Wireless Networks (2021) 27:5029–5038
https://doi.org/10.1007/s11276-021-02788-8

ORIGINAL PAPER

Sparse channel estimation for OFDM based on IOMP algorithm under unknown sparsity

Zhian Zheng[1] · Jianjian Liu[1] · Junjie Zhu[1] · Haiyan Huang[1]

Accepted: 3 September 2021 / Published online: 16 September 2021

Abstract
Compressive sensing (CS) based channel estimation (CE) improves the utilization of spectrum resources effectively in orthogonal frequency division multiplexing (OFDM) system. Generally, the channel sparsity is assumed to be known in sparse reconstruction algorithms such as Basis Pursuit (BP) and Orthogonal Matching Pursuit (OMP). However, since the uncertainty of the environment, the channel sparsity is usually tricky to obtain. This paper proposes an improved OMP algorithm based on intersect operation (IOMP), which does not need a priori knowledge of channel sparsity. Initially, we use the OMP algorithm twice to estimate two different partial sparse delay path supports at partially different pilot positions, respectively. Then introduce the intersect operation to process these two partial supports. Besides, we discussed an optimized pilot allocation scheme based on discrete stochastic approximation and Iterative Exhaustive Search, both of which are employed to build the framework of CS-based channel estimation. Simulation results demonstrate that the proposed IOMP algorithm provides a better bit error ratio (BER) performance than the traditional OMP algorithm under unknown channel sparsity.

Keywords Compressive sensing · Sparse recovery · Orthogonal matching pursuit · Orthogonal frequency division multiplexing

1 Introduction

Orthogonal frequency division multiplexing (OFDM) is widely adopted in various wireless communication systems due to its remarkable robustness against frequency selective fading [5]. To resist the multipath effect of the wireless channel, OFDM receivers require precise channel state information (CSI) to demodulate user data more accurately [11]. In general, the channel estimator acquires CSI.

Sparse channel estimation has attracted considerable interest in the past decade [2, 5, 6, 9, 20], owing to its significant spectrum utilization advantages over the traditional Nyquist Shannon sampling theorem based channel estimation method. Massive experimental data show that wireless multipath channel has sparse characteristics, which gives rise to the sparse channel estimation under the framework of CS theorem [1, 5]. Correspondingly, developing a signal reconstruction algorithm for sparse channel estimation is an important research issue: recovering the sparse signal from the received observation vector based on a known measurement matrix. Some preliminary work on sparse signal reconstruction in the CS process has been gradually carried out, among which the most popular algorithm for sparse channel estimation is based on the strategy of iterative greedy pursuits, such as Orthogonal Matching Pursuit (OMP) [21], Orthogonal Least Squares (OLS) [19] and Compressive Sampling Matching Pursuit (CoSaMP) [13]. These greedy algorithms recover the signal iteratively, searching for a locally optimal solution through the atoms selection of a known sensing matrix in each iteration and striving to find the optimal global solution at the end of the algorithm. Specifically, the OMP algorithm selects the best matching non-repeating atom at each iteration to construct a sparse approximation. OLS algorithm differs from the OMP algorithm in selecting atoms, where the former adopts a more complex rule than

✉ Jianjian Liu
314504964@qq.com

[1] College of Computer and Information Engineering, Central South University of Forestry and Technology, Changsha 410004, China

the latter. CoSaMP algorithm has lower computational complexity than OMP due to the pruning technique introduced in the greedy iteration of the OMP algorithm. However, the performance of the CoSaMP depends on the accuracy of the channel sparsity. The OMP algorithm is the most suitable among the above algorithms to acquire CSI due to its stability and allowable computational complexity. In [2], a channel estimation method based on OMP is proposed to estimate the path delay and complex gain, which is superior to the traditional one based on the Nyquist–Shannon-sampling theorem for time-varying channels.

