE), defined as follows:

$$\text{RIVRMSE} = \sqrt{\left(\frac{1}{\sum_k O_k}\right)\sum_k \sum_{i=1}^{O_k}\left(\frac{IV_{k\tau,i}(Y_{\tau k}, \hat{V}_{\tau k}) - \sigma^{\text{BS}}_{k\tau,i}}{\sigma^{\text{BS}}_{k\tau,i}}\right)^2}, \tag{12}$$

where IV is the one-day-ahead model implied volatility, $\hat{V}_{\tau k}$ is the one-day-ahead instantaneous variance on day k, and O_k represents the number of options in a subset of all the options available on day k.[32]

[31] We also perform an in-sample analysis that employs data over the same period in which the parameters were estimated. These results are presented in the Supplementary Material (Table SM.12).

[32] We run the filter based on the parameters of Table 6 along with option data for 2013 (OTM or ATM options with maturities closest to 30 and 90 days and call-equivalent deltas closest to 0.2, 0.35, 0.5, 0.65 and 0.8, as used in the estimation of Section 5.1). Then, we compute the filtered values of the instantaneous variance on each day, and the one-day-ahead estimate

Here, σ^{BS} denotes the Black and Scholes implied volatility associated with the observed option price. Regarding the sample, we employ all ATM and OTM options available in OptionMetrics for 2013 to compute the RIVRMSE, yielding a total of 77,310 contracts.[33]

The leftmost columns of Table 8 show the RIVRMSE for the panel of 2013 options. We observe that the average fit provided by the parameter set with *ORV* is better than the average fit that excludes these variances when we use the RIVRMSE—an RIVRMSE of 22.66 with *ORV* and 25.66 without.

We use the Diebold and Mariano (1995, henceforth DM) test to see if the apparent predictive superiority of *ORV*-based forecasts is not unique to this sample. Using both RIVRMSE time series—on each day, we compute the RIVRMSE across our daily sets of option to obtain a time series—we compare their forecasting accuracy and test for:[34]

$$H_0 : \mathrm{E}\left[d_t\right] = 0,\ \forall t \qquad H_1 : \mathrm{E}\left[d_t\right] > 0,\ \forall t$$

where $d_t = \text{RIVRMSE}_{\text{without},t} - \text{RIVRMSE}_{\text{with},t}$ is the time-t loss differential between the forecast produced without *ORV* and the one including it. The DM test statistic is 8.09 and is significant at a 95% level, confirming that there exists a differential between the two forecasts and that the one based on *ORV* information produces more accurate results on average, in terms of RIVRMSE.

In order to identify which contracts would benefit more from the inclusion of the *ORV*, we assess the goodness of fit of both parameter sets for these variances. To this end, we use a criterion similar to the RIVRMSE in Equation (12), and compute the relative option realized variance root mean square error (RORVRMSE), defined as:

$$\text{RORVRMSE} = \sqrt{\left(\frac{1}{\sum_k O_k}\right)\sum_k \sum_{i=1}^{O_k}\left(\frac{\Delta OQV_{(k-1)\tau,k\tau,i} - ORV_{(k-1)\tau,k\tau,i}}{ORV_{(k-1)\tau,k\tau,i}}\right)^2},$$

of this quantity, i.e., $\mathrm{E}_{t-\tau}\left[V_t\right]$ that will be used for pricing purposes. Finally, we compute model-predicted implied volatilities and *ORV*s on day t based on these one-day-ahead estimates.

[33]We restrict our analysis to maturities of at least one week and at most one year. As before, observations violating no-arbitrage restrictions are excluded.

[34]The lag in the Diebold and Mariano (1995) is selected as the first partial autocorrelation that is within confidence bounds.

where ΔOQV is the option quadratic variation increment computed from the model and ORV is the observed option realized variance. To compute the RORVRMSE, we employ a dataset of 68,857 options for which ORVs were available.[35]

The rightmost columns of Table 8 shows the RORVRMSE by moneyness and maturity. The parameter set that uses the ORV produces the lowest forecast errors for both measures, as shown by the average means for the whole sample. As expected, the average fit of these quantities is lower since ORVs are included in the information set. Nonetheless, it is remarkable to observe that the overall RORVRMSE is about 1.35 times lower when ORV is included and that there are significant differences across contracts. Similar to the RIVRMSE case, we apply the DM test to both RORVRMSE series and obtain a value of 16.17, statistically confirming the significant differences between the two parameter sets.

The above evidence is consistent with the view of ORV as a new source of information to disentangle jumps in the return and variance processes. The parameter set obtained with the addition of ORV supports a variance process that has less frequent jumps with higher magnitudes and a return process that has more frequent negative jumps with lower magnitudes. This set also has a different attribution of risk premiums between diffusive and discontinuous innovations. It seems most likely that these features are important in the pricing of options, as they consistently produce lower (out-of-sample) forecasting errors on implied volatilities and option realized variances.

6 Concluding Remarks

This paper uses the option realized variance as a new observable quantity to summarize the information from intraday option prices. Recent nonparametric studies have shown that high-frequency asset prices can provide important information about the data generating process.

This paper contributes to the literature in several important ways. First, we study an intraday measure capturing high-frequency variations of option prices—the option realized variance—and provide evidence that microstructure-noise robust methodologies like that proposed by Zhang et al. (2005) can be used to obtain reliable estimates of this measure.

[35]The OptionMetrics data contain more options than the Tick Data dataset. Indeed, computing ORV requires more than one price per day, whereas obtaining an end-of-the-day price requires only one transaction on a given day.

Table 8: **One-Day-Ahead Out-of-Sample Performances, in Terms of RIVRMSE and RORVRMSE (2013).**

	RIVRMSE		**RORVRMSE**	
	With *ORV*	Without *ORV*	With *ORV*	Without *ORV*
DTM < 60	19.673	19.713	512.959	635.928
60 ≤ DTM < 120	19.433	21.086	559.929	810.199
120 ≤ DTM < 180	24.117	28.019	431.521	691.705
180 ≤ DTM	29.822	36.533	390.096	585.188
$\Delta^e < 0.20$	33.011	45.881	871.918	1214.417
$0.20 \leq \Delta^e < 0.35$	24.267	35.424	465.522	617.280
$0.35 \leq \Delta^e < 0.50$	23.603	31.923	275.847	361.764
$0.50 \leq \Delta^e < 0.65$	21.349	19.861	357.394	464.947
$0.65 \leq \Delta^e < 0.80$	16.845	12.815	526.855	701.610
$0.80 \leq \Delta^e$	19.015	13.051	433.913	573.805
All	22.662	25.660	504.304	680.054

The leftmost columns of this table show the implied volatility RMSE (IVRMSE) and the rightmost show the relative implied volatility RMSE (RIVRMSE) computed for options in 2013 using parameters obtained from 2004 to 2012. We compute IVRMSE as:

$$\text{RIVRMSE} = \sqrt{\left(\frac{1}{\sum_k O_k}\right)\sum_k \sum_{i=1}^{O_k}\left(\frac{IV_{k\tau,i}(Y_{\tau k}, \hat{V}_{\tau k}) - \sigma^{\text{BS}}_{k\tau,i}}{\sigma^{\text{BS}}_{k\tau,i}}\right)^2},$$

where IV is the one-day-ahead model implied volatility, $\hat{V}_{\tau k}$ is the one-day-ahead instantaneous variance on day k, and O_k represents the number of options in a subset of all options available on day k. The RIVRMSE is given by moneyness and maturity. The total number of options with implied volatilties in 2013 is 77,310. The relative option realized variance RMSE (RORVRMSE) computed by moneyness and maturity is defined as:

$$\text{RORVRMSE} = \sqrt{\left(\frac{1}{\sum_k O_k}\right)\sum_k \sum_{i=1}^{O_k}\left(\frac{\Delta OQV_{(k-1)\tau,k\tau,i} - ORV_{(k-1)\tau,k\tau,i}}{ORV_{(k-1)\tau,k\tau,i}}\right)^2},$$

where ΔOQV is the option quadratic variation increment computed from the model and ORV is the observed option realized variance. The total number of options with ORV in 2013 is 68,857. Model prices and ORVs are calculated with parameters of Table 6 using the one-day-ahead model instantaneous variance on day k. RIVRMSEs and RORVRMSEs are given in percentages.

Second, the paper empirically documents how the option realized variance provides new information in forecasting future realized variances as well as equity and variance risk premiums. We show that adding ORV in HAR-type regressions leads to statistically significant coefficients across the specifications and to higher adjusted R^2s for the different forecasting horizons.

Third, the paper further explores the economic rationale behind the option realized variance. Specifically, within the context of a jump-diffusion model, we show that variance jumps are a paramount component of the option realized variance not readily available in other commonly used variables. When we estimate this type of model, the parameters inferred without ORV are not able to disentangle diffusive and discontinuous innovations correctly, making the filtered variance jump series extremely noisy. When ORV is included in the set of observable variables, however, the variance jump estimation is steadier and consistent with our economic intuition.

The option realized variance could be generalized to any financial derivative, as long as sufficient market activity is available. The type of information underlying these realized variances will depend on the characteristics of the underlying asset, as well as on the risks captured by the derivative under study. In the specific context of options, using these realized variances for stock options constitute an interesting avenue of research to further explore and understand idiosyncratic and systematic risk factors in these contracts. Using high-frequency data in such applications is a daunting task, which can be alleviated by synthesizing information with option realized variances. We leave these interesting questions for future research.

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A Robust Measurement of Option Realized Variance

A.1 Microstructure Noise and Hausman Tests

To assess the extent of microstructure biases in the naive option realized variance introduced in Equation (1), we first construct signature plots (e.g., Andersen et al., 2001) of this quantity for selected option contracts across the sample period. Then, we conduct a study to formally test for the presence of this noise in high-frequency option data.

Figure 6 presents examples of signature plots, that is, estimates of ORV^{Naive} as a function of the sampling frequency Δ_N for two different option contracts (solid lines). From the figure, it is evident that sampling at lower frequencies (large values of Δ_N) produce ORV estimates closer to the quasi-likelihood robust estimates (dashed lines; see Section A of the Supplementary Material for more details on this estimate) than those computed at higher frequencies.

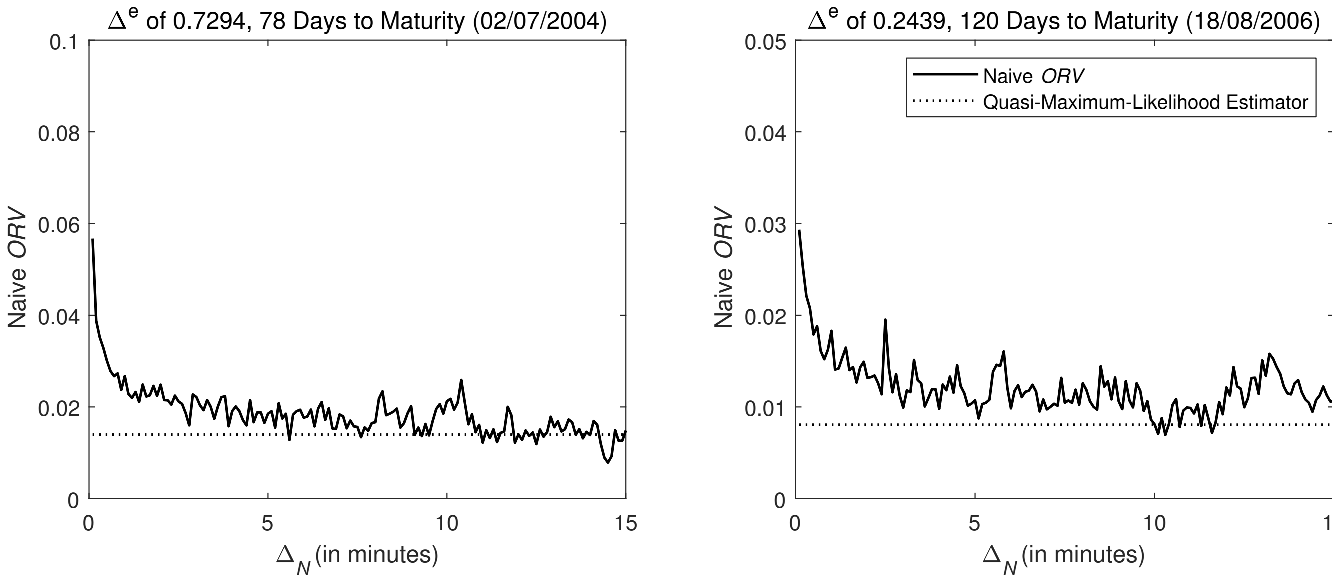

Figure 6: **Examples of Signature Plots.**
This figure presents two examples of signature plots for the option realized variance. Specifically, we consider two options selected at random in our dataset (two options among the 423,844 available). Each panel considers a specific OTM option with a specific call-equivalent delta, and a time to maturity (in days) for a given day (in brackets in the title of each subplot). The naive option realized variance estimator of Equation (1) is used with different values of Δ_N (time between two observations). To compare these naive estimators (solid line), we compute the quasi-maximum-likelihood estimator of the variation as proposed in Ait-Sahalia and Xiu (2019) (dashed line). We assume that the observed option price at each time step is the sum of the (latent) option price and an iid noise term with standard deviation a. Following Ait-Sahalia and Xiu (2019), we define the observed difference in the option price as a moving-average process of order 1, MA(1), and estimate the intraday variation estimator under this assumption (via a Kalman filter).

Ait-Sahalia and Xiu (2019) propose multiple tests for the naive estimator to assess the importance of microstructure noise, depending on the underlying assumptions for the asset dynamics. We focus on three different cases: the Hausman test that is robust to jumps (i.e., H_{3N}), testing for $a^2 = 0$ (i.e., T_N), and testing for the presence of first-order autocorrelation in the option prices (i.e., AC_N). For more details on these tests, please refer to Section A of the Supplementary Material and Ait-Sahalia and Xiu (2019).

For each statistic, we determine if the null hypothesis is rejected (at a confidence level of 95%). Then, we compute the proportion of options for which we reject the null hypothesis on a monthly basis. Figure 7 summarizes the proportion of time we reject the null hypothesis for the three tests mentioned above on a monthly basis. These tests are constructed for the naive version of ORV, i.e., $ORV^{\text{Naive}}(\Delta_N)$. At the beginning of the sample, we conclude that there is a difference between the estimated increment of the

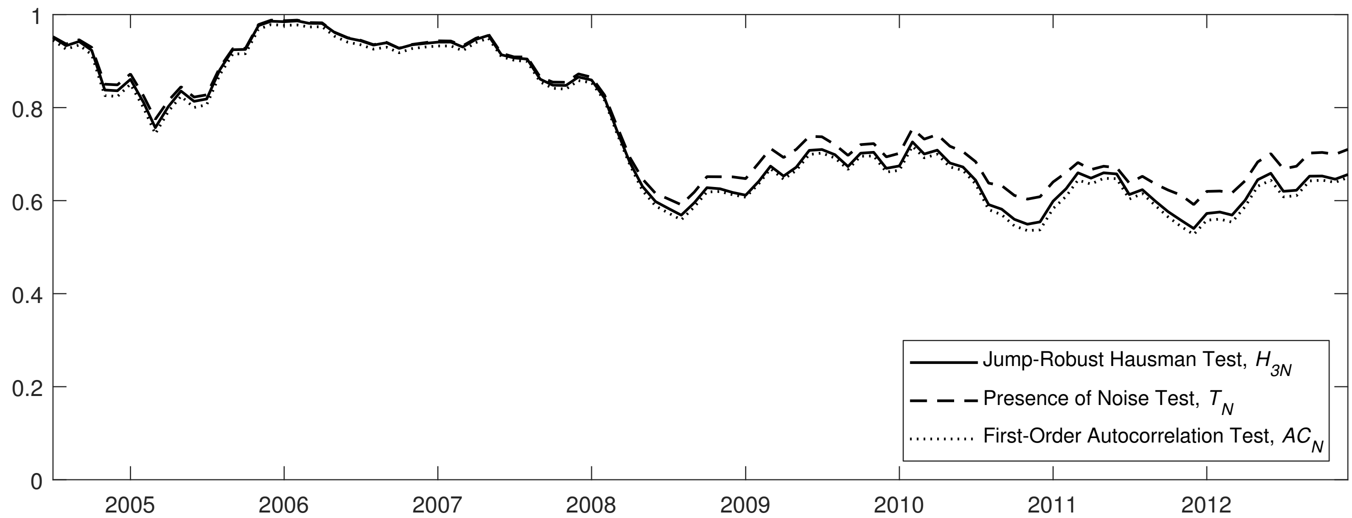

Figure 7: **Proportion of Rejected Null Hypotheses for Hausman Test Statistics and Presence of Noise.** For each of the 423,844 options of the sample, we compute statistic values for three tests. The first one is a Hausman test for whether the option realized variance is consistent with the option quadratic variation obtained via MLE—which is robust to jumps (i.e., H_{3N}). The second one tests for $a^2 = 0$ (i.e., T_N). The third one tests for the presence of first-order autocorrelation in the option prices (i.e., AC_N). For each statistic, we determine if the null hypothesis is rejected (at a confidence level of 95%). Finally, we compute the proportion of options for which we reject the null hypotheses on a monthly basis. These tests are performed on $ORV^{\text{Naive}}(\Delta_N)$, which does not account for microstructure errors.

option quadratic variation ΔOQV_{MLE} and $ORV^{\text{Naive}}(\Delta_N)$. In the second half of the sample, the difference is statistically significant about 60% of the time. Similar conclusions are applicable for the two other Hausman tests performed in this study. It therefore means that high-frequency measures that do not account for microstructure noise will most definitely be positively biased—especially at the beginning of the sample.

Table 9 shows the proportion of rejected null hypotheses for the jump-robust Hausman test on the first-order moving average of the observed price error for three periods (2004–2006, 2007–2009, 2010–2012). Again, it seems that options before 2007 had many more microstructure biases than those after 2007. Generally speaking, deep in-the-money and deep out-of-the-money options seem to be more impacted by microstructure noise, whereas at-the-money options exhibit fewer microstructure issues, on average. Options with shorter time-to-maturity are more likely to have microstructure noise.

A.2 Microstructure-Noise Robust Estimates

This section compares three microstructure-noise robust estimators proposed in the econometric literature. We implement the subsampling approach of Zhang et al. (2005), the kernel-based estimator (Barndorff-Nielsen et al., 2008) and the pre-average method (Hansen and Lunde, 2006; Jacod et al., 2009).

Table 9: **Proportion of Rejected Null Hypotheses for Hausman Test Statistics (T_N).**

Panel A: Years 2004–2006.						
Maturity \ Moneyness	[0.0, 0.2)	[0.2, 0.4)	[0.4, 0.5)	[0.5, 0.6)	[0.6, 0.8)	[0.8, 1.0]
[0, 30)	0.918	0.973	0.979	0.970	0.974	0.900
[30, 90)	0.888	0.949	0.932	0.924	0.941	0.906
[90, 180)	0.889	0.916	0.902	0.901	0.908	0.890
[180, 270)	0.894	0.852	0.889	0.876	0.866	0.864
[270, 365]	0.842	0.827	0.844	0.858	0.825	0.835
Panel B: Years 2007–2009.						
Maturity \ Moneyness	[0.0, 0.2)	[0.2, 0.4)	[0.4, 0.5)	[0.5, 0.6)	[0.6, 0.8)	[0.8, 1.0]
[0, 30)	0.852	0.774	0.689	0.677	0.758	0.820
[30, 90)	0.752	0.676	0.643	0.665	0.708	0.747
[90, 180)	0.707	0.649	0.644	0.684	0.658	0.691
[180, 270)	0.659	0.631	0.671	0.672	0.601	0.651
[270, 365]	0.698	0.578	0.606	0.633	0.613	0.635
Panel C: Years 2010–2012.						
Maturity \ Moneyness	[0.0, 0.2)	[0.2, 0.4)	[0.4, 0.5)	[0.5, 0.6)	[0.6, 0.8)	[0.8, 1.0]
[0, 30)	0.767	0.736	0.676	0.666	0.681	0.736
[30, 90)	0.682	0.576	0.545	0.576	0.537	0.645
[90, 180)	0.626	0.499	0.498	0.527	0.458	0.563
[180, 270)	0.598	0.442	0.461	0.479	0.417	0.548
[270, 365]	0.571	0.420	0.434	0.438	0.406	0.520

This Table presents the proportion of hypotheses rejected for the null of no-microstructure error. For each of the 423,844 options of the sample, we compute the statistic values for the following test: the Hausman test for the presence of noise (i.e., T_N). Then, we determine if the null hypothesis is rejected (at a confidence level of 95%). Moneyness is defined as the call-equivalent delta. Maturity is defined in days-to-maturity.

In this study, the pre-average method is implemented following Oomen (2005), as used by Bajgrowicz et al. (2015). More details on the implementation of these two additional estimators are provided in Section B of the Supplementary Material. For comparison, we add the naive estimator of Equation (1) to this analysis. Figure 8 compares estimates of $ORV^{\text{Subsampling}}$ in Equation (2) to the aforementioned estimators. The top panels of this figure report a high dispersion of points above the diagonal, indicating a positive bias in the naive estimator. The middle and bottom panels, however, show a high concentration of points around the diagonal, suggesting that the bias is well-accounted for by these methods. To further assess the similarities between noise-robust approaches, Table 10 reports correlations between these estimators for different sampling frequencies Δ_N computed over the sample period. Regardless of the sampling frequency, these correlations tend to be above 85% for noise-robust estimates, showing that these estimates display similar properties in our sample. For the naive method, on the other hand, correlations are drastically affected by increases in the sampling period, especially at higher frequencies (e.g., one minute).

