AI1110 ASSIGNMENT-1
PROBABILITY AND RANDOM VARIABLES

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***Abstract*—This document provides the solution to question 1 in Chapter 15 of the 10th grade NCERT textbook, Exercise 2.1.**

QUESTION: Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on
(i) the same day?
(ii) consecutive days?
(iii) different days?

Solution:
To analyze the probabilities, we define the following random variables:

Parameter	Value	Description
X	{2,3,4,5,6}	Day of the week shyam visits shop
Y	{2,3,4,5,6}	Day of the week ekta visits shop

PMF of X and Y:

$$P(X = i) = P(Y = i) = \frac{1}{5}, \quad i = 2, 3, 4, 5, 6 \tag{1}$$

CDF of X and Y:

$$F_X(k) = F_Y(k) = \sum_{i=2}^{k} P(X = i) = \sum_{i=2}^{k} P(Y = i), \quad k = 2, 3, 4, 5, 6 \tag{2}$$

The random variable Z represents the difference between the days Shyam and Ekta visit the shop. Probability Mass Function (PMF) of Z:

$$P(Z = k) = P(X - Y = k) = \sum_{i=2}^{6} P(X = i, Y = i - k), \quad k = -4, -3, \tag{3}$$

Cumulative Distribution Function (CDF) of Z:

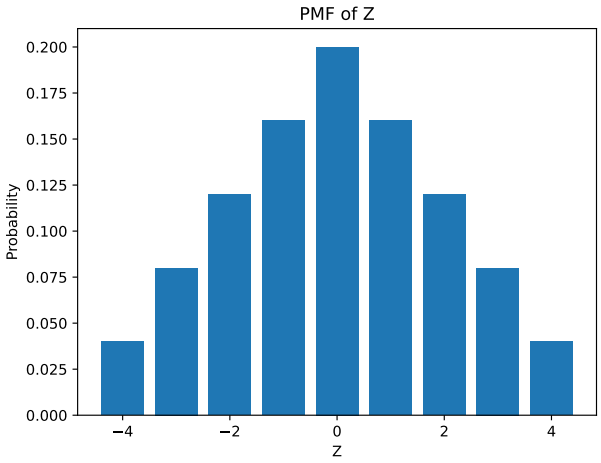
$$F_Z(k) = \sum_{i=-\infty}^{k} P(Z = i), \quad k = -4, -3, -2, -1, 0, 1, 2, 3, 4 \tag{4}$$

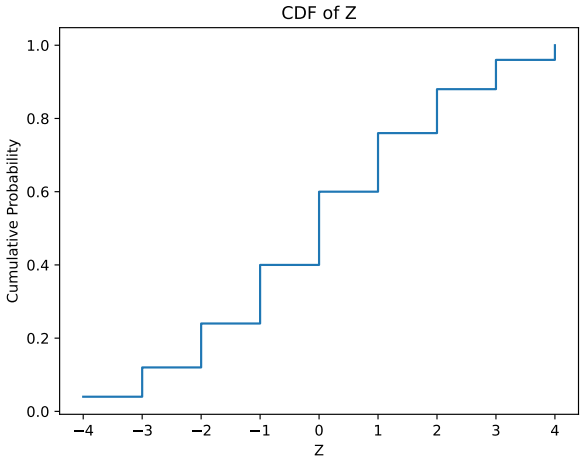
Distribution of Z using Z-transform

since x and y are independent
by knowing pmf of x and y
we can find the pmf of Z by using z transform

$$\Pr(Z = z) = \sum_{x} \sum_{y} \Pr(X = x, Y = y) \cdot z^{-(x-y)} \tag{5}$$

Now, using conditioning and unconditioning, we can solve the problem:





- The probability that both will visit the shop on the same day is given by:

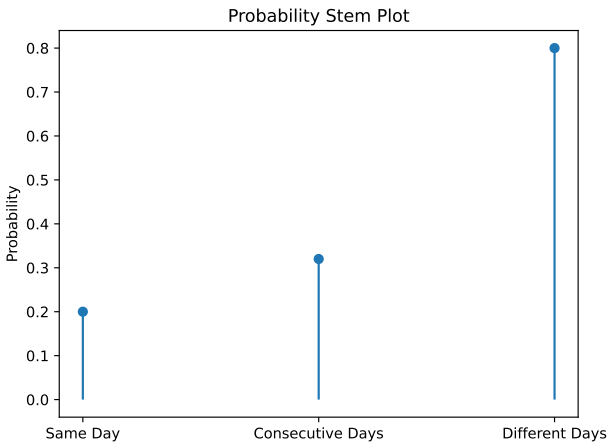
$$\Pr(X = Y) = \begin{cases} \frac{1}{5} & \text{if } X = Y \end{cases}$$

- The probability that both will visit the shop on consecutive days is given by:

$$\Pr(|X - Y| = 1) = \begin{cases} \frac{8}{25} & \text{if } |X - Y| = 1 \end{cases}$$

- The probability that both will visit the shop on different days is given by:

$$\Pr(X \neq Y) = \begin{cases} \frac{4}{5} & \text{if } X \neq Y \end{cases}$$


Probability Stem Plot
Probability
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Same Day
Consecutive Days
Different Days