The main disadvantage of these algorithms is that they all need a priori knowledge of channel sparsity. However, since the uncertainty of the environment, the channel sparsity is usually tricky to obtain. Therefore, the Sparsity Adaptive Matching Pursuit (SAMP) algorithm without the demand of a priori knowledge of channel sparsity has been proposed [7], it uses a stage-based method to estimate the channel sparsity where a fixed step size increases its level in each iteration. In [18], the SAMP algorithm has been adopted in sparse channel estimation for OFDM systems. Although the performance of the SAMP algorithm is slightly better than the OMP algorithm in some channel models, it is also unstable in different channels. In addition, another disadvantage of SAMP is that it needs to estimate the channel noise power. However, the noise power at the receiver is usually tricky to estimate. The complexity of SAMP is also high, which interferes with its implementation. In [10], the new adaptive matching pursuit (NAMP) based on a Singular Entropy order determination mechanism has been proposed, reducing computation complexity and obtaining more accurate results than the SAMP. Nevertheless, none of these algorithms can offer a better tradeoff between accuracy and complexity.

Another critical point of sparse channel estimation is the pilot allocation scheme. As described by the restricted isometry property (RIP) [3], a random matrix is measured to guarantee a high probability of sparse recovery. That is to say, the pilot allocation in sparse channel estimation should be random. However, the random pilot pattern is suboptimal and challenging to implement in practical application. Moreover, verifying whether the measurement matrix satisfies the RIP is also an NP-hard problem. Alternatively, in [8], the mutual incoherence property (MIP) has been proposed to minimize the coherence of the measurement matrix. The MIP can replace the RIP in practice but vice-versa [4]. Therefore, we consider the MIP as the pilot allocation design criterion in this paper. In fact, many OFDM systems have employed deterministic pilot allocation schemes, the research of deterministic pilot design has been shown in [12, 14, 17]. The primary strategy in these studies is based on the MIP, which is much easier to implement and more intuitive than RIP. The MIP-based optimal pilot allocation scheme generated by cyclic different sets (CDS) has been considered, whereas the most practical OFDM system does not include the CDS locations such as IEEE 802.22 standard. In this paper, pilot allocation optimization schemes founded on discrete stochastic approximation (DSA) [16] and Iterative Exhaustive Search (IES) based method [17] has been adopted.

At present, the most common reconstruction algorithms for sparse channel estimation require the sparsity of the channel, but the variability of the communication environment makes it difficult to obtain it accurately. Although the existing adaptive reconstruction algorithm (SAMP) does not need to know the sparsity of the channel, the computation is quite complex, and the performance is unstable in different channels. The objectives of this work are to improve the above drawbacks. Our contributions in this paper can be summarized as follows.

(1) This paper proposes an improved OMP algorithm by introducing intersect operation onto two different partial sparse delay path supports. The main idea of this algorithm is grounded on the fact that the pilot sequence can affect the selected atom result due to the cross-correlation property of the conventional OMP algorithm. Initially, we use the OMP algorithm twice to estimate two different partial sparse delay path supports at partially different pilot positions, respectively. Then introduce the intersect operation to process these two partial supports. Simulation results show that our IOMP algorithm proposed in this paper can obtain a tradeoff between accuracy and complexity and balance the performance in different channels without the demand of a prior knowledge of the channel sparsity.

(2) A joint DSA based optimal pilot allocation scheme and an IES-based optimal pilot allocation scheme are employed in the IOMP algorithm to guarantee its performance. In addition, the two optimal pilot allocation algorithms above have also been applied in this paper to verifying the proposed IOMP algorithm satisfies the MIP.

The rest of this paper is organized as follows. Section 2, introduces the compressive sensing theory, OFDM system model, and sparse channel model, respectively. Section 3 presents the proposed algorithm with pilot allocation methods and a modified sparse channel reconstruction algorithm based on the OMP algorithm. In Sect. 4, simulation result and performance analysis are presented. Section 5 concludes this paper.