We complement the previous evidence with that of Figure 9, which shows average values of estimators

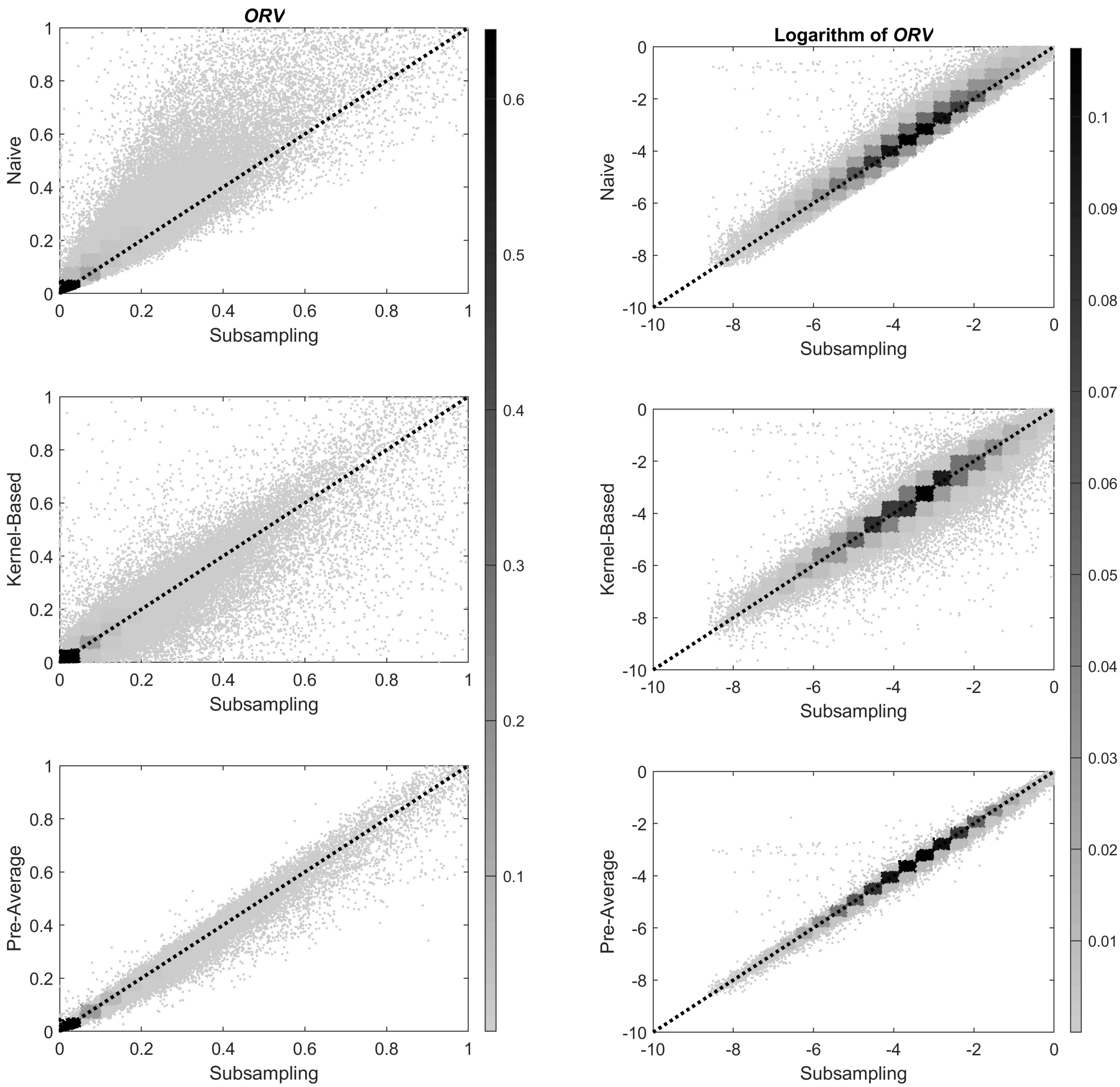

Figure 8: **Scatterplot of Option Realized Variance Estimates Against Subsampling.**
For all of the ATM and OTM options in the sample, we compute the four estimates at a five-minute frequency ($\Delta_N = 5$ minutes, with $k = 3$ for the subsampling estimator in Equation (2)). Next, scatterplots for the option realized variances (left panels) and the logarithm of the option realized variances (right panels) are constructed for these estimates. For comparison, we use the naive *ORV* estimator of Equation (1), the kernel-based method of Barndorff-Nielsen et al. (2008), and the pre-average method (Hansen and Lunde, 2006; Jacod et al., 2009); all these estimators use a five-minute sampling frequency, i.e., $\Delta_N = 5$ minutes). Colours are added to the scatter plot to show the density of points in each bin—dark grey being the most populated areas, and light grey being the least populated areas.

at various sampling frequencies. The sampling at the highest possible frequency—one-minute in our case—leads to more microstructure noise, even when robust methodologies are employed. Moreover, robust estimators based on five-minute data show similar average values across maturities and moneyness levels, illustrated by overlapping confidence intervals. These estimators should be equivalent up to a first order: "when the object to be estimated is quadratic variation, the pre-averaging approach is to first-order equivalent to the realized kernel-based estimator of Barndorff-Nielsen et al. (2008) and the two-scale or multi-scale subsampler of Zhang et al. (2005) or Zhang (2006)" (Christensen et al., 2014). Finally, the 15-minute frequency can yield estimates that differ from one another, suggesting that a small-sample issue arises at lower frequencies. This issue is well illustrated in the context of the subsampling method as, with 15-minute data, the quadratic variation is computed by averaging two grids of only 13 observations each.

Overall, the above results indicate that the five-minute subsampling estimator provides a compromise between sample size, microstructure bias reduction and consistency with other estimators. Moreover, naive estimators at a 15-minute frequency are consistent with the five-minute subsampling estimator, which is in line with the signature plots of Figure 6. The five-minute estimate we selected is also consistent with the work of Liu et al. (2015) as mentioned above.

B More Details on the Framework

B.1 Radon-Nikodym Derivative

Let $\mathcal{F}_t = \sigma \{W_{V,u}, W_{\perp,u}, J_{Y,u}, J_{V,u}\}_{0\leq u\leq t}$ be the $\sigma-$field generated by the past and actual noise terms. The market being incomplete, there are infinitely many equivalent martingale measures. We restrict the choice to those that have a Radon-Nikodym derivative of the form[36]

$$\left.\frac{d\mathbb{Q}}{d\mathbb{P}}\right|_{\mathcal{F}_t} = \frac{\exp\left(-\int_0^t \Lambda_{V,u^-} dW_{V,u} - \int_0^t \Lambda_{\perp,u^-} dW_{\perp,u} + \Gamma_Y J_{Y,t} + \Gamma_V J_{V,t}\right)}{\exp\left(\frac{1}{2}\int_0^t \left(\Lambda^2_{V,u^-} + \Lambda^2_{\perp,u^-}\right) du + \left(\varphi^{\mathbb{P}}_{Z_Y}(\Gamma_Y) - 1\right)\int_0^t \lambda_{Y,u^-} du + \left(\varphi^{\mathbb{P}}_{Z_V}(\Gamma_v) - 1\right)\int_0^t \lambda_{V,u^-} du\right)},$$

where $\varphi^{\mathbb{P}}_{Z_Y}(\Gamma_Y)$ and $\varphi^{\mathbb{P}}_{Z_V}(\Gamma_V)$ represent the moment generating functions of the return and variance jump size,[37]

$$\varphi^{\mathbb{P}}_{Z_Y}(\Gamma_Y) = \exp\left(\mu_Y \Gamma_Y + \frac{1}{2}\sigma_Y^2 \Gamma_Y^2\right) \quad \text{and} \quad \varphi^{\mathbb{P}}_{Z_V}(\Gamma_V) = (1 - \Gamma_V \mu_v)^{-1}.$$

[36] It is required that $\mathrm{E}^{\mathbb{P}}\left[\int_0^t \Lambda^2_{V,u^-} du\right] < \infty$ and $\mathrm{E}^{\mathbb{P}}\left[\int_0^t \Lambda^2_{\perp,u^-} du\right] < \infty$.

[37] Provided that $\Gamma_V < 1/\mu_V$.

Table 10: **Correlation Matrix of the Option Realized Variance over Different Sampling Frequencies and for Different Methods.**

Δ_N		One-Minute Frequency				Five-Minute Frequency				15-Minute Frequency			
		Subsampling	Naive	Kernel	Pre-Average	Subsampling	Naive	Kernel	Pre-Average	Subsampling	Naive	Kernel	Pre-Average
One-Minute Frequency	Subsampling	1.000											
	Naive	0.915	1.000										
	Kernel	0.963	0.830	1.000									
	Pre-Average	0.987	0.849	0.972	1.000								
Five-Minute Frequency	Subsampling	0.937	0.795	0.936	0.963	1.000							
	Naive	0.954	0.861	0.936	0.946	0.934	1.000						
	Kernel	0.858	0.708	0.895	0.898	0.949	0.848	1.000					
	Pre-Average	0.850	0.705	0.870	0.886	0.931	0.831	0.936	1.000				
15-Minute Frequency	Subsampling	0.937	0.795	0.936	0.963	1.000	0.934	0.949	0.931	1.000			
	Naive	0.893	0.764	0.907	0.918	0.937	0.878	0.925	0.944	0.937	1.000		
	Kernel	0.742	0.605	0.782	0.782	0.841	0.736	0.891	0.945	0.841	0.845	1.000	
	Pre-Average	0.850	0.705	0.870	0.886	0.931	0.831	0.936	1.000	0.931	0.944	0.945	1.000

For all of the ATM and OTM options in the sample, we compute the four estimates at one-, five- and 15-minute frequencies: (1) for one-minute, we use $\Delta_N = 1$ minute, with $k = 5$ for the subsampling estimator; (2) for five-minute, we use $\Delta_N = 5$ minutes, $k = 3$ for the subsampling estimator; (3) for 15-minute, we use $\Delta_N = 15$ minutes, $k = 2$ for the subsampling estimator. The four *ORV* estimates are: the naive *ORV* estimator of Equation (1), the subsampling method explained in Equation (2), the kernel-based method of Barndorff-Nielsen et al. (2008), and the pre-average method (Hansen and Lunde, 2006; Jacod et al., 2009). We then winsorize the sample to remove potential outliers (the 0.5^{th} and 99.5^{th} quantiles). From the remaining *ORV* estimates, we obtain the sample correlation estimates.

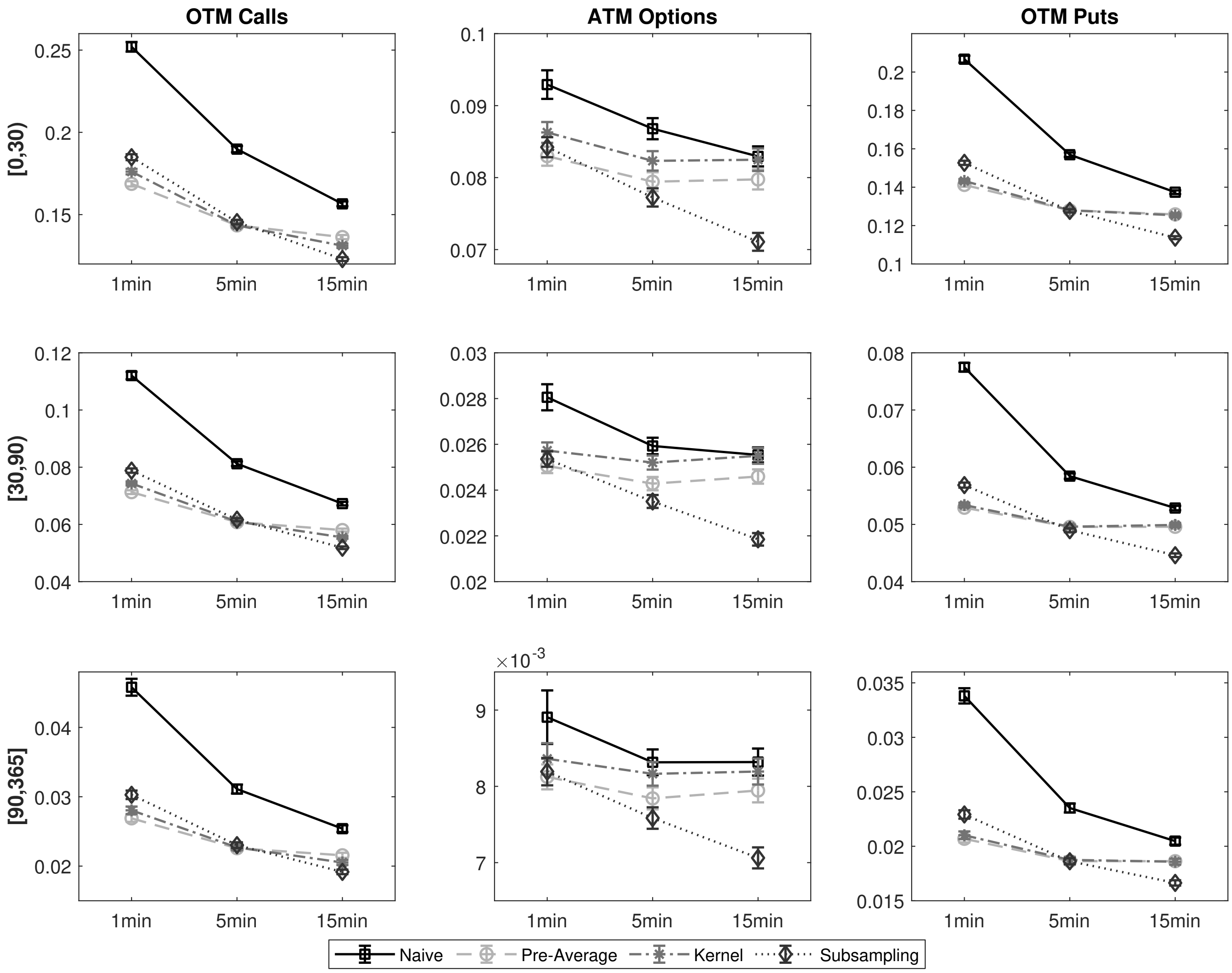

Figure 9: **Option Realized Variance Estimates for Different Sampling Frequencies.**
Each figure presents the average value of a given estimator at one-, five- and 15-minute frequencies for a given group of options. The four estimators under consideration are: the naive *ORV* estimator of Equation (1), the subsampling method explained in Equation (2), the kernel-based method of Barndorff-Nielsen et al. (2008) and the pre-average method (Hansen and Lunde, 2006; Jacod et al., 2009). The top, middle and bottom figures are for short, middle and long maturities; respectively. OTM calls are options with Δ^e below 0.4, ATM options have a Δ^e between 0.4 and 0.6, and OTM puts have a Δ^e above 0.6. Each plot presents the average of a given estimate as well as its 95% confidence interval.

This change of measure is indeed an extended version of the Girsanov theorem. The predictable process $\{\Lambda_{V,t^-}\}_{t\geq 0}$ and $\{\Lambda_{\perp,t^-}\}_{t\geq 0}$ and the constants Γ_Y and Γ_V characterize the risk premiums embedded in this framework by linking the $\mathbb{P}-$ and $\mathbb{Q}-$parameters together.

This approach is different from the one proposed in Duffie et al. (2000), Pan (2002), and Broadie et al. (2007) in which the $\mathbb{P}$- and $\mathbb{Q}$-parameters are allowed to vary independently. However, it shares similarities with Christoffersen et al. (2012) and Ornthanalai (2014), which consider GARCH models with jumps. It is also related to Bates (2000): in the latter, the author restricts the value of certain parameters to be consistent with the time series behaviour of returns.

B.2 Model under the Risk-Neutral Measure $\mathbb{Q}$

For the model's diffusion components, the risk-neutral Brownian motions are constructed in the usual way:

$$W^{\mathbb{Q}}_{V,t} = W_{V,t} + \int_0^t \Lambda_{V,u^-} du, \qquad W^{\mathbb{Q}}_{\perp,t} = W_{\perp,t} + \int_0^t \Lambda_{\perp,u^-} du$$

and $W^{\mathbb{Q}}_{Y,t} = \rho W_{V,t} + \sqrt{1-\rho^2} W_{\perp,t} = W_{Y,t} + \int_0^t \Lambda_{Y,u^-} du$ where $\Lambda_{Y,u^-} = \rho \Lambda_{V,u^-} + \sqrt{1-\rho^2}\Lambda_{\perp,u^-}$.

The risk-neutral jump components are obtained from a direct comparison of the $\mathbb{P}$- and $\mathbb{Q}$-versions of the moment generating functions of the jump increments ($J_{Y,t} - J_{Y,s}$ and $J_{V,t} - J_{V,s}$). It can be shown that the change of measure affects the following parameters:

$$J^{\mathbb{Q}}_{Y,t} = \sum_{n=1}^{N^{\mathbb{Q}}_{Y,t}} Z^{\mathbb{Q}}_{Y,n}, \text{ and } J^{\mathbb{Q}}_{V,t} = \sum_{n=1}^{N^{\mathbb{Q}}_{V,t}} Z^{\mathbb{Q}}_{V,n},$$

where $\left\{N^{\mathbb{Q}}_{Y,t}\right\}_{t\geq 0}$ is a Cox process with predictable intensity $\{\lambda_{Y,t^-}\}_{t\geq 0}$, $\left\{N^{\mathbb{Q}}_{V,t}\right\}_{t\geq 0}$ is a Poisson process of intensity $\lambda^{\mathbb{Q}}_{V,0}$ and

$$\begin{aligned} \lambda^{\mathbb{Q}}_{Y,t^-} &= \varphi^{\mathbb{P}}_{Z_Y}(\Gamma_Y)\,\lambda_{Y,t^-}, \qquad \mu^{\mathbb{Q}}_Y = \mu_Y + \Gamma_Y \sigma^2_Y, \qquad \sigma^{\mathbb{Q}}_Y = \sigma_Y, \\ \lambda^{\mathbb{Q}}_{V,0} &= \varphi^{\mathbb{P}}_{Z_V}(\Gamma_V)\,\lambda_{V,0}, \qquad \mu^{\mathbb{Q}}_V = \varphi^{\mathbb{P}}_{Z_V}(\Gamma_V)\,\mu_V. \end{aligned}$$

The risk-neutral dynamics of the log-price process is established by imposing the discounted price process $\{\exp((q-r)t)\exp(Y_t)\}_{t\geq 0}$ to be a $\mathbb{Q}-$martingale where r is the risk-free rate and q is the dividend rate. To get a semi-closed form for option prices, the risk-neutral stochastic differential equation of the variance process is assumed to have a mean reverting behaviour, as in Heston (1993), among others. Implicitly, it constrains

$$\Lambda_{V,t^-} = \eta_V \sqrt{V_{t^-}}$$

and the model under the risk-neutral measure is thus

$$
\begin{aligned}
dY_t &= \alpha_{t^-}^{\mathbb{Q}} dt + \sqrt{V_{t^-}} dW_{Y,t}^{\mathbb{Q}} + dJ_{Y,t}^{\mathbb{Q}}, \\
dV_t &= \kappa^{\mathbb{Q}} \left(\theta^{\mathbb{Q}} - V_{t^-}\right) dt + \sigma \sqrt{V_{t^-}} dW_{V,t}^{\mathbb{Q}} + dJ_{V,t}^{\mathbb{Q}},
\end{aligned}
$$

where the correspondence between the $\mathbb{P}-$ and $\mathbb{Q}-$parameters is

$$
\begin{aligned}
\alpha_{t^-}^{\mathbb{Q}} &= r - q - \tfrac{1}{2} V_{t^-} - \left(\varphi_{Z_Y}^{\mathbb{Q}}(1) - 1\right) \lambda_{Y,t^-}^{\mathbb{Q}}, & \kappa^{\mathbb{Q}} &= \kappa + \sigma \eta_V, \\
\lambda_{Y,t^-}^{\mathbb{Q}} &= \varphi_{Z_Y}^{\mathbb{P}}(\Gamma_Y) \lambda_{Y,t^-}, & \theta^{\mathbb{Q}} &= \tfrac{\kappa\theta}{\kappa + \sigma\eta_V}.
\end{aligned}
$$

B.3 Pricing

As shown in the companion paper, the price of a European call option with strike price K and maturity T is

$$
C_t(Y_t, V_t) = \exp(Y_t) \exp(-q(T-t)) P_1(Y_t, V_t) - K \exp(-r(T-t)) P_2(Y_t, V_t)
$$

where

$$
P_j(y, v) = \frac{1}{2} + \frac{1}{\pi} \int_0^{\infty} \mathrm{Re}\left(\frac{\exp(-\mathrm{i}uk - y(2-j))\, \varphi_{Y_T|Y_t,V_t}^{\mathbb{Q}}(u\mathrm{i} + (2-j), y, v)}{u\mathrm{i}} \right) du,
$$

$k = \log(K)$, and $\varphi_{Y_T|Y_t,V_t}^{\mathbb{Q}}(u, y, v) = \mathrm{E}_t^{\mathbb{Q}}[\exp(uY_T) | Y_t = y, V_t = v]$ is the moment generating function of Y_T conditional on time t information. More precisely,

$$
\varphi_{Y_T|Y_t,V_t}^{\mathbb{Q}}(u, Y_t, V_t) = \exp(\mathcal{A}(u,t,T) + uY_t + C(u,t,T) V_t)
$$

where

$$
\begin{aligned}
C(u,t,T) &= \frac{2C_0(\exp(-C_2(T-t)) - 1)}{C_2(\exp(-C_2(T-t)) + 1) - C_1(\exp(-C_2(T-t)) - 1)}, \\
C_0 &= \lambda_{Y,1}^{\mathbb{Q}} \left(\varphi_{Z_Y}^{\mathbb{Q}}(1) - 1\right) u - \lambda_{Y,1}^{\mathbb{Q}} \left(\varphi_{Z_Y}^{\mathbb{Q}}(u) - 1\right) + \frac{u - u^2}{2},
\end{aligned}
$$

$$C_1 = \kappa^{\mathbb{Q}} - \rho\sigma u,$$
$$C_2 = \sqrt{C_1^2 + 2\sigma^2 C_0},$$

and

$$\mathcal{A}(u;t,T) = D_0(T-t) + \theta^{\mathbb{Q}}\kappa^{\mathbb{Q}} g_1(t,T) + \lambda_{V,0}^{\mathbb{Q}} g_2(t,T)$$
$$D_0 = -r + (r-q)u + \lambda_{Y,0}^{\mathbb{Q}}\left(\varphi_{Z_Y}^{\mathbb{Q}}(u) - 1\right) - \lambda_{Y,0}^{\mathbb{Q}}\left(\varphi_{Z_Y}^{\mathbb{Q}}(1) - 1\right)u$$
$$g_1(t,T) = \frac{2\log(2C_2) - 2\log\left(C_1\left(e^{C_2(T-t)} - 1\right) + C_2\left(e^{C_2(T-t)} + 1\right)\right) + (C_1 + C_2)(T-t)}{-\sigma^2}$$
$$g_2(t,T) = -\frac{\mu_V^{\mathbb{Q}}}{2}\frac{2\log(2C_2) - 2\log\left(C_1\left(e^{C_2(T-t)} - 1\right) + C_2\left(e^{C_2(T-t)} + 1\right) + 2C_0\mu_V^{\mathbb{Q}}\left(e^{C_2(T-t)} - 1\right)\right)}{C_0\left(\mu_V^{\mathbb{Q}}\right)^2 + C_1\mu_V^{\mathbb{Q}} - \frac{\sigma^2}{2}}$$
$$-\frac{\mu_V^{\mathbb{Q}}}{2}\frac{2C_0\mu_V^{\mathbb{Q}} + C_1 + C_2}{C_0\left(\mu_V^{\mathbb{Q}}\right)^2 + C_1\mu_V^{\mathbb{Q}} - \frac{\sigma^2}{2}}(T-t)$$

The approach, inspired from Heston (1993), relies on an inversion similar to that of Gil-Pelaez (1951). The explicit form of the moment generating function is similar to those found by Filipovic and Mayerhofer (2009) and Duffie et al. (2000).

