AI1110 ASSIGNMENT-1
PROBABILITY AND RANDOM VARIABLES

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NCERT(10.15.2.1)

QUESTION: Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on
(i) the same day?
(ii) consecutive days?
(iii) different days?
Solution:

Let X be the day on which Shyam visits the shop and Y be the day on which Ekta visits the shop. Here, $X, Y \in \{2,3,4,5,6\}$ since they are visiting the shop between Tuesday (day 2) and Saturday (day 6). Each day is equally likely, so $P(X=i) = P(Y=i) = \frac{1}{5}$ for $i = 2,3,4,5,6$

1) The probability that both will visit the shop on the same day is given by:

$$P(X-Y=0) = \sum_{i=2}^{6} P(X=i, Y=i) \tag{1}$$
$$= \sum_{i=2}^{6} P(X=i)P(Y=i) \tag{2}$$
$$= \sum_{i=2}^{6} \left(\frac{1}{5}\right)^2 \tag{3}$$
$$= \frac{1}{5} \tag{4}$$

2) The probability that both will visit the shop on consecutive days is given by:

$$P(|X-Y|=1) = P((X=Y+1) \cup (Y=X+1)) \tag{5}$$
$$= P(X=Y+1) + P(Y=X+1) \tag{6}$$
$$= \sum_{i=2}^{5} P(X=i, Y=i+1) + \sum_{i=3}^{6} P(X=i, Y=i-1) \tag{7}$$
$$= \sum_{i=2}^{5} P(X=i)P(Y=i+1) + \sum_{i=3}^{6} P(X=i)P(Y=i-1) \tag{8}$$
$$= \sum_{i=2}^{5} \frac{1}{5} \times \frac{1}{5} + \sum_{i=3}^{6} \frac{1}{5} \times \frac{1}{5} \tag{9}$$
$$= \frac{8}{25} \tag{10}$$

3) The probability that both will visit the shop on different days and differ by 2:

$$P(|X-Y|=2) = P((X=Y+2) \cup (Y=X+2)) \tag{11}$$
$$= P(X=Y+2) + P(Y=X+2) \tag{12}$$
$$= \sum_{i=2}^{4} P(X=i, Y=i+2) + \sum_{i=4}^{6} P(X=i, Y=i-2) \tag{13}$$
$$= \sum_{i=2}^{4} P(X=i)P(Y=i+2) + \sum_{i=4}^{6} P(X=i)P(Y=i-2) \tag{14}$$
$$= \sum_{i=2}^{4} \frac{1}{5} \times \frac{1}{5} + \sum_{i=4}^{6} \frac{1}{5} \times \frac{1}{5} \tag{15}$$
$$= \frac{6}{25} \tag{16}$$

4) The probability that both will visit the shop on different days differ by 3:

$$P(|X-Y|=3) = P((X=Y+3) \cup (Y=X+3)) \tag{17}$$
$$= P(X=Y+3) + P(Y=X+3) \tag{18}$$
$$= \sum_{i=2}^{3} P(X=i, Y=i+3) + \sum_{i=5}^{6} P(X=i, Y=i-3) \tag{19}$$
$$= \sum_{i=2}^{3} P(X=i)P(Y=i+3) + \sum_{i=5}^{6} P(X=i)P(Y=i-3) \tag{20}$$
$$= \sum_{i=2}^{3} \frac{1}{5} \times \frac{1}{5} + \sum_{i=5}^{6} \frac{1}{5} \times \frac{1}{5} \tag{21}$$
$$= \frac{4}{25} \tag{22}$$

5) The probability that both will visit the shop on different days differ by 4 :

$$P(|X-Y|=4) = P((X=Y+4) \cup (Y=X+4)) \tag{23}$$
$$= P(X=Y+4) + P(Y=X+4) \tag{24}$$
$$= \frac{1}{5} \times \frac{1}{5} + \frac{1}{5} \times \frac{1}{5} \tag{25}$$
$$= \frac{2}{25} \tag{26}$$

6) The probability that both will visit the shop on different days is given by:

$$P(X \neq Y) = p(|X - Y| = 1) + p(|X - Y| = 2) + p(|X - Y| = 3) + p(|X - Y| = 4) \tag{27}$$

$$= \frac{8}{25} + \frac{6}{25} + \frac{4}{25} + \frac{2}{25} \tag{28}$$

$$= \frac{4}{5} \tag{29}$$