Notations In this letter, matrices are denoted by upper case, e.g.$\mathbf{X}$. Transpose and the Hermitian, the complex conjugate transpose are denoted by $\mathbf{A}^T$, $\mathbf{A}^H$, respectively.

The absolute value of Hermitian inner product is $|\langle\cdot,\cdot\rangle|$. $|\mathbf{a}|$ is the l_2 norm of the vector $\mathbf{a}$.

2 Brief introduction of CS and system model

2.1 Brief introduction of CS

The emerging theory of CS implies that sparse vector $x \in R^N$ can be reconstructed with high probability from an observation vector y with a relatively small number of elements, which obeys the underdetermined linear measurement model

$$y = \psi x + v \tag{1}$$

where $\psi \in R^{M\times N}$(with $M \ll N$) denotes sensing matrix, $y \in R^M$ represents the received measurement vector and v is the additive random noise. For this model, the following principles should be noted.

(1) CS linear system model (1) is an underdetermined equation.
(2) The measurement vector y is the linear projection of the sparse signal x.
(3) The Sparsity level k is far less than the dimension of the original signal.

In addition, the sensing matrix satisfies the restricted isometry property (RIP) so that it can be used to reconstruct the sparse signal. The RIP can be described as:

$$(1-\delta_k)\|x\|_2^2 \le |\Psi x|_2^2 \le (1+\delta_k)\|x\|_2^2 \tag{2}$$

where $\delta_k \in (0,1)$ denote the restricted isometry constant. However, verifying whether the sensing matrix satisfies the RIP is a non-deterministic polynomial (NP) hard problem. On the other hand, due to the requirement of algorithm complexity, greedy pursuit algorithms based on iterative are generally adopted in channel estimation, such as the OMP algorithm.

2.2 System model

We consider a cyclic prefix (CP) orthogonal frequency division multiplexing (OFDM) system with K subcarriers in each OFDM symbol. Let T_{ofdm} and T_{cp} denote the OFDM symbol duration and cyclic prefix length, respectively. Thus, the signal bandwidth satisfies $B = K/T$ where $T = T_{ofdm} + T_{cp}$ is the total length of one OFDM symbol. Assuming that f_c is the carrier frequency then the frequency of kth subcarrier can be expressed as $f_k = f_c + k/T,\ k = -K/2,\ldots,K/2-1$ referred as $\mathbf{S}_D$, $\mathbf{S}_P$ and $\mathbf{S}_N$ to the set of data subcarriers, pilot subcarriers, and null subcarriers, which satisfy $\mathbf{S}_D \cup \mathbf{S}_P \cup \mathbf{S}_N = \{-K/2,\ldots,K/2-1\}$.

The symbol of the information transmitted on the kth subcarrier is $s(k)$. According to the above definition, the transmitted passband signal can be described as

$$\tilde{x}(t) = \sum_{k=-K/2}^{K/2-1} s(k)e^{j2\pi f_k t}q(t) \tag{3}$$

in which $t \in [0,T]$ and the pulse shaping filter

$$q(t) = \begin{cases} 1, & t \in [-T_{cp}, T_{ofdm}] \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

We continue to consider the frequency selective fading channel and express the channel impulse response as

$$h(\tau) = \sum_{p=1}^{N_p} A_p \delta(\tau - \tau_p) \tag{5}$$

where A_p and τ_p denote path amplitude and path delay of the pth path, respectively, N_p denote the total number of discrete paths.

At the receiver, the bandpass signal can be expressed as

$$y(t) = \sum_{p=1}^{N_p} A_p \tilde{x}(t-\tau_p) + v(t) \tag{6}$$

Here, $v(t)$ denote the additive white Gaussian noise.