C Filtering and Estimation

C.1 Option Quadratic Variances in the Filter

The estimation of a model using as observable *ORV* requires an approximation of Equation (6). The first factor impacting the time complexity of the approximation is the evaluation of option price O_{u^-}, which is done in semi-closed form through a Fourier transform and partial derivative calculations. The second factor corresponds to the fact that the quadratic variation involves a division by the square of the option price, leading to potential instabilities in the calculation as this price could be very small (e.g., deep out-of-the-money options).

One way to circumvent this problem is by using a price-based version of the option realized variance—a version that uses the price-level instead of the logarithmic-transformed price. This would lead to a different quadratic variation that is more convenient for numerical calculations, that is:

$$\Delta OQV_{t-\tau,t} = \int_{t-\tau}^{t} \left(S_{u^-}^2 \Delta_{u^-}^2 + 2\sigma\rho\Delta_{u^-}\mathcal{V}_{u^-}S_{u^-} + \sigma^2\mathcal{V}_{u^-}^2 \right) V_{u^-}\, du$$
$$+ \sum_{t-\tau<u\leq t} \left(O_u\left(Y_u, V_u\right) - O_u\left(Y_{u^-}, V_{u^-}\right)\right)^2 . \tag{13}$$

We can observe that this expression does not involve the option price, speeding up its computation and making it more stable. In the parametric application of Section 5, we therefore use the price-level version of ORV. In addition to being reliable and efficient, the price-based ORV has the same informational content when compared to the return-based ORV; specifically, it preserves the variance jump term explained above. In Section H of the Supplementary Material, we use ORV based on price changes to conduct most of the tests previously reported in Sections 2 and 3 and reach similar conclusions.

C.2 Particle Filter

To estimate the model presented in Section 4, we propose an implementation of the SIR-type filter (Gordon et al., 1993). This filter is constructed over samples of daily observations, so we define the elapsed time between two consecutive time steps by $\tau = 1/252$ and denote the observed sample by $\{z_{k\tau}\}_{k=1}^{T}$, where

$$z_{k\tau} = \left[Y_{k\tau}, RV_{(k-1)\tau,k\tau}, BV_{(k-1)\tau,k\tau}, \boldsymbol{\sigma}_{k\tau}^{\mathrm{BS}}, \boldsymbol{ORV}_{(k-1)\tau,k\tau} \right].$$

We define $\boldsymbol{\sigma}_{k\tau}^{\mathrm{BS}} = \left[\sigma_{k\tau,1}^{\mathrm{BS}}, ..., \sigma_{k\tau,n_{k\tau}}^{\mathrm{BS}}\right]$ as the implied volatility vector of the $n_{k\tau}$ options available on day k and $\boldsymbol{ORV}_{(k-1)\tau,k\tau} = [ORV_{(k-1)\tau,k\tau,1}, ..., ORV_{(k-1)\tau,k\tau,n_{k\tau}}]$ the corresponding vector of option realized variances.

The first step of our particle filter is to simulate particles (i.e., intraday paths) of the latent variables. Then, to obtain end-of-day quantities, we aggregate the intraday simulated values which produces daily simulated particles:

$$x_{k\tau} = \left[Y_{k\tau}, V_{k\tau}, \Delta I_{(k-1)\tau,k\tau}, \Delta QV_{(k-1)\tau,k\tau}, \boldsymbol{IV}_{k\tau}\left(Y_{k\tau}, V_{k\tau}\right), \boldsymbol{\Delta OQV}_{(k-1)\tau,k\tau}\right],$$

where $\Delta I_{(k-1)\tau,k\tau}$ is the integrated variance generated as a by-product of the simulation stage, $\Delta QV_{(k-1)\tau,k\tau}$, is the simulated quadratic variation, $\boldsymbol{IV}_{k\tau}\left(Y_{k\tau}, V_{k\tau}\right)$ represents the vector of the $n_{k\tau}$ model implied volatilities based on the simulated log-price and variance values, and $\boldsymbol{\Delta OQV}_{(k-1)\tau,k\tau}$ is the vector of the corresponding option quadratic variation calculated from Equation (13).

Further distributional assumptions about the measurement errors are required to connect the observed variables to the state variables; in this study, we assume that the relative error on each observation is normally distributed.[38] Also, to ensure that the joint estimation is not dominated by one particular source, the daily likelihood contribution associated with each observation receives a weight inversely proportional to the number of sources of a given type.[39] That is, log-equity price, realized variance, and bipower variation receive a weight of 1, and implied volatilities and option realized variances a weight of one over $n_{k\tau}$.

Each daily likelihood contribution—a by-product of the filtering method used in this study—is then multiplied to obtain a final likelihood function that can be maximized numerically to find the model parameters along with the posterior distributions of the latent factors.

[38]The standard deviations of these Gaussian random variables are given by η_1, η_2, η_3 and η_4 for RV, BV, IV and ORV, respectively. All of the relative errors are assumed to be independent.

[39]This idea is similar to the weighted likelihood estimator. Hu and Zidek (2002) study the properties of the weighted likelihood estimator and show that the key asymptotic results continue to hold.

Supplementary Material

A Microstructure Noise and Hausman Tests

This section presents, among other things, the Ait-Sahalia and Xiu's (2019) Hausman test used in Section 2.2.

Suppose that the logarithm of the option price, $o_t = \log(O_t)$, follows a discontinuous Itō semimartingale. Using a notation that is slightly different from the one used in the manuscript (but closer to the literature on microstructure noise), the observed option price is given by

$$o_{i\Delta_N} = \widetilde{o}_{i\Delta_N} + aU_i, \quad 0 \leq i \leq N,$$

where U_i is an iid noise with mean 0, variance 1 and a finite fourth moment, and $\widetilde{o}_{i\Delta_N}$ is the (latent) log-option price.

The observed difference in option price,

$$D_{i\Delta_N} = o_{i\Delta_N} - o_{(i-1)\Delta_N},$$

are such that

$$\begin{aligned} D_{i\Delta_N} &= \widetilde{o}_{i\Delta_N} - \widetilde{o}_{(i-1)\Delta_N} + aU_i - aU_{i-1}, \\ D_{(i+1)\Delta_N} &= \widetilde{o}_{(i+1)\Delta_N} - \widetilde{o}_{i\Delta_N} + aU_{i+1} - aU_i. \end{aligned}$$

Therefore, $\mathrm{Cov}(D_{i\Delta_N}, D_{(i+1)\Delta_N}) = -a^2$, $\mathrm{Cov}(D_{i\Delta_N}, D_{(i+2)\Delta_N}) = 0$, etc. This implies that the observed option prices follows an MA(1) process with

$$\mathrm{Var}(D_{i\Delta_N}) = \Delta OQV + 2a^2 \quad \text{and} \quad \mathrm{Cov}(D_{i\Delta_N}, D_{(i-1)\Delta_N}) = -a^2.$$

So the quasi-log lokelihood function for the option price is given by the following expression:

$$\mathcal{L}(\Delta OQV, a^2) = -\frac{N}{2}\log(2\pi) - \frac{1}{2}\log\det(\mathbf{\Sigma}) - \frac{1}{2}\mathbf{V}^\top\mathbf{\Sigma}^{-1}\mathbf{V},$$

where $\mathbf{\Sigma} = (\Delta OQV)\Delta_N \mathbb{I}_N + a^2\mathbb{J}_N$, $\mathbb{I}_N$ denotes the $N \times N$ identify matrix and $(\mathbb{J}_N)_{ij} = -\mathbf{1}_{\{i=j\pm1\}} + 2\mathbf{1}_{\{i=j\}}$ with $\mathbf{1}_{\{\cdot\}}$ denoting the indicator function.[1]

By using the quasi-likelihood function obtained as a by-product of the Kalman filter, we can get quasi-likelihood estimators for ΔOQV (i.e., the increment of the option quadratic variance of Equation 13) and a^2, i.e., $\Delta OQV_{\mathrm{MLE}}$ and a^2_{MLE}, respectively. These two estimators are useful to test whether ORV is a consistent estimator of ΔOQV or to test for the presence of noise.

Ait-Sahalia and Xiu (2019) propose multiple tests, depending on the underlying assumptions for the asset dynamics. We focus on three different cases: the Hausman test that is robust to jumps (i.e., H_{3N}), testing whether $a^2 = 0$ (i.e., T_N), and testing for the presence of first-order autocorrelation in the option prices

[1] Note that the ΔOQV here is not exactly the one discussed in Equation (13). The one in this section is based on log-returns whereas the one in Sections 4 and 5 of the paper is based on actual returns.

(i.e., AC_N):

- The Hausman test statistics for the null hypothesis—robust to jumps—is as follows:

$$H_{3N} = \Delta_N^{-1} \frac{\left(\Delta OQV_{\text{MLE}} - ORV^{\text{Naive}}(\Delta_N)\right)^2}{\widehat{W}_{3N}},$$

  where $\widehat{W}_{3N}$ is related to the quarticity. We can then show that

$$H_{3N} \xrightarrow{\mathcal{L}} \chi_1^2,$$

  under the null hypothesis

$$H_0 : \text{There is no difference between } \Delta OQV_{\text{MLE}} \text{ and } ORV^{\text{Naive}}(\Delta_N).$$

- The presence of noise can be tested by verifying that $a^2 = 0$. We can construct a Student-t test using a^2_{MLE}, standardized by a quarticity estimator:

$$T_N = \Delta_N^{-3/2} \frac{|a^2_{\text{MLE}}|}{\widehat{Q}_{3N}^{1/2}},$$

  where $\widehat{Q}_{3N}$ is a quarticity estimator that is robust to jumps. We therefore have that, under $H_0 : a^2 = 0$,

$$T_N \xrightarrow{\mathcal{L}} \mathcal{N}(0, 1),$$

  under the null hypothesis.
- Finally, we can test for the presence of first-order autocorrelation in option prices by calculating the following statistic:

$$AC_N = \Delta_N^{-1/2} \frac{\sum_{i=1}^{j} D_j D_{j+1}}{\widehat{Q}_{3N}^{1/2}}$$

  and we have

$$AC_N \xrightarrow{\mathcal{L}} \mathcal{N}(0, 1),$$

  under the null hypothesis.

The results of these three tests are given in Appendix A.1 of the paper.

B Robust Estimation of the Option Realized Variance

To assess the adequacy of the subsampling method of Zhang et al.'s (2005) described in Section 2.2 of the manuscript, we now use two other estimators for ORV that are based on microstructure-robust estimator in the literature for RV, in addition to the naive estimator of Equation (1).

- Kernel-based ORV: This estimator is inspired by Barndorff-Nielsen et al. (2008), i.e.,

$$ORV^{\text{Kernel}}_{t-\tau,t}(\Delta_N) = \gamma_0(\mathbf{o}_{t-\tau,t}) + \sum_{h=1}^{H} k\left(\frac{h-1}{H}\right)\left(\gamma_h(\mathbf{o}_{t-\tau,t}) + \gamma_{-h}(\mathbf{o}_{t-\tau,t})\right),$$

  where γ_h is the realized autocovariation process

$$\gamma_h(\mathbf{o}_{t-\tau,t}) = \sum_{j=1}^{N}\left(o_{t-\tau+j\Delta_N} - o_{t-\tau+(j-1)\Delta_N}\right)\left(O_{t-\tau+(j-h)\Delta_N} - o_{t-\tau+(j-h-1)\Delta_N}\right),$$

  and the function k is the kernel function—in our case given given by the Tukey-Hanning kernel of order 2:

$$k(x) = \sin^2\left(\frac{\pi}{2(1-x)^2}\right).$$

  The parameter H is determined as a by-product of a_{MLE} (see Section A of this Supplementary Material) such that

$$H = 5.74\, a_{\text{MLE}} \sqrt{\frac{N}{\gamma^*_h(\mathbf{o}_{t-\tau,t})}},$$

  where $\gamma^*_h(\mathbf{o}_{t-\tau,t})$ is the realized variance estimator based on low frequency data (i.e., larger Δ_N).
- Pre-average ORV: It is a special case of the kernel-based estimator that has been studied by Hansen and Lunde (2006) and Jacod et al. (2009). In here, we use the implementation of Oomen (2005), as used by Bajgrowicz et al. (2015).

The last two methods—kernel-based and pre-averaged—are equivalent, up to the first order, to that proposed by Zhang et al. (2005).[2]

C Use of Trades Instead of Quotes

To understand the difference between quote-based ORVs and trade-based ORVs, we retrieve all intraday prices associated with our option sample (in addition to midquote prices). We apply the following filters to trade prices:

1. A trade price cannot exceed the prevailing ask price by more than 50%.

2. A trade price cannot be less than 50% the prevailing bid price.

3. A trade does not have any special OPRA condition.

For each option in the sample, we use one-minute returns (either midquotes or trades) to compute an estimate of the option realized variance using the subsampling methodology. Table 1 provides average values for different quantities of interest. We observe that ORV values are similar for options when short maturities are considered. Yet, ORV estimates based on trades becomes smaller as the maturity decreases.

[2]"When the object to be estimated is quadratic variation, the pre-averaging approach is to first-order equivalent to the realized kernel-based estimator of Barndorff-Nielsen et al. (2008) and the and the two-scale or multi-scale subsampler of Zhang et al. (2005) or Zhang (2006)." (Christensen et al., 2014)

This effect is due mainly to the number of trades employed in the computation of ORV, as options with higher maturities are less frequently traded in the market. Comparing the number of one-minute intervals with at least one quote, we observe that a large number of intervals will have zero—stale—returns, which has the ultimate effect of lowering our estimates of trade-based ORV. Estimates of ORV using quotes are less susceptible to this effect as they are updated more frequent as a function of changes in market conditions and the underlying value.

D Option Realized Variance with Different Proxies of Efficient Option Prices

This section presents results regarding the effect of different proxies of the option's fundamental value on estimates of the ORV. The first proxy, which is employed in all our empirical results, comes from bid-ask midpoints. The second is based on the methodology proposed by Muravyev and Pearson (2016), which computes the efficient option price with the Black and Scholes (1973) model.

To construct the efficient price proxy using the Muravyev and Pearson methodology, we first compute the implied volatility every minute for each option in our sample. Then, for a given point in time, we construct a conditional implied volatility estimate using the average of the previous 30 observations. This estimate, together with the S&P 500 index[3], provides the two inputs required to compute the value of the option using the Black and Scholes formula.[4] This procedure yields 360 one-minute option prices per day, from which ORV estimates are computed using the subsampling methodology of Zhang et al. (2005). We refer to these values as BS ORV estimates. For the sake of comparison, ORV estimates based on midquote prices also use 360 one-minute option prices. Throughout the following empirical analyses, we scale ORV values by the squared of the end-of-day option value, and work with log values of these quantities.

Figure SM.1 presents a scatter plot of daily midquote ORV estimates against BS ORV estimates for all options in our sample. The highest density of points is concentrated on the main diagonal, suggesting that estimates of ORV are similar for both definitions. The distribution of the daily correlation between these two estimates is characterized by the quartiles: the first quartile is 87.98%, the median 91.08%, and the third quartile 93.56%. To study the linear relationship between these two estimates in more detail, we regress midquote ORV estimates onto BS ORV values each day, and store the slope of this regression. The distribution of this parameter is characterized by the quartiles: the first quartile is 0.94, the median 0.99, and the third quartile 1.05. A test of this parameter being equal to one has a t-statistic of 1.14, confirming that both proxies for the option value yield similar ORV estimates.[5] In unreported results, we compute the ORV using the pre-average approach and they lead to similar conclusions.

E Predicting Index Return Variations: Additional Results

In this section, we present some of the additional results related to the HAR regressions done in Section 3 of the paper.

[3]We obtain from Tick Data all S&P 500 index values disseminated for every day. From these tick data, we construct one-minute time-series by recording the last observation over a given interval. These values are used as the underlying value at a given moment in time. Because certain values of the index were unavailable, 29 days in the sample were removed.

[4]Other parameters such as maturity, strike price, dividend yield, and interest rate are kept constant throughout the day

[5]Cross-sectional regressions include a fixed effect associated to the option type. Newey-West robust errors are employed in the calculation of the t-statistic. The lag for this procedure is set at 30.

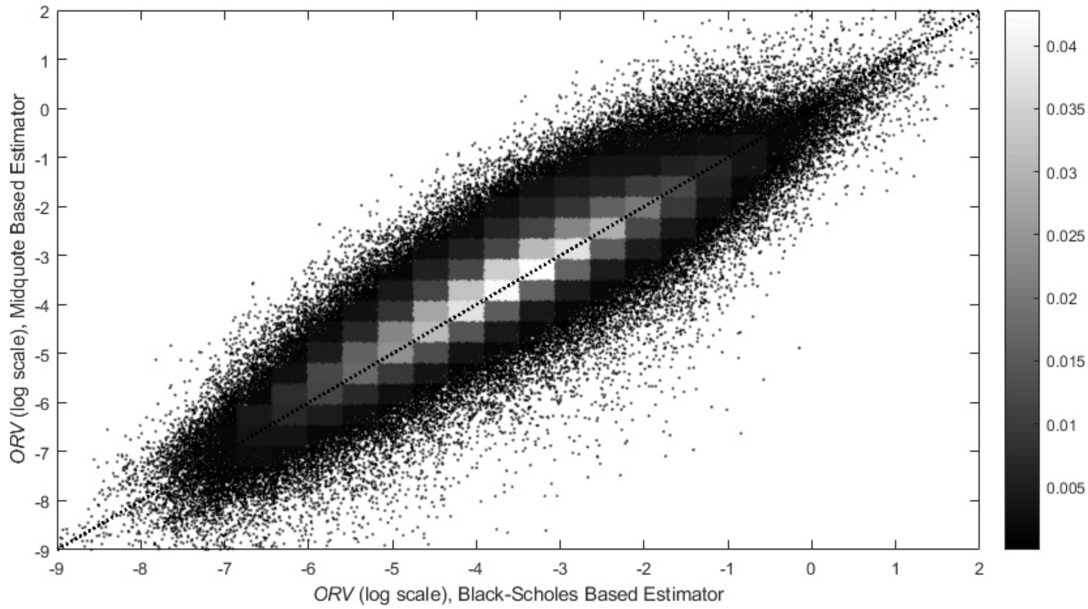

Figure SM.1: **Scatterplot of Scaled *ORV* Estimates from MidQuotes and Black-Scholes Prices.**
For each option in our sample, we estimate daily values of *ORV* using the subsampling method with one-minute option prices. The *ORV* estimates are scaled by the square of the end-of-day option midquote price. The figure compares log scaled values for estimates based on midquote option prices against estimates based on Black-Scholes option prices as proposed in Muravyev and Pearson (2016). Grey levels in the figure encode the density of points associated with a given region: light (dark) grey represents the highest (lowest) concentration regions.

E.1 Other HAR Specifications

Table SM.1 reports the results for three additional HAR regressions: (1) considering only lagged realized variances and the ATM implied variance (as in Busch et al., 2011), (2) considering bipower variations and jump variations over different lags (similar to one of the regressions proposed in Andersen et al., 2007), and (3) considering lagged values of RV based on intraday absolute returns, RAV, as discussed in Forsberg and Ghysels (2007). Generally speaking, the new variable, ORV, increases the adjusted R^2 in most cases, and is statistically significant in all cases but one.

These regressions are redone using the equity risk premium and the variance risk premium as the dependent variable instead of the realized variance. Table SM.2 reports the results for the equity risk premium. In that case, the coefficients associated with ORV are still negative. They are statistically significant for the OTM call ORV measures in most cases. Results over longer time horizons are stronger and lead to more important increases in the adjusted R^2 generally speaking.

Regarding the variance risk premium, the results are again very robust to other specifications, (see Table SM.3). Overall, the ORV measure coefficients are all statistically significant and adding the option realized variance to the mix increases the adjusted R^2 in all the cases.

E.2 Impact of the Moneyness

In this section, we assess the impact of using ORV measures than differ from those used in the main part of the paper (i.e., call-equivalent deltas of 0.2 and 0.8). Specifically, we redo the regression of Panel A in Table 3 using the option realized variance measure with different call-equivalent deltas. Table SM.4 reports

Table SM.1: **Predictive Heterogeneous Autoregressive Regressions on the Realized Variance.**

Panel A: Regression HAR-*RV-IV-ORV*.

$RV_{t,t+h} = \beta_0 + \beta_1 RV_t^{(d)} + \beta_2 RV_t^{(w)} + \beta_3 RV_t^{(m)} + \beta_4 IV_t + \beta_5 ORV_t^{\Delta^e=0.2} + \beta_6 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_0	**-0.0135**	**-0.0225**	**-0.0301**	**-0.0064**	**-0.0146**	**-0.0186**	0.0032	-0.0061	-0.0078	**0.0202**	**0.0071**	**0.0116**
	(0.0063)	(0.0091)	(0.0129)	(0.0031)	(0.0057)	(0.0073)	(0.0024)	(0.0050)	(0.0051)	(0.0019)	(0.0022)	(0.0020)
β_1	0.0301	-0.0213	-0.0979	0.1103	0.0631	0.0163	0.0802	0.0270	-0.0049	0.0097	**-0.0649**	**-0.0558**
	(0.1893)	(0.1852)	(0.1907)	(0.0811)	(0.0740)	(0.0676)	(0.0579)	(0.0494)	(0.0456)	(0.0161)	(0.0264)	(0.0220)
β_2	**0.4808**	**0.4562**	**0.4445**	**0.3158**	**0.2933**	**0.2892**	**0.2702**	**0.2448**	**0.2461**	0.0691	0.0336	0.0506
	(0.1987)	(0.1880)	(0.1724)	(0.1186)	(0.1130)	(0.1071)	(0.1259)	(0.1163)	(0.1169)	(0.0375)	(0.0319)	(0.0351)
β_3	-0.4086	-0.4124	-0.3541	-0.0880	-0.0914	-0.0479	0.1889	0.1850	0.2251	0.0177	0.0123	0.0457
	(0.2789)	(0.2817)	(0.2489)	(0.1835)	(0.1816)	(0.1657)	(0.1822)	(0.1806)	(0.1655)	(0.0494)	(0.0453)	(0.0442)
β_4	**0.8627**	**0.9157**	**0.9027**	**0.5559**	**0.6045**	**0.5853**	0.2124	**0.2672**	**0.2390**	**0.0998**	**0.1766**	**0.1203**
	(0.3230)	(0.3312)	(0.3081)	(0.1556)	(0.1629)	(0.1456)	(0.1170)	(0.1223)	(0.1050)	(0.0463)	(0.0468)	(0.0418)
β_5 / β_6	–	**0.1494**	**0.4375**	–	**0.1372**	**0.3213**	–	**0.1546**	**0.2908**	–	**0.2165**	**0.2239**
	–	(0.0573)	(0.1930)	–	(0.0517)	(0.1310)	–	(0.0729)	(0.1156)	–	(0.0394)	(0.0469)
Adjusted R^2	0.5614	0.5665	0.5884	0.6681	0.6746	0.6896	0.5775	0.5881	0.6000	0.1108	0.1711	0.1491

Panel B: Regression HAR-*BV-JV-IV-ORV*.

