AI1110 ASSIGNMENT-1
PROBABILITY AND RANDOM VARIABLES

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NCERT(10.15.2.1)

QUESTION: Two customers Shyam and Ekta are visiting a particular shop in the same week (Tuesday to Saturday). Each is equally likely to visit the shop on any day as on another day. What is the probability that both will visit the shop on
(i) the same day?
(ii) consecutive days?
(iii) different days?

Solution:
Let X be the day on which Shyam visits the shop and Y be the day on which Ekta visits the shop. Here, $X, Y \in \{2, 3, 4, 5, 6\}$ since they are visiting the shop between Tuesday (day 2) and Saturday (day 6). Each day is equally likely, so $P(X = i) = P(Y = i) = \frac{1}{5}$ for $i = 2, 3, 4, 5, 6$

The possible values of $X - Y$ can be computed by subtracting the values of Y from X. The resulting values can be expressed as $-4, -3, -2, -1, 0, 1, 2, 3, 4$.

Here, $P(|X - Y| = k)$ represents the probability of visiting shop by shyam and ekta differ by k days
let's first find the distribution of $X - Y$:

$$for \tag{1}$$
$$x - y \neq 0 \tag{2}$$
$$Here, k \in \{1, 2, 3, 4\} \tag{3}$$
$$P(|X - Y| = k) = P(X - Y = k \cup X - Y = -k) \tag{4}$$
$$P(|X - Y| = k) = P(X - Y = k) + P(X - Y = -k) \tag{5}$$
$$since, P(X - Y = k) = P(X - Y = -k) \tag{6}$$
$$P(|X - Y| = k) = 2 \times P(X - Y = k) \tag{7}$$
$$P(|X - Y| = k) = 2 \times \sum_{i=2}^{6-k} P(X = i + k, Y = i) \tag{8}$$
$$= 2 \times \sum_{i=2}^{6-k} P(X = i + k)P(Y = i) \tag{9}$$
$$= 2 \times \sum_{i=2}^{6-k} \left(\frac{1}{5}\right)\left(\frac{1}{5}\right) \tag{10}$$
$$= \frac{2}{25} \sum_{i=2}^{6-k} 1 \tag{11}$$
$$= \frac{2}{25} \times 5 - k \tag{12}$$
$$= \frac{10 - 2k}{25} \tag{13}$$

k	$P(|X - Y| = k)$
0	$\frac{1}{5}$
1	$\frac{8}{25}$
2	$\frac{6}{25}$
3	$\frac{4}{25}$
4	$\frac{2}{25}$

Now, using conditioning and unconditioning, we can solve the problem:

1) The probability that both will visit the shop on the same day is given by:
$$P(X = Y) = P(X - Y = 0) \tag{14}$$
$$= \frac{1}{5} \tag{15}$$

2) The probability that they will visit on consecutive days is given by:
$$P(|X - Y| = 1) = P(X - Y = 1 \cup X - Y = -1) \tag{16}$$
$$= P(X - Y = 1) + P(X - Y = -1) \tag{17}$$
$$= \frac{4}{25} + \frac{4}{25} \tag{18}$$
$$= \frac{8}{25} \tag{19}$$

3) The probability that they will visit on different days is given by:
$$P(X \neq Y) = 1 - P(X = Y) \tag{20}$$
$$= 1 - \frac{1}{5} \tag{21}$$
$$= \frac{4}{5} \tag{22}$$

Therefore, the probabilities are:

$$P(X = Y) = \frac{1}{5} \tag{23}$$
$$P(|X - Y| = 1) = \frac{8}{25} \tag{24}$$
$$P(X \neq Y) = \frac{4}{5} \tag{25}$$
$$\tag{26}$$