After CP removal (here it is assumed that the CP is longer than the delay spread) and OFDM demodulation, the output $\mathbf{Z}[m]$ of the mth subcarrier can be written simply as

$$\mathbf{Z}[m] = \mathbf{H}[m]\mathbf{S}[m] + \mathbf{V}[m] \tag{7}$$

in which $\mathbf{H}[m]$ is the frequency response, and can be expressed as

$$\mathbf{H}[m] = \sum_{p=1}^{N_p} A_p e^{-j2\pi \frac{m}{T}\tau_p} \tag{8}$$

2.3 Compressive sensing based sparse signal reconstruction model

Referring to the form of the formula (1), we can rewrite (7) into the following matrix form as

$$\begin{aligned} \mathbf{Z} &= [\boldsymbol{\Lambda}_1\mathbf{s}\ldots\boldsymbol{\Lambda}_{N_p}\mathbf{s}]\begin{bmatrix} A_1 \\ \vdots \\ A_{N_p} \end{bmatrix} + \mathbf{v} \\ &= \boldsymbol{\Psi}\mathbf{A} + \mathbf{v} \end{aligned} \tag{9}$$

Here, $\boldsymbol{\Psi}$ is described as an over-complete dictionary matrix and $\mathbf{A} = \{A_1,\ldots,A_{N_p}\}$ is named as a coefficient vector.

Without loss of generality, assume that the maximum path delay falls into the guard interval, and the discrete multipath delay set (DS) can be written as $\tau' \in \left\{0, \frac{T}{\lambda K}, \frac{2T}{\lambda K}, \ldots, T_g\right\}$, where λ is an oversampling factor and $N_\tau := |\tau'|$ is the length of the discrete multipath DS.

Combine with the (7) and (9), we have

$$\begin{bmatrix} z(q_1) \\ \vdots \\ z(q_{K_p}) \end{bmatrix} = \begin{bmatrix} s(q_1) & & \\ & \ddots & \\ & & s(q_{K_p}) \end{bmatrix} \times \begin{bmatrix} e^{-j2\pi\frac{q_1}{T}\tau_1} & \cdots & e^{-j2\pi\frac{q_1}{T}\tau_{N_\tau}} \\ \vdots & \ddots & \vdots \\ e^{-j2\pi\frac{q_{K_p}}{T}\tau_1} & \cdots & e^{-j2\pi\frac{q_{K_p}}{T}\tau_{N_\tau}} \end{bmatrix} \begin{bmatrix} A_1 \\ \vdots \\ A_{N_\tau} \end{bmatrix} + \begin{bmatrix} V(q_1) \\ \vdots \\ V(q_{K_p}) \end{bmatrix} \tag{10}$$

In matrix form, the (10) can be expressed as:

$$\mathbf{Z} = \mathbf{SH} + \mathbf{V} = \mathbf{SFA} + \mathbf{V} \tag{11}$$

where $\mathbf{Z} = [z(q_1), z(q_2), \ldots, z(q_{K_p})]$ is the received symbol of the pilot location. The pilot subcarriers set is $S_p = \left\{q_1, \cdots, q_{K_p}\right\}$ with $K_P := |S_P|$. $\mathbf{F}$ is the $\mathrm{K_p} \times N_\tau$ DFT matrix with $\mathbf{F}_{K_p N_\tau} = e^{-j2\pi q_{K_p}\tau_{N_\tau}/T}$. Once obtain the coefficient matrix, the sparse signal can be reconstructed by (8). Similar to (1), the aim of (11) is to reconstruct $\mathbf{A}$ through one of sparse signal recovery algorithms, such as OMP.

3 Proposed algorithms

This section describes the signal reconstruction algorithms and pilot allocation method used in channel estimation for the OFDM system. Firstly, we present the proposed IOMP algorithm for channel estimation over the OFDM system. Secondly, we further consider two pilot allocation methods based on the DSA algorithm and Iterative Exhaustive Search (IES) strategy and discuss their respective advantages.