$RV_{t,t+h} = \beta_0 + \beta_1 BV_t^{(d)} + \beta_2 BV_t^{(w)} + \beta_3 BV_t^{(m)} + \beta_4 JV_t^{(d)} + \beta_5 JV_t^{(w)} + \beta_6 JV_t^{(m)} + \beta_7 IV_t + \beta_8 ORV_t^{\Delta^e=0.2} + \beta_9 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_0	**-0.0123**	**-0.0185**	**-0.0265**	**-0.0077**	**-0.0139**	**-0.0177**	-0.0030	**-0.0123**	**-0.0132**	**0.0175**	0.0034	**0.0086**
	(0.0049)	(0.0057)	(0.0094)	(0.0024)	(0.0036)	(0.0053)	(0.0022)	(0.0050)	(0.0050)	(0.0019)	(0.0025)	(0.0021)
β_1	0.2253	0.1793	0.0779	0.2490	0.2025	0.1449	0.1394	0.0700	0.0336	0.0151	**-0.0896**	**-0.0765**
	(0.1292)	(0.1323)	(0.1328)	(0.1334)	(0.1354)	(0.1151)	(0.0783)	(0.0808)	(0.0698)	(0.0213)	(0.0294)	(0.0253)
β_2	0.5862	0.5737	**0.5734**	**0.4060**	**0.3935**	**0.3970**	0.1817	0.1630	0.1725	0.0078	-0.0205	-0.0002
	(0.3063)	(0.3040)	(0.2918)	(0.1466)	(0.1434)	(0.1375)	(0.0995)	(0.0917)	(0.0938)	(0.0512)	(0.0473)	(0.0524)
β_3	**-0.7118**	**-0.7191**	**-0.6560**	**-0.5649**	**-0.5723**	**-0.5255**	**-0.4597**	**-0.4707**	**-0.4197**	**-0.1925**	**-0.2092**	-0.1579
	(0.3398)	(0.3402)	(0.3103)	(0.1980)	(0.1988)	(0.1804)	(0.1815)	(0.1826)	(0.1611)	(0.0964)	(0.0926)	(0.0914)
β_4	-1.5124	-1.4587	-1.3199	-1.0791	-1.0248	-0.9431	-0.5893	-0.5084	-0.4512	-0.1090	0.0132	0.0107
	(0.9329)	(0.8987)	(0.7404)	(0.6314)	(0.6010)	(0.5214)	(0.4534)	(0.4016)	(0.3482)	(0.1219)	(0.0959)	(0.0918)
β_5	-1.7376	-1.7855	-1.8649	**-1.9770**	**-2.0255**	**-2.0670**	-0.3305	-0.4028	-0.4219	0.2549	0.1458	0.1757
	(1.4899)	(1.4759)	(1.4655)	(0.9302)	(0.8947)	(0.8424)	(0.9393)	(0.8781)	(0.8248)	(0.4524)	(0.3824)	(0.3867)
β_6	**4.8819**	**4.9628**	**4.7480**	**7.9704**	**8.0522**	**7.8758**	**10.9508**	**11.0729**	**10.8547**	**3.4975**	**3.6817**	**3.4143**
	(1.5811)	(1.5739)	(1.4618)	(1.6849)	(1.6642)	(1.5856)	(1.7595)	(1.7541)	(1.6644)	(1.2056)	(1.1988)	(1.2163)
β_7	**0.7732**	**0.8146**	**0.8198**	**0.5372**	**0.5790**	**0.5701**	**0.3501**	**0.4125**	**0.3835**	**0.1675**	**0.2618**	**0.1965**
	(0.2498)	(0.2504)	(0.2480)	(0.1145)	(0.1200)	(0.1140)	(0.0967)	(0.1087)	(0.0970)	(0.0574)	(0.0605)	(0.0551)
β_8 / β_9	–	**0.0991**	**0.3669**	–	**0.1003**	**0.2593**	–	**0.1496**	**0.2633**	–	**0.2258**	**0.2282**
	–	(0.0347)	(0.1302)	–	(0.0315)	(0.0884)	–	(0.0679)	(0.1007)	–	(0.0391)	(0.0455)
Adjusted R^2	0.5967	0.5988	0.6149	0.7165	0.7198	0.7299	0.6374	0.6470	0.6551	0.1298	0.1936	0.1680

Panel C: Regression HAR-*RAV-IV-ORV*.

$RV_{t,t+h} = \beta_0 + \beta_1 RAV_t^{(d)} + \beta_2 RAV_t^{(w)} + \beta_3 RAV_t^{(m)} + \beta_4 IV_t + \beta_5 ORV_t^{\Delta^e=0.2} + \beta_6 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

	One Day			One Week			One Month			One Year		
β_0	-0.0028	-0.0068	**-0.0112**	**-0.0138**	**-0.0162**	**-0.0184**	**-0.0215**	**-0.0234**	**-0.0249**	-0.0059	**-0.0072**	-0.0068
	(0.0082)	(0.0069)	(0.0043)	(0.0037)	(0.0040)	(0.0046)	(0.0062)	(0.0068)	(0.0070)	(0.0038)	(0.0036)	(0.0037)
β_1	0.0553	-0.0901	-0.3007	0.1612	0.0756	-0.0341	**0.1533**	0.0868	0.0126	**0.1626**	**0.1173**	**0.1259**
	(0.1579)	(0.1958)	(0.2955)	(0.0935)	(0.1017)	(0.0834)	(0.0628)	(0.0623)	(0.0515)	(0.0329)	(0.0380)	(0.0446)
β_2	0.2996	0.2908	0.3275	0.2037	0.1984	0.2190	0.2238	0.2197	0.2348	0.0799	0.0771	0.0828
	(0.1835)	(0.1708)	(0.1759)	(0.1241)	(0.1197)	(0.1165)	(0.1229)	(0.1195)	(0.1205)	(0.0454)	(0.0442)	(0.0453)
β_3	**-0.5768**	**-0.5797**	**-0.5200**	**-0.2901**	**-0.2918**	**-0.2589**	-0.0135	-0.0148	0.0090	**0.2499**	**0.2490**	**0.2557**
	(0.2280)	(0.2276)	(0.1909)	(0.1175)	(0.1165)	(0.1063)	(0.1217)	(0.1209)	(0.1171)	(0.0684)	(0.0678)	(0.0673)
β_4	**1.2723**	**1.4863**	**1.6112**	**0.7352**	**0.8612**	**0.9212**	0.1413	0.2393	0.2753	-0.5580	-0.4913	-0.5231
	(0.3956)	(0.4613)	(0.5287)	(0.1309)	(0.1462)	(0.1660)	(0.1481)	(0.1367)	(0.1398)	(0.1214)	(0.1268)	(0.1284)
β_5 / β_6	–	**0.2193**	**0.6178**	–	**0.1290**	**0.3389**	–	**0.1003**	**0.2442**	–	**0.0683**	0.0636
	–	(0.0859)	(0.3146)	–	(0.0413)	(0.1264)	–	(0.0457)	(0.0976)	–	(0.0301)	(0.0476)
Adjusted R^2	0.5748	0.5822	0.6068	0.6686	0.6723	0.6827	0.5703	0.5731	0.5796	0.2661	0.2698	0.2676

Various linear regressions that include *ORV*—similar to those of Table 3—are performed, except that other specifications of the HAV framework are used. We compute Newey-West standard errors; these are in parentheses in the above table. Values in bold are statistically significant at a confidence level of 95%.

the results for four different horizons, i.e., one day, one week, one month, and one year. In summary, the results are robust for every ORV measures used: the ORV is statistically significant for all the moneyness levels and the adjusted R^2 increases in all cases with respect to the base case. This does not mean, however, than all the ORV measures contain the same information as some of the regressors are not statistically significant in all the regressions. The significance of the other regressors changes as the information in ORV differs for various levels of moneyness.

E.3 Out-of-Sample Analysis of the Predictive Regressions

This section presents out-of-sample studies of the informational content of ORV measures for predicting RV, the equity risk premium (*ERP*), and the variance risk premium (*VRP*). The objective is to see whether the in-sample gains documented in Sections 3.1 and 3.2 lead to better forecasts in an out-of-sample exer-

Table SM.2: **Predictive Heterogeneous Autoregressive Regressions on the Equity Risk Premium.**

Panel A: Regression HAR-*RV-IV-ORV*.

$ERP_{t,t+h} = \beta_0 + \beta_1 RV_t^{(d)} + \beta_2 RV_t^{(w)} + \beta_3 RV_t^{(m)} + \beta_4 IV_t + \beta_5 ORV_t^{\Delta^e=0.2} + \beta_6 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_0	-0.0009	0.0015	0.0004	-0.0025	0.0004	-0.0007	-0.0033	0.0058	0.0036	-0.0047	**0.0810**	**0.0485**
	(0.0006)	(0.0011)	(0.0011)	(0.0016)	(0.0022)	(0.0019)	(0.0035)	(0.0057)	(0.0045)	(0.0129)	(0.0158)	(0.0139)
β_1	0.0294	**0.0427**	0.0394	0.0220	0.0385	0.0354	-0.0185	0.0334	0.0345	**-0.2252**	0.2634	0.1840
	(0.0231)	(0.0208)	(0.0239)	(0.0305)	(0.0329)	(0.0324)	(0.0447)	(0.0403)	(0.0387)	(0.1052)	(0.1775)	(0.1417)
β_2	-0.0272	-0.0209	-0.0244	-0.0613	-0.0534	-0.0575	-0.1787	-0.1540	-0.1637	-0.3067	-0.0738	-0.1910
	(0.0236)	(0.0223)	(0.0220)	(0.0528)	(0.0528)	(0.0522)	(0.0945)	(0.0863)	(0.0895)	(0.2098)	(0.1841)	(0.2029)
β_3	**-0.0623**	**-0.0613**	**-0.0665**	**-0.2024**	**-0.2012**	**-0.2081**	**-0.4067**	**-0.4029**	**-0.4293**	-0.4345	-0.3986	**-0.6088**
	(0.0269)	(0.0263)	(0.0262)	(0.0909)	(0.0895)	(0.0891)	(0.1202)	(0.1200)	(0.1155)	(0.3130)	(0.3011)	(0.3076)
β_4	0.0611	0.0473	0.0579	**0.2212**	**0.2042**	**0.2170**	**0.5143**	**0.4608**	**0.4977**	**1.6337**	**1.1307**	**1.5058**
	(0.0313)	(0.0320)	(0.0315)	(0.0850)	(0.0865)	(0.0848)	(0.1503)	(0.1559)	(0.1484)	(0.3549)	(0.3610)	(0.3351)
β_5 / β_6	–	**-0.0387**	-0.0342	–	**-0.0479**	-0.0457	–	**-0.1509**	-0.1813	–	**-1.4194**	**-1.3989**
	–	(0.0147)	(0.0234)	–	(0.0221)	(0.0338)	–	(0.0769)	(0.0984)	–	(0.2560)	(0.2984)
Adjusted R^2	0.0231	0.0325	0.0272	0.0427	0.0461	0.0443	0.0675	0.0782	0.0766	0.0482	0.1095	0.0835

Panel B: Regression HAR-*BV-JV-IV-ORV*.

$ERP_{t,t+h} = \beta_0 + \beta_1 BV_t^{(d)} + \beta_2 BV_t^{(w)} + \beta_3 BV_t^{(m)} + \beta_4 JV_t^{(d)} + \beta_5 JV_t^{(w)} + \beta_6 JV_t^{(m)} + \beta_7 IV_t + \beta_8 ORV_t^{\Delta^e=0.2} + \beta_9 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_0	-0.0009	0.0011	0.0000	-0.0004	0.0025	0.0010	0.0004	0.0103	0.0075	0.0060	**0.0970**	**0.0610**
	(0.0007)	(0.0009)	(0.0009)	(0.0019)	(0.0025)	(0.0024)	(0.0037)	(0.0063)	(0.0051)	(0.0137)	(0.0176)	(0.0145)
β_1	-0.0068	0.0081	0.0024	-0.0076	0.0139	0.0069	-0.0292	0.0445	0.0439	-0.2495	**0.4299**	0.3215
	(0.0297)	(0.0299)	(0.0302)	(0.0543)	(0.0572)	(0.0572)	(0.0605)	(0.0639)	(0.0604)	(0.1285)	(0.1922)	(0.1689)
β_2	-0.0221	-0.0181	-0.0213	0.0206	0.0264	0.0219	-0.0511	-0.0313	-0.0448	-0.1502	0.0330	-0.1007
	(0.0273)	(0.0272)	(0.0272)	(0.0683)	(0.0690)	(0.0684)	(0.1147)	(0.1113)	(0.1123)	(0.3696)	(0.3479)	(0.3772)
β_3	-0.0237	-0.0214	-0.0272	-0.0429	-0.0394	-0.0484	-0.1396	-0.1279	-0.1672	0.5634	0.6714	0.3475
	(0.0322)	(0.0321)	(0.0312)	(0.1034)	(0.1038)	(0.1014)	(0.2224)	(0.2232)	(0.2130)	(0.7302)	(0.7139)	(0.7130)
β_4	**0.3185**	**0.3011**	**0.3064**	0.2736	0.2485	0.2547	0.1592	0.0733	0.0638	0.3409	-0.4514	-0.4048
	(0.1423)	(0.1303)	(0.1322)	(0.1801)	(0.1779)	(0.1777)	(0.3054)	(0.2674)	(0.2599)	(0.6421)	(0.6533)	(0.6159)
β_5	0.0376	0.0531	0.0456	-0.5537	-0.5312	-0.5411	-1.0171	-0.9403	-0.9539	0.2643	0.9725	0.7579
	(0.1964)	(0.1946)	(0.1965)	(0.6321)	(0.6206)	(0.6224)	(1.1873)	(1.1319)	(1.1094)	(2.8264)	(2.5781)	(2.6450)
β_6	-0.6644	**-0.6906**	-0.6560	**-2.7887**	**-2.8266**	**-2.7755**	-4.6438	-4.7734	-4.5774	-17.0278	-18.2226	-16.5091
	(0.3500)	(0.3470)	(0.3466)	(1.2395)	(1.2272)	(1.2310)	(2.4253)	(2.4627)	(2.3950)	(9.4418)	(9.4305)	(9.5800)
β_7	**0.0691**	0.0557	**0.0662**	0.1705	0.1511	0.1659	**0.4142**	**0.3478**	**0.3911**	**1.3834**	0.7719	**1.2029**
	(0.0301)	(0.0309)	(0.0307)	(0.0942)	(0.0980)	(0.0955)	(0.1639)	(0.1745)	(0.1644)	(0.4494)	(0.4692)	(0.4274)
β_8 / β_9	–	**-0.0322**	-0.0230	–	**-0.0464**	-0.0361	–	**-0.1589**	-0.1819	–	**-1.4643**	**-1.4217**
	–	(0.0112)	(0.0165)	–	(0.0216)	(0.0324)	–	(0.0768)	(0.0960)	–	(0.2574)	(0.2996)
Adjusted R^2	0.0417	0.0479	0.0433	0.0616	0.0648	0.0625	0.0824	0.0940	0.0912	0.0565	0.1199	0.0915

Panel C: Regression HAR-*RAV-IV-ORV*.

$ERP_{t,t+h} = \beta_0 + \beta_1 RAV_t^{(d)} + \beta_2 RAV_t^{(w)} + \beta_3 RAV_t^{(m)} + \beta_4 IV_t + \beta_5 ORV_t^{\Delta^e=0.2} + \beta_6 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

	One Day			One Week			One Month			One Year		
β_0	**0.0039**	**0.0043**	**0.0040**	**0.0114**	**0.0118**	**0.0114**	**0.0269**	**0.0293**	**0.0288**	**0.1457**	**0.1604**	**0.1549**
	(0.0013)	(0.0013)	(0.0013)	(0.0029)	(0.0030)	(0.0030)	(0.0056)	(0.0063)	(0.0063)	(0.0251)	(0.0232)	(0.0239)
β_1	-0.0029	0.0118	0.0014	-0.0141	0.0000	-0.0127	-0.0699	0.0140	0.0078	**-0.9923**	-0.4610	**-0.6034**
	(0.0257)	(0.0319)	(0.0399)	(0.0360)	(0.0387)	(0.0410)	(0.0566)	(0.0543)	(0.0602)	(0.2110)	(0.2504)	(0.2740)
β_2	-0.0226	-0.0217	-0.0230	-0.0671	-0.0663	-0.0673	-0.1923	-0.1871	-0.1983	-0.3848	-0.3523	-0.4153
	(0.0244)	(0.0235)	(0.0250)	(0.0563)	(0.0563)	(0.0563)	(0.1099)	(0.1068)	(0.1094)	(0.2935)	(0.2848)	(0.2939)
β_3	**-0.0555**	**-0.0552**	**-0.0562**	**-0.1339**	**-0.1337**	**-0.1342**	**-0.1897**	**-0.1881**	**-0.2021**	**-1.3634**	**-1.3527**	**-1.4254**
	(0.0183)	(0.0181)	(0.0179)	(0.0543)	(0.0543)	(0.0546)	(0.0956)	(0.0949)	(0.0944)	(0.4580)	(0.4476)	(0.4488)
β_4	**0.1306**	**0.1089**	**0.1265**	**0.3288**	**0.3080**	**0.3274**	**0.6547**	**0.5313**	**0.5807**	**4.9067**	**4.1247**	**4.5364**
	(0.0474)	(0.0551)	(0.0554)	(0.0739)	(0.0811)	(0.0756)	(0.1654)	(0.1566)	(0.1500)	(0.8984)	(0.9081)	(0.9237)
β_5 / β_6	–	-0.0223	-0.0075	–	-0.0213	-0.0025	–	**-0.1265**	-0.1349	–	**-0.8010**	**-0.6750**
	–	(0.0183)	(0.0346)	–	(0.0248)	(0.0407)	–	(0.0579)	(0.0892)	–	(0.2313)	(0.2969)
Adjusted R^2	0.0254	0.0271	0.0251	0.0437	0.0437	0.0432	0.0562	0.0609	0.0589	0.1558	0.1684	0.1603

Various linear regressions that include *ORV*—similar to those of Table 4—are performed. We compute Newey-West standard errors; these are in parentheses in the above table. Values in bold are statistically significant at a confidence level of 95%.

cise. We consider the two classes of HAR models used in Sections 3.1 and 3.2, each with 18 variations depending on the ORV variable employed in the specification. The first model corresponds to the HAR-*RV-JV-IV-ORV* and the second to the LHAR-*RV-IV-ORV*. We use four forecasting horizons: one day, one week, one month, and one year. All forecasts are generated using rolling regressions based on the previous two years of data (500 observations) and the parameter estimates are updated daily.

The average root mean square error (RMSE) for both base models and their variations are reported for both specifications in Tables SM.5 and SM.6. Panel A of both tables shows the RMSE for the base model (i.e., excluding *ORV*), while Panel B does so for each of the 18 variations. For all three variables under consideration—the future realized variance, the equity risk premium, and the variance risk premium—it is evident that *ORV*-based models produce forecasts that at least are equivalent to those of the base case specification and, in several cases, these forecasts are significantly superior. We only observe few cases in

Table SM.3: **Predictive Heterogeneous Autoregressive Regressions on the Variance Risk Premium.**

Panel A: Regression HAR-*RV*-*IV*-*ORV*.

$VRP_{t,t+h} = \beta_0 + \beta_1 RV_t^{(d)} + \beta_2 RV_t^{(w)} + \beta_3 RV_t^{(m)} + \beta_4 IV_t + \beta_5 ORV_t^{\Delta^e=0.2} + \beta_6 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_0	-0.0107	**-0.0207**	**-0.0253**	-0.0035	**-0.0128**	**-0.0137**	0.0045	-0.0055	-0.0059	-0.0020	**-0.0213**	**-0.0166**
	(0.0057)	(0.0079)	(0.0113)	(0.0027)	(0.0050)	(0.0061)	(0.0023)	(0.0046)	(0.0048)	(0.0030)	(0.0035)	(0.0035)
β_1	-0.1510	-0.2082	-0.2633	-0.0709	-0.1238	-0.1492	0.0177	-0.0391	-0.0622	**0.1920**	**0.0820**	**0.0800**
	(0.1974)	(0.1877)	(0.1992)	(0.0871)	(0.0770)	(0.0766)	(0.0498)	(0.0434)	(0.0422)	(0.0578)	(0.0283)	(0.0301)
β_2	**0.4838**	0.4565	**0.4520**	**0.3193**	**0.2941**	**0.2972**	**0.2849**	**0.2578**	**0.2623**	**0.2111**	0.1586	**0.1794**
	(0.2435)	(0.2338)	(0.2206)	(0.1549)	(0.1492)	(0.1440)	(0.1285)	(0.1179)	(0.1173)	(0.0748)	(0.0656)	(0.0681)
β_3	-0.5109	-0.5151	-0.4630	-0.1902	-0.1941	-0.1568	0.0057	0.0015	0.0397	**0.3184**	**0.3103**	**0.3661**
	(0.2692)	(0.2714)	(0.2446)	(0.1883)	(0.1854)	(0.1751)	(0.1906)	(0.1876)	(0.1739)	(0.1020)	(0.0993)	(0.0927)
β_4	-0.2220	-0.1631	-0.1869	**-0.5299**	**-0.4755**	**-0.5055**	**-0.8718**	**-0.8134**	**-0.8468**	**-1.0847**	**-0.9714**	**-1.0497**
	(0.2855)	(0.2889)	(0.2726)	(0.1527)	(0.1583)	(0.1487)	(0.1130)	(0.1159)	(0.1039)	(0.1162)	(0.1076)	(0.1020)
β_5 / β_6	–	**0.1662**	**0.3840**	–	**0.1536**	**0.2679**	–	**0.1649**	**0.2731**	–	**0.3196**	**0.3829**
	–	(0.0488)	(0.1680)	–	(0.0467)	(0.1093)	–	(0.0706)	(0.1120)	–	(0.0421)	(0.0573)
Adjusted R^2	0.1997	0.2118	0.2389	0.3656	0.3811	0.3939	0.4621	0.4769	0.4864	0.3397	0.3956	0.3875

Panel B: Regression HAR-*BV*-*JV*-*IV*-*ORV*.

