AI1110 ASSIGNMENT-1 PROBABILITY AND RANDOM VARIABLES

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NCERT(10.15.2.1)

QUESTION: Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on
(i) the same day?
(ii) consecutive days?
(iii) different days?
Solution:
To analyze the probabilities, we define the following random variables:

Parameter	Value	Description
X	$\{2,3,4,5,6\}$	Day of the week Shyam visits the shop
Y	$\{2,3,4,5,6\}$	Day of the week Ekta visits the shop

since they are visiting the shop between Tuesday (day 2) and Saturday (day 6). Each day is equally likely, so $P(X = i) = P(Y = i) = \frac{1}{5}$ for $i = 2, 3, 4, 5, 6$

The possible values of $X - Y$ can be computed by subtracting the values of Y from X. The resulting values can be expressed as $-4, -3, -2, -1, 0, 1, 2, 3, 4$.

let's first find the distribution of $X - Y$:

$$Here, k \in \{-1, -2, -3, -4\} \tag{1}$$

$$P(X - Y = k) = \sum_{i=2}^{6+k} P(X = i + k, Y = i) \tag{2}$$
$$= \sum_{i=2}^{6+k} P(X = i + k)P(Y = i) \tag{3}$$
$$= \sum_{i=2}^{6+k} \left(\frac{1}{5}\right)\left(\frac{1}{5}\right) \tag{4}$$
$$= \frac{1}{25} \sum_{i=2}^{6+k} 1 \tag{5}$$
$$= \frac{1}{25} \times 5 + k \tag{6}$$
$$= \frac{5 + k}{25} \tag{7}$$

1) To find the Cumulative Distribution Function (CDF) of $Z = X - Y$:

$$F_Z(k) = P(Z \le k) \tag{8}$$
$$= \begin{cases} P(Z \le 0), & \text{if } k = 0 \\ P(Z \le 1), & \text{if } k = 1 \\ P(Z \le -1), & \text{if } k = -1 \\ P(Z \le 2), & \text{if } k = 2 \\ P(Z \le -2), & \text{if } k = -2 \\ P(Z \le 3), & \text{if } k = 3 \\ P(Z \le -3), & \text{if } k = -3 \\ P(Z \le 4), & \text{if } k = 4 \\ P(Z \le -4), & \text{if } k = -4 \end{cases} \tag{9}$$

Now, let's calculate the CDF for our required value of k: by keeping the values of k in equation 7 .

$$F_Z(-2) = P(Z \le -2) = P(Z = -4) + P(Z = -3) + P(Z = -2) \tag{10}$$
$$= \frac{1}{25} + \frac{2}{25} + \frac{3}{25} = \frac{6}{25} \tag{11}$$
$$F_Z(-1) = P(Z \le -1) = F_Z(-2) + P(Z = -1) \tag{12}$$
$$= \frac{6}{25} + \frac{4}{25} = \frac{10}{25} \tag{13}$$
$$F_Z(0) = P(Z \le 0) = F_Z(-1) + P(Z = 0) \tag{14}$$
$$= \frac{10}{25} + \frac{5}{25} = \frac{15}{25} \tag{15}$$
$$F_Z(1) = P(Z \le 1) = F_Z(0) + P(Z = 1) \tag{16}$$
$$= \frac{15}{25} + \frac{4}{25} = \frac{19}{25} \tag{17}$$
$$\tag{18}$$

2) To find the Probability Mass Function (PMF) of Z, we can use the CDF as follows:

$$p_Z(k) = P(Z = k) \tag{19}$$
$$= F_Z(k) - F_Z(k - 1) \tag{20}$$
$$= \begin{cases} \frac{15}{25} - \frac{10}{25} = \frac{1}{5}, & \text{if } k = 0 \\ \frac{19}{25} - \frac{15}{25} = \frac{4}{25}, & \text{if } k = 1 \\ \frac{10}{25} - \frac{6}{25} = \frac{4}{25}, & \text{if } k = -1 \end{cases} \tag{21}$$
$$\tag{22}$$

Now, using conditioning and unconditioning, we can solve the problem:

- The probability that both will visit the shop on the same day is given by:

$$P(X = Y) = \begin{cases} \frac{1}{5} & \text{if } X = Y \end{cases}$$

- The probability that both will visit the shop on consecutive days is given by:

$$P(|X - Y| = 1) = \begin{cases} \frac{8}{25} & \text{if } |X - Y| = 1 \end{cases}$$

- The probability that both will visit the shop on different days is given by:

$$P(X \neq Y) = \begin{cases} \frac{4}{5} & \text{if } X \neq Y \end{cases}$$