3.1 Recovery algorithms

The OMP algorithm [21],iteratively recovers the signal as summarized in Algorithm 1, has been widely implemented for sparse signal reconstruction to solve the problem (11). It contains two main steps at each iteration: the atom selection step of a known sensing matrix and the coefficients calculation step. The atom selection step at the ith iteration corresponds to fulfill the search of one path delay in the scenario of sparse channel estimation, i.e., estimates the active atom index with the help of cross-correlation of residual vector and each column vector of a sensing matrix as follows

$$l = \max_{l} \frac{a_l^H b_{i-1}}{\|a_l\|_2^2} \tag{12}$$

where a_l represents the lth column vector of sensing matrix, b_{i-1} is the residual vector after the previous iteration, and l denotes the searched discrete path delay (also referred to as the index of a selected atom). The set of all l found in each iteration constitutes the estimated multipath DS. As shown in the 5th row of Algorithm 1, the path complex gains are reconstructed from the updated DS and measurement vector based on the least-squares criterion in the coefficients calculation step. In this paper, we assume that the channel sparsity is unknown as a prerequisite. With the increase of iteration, the size of the atom index set and the inaccuracy of the estimated coefficients will also increase.

The proposed IOMP algorithm is summarized in Algorithm 2 and sketched in Fig. 2. The main idea of this algorithm is grounded on the fact that the pilot sequence used can affect the selected atom result due to the cross-correlation property of (12). We introduce two sensing matrices Ψ_{P_1} and Ψ_{P_2} for formula (9), which is generated by two pilot sequences with partially identical elements $P_1, P_2 (P_2 \subset P_1)$. As the generation of the two pilot sequences, please refer to Sect. 3.2. Two different sparse support (i.e., two different DSs) can be obtained from these two sensing matricesusing OMP. Assume that two partial sparse support DSs τ^1, τ^2 (τ^1 is set of τ_i^1, τ^2 is set of τ_i^2) are the estimation results from the sensing matrixes constructed by the two pilots sequences P_1, P_2, respectively. Similar to the channel impulse response model (5), the corresponding impulse response $h^1(\tau)$ and $h^1(\tau)$ can be expressed as

$$\begin{cases} h^1(\tau) = \sum_{i=1}^{I} A_i^1 \delta(\tau - \tau_i^1) \\ h^2(\tau) = \sum_{i=1}^{I} A_i^2 \delta(\tau - \tau_i^2) \end{cases} \tag{13}$$

Through intersect the two DSs τ^1, τ^2, the final result can be obtained as $\tau^f = \tau^1 \cap \tau^2$, and the corresponding channel impulse response result can be indicated as $h^f(\tau) = \sum_{i=1}^{R} A_i{}^f \delta(\tau - \tau_i)$, where $R = \left|\tau^f\right|$ is the number of elements in the set τ^f. Even if the number of iterations differs significantly from the actual sparsity, IOMP can still save the primary energy of the channel. More pertinently, our proposed IOMP algorithm is found on the theory of the OMP algorithm, and the intersect operation to use the properties of different pilot sequences to obtain their different DS. In brief, the proposed IOMP algorithm can be summarized as two main steps:

General steps First, set the pilot sequence 1 (PS1) with an optimal pilot allocation method. The OMP algorithm is used for channel estimation with PS1 and obtain the DS1. Further, search the pilot sequence 2 (PS2) based on MIP from PS1 (here, the size of the two sets PS 1 and PS 2 is different), and then use the OMP algorithm with PS2 to obtain DS2.

Intersect steps Consider the impact of the noise effect and two different pilot patterns on OMP channel estimation. The intersect operation is used to perform the intersect result of DS1 and DS2. The performance of the estimation can be verified through the comparison with the CIR.