$VRP_{t,t+h} = \beta_0 + \beta_1 BV_t^{(d)} + \beta_2 BV_t^{(w)} + \beta_3 BV_t^{(m)} + \beta_4 JV_t^{(d)} + \beta_5 JV_t^{(w)} + \beta_6 JV_t^{(m)} + \beta_7 IV_t + \beta_8 ORV_t^{\Delta^e=0.2} + \beta_9 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_0	**-0.0096**	**-0.0173**	**-0.0222**	**-0.0049**	**-0.0127**	**-0.0133**	-0.0021	**-0.0124**	**-0.0119**	-0.0053	**-0.0250**	**-0.0194**
	(0.0047)	(0.0054)	(0.0086)	(0.0022)	(0.0033)	(0.0045)	(0.0021)	(0.0049)	(0.0049)	(0.0030)	(0.0036)	(0.0032)
β_1	0.0494	-0.0077	-0.0805	0.0715	0.0140	-0.0154	0.0710	-0.0056	-0.0305	**0.2247**	0.0775	0.0783
	(0.1350)	(0.1379)	(0.1499)	(0.1234)	(0.1250)	(0.1128)	(0.0877)	(0.0907)	(0.0819)	(0.0517)	(0.0487)	(0.0459)
β_2	0.4986	0.4832	0.4874	**0.3223**	0.3068	**0.3148**	0.1457	0.1251	0.1369	**0.2535**	**0.2138**	**0.2408**
	(0.2993)	(0.2958)	(0.2861)	(0.1613)	(0.1578)	(0.1543)	(0.1076)	(0.0999)	(0.0999)	(0.1069)	(0.0972)	(0.1026)
β_3	**-0.6980**	**-0.7070**	**-0.6488**	**-0.5521**	**-0.5613**	**-0.5193**	**-0.5923**	**-0.6045**	**-0.5540**	-0.1626	-0.1860	-0.1072
	(0.3250)	(0.3254)	(0.2993)	(0.1963)	(0.1971)	(0.1820)	(0.1952)	(0.1958)	(0.1742)	(0.1430)	(0.1366)	(0.1344)
β_4	**-1.7069**	**-1.6403**	**-1.5372**	**-1.2632**	**-1.1961**	**-1.1497**	-0.6003	-0.5110	-0.4678	-0.2494	-0.0778	-0.0582
	(0.8454)	(0.8047)	(0.6851)	(0.5623)	(0.5275)	(0.4813)	(0.4507)	(0.4168)	(0.3912)	(0.2278)	(0.1638)	(0.1532)
β_5	-0.4418	-0.5013	-0.5541	-0.7169	-0.7768	-0.7920	0.4015	0.3217	0.3138	**-1.3801**	**-1.5335**	**-1.5067**
	(1.3320)	(1.3292)	(1.3389)	(0.8981)	(0.8724)	(0.8616)	(0.9671)	(0.8980)	(0.8562)	(0.6655)	(0.5958)	(0.6088)
β_6	2.8130	2.9134	2.6949	**5.9188**	**6.0199**	**5.8398**	**9.9399**	**10.0746**	**9.8477**	**8.2585**	**8.5174**	**8.1254**
	(1.5953)	(1.5850)	(1.5025)	(1.8640)	(1.8323)	(1.7960)	(2.0058)	(1.9750)	(1.8985)	(1.5813)	(1.5706)	(1.6038)
β_7	-0.2944	-0.2430	-0.2533	**-0.5319**	**-0.4801**	**-0.5044**	**-0.7174**	**-0.6485**	**-0.6853**	**-1.0274**	**-0.8949**	**-0.9811**
	(0.2278)	(0.2266)	(0.2262)	(0.1189)	(0.1249)	(0.1196)	(0.1042)	(0.1157)	(0.1053)	(0.1242)	(0.1199)	(0.1125)
β_8 / β_9	–	**0.1231**	**0.3236**	–	**0.1239**	**0.2165**	–	**0.1650**	**0.2527**	–	**0.3173**	**0.3646**
	–	(0.0367)	(0.1187)	–	(0.0323)	(0.0769)	–	(0.0679)	(0.1006)	–	(0.0421)	(0.0530)
Adjusted R^2	0.2445	0.2508	0.2711	0.4231	0.4328	0.4407	0.5293	0.5437	0.5493	0.3799	0.4334	0.4214

Panel C: Regression HAR-*RAV*-*IV*-*ORV*.

$VRP_{t,t+h} = \beta_0 + \beta_1 RAV_t^{(d)} + \beta_2 RAV_t^{(w)} + \beta_3 RAV_t^{(m)} + \beta_4 IV_t + \beta_5 ORV_t^{\Delta^e=0.2} + \beta_6 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

	One Day			One Week			One Month			One Year		
β_0	0.0073	0.0024	-0.0011	-0.0036	-0.0069	-0.0082	**-0.0134**	**-0.0158**	**-0.0171**	**-0.0394**	**-0.0450**	**-0.0440**
	(0.0077)	(0.0059)	(0.0035)	(0.0047)	(0.0045)	(0.0051)	(0.0052)	(0.0057)	(0.0060)	(0.0058)	(0.0051)	(0.0054)
β_1	-0.1106	-0.2887	-0.4656	-0.0060	-0.1245	-0.2013	0.0876	-0.0002	-0.0689	**0.2731**	0.0717	0.0783
	(0.1952)	(0.2401)	(0.3371)	(0.0972)	(0.1129)	(0.1150)	(0.0686)	(0.0718)	(0.0638)	(0.0513)	(0.0665)	(0.0628)
β_2	0.3031	0.2923	0.3310	0.2085	0.2013	0.2238	0.2264	0.2210	0.2386	**0.1811**	**0.1688**	**0.1963**
	(0.2210)	(0.2054)	(0.2136)	(0.1489)	(0.1413)	(0.1412)	(0.1307)	(0.1255)	(0.1267)	(0.0884)	(0.0796)	(0.0833)
β_3	**-0.4910**	**-0.4946**	**-0.4344**	**-0.2057**	**-0.2081**	**-0.1746**	-0.0323	-0.0340	-0.0073	0.1098	0.1058	0.1409
	(0.2090)	(0.2078)	(0.1726)	(0.0965)	(0.0944)	(0.0847)	(0.1121)	(0.1107)	(0.1050)	(0.1312)	(0.1243)	(0.1238)
β_4	0.0533	0.3155	0.3913	**-0.4823**	-0.3079	-0.2964	**-1.0249**	**-0.8957**	**-0.8759**	**-1.2900**	**-0.9935**	**-1.1045**
	(0.3701)	(0.4453)	(0.4990)	(0.1336)	(0.1621)	(0.1658)	(0.1462)	(0.1396)	(0.1400)	(0.2514)	(0.2526)	(0.2494)
β_5 / β_6	–	**0.2685**	**0.6161**	–	**0.1787**	**0.3389**	–	**0.1324**	**0.2717**	–	**0.3037**	**0.3382**
	–	(0.1021)	(0.3136)	–	(0.0563)	(0.1293)	–	(0.0501)	(0.1028)	–	(0.0548)	(0.0659)
Adjusted R^2	0.2022	0.2233	0.2623	0.3595	0.3734	0.3864	0.4660	0.4722	0.4802	0.3357	0.3689	0.3576

Various linear regressions that include *ORV*—similar to those of Table 4—are performed. We compute Newey-West standard errors; these are in parentheses in the above table. Values in bold are statistically significant at a confidence level of 95%.

which the base model outperforms *ORV*-based forecasts, all of them associated to the HAR-*RV*-*JV*-*IV* in Table SM.5 for one-day-ahead forecasts of the equity risk premium. We thus conclude that the in-sample gains associated to the inclusion of *ORV* measures are also present out-of-sample.

F Summarizing the Option Realized Variance Panel

Table 2 provides evidence suggesting that the *ROV* measures are linearly related to one another. However, this evidence is not sufficient to conclude that only a few variables of the panel are needed to summarize the complete panel for model estimation. Thus, we follow Andersen et al. (2015b) and study the information contained in the *ROV* surface by looking at its principal characteristics and documenting how they interact with variables capturing the surface's dynamics and market activity.

The daily *ROV* surface is summarized via four different characteristics: level, term structure, skew, and

Table SM.4: **Predictive Heterogeneous Autoregressive Regressions on the Realized Variance Using Different Moneyness.**

Regression HAR-*RV*-*JV*-*IV*-*ORV*.

$RV_{t,t+h} = \beta_0 + \beta_1 RV_t^{(d)} + \beta_2 RV_t^{(w)} + \beta_3 RV_t^{(m)} + \beta_4 JV_t^{(d)} + \beta_5 IV_t + \beta_6 ORV_t + \varepsilon_{t,t+h}$

Panel A: One Day.

Δ^e		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
β_0	**-0.0106**	**-0.0163**	**-0.0167**	**-0.0158**	**-0.0159**	**-0.0187**	**-0.0232**	**-0.0244**	**-0.0252**	**-0.0229**
	(0.0048)	(0.0053)	(0.0056)	(0.0055)	(0.0055)	(0.0064)	(0.0085)	(0.0091)	(0.0097)	(0.0087)
β_1	**0.2918**	0.2614	0.2452	0.2454	0.2430	0.2182	0.1834	0.1672	0.1401	0.1356
	(0.1293)	(0.1337)	(0.1373)	(0.1400)	(0.1419)	(0.1382)	(0.1298)	(0.1278)	(0.1286)	(0.1289)
β_2	**0.4374**	**0.4197**	**0.4236**	**0.4280**	**0.4286**	**0.4277**	**0.4230**	**0.4186**	**0.4139**	**0.3983**
	(0.1686)	(0.1694)	(0.1674)	(0.1675)	(0.1672)	(0.1651)	(0.1619)	(0.1595)	(0.1564)	(0.1505)
β_3	-0.4029	-0.4001	-0.4057	-0.4066	-0.4055	-0.3979	-0.3786	-0.3709	-0.3576	-0.3438
	(0.2757)	(0.2768)	(0.2772)	(0.2773)	(0.2772)	(0.2765)	(0.2675)	(0.2629)	(0.2546)	(0.2497)
β_4	-2.1064	-2.0418	-2.0010	-1.9972	-1.9884	-1.9326	-1.8577	**-1.8185**	**-1.7597**	**-1.7253**
	(1.1028)	(1.0924)	(1.0852)	(1.0940)	(1.0989)	(1.0546)	(0.9599)	(0.9250)	(0.8751)	(0.8784)
β_5	**0.7482**	**0.7772**	**0.7885**	**0.7838**	**0.7842**	**0.7936**	**0.7985**	**0.8005**	**0.8010**	**0.7890**
	(0.2556)	(0.2545)	(0.2561)	(0.2555)	(0.2551)	(0.2591)	(0.2601)	(0.2586)	(0.2568)	(0.2521)
β_6	–	**0.0604**	**0.0973**	**0.1176**	**0.1563**	**0.2750**	**0.4113**	**0.3942**	**0.3712**	**0.2583**
	–	(0.0237)	(0.0349)	(0.0485)	(0.0662)	(0.0829)	(0.1338)	(0.1326)	(0.1362)	(0.0916)
Adjusted R^2	0.5867	0.5881	0.5887	0.5883	0.5882	0.5903	0.5972	0.6008	0.6054	0.6063

Panel B: One Week.

Δ^e		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
β_0	**-0.0042**	**-0.0102**	**-0.0102**	**-0.0095**	**-0.0093**	**-0.0111**	**-0.0137**	**-0.0148**	**-0.0148**	**-0.0129**
	(0.0020)	(0.0032)	(0.0032)	(0.0031)	(0.0031)	(0.0035)	(0.0045)	(0.0050)	(0.0051)	(0.0044)
β_1	**0.3094**	0.2772	0.2623	0.2613	0.2621	0.2460	0.2267	0.2135	0.1989	0.1977
	(0.1428)	(0.1459)	(0.1457)	(0.1476)	(0.1488)	(0.1452)	(0.1348)	(0.1300)	(0.1256)	(0.1268)
β_2	**0.2829**	**0.2640**	**0.2689**	**0.2731**	**0.2744**	**0.2745**	**0.2719**	**0.2684**	**0.2657**	**0.2549**
	(0.0989)	(0.0980)	(0.0980)	(0.0981)	(0.0981)	(0.0975)	(0.0965)	(0.0956)	(0.0950)	(0.0928)
β_3	-0.0837	-0.0807	-0.0864	-0.0874	-0.0862	-0.0793	-0.0651	-0.0590	-0.0507	-0.0414
	(0.1650)	(0.1639)	(0.1648)	(0.1655)	(0.1655)	(0.1645)	(0.1601)	(0.1575)	(0.1541)	(0.1513)
β_4	**-1.6024**	-1.5339	-1.4960	-1.4892	-1.4881	-1.4527	**-1.4129**	**-1.3808**	**-1.3499**	**-1.3300**
	(0.8024)	(0.7855)	(0.7785)	(0.7838)	(0.7890)	(0.7676)	(0.7179)	(0.6925)	(0.6701)	(0.6718)
β_5	**0.4688**	**0.4996**	**0.5094**	**0.5057**	**0.5036**	**0.5079**	**0.5071**	**0.5090**	**0.5072**	**0.4980**
	(0.1017)	(0.1059)	(0.1069)	(0.1071)	(0.1069)	(0.1074)	(0.1053)	(0.1044)	(0.1029)	(0.0998)
β_6	–	**0.0641**	**0.0983**	**0.1219**	**0.1515**	**0.2369**	**0.3135**	**0.3034**	**0.2704**	**0.1847**
	–	(0.0221)	(0.0320)	(0.0430)	(0.0551)	(0.0801)	(0.1066)	(0.1046)	(0.0965)	(0.0652)
Adjusted R^2	0.6897	0.6923	0.6929	0.6924	0.6920	0.6938	0.6988	0.7021	0.7044	0.7045

Panel C: One Month.

Δ^e		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
β_0	**0.0039**	-0.0057	-0.0051	-0.0041	-0.0038	-0.0052	-0.0063	-0.0071	-0.0072	-0.0059
	(0.0019)	(0.0041)	(0.0045)	(0.0044)	(0.0044)	(0.0045)	(0.0045)	(0.0047)	(0.0046)	(0.0040)
β_1	0.1427	0.0910	0.0730	0.0706	0.0711	0.0590	0.0542	0.0431	0.0275	0.0181
	(0.0780)	(0.0803)	(0.0830)	(0.0854)	(0.0870)	(0.0841)	(0.0760)	(0.0737)	(0.0727)	(0.0730)
β_2	**0.2598**	**0.2296**	**0.2392**	**0.2452**	**0.2469**	**0.2487**	**0.2480**	**0.2447**	**0.2419**	0.2286
	(0.1225)	(0.1145)	(0.1149)	(0.1163)	(0.1169)	(0.1179)	(0.1175)	(0.1164)	(0.1164)	(0.1140)
β_3	0.1902	0.1950	0.1861	0.1846	0.1864	0.1960	0.2101	0.2158	0.2246	0.2374
	(0.1803)	(0.1759)	(0.1794)	(0.1811)	(0.1810)	(0.1770)	(0.1708)	(0.1681)	(0.1653)	(0.1607)
β_4	-0.5027	-0.3929	-0.3451	-0.3330	-0.3298	-0.3051	-0.3000	-0.2727	-0.2396	-0.1988
	(0.5580)	(0.5194)	(0.5130)	(0.5177)	(0.5230)	(0.5026)	(0.4673)	(0.4478)	(0.4291)	(0.4233)
β_5	0.1851	**0.2344**	**0.2452**	**0.2403**	**0.2378**	**0.2366**	**0.2261**	**0.2268**	**0.2251**	**0.2176**
	(0.0988)	(0.1015)	(0.1072)	(0.1083)	(0.1083)	(0.1052)	(0.0993)	(0.0979)	(0.0958)	(0.0926)
β_6	–	**0.1027**	**0.1456**	0.1826	0.2291	**0.3127**	**0.3353**	**0.3149**	**0.2817**	**0.2060**
	–	(0.0448)	(0.0712)	(0.0966)	(0.1265)	(0.1512)	(0.1425)	(0.1280)	(0.1109)	(0.0800)
Adjusted R^2	0.5800	0.5888	0.5891	0.5880	0.5869	0.5892	0.5933	0.5971	0.6004	0.6036

Panel D: One Year.

Δ^e		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
β_0	**0.0201**	0.0024	0.0063	**0.0094**	**0.0108**	**0.0123**	**0.0137**	**0.0128**	**0.0109**	**0.0086**
	(0.0019)	(0.0024)	(0.0023)	(0.0022)	(0.0023)	(0.0023)	(0.0022)	(0.0021)	(0.0019)	(0.0018)
β_1	0.0047	**-0.0899**	**-0.1024**	**-0.0916**	**-0.0816**	**-0.0670**	**-0.0509**	**-0.0610**	**-0.0907**	**-0.1415**
	(0.0216)	(0.0288)	(0.0308)	(0.0303)	(0.0300)	(0.0270)	(0.0227)	(0.0227)	(0.0258)	(0.0336)
β_2	0.0699	0.0145	0.0382	0.0503	0.0544	0.0604	0.0625	0.0600	0.0551	0.0332
	(0.0367)	(0.0313)	(0.0310)	(0.0322)	(0.0333)	(0.0346)	(0.0348)	(0.0344)	(0.0345)	(0.0346)
β_3	0.0176	0.0264	0.0113	0.0101	0.0131	0.0226	0.0301	0.0345	0.0462	0.0730
	(0.0494)	(0.0433)	(0.0453)	(0.0469)	(0.0475)	(0.0472)	(0.0466)	(0.0461)	(0.0450)	(0.0434)
β_4	0.0399	**0.2411**	**0.2822**	0.2668	0.2484	0.2092	0.1674	0.1918	**0.2581**	**0.3967**
	(0.1686)	(0.1175)	(0.1302)	(0.1363)	(0.1410)	(0.1356)	(0.1286)	(0.1217)	(0.1152)	(0.1287)
β_5	**0.1020**	**0.1924**	**0.1945**	**0.1759**	**0.1656**	**0.1462**	**0.1278**	**0.1296**	**0.1352**	**0.1402**
	(0.0446)	(0.0464)	(0.0483)	(0.0481)	(0.0479)	(0.0460)	(0.0442)	(0.0437)	(0.0425)	(0.0404)
β_6	–	**0.1882**	**0.2238**	**0.2442**	**0.2763**	**0.2680**	**0.2109**	**0.2080**	**0.2336**	**0.2419**
	–	(0.0296)	(0.0400)	(0.0526)	(0.0688)	(0.0770)	(0.0649)	(0.0545)	(0.0470)	(0.0375)
Adjusted R^2	0.1104	0.1964	0.1731	0.1519	0.1394	0.1298	0.1253	0.1316	0.1507	0.2042

We compute Newey-West standard errors; these are in parentheses in the above table. Values in bold are statistically significant at a confidence level of 95%.

Table SM.5: **Out-of-Sample Tests on the Predictive Regressions.**

Panel A: HAR-*RV*-*JV*-*IV*

	RV				*ERP*				*VRP*			
	One Day	One Week	One Month	One Year	One Day	One Week	One Month	One Year	One Day	One Week	One Month	One Year
	3.55	3.41	3.59	3.46	1.56	3.05	5.70	23.82	3.96	3.56	3.81	4.85

Panel B: HAR-*RV*-*JV*-*IV*-*ORV*.

		RV				*ERP*				*VRP*			
	Delta	One Day	One Week	One Month	One Year	One Day	One Week	One Month	One Year	One Day	One Week	One Month	One Year
30 Days	0.1	3.52	3.39	3.46*	3.11*	1.55	3.04	5.64	21.47*	3.91*	3.56	3.88	4.48*
	0.2	3.54	3.37	3.41*	3.16*	1.54	3.04	5.66	22.00*	3.90*	3.54	3.84	4.49*
	0.3	3.55	3.37	3.46*	3.24*	1.55	3.04	5.68	22.41*	3.87*	3.55	3.86	4.56*
	0.4	3.55	3.37	3.49*	3.30*	1.55	3.04	5.67	22.70*	3.87*	3.56	3.88	4.59*
	0.5	3.52	3.37	3.47*	3.33*	1.55	3.04	5.66	23.03*	3.89*	3.55	3.91	4.65*
	0.6	3.48	3.35	3.49*	3.36*	1.57	3.05	5.66	23.55	3.86*	3.54	3.89	4.71*
	0.7	3.44*	3.34	3.48*	3.37*	1.57^{+}	3.05	5.67	23.53	3.85*	3.54	3.88	4.70*
	0.8	3.42*	3.32*	3.43*	3.31*	1.57^{+}	3.04	5.65	23.21*	3.82*	3.52	3.86	4.63*
	0.9	3.37*	3.30*	3.37*	3.11*	1.56	3.05	5.63	21.68*	3.80*	3.49*	3.78	4.33*
90 Days	0.1	3.52*	3.39	3.52	3.14*	1.55	3.04	5.66	21.49*	3.94*	3.57	3.83	4.49*
	0.2	3.50*	3.36	3.44*	3.20*	1.55*	3.05	5.64*	21.90*	3.94	3.55	3.81	4.51*
	0.3	3.51	3.36	3.45*	3.27*	1.56	3.05	5.63*	22.34*	3.91	3.54	3.81	4.57*
	0.4	3.52	3.36	3.45*	3.30*	1.57	3.04	5.63*	22.42*	3.91	3.54	3.82	4.64*
	0.5	3.50	3.35	3.45*	3.36*	1.58^{+}	3.04	5.66	23.17*	3.89	3.55	3.87	4.70*
	0.6	3.42*	3.34	3.49	3.38*	1.58^{+}	3.04	5.66	23.51	3.88*	3.55	3.89	4.73*
	0.7	3.38*	3.30*	3.48*	3.38*	1.58^{+}	3.04	5.67	23.57	3.86*	3.54	3.87	4.70*
	0.8	3.39*	3.31*	3.46*	3.33*	1.58^{+}	3.04	5.66	23.32*	3.86*	3.53	3.85	4.64*
	0.9	3.46*	3.34	3.47*	3.30*	1.57^{+}	3.05	5.68	22.60*	3.90*	3.53	3.81	4.61*

This table reports root mean square error (RMSE, multiplied by 100) associated with the out-of-sample forecasting of *RV*, *ERP*, and *VRP*, for four different horizons: one day, one week, one month, and one year. Results for the HAR-*RV*-*JV*-*IV* model are presented in the first row. We add different values of *ORV* to this specification, producing a total of 18 specifications (nine levels of moneyness, two maturities). The RMSEs associated with each specification are presented in rows 2 to 19. We compare the forecast of the base model for each of the new specifications using the Diebold and Mariano test. When the *ORV*-based model is significantly more accurate than the base model (out-of-sample), the RMSE is marked with a star. When the base model is significantly more accurate than the *ORV*-based model, the RMSE is marked with a cross. To take into account that 18 test are carried out simultaneous, we apply a Bonferroni correction and work with a 95% confidence level in the tests.

Table SM.6: **Out-of-Sample Tests on the Predictive Regressions, continued.**

Panel A: LHAR-*RV*-*IV*

	RV				*ERP*				*VRP*			
	One Day	One Week	One Month	One Year	One Day	One Week	One Month	One Year	One Day	One Week	One Month	One Year
	3.21	3.38	3.75	3.45	1.56	3.03	5.74	23.30	3.70	3.75	4.21	4.79

Panel B: LHAR-*RV*-*IV*-*ORV*.

