AER 336S: Assignment 2

1. With $[a, b] = [-1, 1]$, $n = 3$, $x_1 = -1$, $x_2 = 0$, $x_3 = 1$, derive the quadrature weights A_1, A_2, and A_3 using Lagrange polynomials. Apply these weights to approximate the integral $\int_{-1}^{1} e^x dx$. Compare your error with that predicted by the error formula given in class for Simpson's rule.

2. Approximate the integral $\int_{-1}^{1} e^x dx$ using the compound Simpson rule with two equal subintervals. What is the percent error in the approximation relative to the exact integral?

3. Approximate the integral $\int_{2.0}^{2.5} e^x dx$ using the Gauss quadrature rule with three points. For the interval [-1,1], the quadrature weights are $A_1 = A_3 \approx 0.555556$, $A_2 \approx 0.888889$ for three-point Gauss quadrature, and the quadrature points are $x_1 = -x_3 \approx -0.774597$, $x_2 = 0$. What is the percent error in the approximation relative to the exact integral?

4. Write a program to solve tridiagonal linear systems using LU factorization specialized to such systems, making it as efficient as possible. Assume that pivoting is not needed. Apply the program to a linear system with -2 on the diagonal and 1 above and below the diagonal, with a random right-hand vector. Show the 2-norm of the difference between the solution from your method and that from the Matlab backslash command (or equivalent). Determine (by hand) the number of floating point operations required for your method for a general tridiagonal matrix (again assume pivoting is not needed) and its dependence on the size of the system. Plot the matrix graph of the L and U factors that result from your program.