Algorithm 1 Orthogonal matching pursuit algorithm

1: Initialization: The residual vector $r_0 = y$, initial iterations i, maximum iteration number I, maximum relative index set $\Lambda_i = \emptyset$.
2: **for** $i = 1 : I$ **do**
3: Calculate the correlation between the residuals and the columns of the dictionary $a_l^H b_{i-1}$
4: Find a maximum relative index $l = \max_l \frac{a_l^H b_{i-1}}{\|a_l\|_2^2}$, update the index set $\Lambda_i = \Lambda_{i-1} \cup \{l\}$
5: Calculate estimated coefficient $h_i = (A_{\Lambda_i}^T A_{\Lambda_i})^{-1} A_{\Lambda_i}^T y$
6: Update residual vector $b_i = y - A_{\Lambda_i} h_i$
7: Set $i = i + 1$ and return to step 2 until the stop condition is reached
8: **end for**

Algorithm 2 Intersect Orthogonal matching pursuit algorithm

General steps:
1: Initialization: The residual vector $r_0 = y$, the number of initial iterations i, maximum iteration number I, non-zero position 1 u_1, non-zero position 2 u_2, $\tau' = [0, \frac{T}{\lambda K}, \frac{2T}{\lambda K}, \cdots, T_g]$
2: Set pilot sequence P_1 by DSA algorithm
3: Set pilot sequence P_2 by IES based on P_1
4: OMP 1 use P_1, $x_1 = OMP\,(Y_{P_1}, A_{P_1}, I)$
save the non-zero position of x_1, $u_1 = find(x_1 \sim= 0)$
5: OMP 2 use P_1, $x_2 = OMP\,(Y_{P_2}, A_{P_2}, I)$
save the non-zero position of x_2, $u_2 = find(x_2 \sim= 0)$
Intersect steps:
6: The result of P_1 intersect with P_2 to obtain the final position set: $u_f = u_1 \cap u_2$
7: initialize final DS, $\tau^f = zeros(length(\tau))$, calculate the final DS, $\tau^f(u_f) = \tau(u_f)$
8: calculate the amplitude, $A_i{}^f = x_2(u_f)$
9: calculate the channel impulse response by $h^f(\tau) = \sum_{i=1}^{R} A_i{}^f \delta\left(\tau - \tau_i{}^f\right)$.

3.2 Pilot allocation methods

According to the CS theory, the measurement matrix must satisfy the RIP to recover the sparse original signal. As a matter of fact, however, the proof of the RIP is an NP-hard problem. Fortunately, the measurement matrix's mutual incoherence property (MIP) is more practical than RIPand can substitute for RIP. Given this consideration, MIP is adopted as the optimal pilot allocation rule in this paper.

For a given pilot sequence $S_p = \left\{q_1, \ldots, q_{K_p}\right\}$ we define the coherence of the measurement matrix A as

$$g(S_p) \triangleq \max_{0 \le m < n \le L-1} |\langle A(m), A(n)\rangle| \tag{14}$$

where m and n are the rows and columns of A, $\langle A(m), A(n)\rangle$ denote the inner product of $A(m)$ and $A(n)$. For the pilot allocation schemes, the objective function can be expressed as

$$S_{opt} = \arg\min_{S_p} g(S_p) \tag{15}$$

The discrete stochastic approximation (DSA) [16] based optimized pilot allocation method is adopted in this paper. The main idea of the DSA method can be summarized as two steps: first, generate a random pilot sequence and then gradually update each component of the sequence in the optimal global direction. The steps of DSA can be stated as follows:

Algorithm 3 DSA algorithm

1: Initialization: Randomly generate a pilot sequence p_0, set $P = 0_{N \times N_p}$,$P[1] = p_0$,
set occupation probability vector $\pi[0] = 0_N$, $\pi[0, 0] = 1$,
set $u = 0$,$v = 0$
2: **for** $n = 0 : M - 1$ **do**
3: **for** $k = 0 : N_p - 1$ **do**
4: Sampling and evaluation: $m = n * N_p + k$,
Generate $\tilde{p}_m / p_m$
5: Acceptance:
6: **if** $g(\tilde{p}_m) < g(p_m)$ **then**
7: $p_{m+1} = \tilde{p}_m, u = m + 1$
8: **else**
9: $p_{m+1} = p_m$
10: **end if**
11: Updating occupation probability vector
12: searching p_{m+1} in P;
13: **if** found **then**
14: q_m = the found row index in P
15: **else**
16: $q_m = m + 1$, store P_{m+1} into $P[m + 1]$
17: **end if**
18: $\pi[m + 1] = \pi[m] + (r[q_m] - \pi[m]/(m + 1))$
19: Selection
20: **if** $\pi[m + 1, u] > \pi[m + 1, v]$ **then**
21: $\hat{p}_{m+1} = p_{m+1}, v = u$
22: **else**
23: $\hat{p}_{m+1} = \hat{p}_m$
24: **end if**
25: **end for**
26: **end for**