		RV				*ERP*				*VRP*			
	Delta	One Day	One Week	One Month	One Year	One Day	One Week	One Month	One Year	One Day	One Week	One Month	One Year
30 Days	0.1	3.21	3.38	3.66*	3.11*	1.55	3.03	5.67	20.93*	3.66*	3.73	4.19	4.47*
	0.2	3.23	3.32*	3.67*	3.14*	1.55	3.03	5.68	21.41*	3.63*	3.72	4.19	4.48*
	0.3	3.23	3.32*	3.68	3.18*	1.55	3.03	5.71	22.12*	3.64*	3.74	4.19	4.54*
	0.4	3.22	3.34	3.69	3.21*	1.54	3.02	5.71	22.38*	3.64*	3.76	4.21	4.58*
	0.5	3.23	3.35	3.73	3.27*	1.54	3.02	5.71	22.74*	3.64*	3.76	4.22	4.65*
	0.6	3.23	3.36	3.76	3.35*	1.55	3.02	5.72	22.98*	3.64*	3.75	4.23	4.70*
	0.7	3.22	3.35	3.79	3.34*	1.55	3.03	5.72	22.95*	3.64*	3.73	4.25	4.70*
	0.8	3.24	3.34	3.74	3.26*	1.55	3.03	5.71	22.60*	3.63*	3.72	4.21	4.65*
	0.9	3.21	3.33	3.64*	3.06*	1.57	3.04	5.68	20.89*	3.59*	3.71	4.15	4.42*
90 Days	0.1	3.20	3.38	3.70	3.11*	1.56	3.03	5.70	21.43*	3.67*	3.76	4.19	4.51*
	0.2	3.23	3.34*	3.67*	3.13*	1.55	3.02	5.64*	21.83*	3.64*	3.74	4.15	4.50*
	0.3	3.24	3.32*	3.68*	3.21*	1.55	3.02	5.66*	22.20*	3.66	3.73	4.18	4.55*
	0.4	3.24	3.34	3.70	3.26*	1.55	3.02	5.68*	22.32*	3.67	3.73	4.19	4.61*
	0.5	3.23	3.32	3.78	3.35*	1.56	3.02	5.71	22.86*	3.69	3.74	4.26	4.70*
	0.6	3.22	3.32	3.83	3.40	1.57	3.02	5.73	23.16	3.67	3.71	4.29	4.74
	0.7	3.21	3.32	3.81	3.38*	1.56	3.02	5.74	23.20	3.66	3.70	4.24	4.71
	0.8	3.23	3.30*	3.73	3.30*	1.56	3.02	5.74	22.91*	3.64*	3.68*	4.18	4.66*
	0.9	3.22	3.35	3.69	3.26*	1.55	3.02	5.76	22.20*	3.64*	3.71	4.19	4.63*

This table reports root mean square error (RMSE, multiplied by 100) associated with the out-of-sample forecasting of *RV*, *ERP*, and *VRP*, for four different horizons: one day, one week, one month, and one year. Results for the LHAR-*RV*-*IV* model are presented in the first row. We add different values of *ORV* to this specification, producing a total of 18 specifications (nine levels of moneyness, two maturities). The RMSEs associated with each specification are presented in lines 2 to 19. We compare the forecast of the base model for each of the new specifications using the Diebold and Mariano test. When the *ORV*-based model is significantly more accurate than the base model (out-of-sample), the RMSE is marked with a star. When the base model is significantly more accurate than the *ORV*-based model, the RMSE is marked with a cross. To take into account that 18 test are carried out simultaneous, we apply a Bonferroni correction and work with a 95% confidence level in the tests.

skew term structure. The *ROV* level comes from the ATM option ($\Delta^e = 0.5$) with 30 days to maturity. The *ROV* term structure (TS) is defined as the difference between the *ROV* of the ATM with 90 days to maturity minus the *ROV* of the ATM option with 30 days to maturity. The *ROV* skew represents the difference between shorter dated OTM put options (i.e., $\Delta^e = 0.9$) and OTM call options (i.e., $\Delta^e = 0.1$), both with 30 days to maturity. The *ROV* skew term structure (Skew TS) is the difference between longer- and shorter-dated skew, with the longer-dated (i.e., 90 days to maturity) skew defined analogously to the shorter one. These characteristics are presented in Figure SM.2.

The surface level presents sporadic spikes that generate some persistence after their occurrence (top-left panel of Figure SM.2). ATM options exhibit realized variations that somewhat mimic the behaviour of the underlying realized variance, as evidenced by a correlation coefficient of about 50% between these two variables. As for other surface characteristics, they generally fluctuate close to zero and are particularly sizeable during turbulent market periods, revealing some interesting patterns. For instance, large negative values in the *ROV* term structure (top-right panel of Figure SM.2) show that shocks are more substantial for short-dated options. The modest correlation of this characteristic with *RV* (−0.319) and with the realized jump variation (−0.244) indicates that the *ROV* term structure could be a non-redundant source of information. The *ROV* skew (bottom-left panel of Figure SM.2) is on average close to zero with negative values during turbulent times, meaning that OTM puts tend to be more responsive to negative shocks than OTM calls, as expected. This type of responsiveness is specifically observed for short-dated options, as evidenced by the negative sign associated with values of the skew term structure (bottom-right panel of Figure SM.2).[6]

We now use a PCA to summarize the entire *ROV* panel so that we can more closely scrutinize different aspects of the commonality between the *ROV* surface characteristics, the realized variance and the jump activity of the S&P index. Specifically, we use the balanced panel of options and conduct a PCA over these values for the full sample.[7] The PCA analysis shows that the *ROV* surface displays a dominant level-type effect, as the first principal component (PC) accounts for 82.60% of the total variation and displays a high degree of commonality with the surface level. The second PC captures 8.91% of the total variation, while those that follow account for 3.11% and 2.95%, respectively.

Following Andersen et al. (2015b), we now run in-sample regressions of surface characteristics on PCs to determine whether the *ROV* surface can be summarized in one simple specification (e.g., the first component of a PCA) or whether different daily *ROV* measures are needed to appropriately capture the nonlinear changes in the surface. Thus, we run the following regression:

$$\text{Char}_t = \beta_0 + \beta_1 \text{PC}_{1,t} + \beta_2 \text{PC}_{2,t} + \beta_3 \text{PC}_{3,t} + \beta_4 \text{PC}_{4,t} + \varepsilon_t,$$

where Char_t is the characteristic of interest at time t (i.e., level, term structure, skew and skew term structure) and $\text{PC}_{n,t}$ represents the n^{th} principal component at time t. In addition, we also regress *RV* and *JV* on the PCs to further understand the relationship between *ROV* and realized variations of the index. Table SM.7 reports the results of these regressions.

The *ROV* characteristics load differently on PCs, despite the fact that the first four PCs jointly explain

[6] Both the *ROV* skew and skew term structure change drastically on February 27, 2007, which at that time was the day of the biggest one-day point loss since 2001, which could explain these changes to some extent.

[7] Again, we take nine equally spaced points from a given surface over the call-equivalent delta dimension (between 0.1 and 0.9) for maturities of 30 and 90 days.

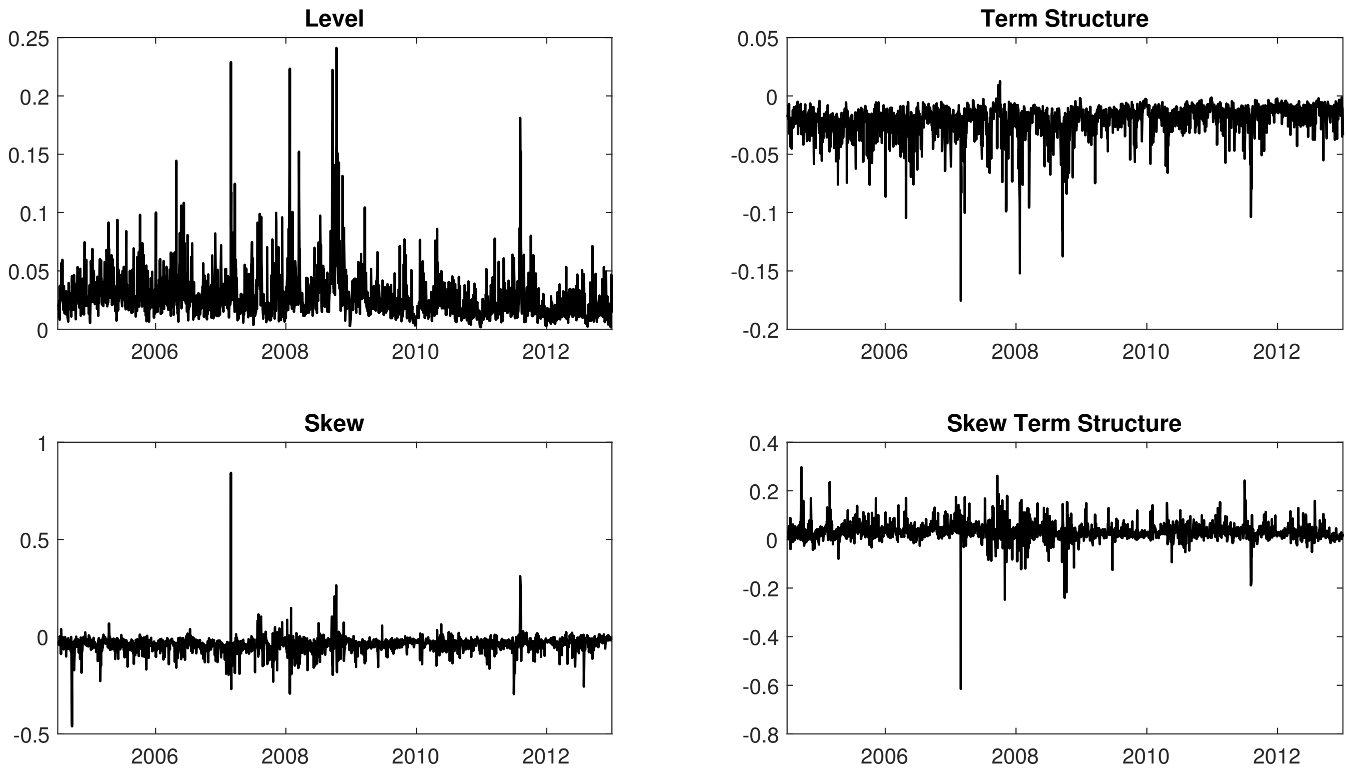

Figure SM.2: **Realized Option Variance Surface Characteristics.**
From July 2004 to December 2012, we collect the *ROV*s for all the ATM and OTM options in our dataset and perform a daily locally weighted scatter plot smoothing across moneyness and maturities. Realized option variance characteristics are then extracted from the fitted surfaces. The realized option variance level is the ATM option ($\Delta^e = 0.5$) with 30 days to maturity. The realized option variance term structure is defined as the difference between the realized option variance of the ATM with 90 days to maturity minus the *ROV* of the ATM option with 30 days to maturity. The realized option variance skew represents the difference between shorter-dated OTM put options (i.e., $\Delta^e = 0.9$) and OTM call options (i.e., $\Delta^e = 0.1$), both with 30 days to maturity. Finally, the *ROV* skew term structure is the difference between longer- and shorter-dated skew, with the longer-dated (i.e., 90 days to maturity) skew defined analogously to the shorter one.

95.6% of the total variation in the *ROV* surface and that these PCs are successful at explaining each of the characteristics with high R^2 values. For instance, the level of the surface is positively impacted by the first PC but the other three factors load negatively on this quantity. Moreover, the term structure does not significantly load on the second PC, but others do. These differences demonstrate that the information contained in the panel, as summarized by the first four PCs, impacts the various characteristics in different ways.

Notice also how successful PCs are at capturing the behaviour of index price variation, as evidenced by the R^2 values of 60% for the realized variance and 30% for the realized jump variation. Yet, the PCs impact these variables differently, suggesting that the information contained in the surface is impounded differently in these variations. This difference is also evidenced by examining the persistence of the regression residuals, which have different autocorrelation patterns. Persistent residuals in the realized variance regression suggest that either the relationship is nonlinear, or that there are missing factors in the model (e.g. lagged values of *RV*). On the other hand, the jump realized variation regression exhibits low persistence in the residuals.

Table SM.7: **Principal Component Regressions of the Option Realized Variance Surface and Realized Measures.**

	RV	*JV*
PC_1	**0.365**	**0.020**
	(0.084)	(0.004)
PC_2	0.121	-0.002
	(0.068)	(0.005)
PC_3	**0.755**	**0.046**
	(0.345)	(0.022)
PC_4	**-1.396**	**-0.064**
	(0.473)	(0.027)
R^2	0.559	0.286
AC(1)	0.558	0.153
AC(2:10)	0.415	0.129
AC(11:20)	0.279	0.111

We extract from the *ROV* surface 18 values per day, from July 2004 to December 2012, and conduct a PCA over these values for the full sample. More precisely, we take nine equally spaced points over the call-equivalent delta dimension between 0.1 and 0.9 for maturities of 30 and 90 business days. Then, the following linear regressions are performed:

$$RM_t = \beta_0 + \beta_1 \mathrm{PC}_{1,t} + \beta_2 \mathrm{PC}_{2,t} + \beta_3 \mathrm{PC}_{3,t} + \beta_4 \mathrm{PC}_{4,t} + \varepsilon_t,$$

where RM_t is the realized measures of interest at time t (i.e., RV and JV) and $\mathrm{PC}_{n,t}$ represents the n^{th} principal component at time t. R^2 and Newey and West (1987) standard errors (in parentheses) are reported. Values in bold are statistically significant at a confidence level of 95%. The first sample autocorrelation coefficient and the average sample autocorrelation over two to 10 and 11 to 20 lags of the regression residuals are exhibited.

Two main findings can be derived from these regressions. First, there is no simple mapping between the principal components and characteristics: they are intertwined in a nontrivial way. Second, the *ROV* surface has pronounced nonlinearities, as it is difficult to separate the forces driving the characteristics of the surface. The fact that the different characteristics of the *ROV* surface cannot be summarized in one simple specification of PCs suggests that various realized option variances should be used when extracting information from the panel. In other words, using only one *ROV* (or summarized versions of the *ROV* measures) would not capture all the information contained in the cross-section of the realized option variance.

G Simulation-Based Analysis of Variance Jumps

This section replicates the results of Section 4.3 of the main paper, but this time considering ITM options instead of OTM options. Figure SM.3 resembles Figure 3 although we consider ITM put and call options with call-equivalent deltas of 0.2 and 0.8, respectively. In this example—same as the one investigated in Section 4.3—the variance jump does not have a significant impact on the option returns for ITM options. In other words, ITM options lead to very small option jumps and are, therefore, not informative with respect to the variance jump occurrences. This result further justifies our choice of using OTM and ATM options in this study.

Table SM.8 mimics Table 5 of the main paper but, again, considering ITM options instead of OTM options. Once more, using ITM options seem to reduce the informational content—especially in terms of variance jumps—which is consistent the Itō decomposition of Equation (6).

H Information Content: Using Price-Based Option Realized Variance Measures

In this section, we consider the predictive regressions of Section 3 with price-based option realized variance measures. Table SM.9 reports the results of the predictive regressions on the future realized variance, in the spirit of Table 3 of the paper. Generally speaking, the results are slightly weaker than those presented

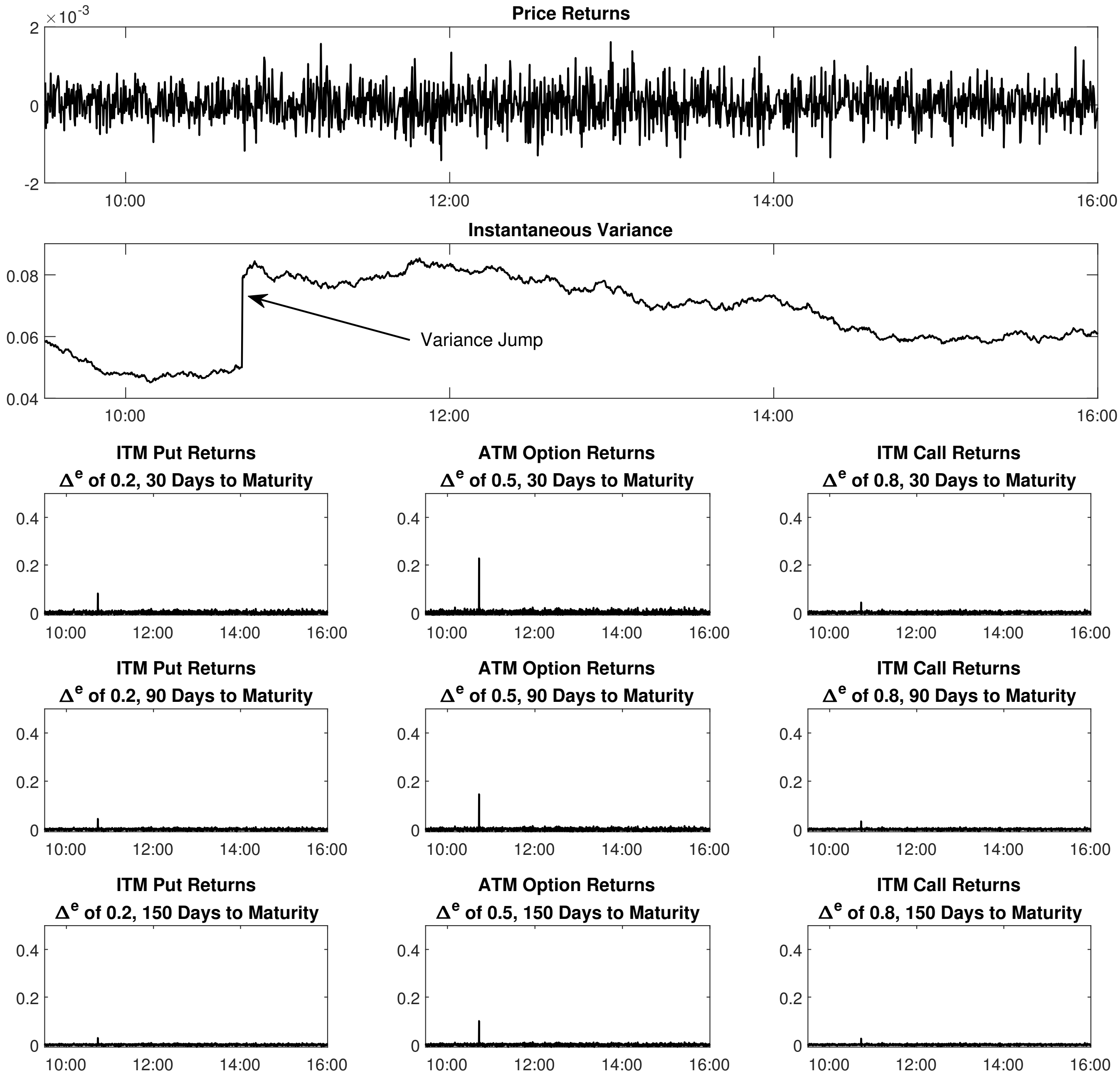

Figure SM.3: **Example of Option Returns for ITM and ATM Options.**
This figure reports the intraday equity returns, the instantaneous variance, and the intraday returns associated with nine different option contracts, for the same day as that investigated in Section 4.3. We generate a one-day path of the data generating process at sampling frequency of 1/1,560 (every 15 seconds during a 6.5-hour trading day) during which a variance jump occurred. We then compute model option prices for multiple different contracts. Parameters used in the simulation are presented in Table 6. We consider ITM put, ATM, and OTM call options with call-equivalent deltas 0.2, 0.5 and 0.8, respectively, as well as maturities of 30, 90, and 150 days.

in the paper. Nonetheless, the *ORV* increases the adjusted R^2s in most case, and most of the coefficients are statistically significant—especially over the longer horizons.

Tables SM.10 and SM.11 report the risk premium regressions using the price-based *ORV* instead of the

Table SM.8: **Mean of the Average Option Jump Size Relative to the Variance Jump for ITM and ATM Options.**

	$\Delta^e = 0.2$	$\Delta^e = 0.5$	$\Delta^e = 0.8$
DTM = 30	1.794	7.298	1.918
DTM = 90	0.688	3.243	0.834
DTM = 150	0.410	2.046	0.532

This table reports the mean of the average option jump size relative to the variance jump across 100 paths of the data generating process using Equation (10). Specifically, we generate 100 one-day paths of the data generating process at sampling frequency of 1/1,560 (every 15 seconds during a 6.5-hour trading day). We then compute model option prices for multiple different contracts. Parameters used in the simulation are presented in Table 6. We consider ITM put, ATM, and ITM call options with call-equivalent deltas 0.2, 0.5 and 0.8, respectively, as well as maturities of 30, 90, and 150 days.

return-based one. Again, we see quite significant increases in adjusted R^2 across the board. Regarding the significance of the coefficients, they tend to be less significant when using the price-based ORV. However, more than half of those associated with ORV are still statistically significant when using the price-based version of our measure.

In summary, using the price-based ORV instead of the return-based one does not change our results much in terms of predictive regressions, the only difference being that it slightly reduces the significance of a small number of coefficients. Nonetheless, adding ORV has a positive impact on the adjusted R^2 in all cases.

I Integrated Risk Premiums

I.1 Equity

In the spirit of Bardgett et al. (2019), and consistent with Bollerslev and Todorov (2011) and Aït-Sahalia et al. (2015), we can divide the integrated equity risk premium into two components: diffusive and jump risk premiums. In our framework,

$$IERP_{t,t+h} = \frac{1}{h}\left(\mathrm{E}_t^{\mathbb{P}}\left[Y_{t+h} - Y_t\right] - \mathrm{E}_t^{\mathbb{Q}}\left[Y_{t+h} - Y_t\right]\right)$$

and this $IERP$ can be decomposed into two components:

$$IERP_{t,t+h} = IERP_{t,t+h}^{\text{Diffusion}} + IERP_{t,t+h}^{\text{Jump}}$$

where

$$IERP_{t,t+h}^{\text{Diffusion}} = \frac{1}{h}\eta_Y \mathrm{E}_t^{\mathbb{P}}\left[\int_t^{t+h} V_{s^-}\, ds\right]$$

and

$$IERP_{t,t+h}^{\text{Jump}} = \frac{1}{h}\gamma_Y\left(\lambda_{Y,0}h + \lambda_{Y,1}\mathrm{E}_t^{\mathbb{P}}\left[\int_t^{t+h} V_{s^-}\, ds\right]\right).$$

Under our modelling framework, it is straightforward to compute $\mathrm{E}_t^{\mathbb{P}}\left[\int_t^T V_{s^-}\, ds\right]$. First, we know that

$$\mathrm{E}_t^{\mathbb{P}}\left[\int_t^{t+h} V_{s^-}\, ds\right] = \int_t^{t+h} \mathrm{E}_t^{\mathbb{P}}\left[V_{s^-}\right] ds$$

Table SM.9: **Predictive Regressions on the Realized Variance Using the Price-Based Option Realized Variance.**

Panel A: Regression HAR-*RV-JV-IV-ORV*.

$RV_{t,t+h} = \beta_0 + \beta_1 RV_t^{(d)} + \beta_2 RV_t^{(w)} + \beta_3 RV_t^{(m)} + \beta_4 JV_t^{(d)} + \beta_5 IV_t + \beta_6 ORV_t^{\Delta^e=0.2} + \beta_7 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_0	**-0.0106**	**-0.0137**	**-0.0117**	**-0.0042**	**-0.0071**	**-0.0060**	**0.0039**	0.0002	0.0005	**0.0201**	**0.0155**	**0.0151**
	(0.0048)	(0.0053)	(0.0047)	(0.0020)	(0.0026)	(0.0026)	(0.0019)	(0.0022)	(0.0025)	(0.0019)	(0.0016)	(0.0016)
β_1	**0.2918**	0.2003	0.2514	**0.3094**	0.2206	0.2387	0.1427	0.0301	0.0141	0.0047	**-0.1316**	**-0.1832**
	(0.1293)	(0.1200)	(0.1451)	(0.1428)	(0.1184)	(0.1444)	(0.0780)	(0.0668)	(0.0910)	(0.0216)	(0.0395)	(0.0578)
β_2	**0.4374**	**0.3733**	**0.4199**	**0.2829**	**0.2206**	**0.2522**	0.2598	0.1809	0.2041	0.0699	-0.0256	-0.0116
	(0.1686)	(0.1576)	(0.1707)	(0.0989)	(0.0907)	(0.0948)	(0.1225)	(0.0977)	(0.1113)	(0.0367)	(0.0274)	(0.0364)
β_3	-0.4029	-0.3556	-0.4098	-0.0837	-0.0377	-0.0956	0.1902	0.2485	0.1686	0.0176	0.0882	-0.0140
	(0.2757)	(0.2620)	(0.2744)	(0.1650)	(0.1493)	(0.1683)	(0.1803)	(0.1482)	(0.1815)	(0.0494)	(0.0489)	(0.0540)
β_4	-2.1064	**-1.8574**	-1.9699	**-1.6024**	**-1.3608**	-1.3634	-0.5027	-0.1964	-0.0683	0.0399	**0.4108**	**0.6751**
	(1.1028)	(0.9317)	(1.1409)	(0.8024)	(0.6346)	(0.7507)	(0.5580)	(0.3816)	(0.4980)	(0.1686)	(0.1977)	(0.2390)
β_5	**0.7482**	**0.6661**	**0.7566**	**0.4688**	**0.3891**	**0.4834**	0.1851	0.0840	**0.2115**	**0.1020**	-0.0203	**0.1407**
	(0.2556)	(0.2247)	(0.2515)	(0.1017)	(0.0789)	(0.1049)	(0.0988)	(0.0845)	(0.0996)	(0.0446)	(0.0390)	(0.0460)
β_6 / β_7	–	**0.0024**	0.0003	–	**0.0023**	0.0006	–	**0.0029**	**0.0010**	–	**0.0036**	**0.0015**
	–	(0.0006)	(0.0003)	–	(0.0009)	(0.0003)	–	(0.0011)	(0.0005)	–	(0.0004)	(0.0003)
Adjusted R^2	0.5867	0.6019	0.5873	0.6897	0.7111	0.6933	0.5800	0.6240	0.5954	0.1104	0.2962	0.2057

Panel B: Regression LHAR-*RV-IV-ORV*.

$RV_{t,t+h} = \beta_0 + \beta_1 RV_t^{(d)} + \beta_2 RV_t^{(w)} + \beta_3 RV_t^{(m)} + \beta_4 r_t^{(d)} + \beta_5 r_t^{(w)} + \beta_6 r_t^{(m)} + \beta_7 IV_t + \beta_8 ORV_t^{\Delta^e=0.2} + \beta_9 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_0	**-0.0131**	**-0.0141**	**-0.0145**	**-0.0061**	**-0.0084**	**-0.0081**	0.0035	0.0003	0.0010	**0.0201**	**0.0151**	**0.0167**
	(0.0055)	(0.0055)	(0.0058)	(0.0026)	(0.0034)	(0.0033)	(0.0021)	(0.0024)	(0.0024)	(0.0019)	(0.0015)	(0.0017)
β_1	-0.0253	-0.0448	-0.0584	0.0604	0.0190	0.0149	0.0478	-0.0102	-0.0119	0.0006	-0.0898	-0.0815
	(0.1908)	(0.1855)	(0.1837)	(0.0764)	(0.0671)	(0.0663)	(0.0528)	(0.0374)	(0.0403)	(0.0150)	(0.0485)	(0.0547)
β_2	**0.4204**	**0.3997**	**0.3966**	**0.2827**	**0.2387**	**0.2500**	**0.2611**	**0.1995**	**0.2182**	0.0337	-0.0624	-0.0253
	(0.1746)	(0.1718)	(0.1646)	(0.0829)	(0.0799)	(0.0760)	(0.1166)	(0.0954)	(0.1063)	(0.0404)	(0.0329)	(0.0399)
β_3	-0.1617	-0.1671	-0.1800	0.0778	0.0664	0.0528	**0.3221**	**0.3061**	0.2892	0.0533	0.0283	0.0080
	(0.1448)	(0.1474)	(0.1509)	(0.1454)	(0.1441)	(0.1441)	(0.1486)	(0.1460)	(0.1516)	(0.0500)	(0.0406)	(0.0502)
β_4	**-1.1932**	**-1.0220**	**-1.2297**	**-0.4428**	-0.0797	**-0.4930**	**-0.5575**	-0.0480	**-0.6234**	-0.1633	**0.6311**	**-0.2539**
	(0.4386)	(0.4521)	(0.4582)	(0.1386)	(0.2110)	(0.1496)	(0.2194)	(0.1552)	(0.2323)	(0.0979)	(0.1087)	(0.0935)
β_5	-2.3594	-2.2328	-2.2343	-2.2990	-2.0306	-2.1273	**-1.6970**	-1.3204	-1.4711	-0.0475	0.5398	0.2627
	(1.8967)	(1.8684)	(1.8657)	(1.3341)	(1.2270)	(1.2745)	(0.7868)	(0.7979)	(0.7877)	(0.3952)	(0.3739)	(0.3992)
β_6	**-5.7548**	**-5.2394**	**-5.0866**	-3.2521	-2.1589	-2.3342	-1.4019	0.1319	-0.1948	-2.7308	-0.3391	-1.0731
	(2.3086)	(2.3115)	(2.1747)	(2.5053)	(2.1513)	(2.2131)	(2.0507)	(1.8644)	(1.8024)	(1.8670)	(1.4262)	(1.6674)
β_7	**0.4678**	**0.4729**	**0.4776**	**0.3126**	**0.3234**	**0.3261**	0.0054	0.0206	0.0231	0.0424	0.0661	0.0668
	(0.1365)	(0.1321)	(0.1318)	(0.1025)	(0.0993)	(0.1017)	(0.0875)	(0.0854)	(0.0870)	(0.0491)	(0.0458)	(0.0486)
β_8 / β_9	–	0.0009	0.0005	–	0.0019	0.0007	–	**0.0026**	0.0010	–	**0.0041**	**0.0013**
	–	(0.0005)	(0.0003)	–	(0.0010)	(0.0004)	–	(0.0012)	(0.0005)	–	(0.0005)	(0.0003)
Adjusted R^2	0.6123	0.6137	0.6143	0.6971	0.7078	0.7032	0.6006	0.6277	0.6142	0.1230	0.3137	0.1975

This table presents two HAR regressions in the spirit of Corsi (2009) where the dependent variable is the realized variance over future horizons. Columns 2, 5, 8 and 11 are associated with β_6 (Panel A) and β_8 (Panel B), i.e., the OTM call *ORV*s, and Columns 3, 6, 9 and 12 are associated with β_7 (Panel A) and β_9 (Panel B), i.e., the OTM put *ORV*s. We compute Newey-West standard errors; these are in parentheses in the above table. Values in bold are statistically significant at a confidence level of 95%.

Table SM.10: **Predictive Regression on the Equity Risk Premium Using the Price-Based Option Realized Variance.**

Panel A: Regression HAR-*RV*-*JV*-*IV*-*ORV*.

$ERP_{t,t+h} = \beta_0 + \beta_1 RV_t^{(d)} + \beta_2 RV_t^{(w)} + \beta_3 RV_t^{(m)} + \beta_4 JV_t^{(d)} + \beta_5 IV_t + \beta_6 ORV_t^{\Delta^e=0.2} + \beta_7 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_0	**-0.0013**	**-0.0012**	-0.0008	-0.0025	-0.0020	-0.0015	-0.0031	0.0001	0.0001	-0.0050	**0.0257**	**0.0326**
	(0.0006)	(0.0006)	(0.0006)	(0.0016)	(0.0017)	(0.0016)	(0.0034)	(0.0032)	(0.0033)	(0.0128)	(0.0104)	(0.0110)
β_1	-0.0118	-0.0099	0.0067	0.0144	0.0316	0.0546	0.0000	0.0959	0.1204	-0.2473	**0.6700**	**1.1729**
	(0.0300)	(0.0300)	(0.0303)	(0.0545)	(0.0562)	(0.0601)	(0.0647)	(0.0644)	(0.0880)	(0.1403)	(0.2699)	(0.4188)
β_2	-0.0204	-0.0191	-0.0124	-0.0600	-0.0480	-0.0426	-0.1818	-0.1146	-0.1296	-0.3031	0.3398	0.3130
	(0.0202)	(0.0190)	(0.0208)	(0.0548)	(0.0522)	(0.0543)	(0.0937)	(0.0776)	(0.0856)	(0.2035)	(0.1862)	(0.2130)
β_3	**-0.0631**	**-0.0641**	**-0.0600**	**-0.2026**	**-0.2115**	**-0.1958**	**-0.4063**	**-0.4559**	**-0.3861**	-0.4349	**-0.9096**	-0.1957
	(0.0234)	(0.0229)	(0.0248)	(0.0905)	(0.0890)	(0.0911)	(0.1200)	(0.1173)	(0.1237)	(0.3120)	(0.3849)	(0.3517)
β_4	**0.3319**	**0.3268**	0.2692	0.0611	0.0143	-0.0748	-0.1494	-0.4102	-0.5561	0.1775	-2.3181	**-4.6222**
	(0.1669)	(0.1598)	(0.1520)	(0.2286)	(0.2265)	(0.2445)	(0.4068)	(0.3592)	(0.4698)	(0.9543)	(1.5202)	(1.7198)
β_5	**0.0791**	**0.0808**	**0.0753**	**0.2246**	**0.2400**	**0.2163**	**0.5062**	**0.5922**	**0.4814**	**1.6433**	**2.4663**	**1.3509**
	(0.0299)	(0.0300)	(0.0310)	(0.0879)	(0.0902)	(0.0880)	(0.1506)	(0.1615)	(0.1486)	(0.3561)	(0.3014)	(0.3397)
β_6 / β_7	–	0.0000	**-0.0002**	–	-0.0004	-0.0003	–	**-0.0025**	**-0.0010**	–	**-0.0239**	**-0.0115**
	–	(0.0001)	(0.0001)	–	(0.0004)	(0.0002)	–	(0.0010)	(0.0005)	–	(0.0030)	(0.0020)
Adjusted R^2	0.0401	0.0399	0.0444	0.0424	0.0458	0.0477	0.0674	0.1016	0.0817	0.0478	0.2467	0.1766

Panel B: Regression LHAR-*RV*-*IV*-*ORV*.

$ERP_{t,t+h} = \beta_0 + \beta_1 RV_t^{(d)} + \beta_2 RV_t^{(w)} + \beta_3 RV_t^{(m)} + \beta_4 r_t^{(d)} + \beta_5 r_t^{(w)} + \beta_6 r_t^{(m)} + \beta_7 IV_t + \beta_8 ORV_t^{\Delta^e=0.2} + \beta_9 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_0	-0.0008	-0.0004	-0.0001	-0.0026	-0.0016	-0.0015	-0.0037	0.0000	-0.0010	-0.0051	**0.0287**	0.0223
	(0.0006)	(0.0007)	(0.0007)	(0.0016)	(0.0016)	(0.0015)	(0.0035)	(0.0034)	(0.0033)	(0.0130)	(0.0105)	(0.0117)
β_1	0.0306	0.0384	**0.0464**	0.0189	0.0373	0.0449	-0.0378	0.0289	0.0270	-0.1869	0.4274	0.4669
	(0.0265)	(0.0252)	(0.0212)	(0.0295)	(0.0320)	(0.0320)	(0.0489)	(0.0336)	(0.0373)	(0.1012)	(0.3299)	(0.4410)
β_2	-0.0317	-0.0234	-0.0203	**-0.0994**	-0.0799	-0.0807	**-0.2337**	-0.1627	-0.1871	-0.1872	**0.4658**	0.2827
	(0.0239)	(0.0227)	(0.0229)	(0.0501)	(0.0499)	(0.0502)	(0.1155)	(0.0973)	(0.1070)	(0.2339)	(0.1961)	(0.2328)
β_3	-0.0522	-0.0500	-0.0435	**-0.1972**	**-0.1921**	**-0.1829**	**-0.4153**	**-0.3969**	**-0.3796**	**-0.6799**	-0.5104	-0.3194
	(0.0295)	(0.0292)	(0.0300)	(0.0893)	(0.0883)	(0.0868)	(0.1274)	(0.1267)	(0.1305)	(0.3448)	(0.3063)	(0.3421)
β_4	**-0.1452**	**-0.2142**	-0.1277	-0.0012	-0.1626	0.0275	**0.5856**	-0.0002	**0.6570**	**1.6988**	**-3.6948**	**2.4201**
	(0.0680)	(0.0834)	(0.0673)	(0.1299)	(0.1332)	(0.1279)	(0.2727)	(0.2181)	(0.2811)	(0.6036)	(0.7906)	(0.5630)
β_5	0.0962	0.0452	0.0365	0.3315	0.2122	0.2334	-0.2168	-0.6499	-0.4615	0.7611	-3.2263	-1.7093
	(0.2893)	(0.2799)	(0.2812)	(0.5284)	(0.5186)	(0.5251)	(0.8802)	(0.9159)	(0.8680)	(2.8181)	(2.4971)	(2.6505)
β_6	-0.4014	-0.6092	-0.7207	-2.7488	**-3.2348**	**-3.2733**	-3.7423	-5.5059	-5.0503	10.0923	-6.1451	-3.1101
	(0.6203)	(0.5762)	(0.5874)	(1.5667)	(1.4913)	(1.5411)	(3.1866)	(3.1751)	(3.2403)	(12.1631)	(8.8793)	(10.0916)
β_7	0.0390	0.0370	0.0343	**0.2128**	**0.2080**	**0.2051**	**0.5585**	**0.5411**	**0.5393**	**2.0597**	**1.8989**	**1.8656**
	(0.0348)	(0.0350)	(0.0356)	(0.0893)	(0.0892)	(0.0890)	(0.1775)	(0.1769)	(0.1749)	(0.4064)	(0.3604)	(0.3687)
β_8 / β_9	–	-0.0004	**-0.0003**	–	**-0.0008**	**-0.0004**	–	**-0.0030**	**-0.0010**	–	**-0.0280**	**-0.0104**
	–	(0.0002)	(0.0001)	–	(0.0004)	(0.0002)	–	(0.0012)	(0.0005)	–	(0.0035)	(0.0020)
Adjusted R^2	0.0291	0.0360	0.0428	0.0520	0.0620	0.0615	0.0771	0.1157	0.0942	0.0568	0.2646	0.1688

Various linear regressions that include ORV—similar to those of Table 4—are performed, except that the left-hand side (i.e., dependent variable) is replaced by the equity risk premium over a period of length h, $ERP_{t,t+h}$. Columns 2, 5, 8 and 11 are associated with β_6 (Panel A) and β_8 (Panel B), i.e., the OTM call ORVs, and Columns 3, 6, 9 and 12 are associated with β_7 (Panel A) and β_9 (Panel B), i.e., the OTM put ORVs. We compute Newey-West standard errors; these are in parentheses in the above table. Values in bold are statistically significant at a confidence level of 95%.

Table SM.11: **Predictive Regression on the Variance Risk Premium Using the Price-Based Option Realized Variance.**

Panel A: Regression HAR-*RV*-*JV*-*IV*-*ORV*.

$VRP_{t,t+h} = \beta_0 + \beta_1 RV_t^{(d)} + \beta_2 RV_t^{(w)} + \beta_3 RV_t^{(m)} + \beta_4 JV_t^{(d)} + \beta_5 IV_t + \beta_6 ORV_t^{\Delta^e=0.2} + \beta_7 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_0	-0.0081	**-0.0104**	**-0.0087**	-0.0016	-0.0039	-0.0030	**0.0049**	0.0014	0.0020	-0.0012	**-0.0077**	**-0.0068**
	(0.0044)	(0.0047)	(0.0041)	(0.0017)	(0.0022)	(0.0022)	(0.0019)	(0.0022)	(0.0025)	(0.0029)	(0.0023)	(0.0026)
β_1	0.0835	0.0134	0.0619	0.1004	0.0327	0.0482	0.0551	-0.0502	-0.0557	**0.2679**	0.0714	0.0546
	(0.1315)	(0.1288)	(0.1410)	(0.1232)	(0.1048)	(0.1240)	(0.0832)	(0.0745)	(0.1002)	(0.0516)	(0.0432)	(0.0634)
β_2	**0.4450**	0.3958	**0.4356**	**0.2910**	0.2435	**0.2683**	**0.2787**	**0.2049**	**0.2306**	**0.1985**	0.0608	0.1060
	(0.2186)	(0.2100)	(0.2212)	(0.1395)	(0.1330)	(0.1330)	(0.1253)	(0.1010)	(0.1121)	(0.0706)	(0.0518)	(0.0650)
β_3	-0.5058	-0.4695	-0.5095	-0.1865	-0.1515	-0.1953	0.0065	0.0610	-0.0122	**0.3200**	**0.4217**	**0.2841**
	(0.2648)	(0.2570)	(0.2640)	(0.1711)	(0.1639)	(0.1752)	(0.1887)	(0.1617)	(0.1934)	(0.1000)	(0.0830)	(0.1012)
β_4	**-1.8875**	**-1.6966**	-1.8145	**-1.3787**	**-1.1945**	-1.2022	-0.3013	-0.0147	0.0734	**-0.6107**	-0.0762	0.1102
	(0.9342)	(0.8156)	(0.9808)	(0.6690)	(0.5668)	(0.6420)	(0.5136)	(0.4217)	(0.5393)	(0.3080)	(0.2820)	(0.3207)
β_5	-0.3246	-0.3876	-0.3202	**-0.6049**	**-0.6656**	**-0.5941**	**-0.8882**	**-0.9827**	**-0.8654**	**-1.1179**	**-1.2941**	**-1.0739**
	(0.2244)	(0.2038)	(0.2212)	(0.1075)	(0.1015)	(0.1103)	(0.1007)	(0.0922)	(0.1048)	(0.1109)	(0.0915)	(0.1078)
β_6 / β_7	–	**0.0018**	0.0002	–	**0.0018**	0.0004	–	**0.0027**	0.0009	–	**0.0051**	**0.0017**
	–	(0.0005)	(0.0003)	–	(0.0008)	(0.0003)	–	(0.0011)	(0.0005)	–	(0.0005)	(0.0003)
Adjusted R^2	0.2379	0.2547	0.2380	0.3959	0.4193	0.3994	0.4631	0.5102	0.4770	0.3444	0.5081	0.3963

Panel B: Regression LHAR-*RV*-*IV*-*ORV*.