In Algorithm 3, the occupation probability vector represents the estimated occupation probability of one pilot allocation. The vector $\pi[m, i]$ is defined as the ith component of $\pi[m]$.

The pilot allocation can be determined before transmission, so the DSA is suitable for the frequency selective fading channel. For another pilot allocation method used in this letter, we propose a deterministic pilot allocation method based on the Iterative Exhaustive Search (IES) strategy to reduce the computation complexity. The proposed pilot position search method can increase the number of pilots while keeping the original input pilot position unchanged, i.e. $S_2 \subset S_1$, here $S_1 \in R^{K_1}$ and $S_2 \in R^{K_2}$ denote the pilot index sequences generated by the DSA and IES method, respectively, and some elements of S_1 and S_2 are repeated. The critical point of the IES algorithm is to iteratively select the result with the best mutual incoherence property (MIP) among all the elements in the candidate set. Similarly, the IES algorithm is also a deterministic pilot allocation method. The critical steps of this algorithm are summarized as follows:

Step 1: Generate the candidate difference set s between the active subcarriers index p_{diff}, and ith pilot indices:

$$p_{diff} = N \backslash N_f$$
$$s = p_{diff} \backslash p^{(i)}$$

where N and N_f denote the length of FFT and active subcarriers in each OFDM symbol.

Step 2: For $i = 1, \ldots, N_s$, calculate all the MIP results of the sequence s and save the minimum one, where $N_s = |s|$.

$$g(p) \triangleq \max_{i=1,\ldots,N_s} \left| \left\langle p^{(i)}, D_{dft} \right\rangle \right|$$
$$S_{p_{opt}} = \arg\min_{p \in s} g(p)$$

Step 3: Add $S_{p_{opt}}$ to the original set to form a new pilot set.

$$p = p \cup S_{p_{opt}}$$

Loop Step 1–Step 3 until the stop condition is reached.

Algorithm 4 IES algorithm

Initialization: Length of FFT N_f , number of active subcarriers N, number of pilots N_p, number of supplements N_s,
set of pilot indices p,
set of active subcarriers index D,
set of $gp, M = \emptyset$. DFT matrix $D_{dft}, p_{diff} = N \backslash N_f$,
for $i = 1, \ldots, N_s$ **do**
  $p^{(i)} = p$
  Difference set between the active subcarriers index and ith pilot indices: $s = p_{diff} \backslash p^{(i)}$
  for $j = 1 : (N_f - N_{diff})$ **do**
    $p^{(i)} = p^{(i)} \cup s$
    The coherence of the DFT matrix and $p^{(i)}$
    $gp \triangleq \max \left| \left\langle p^{(i)}, D_{dft} \right\rangle \right|$
    $M^{(k)} = gp$
    $p^{(i)} = p$
  end for
  $[value, index] = min[M^{(k)}]$
  $p = p \cup s(index)$
end for

The schematic diagram of the DSA and IES-based pilot allocation scheme is shown in Fig. 1.

Both methods have a minimum MIP to obtain better performance. Considering the analysis above, we apply the DSA and IES-based optimal pilot allocation algorithm to accelerate the performance of the IOMP algorithm, as shown in Algorithm 3. The schematic representation of the proposed IOMP algorithm with the deterministic pilot allocation method is shown in Fig. 2.