$VRP_{t,t+h} = \beta_0 + \beta_1 RV_t^{(d)} + \beta_2 RV_t^{(w)} + \beta_3 RV_t^{(m)} + \beta_4 r_t^{(d)} + \beta_5 r_t^{(w)} + \beta_6 r_t^{(m)} + \beta_7 IV_t + \beta_8 ORV_t^{\Delta^e=0.2} + \beta_9 ORV_t^{\Delta^e=0.8} + \varepsilon_{t,t+h}$

h	One Day			One Week			One Month			One Year		
β_0	**-0.0103**	**-0.0115**	**-0.0116**	-0.0034	-0.0057	-0.0052	**0.0048**	0.0014	0.0025	-0.0018	**-0.0078**	**-0.0058**
	(0.0051)	(0.0049)	(0.0053)	(0.0025)	(0.0031)	(0.0031)	(0.0021)	(0.0023)	(0.0024)	(0.0028)	(0.0024)	(0.0026)
β_1	-0.1817	-0.2027	-0.2122	-0.0962	-0.1390	-0.1391	0.0010	-0.0605	-0.0536	**0.1644**	0.0561	0.0686
	(0.2040)	(0.1953)	(0.1958)	(0.0887)	(0.0755)	(0.0757)	(0.0461)	(0.0337)	(0.0377)	(0.0442)	(0.0317)	(0.0392)
β_2	**0.4511**	0.4289	0.4292	**0.3139**	**0.2684**	**0.2831**	**0.2987**	**0.2333**	**0.2595**	**0.1475**	0.0325	0.0787
	(0.2300)	(0.2313)	(0.2220)	(0.1337)	(0.1345)	(0.1248)	(0.1190)	(0.1012)	(0.1056)	(0.0682)	(0.0531)	(0.0637)
β_3	**-0.3455**	**-0.3513**	**-0.3623**	-0.1054	-0.1172	-0.1290	0.0946	0.0776	0.0645	**0.4663**	**0.4365**	**0.4136**
	(0.1669)	(0.1682)	(0.1718)	(0.1736)	(0.1723)	(0.1734)	(0.1701)	(0.1668)	(0.1743)	(0.0864)	(0.0778)	(0.0854)
β_4	**-0.9695**	-0.7857	**-1.0031**	-0.2233	0.1526	-0.2706	**-0.4920**	0.0483	**-0.5522**	**-0.8918**	0.0586	**-0.9974**
	(0.4453)	(0.4902)	(0.4588)	(0.1397)	(0.1915)	(0.1515)	(0.2420)	(0.1483)	(0.2564)	(0.1279)	(0.1640)	(0.1260)
β_5	-1.3714	-1.2355	-1.2562	-1.3151	-1.0372	-1.1531	-1.1506	-0.7512	-0.9442	-0.7891	-0.0865	-0.4273
	(1.8213)	(1.8029)	(1.7962)	(1.2377)	(1.1466)	(1.1853)	(0.8879)	(0.9091)	(0.8968)	(0.6845)	(0.6645)	(0.6720)
β_6	-3.3410	-2.7879	-2.7256	-0.8412	0.2905	0.0245	0.4526	2.0790	1.5555	**-5.4470**	-2.5857	-3.5135
	(2.2514)	(2.2688)	(2.1877)	(2.5157)	(2.2486)	(2.3443)	(2.1038)	(1.9148)	(1.9152)	(2.5547)	(2.0374)	(2.2559)
β_7	**-0.4966**	**-0.4911**	**-0.4875**	**-0.6538**	**-0.6426**	**-0.6411**	**-1.0164**	**-1.0003**	**-1.0002**	**-1.3332**	**-1.3048**	**-1.3048**
	(0.1424)	(0.1374)	(0.1393)	(0.1437)	(0.1422)	(0.1449)	(0.1056)	(0.1035)	(0.1076)	(0.1020)	(0.0933)	(0.0980)
β_8 / β_9	–	0.0010	0.0005	–	**0.0020**	0.0007	–	**0.0028**	0.0009	–	**0.0049**	**0.0015**
	–	(0.0006)	(0.0003)	–	(0.0009)	(0.0004)	–	(0.0012)	(0.0005)	–	(0.0006)	(0.0003)
Adjusted R^2	0.2442	0.2474	0.2474	0.3780	0.3998	0.3883	0.4748	0.5122	0.4887	0.3952	0.5110	0.4382

Various linear regressions that include ORV—similar to those of Table 4—are performed, except that the left-hand side (i.e., dependent variable) is replaced by the variance risk premium over a period of length h, $VRP_{t,t+h}$. Columns 2, 5, 8 and 11 are associated with β_6 (Panel A) and β_8 (Panel B), i.e., the OTM call ORVs, and Columns 3, 6, 9 and 12 are associated with β_7 (Panel A) and β_9 (Panel B), i.e., the OTM put ORVs. We compute Newey-West standard errors; these are in parentheses in the above table. Values in bold are statistically significant at a confidence level of 95%.

by Tonelli's theorem. Then, by taking the expectation on both side of the stochastic differential equation, we can find the following ODE:

$$y' = \kappa(\theta - y)\,dt + \lambda_{V,0}\mu_V$$

where $y = \mathrm{E}_t^{\mathbb{P}}[V_{s^-}]$. The solution of the ordinary differential equation is therefore given by

$$\mathrm{E}_t^{\mathbb{P}}[V_{s^-}] = A + \exp(-\kappa(s-t))(V_t - A)\,,\; A = \frac{\lambda_{V,0}\mu_V + \theta\kappa}{\kappa}.$$

Lastly,

$$\mathrm{E}_t^{\mathbb{P}}\left[\int_t^{t+h} V_{s^-}\,ds\right] = Ah + \frac{1}{\kappa}(1 - \exp(-\kappa h))(V_t - A)\,.$$

I.2 Variance

The integrated variance risk premium is given by the following expression:

$$IVRP_{t,t+h} = \frac{1}{h}\left(\mathrm{E}_t^{\mathbb{P}}[\Delta QV_{t,t+h}] - \mathrm{E}_t^{\mathbb{Q}}[\Delta QV_{t,t+h}]\right).$$

Again, we can divide the integrated variance risk premium in two components: diffusive and jump risk premiums,

$$IVRP_{t,t+h} = IVRP_{t,t+h}^{\text{Diffusion}} + IVRP_{t,t+h}^{\text{Jump}}$$

where

$$IVRP_{t,t+h}^{\text{Diffusion}} = \frac{1}{h}\left(\mathrm{E}_t^{\mathbb{P}}\left[\int_t^{t+h} V_{s^-}\,ds\right] - \mathrm{E}_t^{\mathbb{Q}}\left[\int_t^{t+h} V_{s^-}\,ds\right]\right)$$

and

$$\begin{aligned} IVRP_{t,t+h}^{\text{Jump}} = & \frac{1}{h}\left(\lambda_{Y,0}h + \lambda_{Y,1}\mathrm{E}_t^{\mathbb{P}}\left[\int_t^{t+h} V_{s^-}\,ds\right]\right)\left((\mu_Y)^2 + (\sigma_Y)^2\right) \\ & - \frac{1}{h}\left(\lambda_{Y,0}^{\mathbb{Q}}h + \lambda_{Y,1}^{\mathbb{Q}}\mathrm{E}_t^{\mathbb{Q}}\left[\int_t^{t+h} V_{s^-}\,ds\right]\right)\left(\left(\mu_Y^{\mathbb{Q}}\right)^2 + \left(\sigma_Y^{\mathbb{Q}}\right)^2\right). \end{aligned}$$

J Option Quadratic Variation

Lemma 1. *Let $O_t \equiv O_t(Y_t, V_t)$ be an option price at time t. In the proposed framework, we can characterize the option price variation for a European option as*

$$\begin{aligned} [O,O]_t = & \int_0^t \left(\left(\frac{\partial O_u}{\partial y}(Y_{u^-}, V_{u^-})\right)^2 + 2\sigma\rho\frac{\partial O_u}{\partial y}(Y_{u^-}, V_{u^-})\frac{\partial O_u}{\partial v}(Y_{u^-}, V_{u^-}) + \sigma^2\left(\frac{\partial O_u}{\partial v}(Y_{u^-}, V_{u^-})\right)^2\right)V_{u^-}du \\ & + \sum_{0<u\le t}\{O_u(Y_u, V_u) - O_u(Y_{u^-}, V_{u^-})\}^2 \end{aligned}$$

because the option price is a smooth function of Y and V.

Proof of Lemma 1. The option price is a function of time, the log-equity price and the instantaneous variance: $O_t = O_t(Y_t, V_t)$. Assuming that O is twice continuously differentiable, Itō's lemma implies that

$$
\begin{aligned}
&O_t(Y_t, V_t) - O_0(Y_0, V_0) \\
&= \int_0^t \frac{\partial O_u}{\partial u}(Y_{u^-}, V_{u^-})\, du + \int_0^t \frac{\partial O_u}{\partial y}(Y_{u^-}, V_{u^-})\, dY_u + \int_0^t \frac{\partial O_u}{\partial v}(Y_{u^-}, V)\, dV_u \\
&\quad + \frac{1}{2}\int_0^t \frac{\partial^2 O_u}{\partial y^2}(Y_{u^-}, V_{u^-})\, d[Y,Y]_u^{\mathrm{c}} + \frac{1}{2}\int_0^t \frac{\partial^2 O_u}{\partial v^2}(Y_{u^-}, V_{u^-})\, d[V,V]_u^{\mathrm{c}} + \int_0^t \frac{\partial^2 O_u}{\partial v \partial y}(Y_{u^-}, V_{u^-})\, d[Y,V]_u^{\mathrm{c}} \\
&\quad + \sum_{0<u\le t}\left\{O_u(Y_u, V_u) - O_u(Y_{u^-}, V_{u^-})\right\} - \sum_{0<u\le t}\left\{\frac{\partial O_u}{\partial y}(Y_{u^-}, V_{u^-})(Y_u - Y_{u^-})\right\} \\
&\quad - \sum_{0<u\le t}\left\{\frac{\partial O_u}{\partial v}(Y_{u^-}, V_{u^-})(V_u - V_{u^-})\right\}.
\end{aligned}
$$

Replacing Equation (4) in the latter expression leads to

$$
\begin{aligned}
&O_t(Y_t, V_t) - O_0(Y_0, V_0) \\
&= \int_0^t \frac{\partial O_u}{\partial u}(Y_{u^-}, V_{u^-})\, du + \int_0^t \frac{\partial O_u}{\partial y}(Y_{u^-}, V_{u^-})\, \alpha_{u^-} du + \int_0^t \frac{\partial O_u}{\partial y}(Y_{u^-}, V_{u^-})\, \rho\sqrt{V_{u^-}} dW_{V,u} \\
&\quad + \int_0^t \frac{\partial O_u}{\partial y}(Y_{u^-}, V_{u^-})\, \sqrt{1-\rho^2}\sqrt{V_{u^-}} dW_{\perp,u} + \int_0^t \frac{\partial O_u}{\partial y}(Y_{u^-}, V_{u^-})\, dJ_{Y,u} \\
&\quad + \int_0^t \frac{\partial O_u}{\partial v}(Y_{u^-}, V_{u^-})\, \kappa(\theta - V_{u^-})\, du + \int_0^t \frac{\partial O_u}{\partial v}(Y_{u^-}, V_{u^-})\, \sigma\sqrt{V_{u^-}} dW_{V,u} + \int_0^t \frac{\partial O_u}{\partial v}(Y_{u^-}, V_{u^-})\, dJ_{V,u} \\
&\quad + \frac{1}{2}\int_0^t \frac{\partial^2 O_u}{\partial y^2}(Y_{u^-}, V_{u^-})\, V_{u^-} du + \frac{1}{2}\int_0^t \frac{\partial^2 O_u}{\partial v^2}(Y_{u^-}, V_{u^-})\, \sigma^2 V_{u^-} du + \int_0^t \frac{\partial^2 O_u}{\partial v \partial y}(Y_{u^-}, V_{u^-})\, \sigma\rho V_{u^-} du \\
&\quad + \sum_{0<u\le t}\left\{O_u(Y_u, V_u) - O_u(Y_{u^-}, V_{u^-})\right\} - \int_0^t \frac{\partial O_u}{\partial y}(Y_{u^-}, V_{u^-})\, dJ_{Y,u} - \int_0^t \frac{\partial O_u}{\partial v}(Y_{u^-}, V_{u^-})\, dJ_{V,u}.
\end{aligned}
$$

Then,

$$
\begin{aligned}
&O_t(Y_t, V_t) - O_0(Y_0, V_0) \\
&= \int_0^t \left\{\begin{array}{l} \frac{\partial O_u}{\partial u}(Y_{u^-}, V_{u^-}) + \alpha_{u^-}\frac{\partial O_u}{\partial y}(Y_{u^-}, V_{u^-}) + \frac{\partial O_u}{\partial v}(Y_{u^-}, V_{u^-})\, \kappa(\theta - V_{u^-}) \\ + \left(\frac{1}{2}\frac{\partial^2 O_u}{\partial y^2}(Y_{u^-}, V_{u^-}) + \frac{1}{2}\frac{\partial^2 O_u}{\partial v^2}(Y_{u^-}, V_{u^-})\, \sigma^2 + \frac{\partial^2 O_u}{\partial v \partial y}(Y_{u^-}, V_{u^-})\, \sigma\rho\right) V_{u^-} \end{array}\right\} du \\
&\quad + \int_0^t \left(\rho\frac{\partial O_u}{\partial y}(Y_{u^-}, V_{u^-}) + \sigma\frac{\partial O_u}{\partial v}(Y_{u^-}, V_{u^-})\right)\sqrt{V_{u^-}} dW_{V,u} + \int_0^t \frac{\partial O_u}{\partial y}(Y_{u^-}, V_{u^-})\, \sqrt{1-\rho^2}\sqrt{V_{u^-}} dW_{\perp,u} \\
&\quad + \sum_{0<u\le t}\left\{O_u(Y_u, V_u) - O_u(Y_{u^-}, V_{u^-})\right\}.
\end{aligned}
$$

Finally, the quadratic variation is given by the following expression:

$$
\begin{aligned}
[O,O]_t = &\int_0^t \left(\rho\frac{\partial O_u}{\partial y}(Y_{u^-}, V_{u^-}) + \sigma\frac{\partial O_u}{\partial v}(Y_{u^-}, V_{u^-})\right)^2 V_{u^-} du + \int_0^t \left(\frac{\partial O_u}{\partial y}(Y_{u^-}, V_{u^-})\right)^2 \left(1-\rho^2\right) V_{u^-} du \\
&+ \sum_{0<u\le t}\left\{O_u(Y_u, V_u) - O_u(Y_{u^-}, V_{u^-})\right\}^2
\end{aligned}
$$

$$
= \int_0^t \left(\left(\frac{\partial O_u}{\partial y} (Y_{u^-}, V_{u^-}) \right)^2 + 2\sigma\rho \frac{\partial O_u}{\partial y} (Y_{u^-}, V_{u^-}) \frac{\partial O_u}{\partial v} (Y_{u^-}, V_{u^-}) + \sigma^2 \left(\frac{\partial O_u}{\partial v} (Y_{u^-}, V_{u^-}) \right)^2 \right) V_{u^-} du
$$

$$
+ \sum_{0<u\leq t} \{ O_u (Y_u, V_u) - O_u (Y_{u^-}, V_{u^-}) \}^2 .
$$

□

K Option Pricing Implications: Additional Results

K.1 In-Sample Assessment

The leftmost columns of Panel A (Table SM.12) show the IVRMSE and the RIVRMSE for the panel of 21,400 options employed in the estimation. We observe that the fit provided by the parameter set without *ORV*s is more accurate on average than the one that includes these variances—an RIVRMSE of 24.56 with *ORV* vs. 23.45 without. This result stems from the fact that maximization of the likelihood in the former case backs parameters that better fit *IV*s. However, the average fit is not that different, so the addition of *ORV*s does not substantially deteriorate the overall fit of option prices.

Rightmost columns of Panel A (Table SM.12) exhibit the ORVRMSE and the RORVRMSE by moneyness and maturity. As expected, the average fit of these quantities is lower when *ORV*s are included in the information set. Nonetheless, it is interesting to observe that the overall RORVRMSE is about 3 times lower when *ORV* is included and that there are significant differences across contracts. Whereas error differences for call options are almost fourfold, those differences for put options are about twofold. The differences across contracts suggest that *ORV*s bring more information from OTM calls than from OTM puts, which is in line with the importance of the coefficient associated with call contracts in explaining future index variation as discussed in Subsection 3.2.1.

We continue with the in-sample analysis by looking at one-day-ahead predicted option prices from both sets of parameters. To analyze the fit of implied volatilities, we enlarge our estimation sample of 21,400 options to include all ATM and OTM options available for the S&P 500 index in OptionMetrics between July 2004 and December 2012.[8] We restrict our analysis to maturities of at least one week and at most one year. As before, observations violating no-arbitrage restrictions are excluded. Our new sample is composed of a total of 401,081 contracts. Option prices are converted to implied volatilities with the Black and Scholes's (1973) pricing formula. To compute model-predicted implied volatilities on day t, we calculate one-day-ahead expectations of model variables for day t using the filter's predictive distribution resulting from day $t - 1$. Using observed and predicted implied volatilities, we compute the IVRMSE and the RIVRMSE according to maturity, moneyness, and year, which provides a better picture of the fit associated with each parameter set.

The overall IVRMSE and RIVRMSE (leftmost columns of Panel B in Table SM.12) are very close on average for both parameter sets, with lower (higher) values of RIVRMSE (IVRMSE) observed when *ORV*s are included (32.79 against 32.87 for RIVRMSE and 7.08 against 7.37 for IVRMSE). We use the Diebold and Mariano (1995, DM henceforth) test to see if the apparent predictive superiority of *ORV* based forecasts is not particular to this sample. Using both RIVRMSE time series, we compare their

[8] We continue using ATM and OTM options to keep a comparable sample with that employed in our previous analyses.

Table SM.12: **In-Sample Performance (2004–2012).**

Panel A: In-Sample Option Pricing and Option Realized Variance Performance.

	IVRMSE		RIVRMSE		ORVRMSE		RORVRMSE	
	With *ORV*	Without *ORV*	With *ORV*	Without *ORV*	With *ORV*	Without *ORV*	With *ORV*	Without *ORV*
DTM = 30, Δ^e = 0.20	4.090	3.597	19.126	17.623	416.859	2704.734	50.169	152.952
DTM = 30, Δ^e = 0.35	5.580	5.047	25.048	23.036	1252.118	3061.004	36.292	103.587
DTM = 30, Δ^e = 0.50	6.701	6.330	28.509	27.928	2742.551	2957.215	41.809	90.903
DTM = 30, Δ^e = 0.65	7.417	6.924	27.267	25.944	3533.420	3534.923	55.261	106.114
DTM = 30, Δ^e = 0.80	8.112	7.495	25.029	23.165	1987.343	1939.840	61.432	95.191
DTM = 90, Δ^e = 0.20	4.089	3.781	21.965	26.015	1014.670	3358.813	61.024	272.479
DTM = 90, Δ^e = 0.35	5.359	4.518	23.370	22.900	3593.646	5253.486	40.777	145.631
DTM = 90, Δ^e = 0.50	6.474	5.505	25.235	23.158	2918.781	3978.663	40.765	101.021
DTM = 90, Δ^e = 0.65	7.367	6.224	25.165	22.396	3561.931	3699.306	44.922	122.239
DTM = 90, Δ^e = 0.80	7.963	6.647	23.572	20.686	3625.592	3587.348	53.565	137.197
All	6.470	5.749	24.557	23.447	2720.168	3506.473	49.341	142.173

Panel B: One-Day-Ahead In-Sample Performance.

	IVRMSE		RIVRMSE		ORVRMSE		RORVRMSE	
	With *ORV*	Without *ORV*	With *ORV*	Without *ORV*	With *ORV*	Without *ORV*	With *ORV*	Without *ORV*
DTM < 60	8.713	8.289	26.784	25.388	3684.062	3702.479	233.084	286.972
60 ≤ DTM < 120	7.569	6.665	26.202	24.238	3493.978	3527.906	230.462	337.403
120 ≤ DTM < 180	7.344	6.127	26.072	25.078	4357.361	4345.883	203.418	335.046
180 ≤ DTM	7.381	6.174	51.762	55.280	4047.372	4004.356	180.707	270.458
Δ^e < 0.20	4.410	4.405	23.251	28.561	1540.988	1601.840	358.691	512.514
0.20 ≤ Δ^e < 0.35	5.301	4.951	29.545	31.111	2126.193	2160.627	216.983	302.681
0.35 ≤ Δ^e < 0.50	6.188	5.726	34.516	35.511	4220.592	4228.749	152.915	198.512
0.50 ≤ Δ^e < 0.65	8.782	7.960	41.422	41.242	6198.602	6221.710	188.624	239.686
0.65 ≤ Δ^e < 0.80	9.366	8.422	37.413	36.827	3870.019	3851.348	200.039	267.024
0.80 ≤ Δ^e	10.022	9.093	31.737	29.622	2018.164	2007.785	210.692	285.870
2004	2.434	2.665	25.727	28.524	475.761	584.667	204.642	268.174
2005	1.898	2.970	28.440	33.708	517.591	661.165	270.869	348.781
2006	2.200	3.114	26.579	32.049	700.969	875.655	316.429	406.996
2007	4.886	4.829	29.292	31.867	2549.412	2675.179	220.312	290.461
2008	11.012	10.124	31.172	30.561	8184.322	8071.828	77.094	92.507
2009	11.799	10.403	38.471	35.744	2519.262	2529.008	79.206	93.686
2010	8.050	7.127	34.994	33.489	4588.308	4634.097	188.202	284.603
2011	9.512	8.650	35.335	34.376	2827.878	2899.712	193.817	274.963
2012	5.696	5.195	31.679	31.393	1287.920	1417.218	314.548	437.402
All	8.084	7.367	32.788	32.866	3759.446	3767.826	221.94	303.31

The leftmost columns of Panel A show the implied volatility RMSE (IVRMSE) and the relative implied volatility RMSE (RIVRMSE) for the panel of 21,400 options employed in the estimation:

$$\text{IVRMSE} = \sqrt{\left(\frac{1}{\sum_k O_k}\right)\sum_k \sum_{i=1}^{O_k}\left(IV_{k\tau,i}(Y_{\tau k},\hat{V}_{\tau k}) - \sigma^{\text{BS}}_{k\tau,i}\right)^2}, \quad \text{RIVRMSE} = \sqrt{\left(\frac{1}{\sum_k O_k}\right)\sum_k \sum_{i=1}^{O_k}\left(\frac{IV_{k\tau,i}(Y_{\tau k},\hat{V}_{\tau k}) - \sigma^{\text{BS}}_{k\tau,i}}{\sigma^{\text{BS}}_{k\tau,i}}\right)^2},$$

where IV is the one-day-ahead model implied volatility, $\hat{V}_{\tau k}$ is the one-day-ahead instantaneous variance on day k, and O_k represents the number of options in a subset of all the options available on day k. The IVRMSE and RIVRMSE are given by moneyness and maturity. The rightmost columns of Panel A exhibit the option realized variance RMSE (ORVRMSE) and the relative option realized variance RMSE (RORVRMSE) by moneyness and maturity:

$$\text{ORVRMSE} = \sqrt{\left(\frac{1}{\sum_k O_k}\right)\sum_k \sum_{i=1}^{O_k}\left(\Delta OQV_{(k-1)\tau,k\tau,i} - ORV_{(k-1)\tau,k\tau,i}\right)^2}, \quad \text{RORVRMSE} = \sqrt{\left(\frac{1}{\sum_k O_k}\right)\sum_k \sum_{i=1}^{O_k}\left(\frac{\Delta OQV_{(k-1)\tau,k\tau,i} - ORV_{(k-1)\tau,k\tau,i}}{ORV_{(k-1)\tau,k\tau,i}}\right)^2},$$

where ΔOQV is the option quadratic variation increment computed from the model and ORV is the observed option realized variance. To analyze the fit of implied volatilities and ORV, we enlarge our estimation sample of 21,400 options to include all ATM and OTM options available for the S&P 500 index between July 2004 and December 2012 in Panel B. Our new sample is composed of a total of 401,081 contracts for the implied volatilities (leftmost columns). For the option realized variance (rightmost columns), 282,534 contracts were included in our analysis for which ORVs were available in the TickData database.

forecasting accuracy and test for:[9]

$$H_0 : \mathrm{E}[d_t] = 0,\ \forall t \qquad H_1 : \mathrm{E}[d_t] > 0,\ \forall t$$

where $d_t = \text{RIVRMSE}_{\text{without},t} - \text{RIVRMSE}_{\text{with},t}$ is the time-t loss differential between the forecast produced without *ORV* and the one including it. The DM test statistic is 5.32 and is significant at a 95% level, confirming the existence of a differential between the two forecasts and that the one based on *ORV* information produces more accurate results on average. A closer scrutiny of the data reveals that including *ORV*s is especially helpful in the pricing of options with long maturities, OTM calls, and options for the years before 2008.

Lastly, we carry out a similar exercise with option realized variances and analyze one-day-ahead forecasts for both parameter sets. For this exercise, we restrict our enlarged sample to those options for which *ORV* data is available in the TickData database. Out of 401,081 options, 282,534 contracts were included in our analysis. As reported in the rightmost columns of Panel B, Table SM.12, we observe again that, across different option characteristics and years, ORVRMSE and RORVRMSE are lower for the parameter set obtained with *ORV*s. Similar to the RIVRMSE case, we apply the DM test to both RORVRMSE series and obtain a value of 13.63, statistically confirming the significant differences between the two sets of parameters.[10]

[9]The lag in the Diebold and Mariano (1995) is selected as the first partial autocorrelation that is within confidence bounds. The estimated lag in this exercise is 2.

[10]We repeated our analysis of RIVRMSE with the sample of 282,534 contracts and obtained similar results.