3.3 Complexity analysis

According to [22], Assume that the dimension of the measurement matrix is $M \times N$ and the sparsity level is K, the calculation of the SAMP algorithm is mainly concentrated in the step of merging the set of indices and

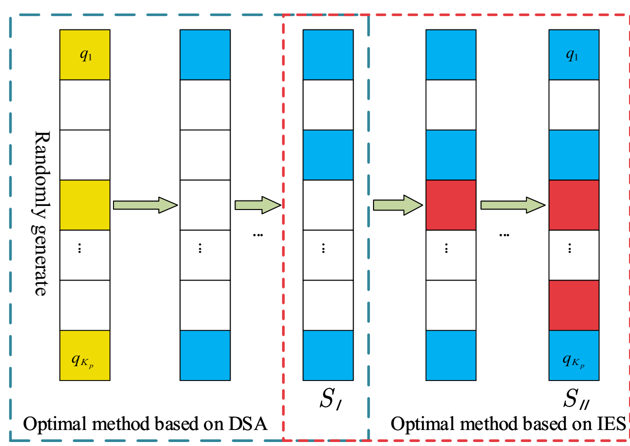


Fig. 1 A joint DSA and IES based optimal pilot allocation scheme

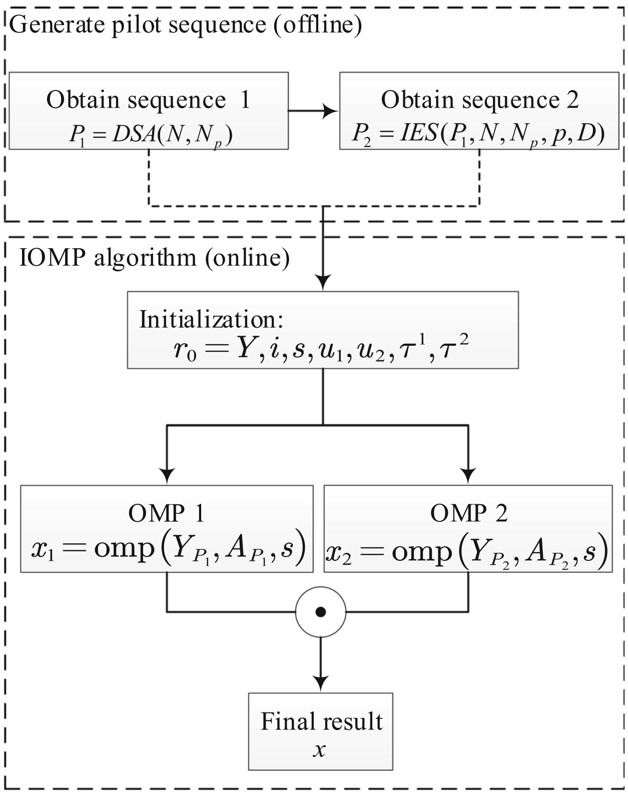


Fig. 2 Schematic representation of the proposed IOMP algorithm

approximating the signal on the merged set of indices by the LS technique, which is roughly $O(MN + MK^2)$. Moreover, the total amount of the computation for the SAMP equals the product of $O(MN + MK^2)$ and the number of iterations N_I. Although the IOMP algorithm used OMP algorithm twice, the P_1 and P_2 are partially repeated, which reduces the computational complexity to some extent. Note that the computational complexity of the OMP algorithms is approximate $O(KMN)$ [15]. Therefore, our proposed IOMP algorithm's total computational complexity is lower than that of the SAMP algorithm. Besides, for the time-invariant channel, the pilot sequences set can be set up before signal transmission. Hence, the proposed IOMP algorithm demonstrates a better tradeoff between the performance and the computation complexity. Table 1 presents the complexity of these algorithms.

Table 1 Computation complexity

Algorithm	Complexity
OMP	$O(KMN)$
CoSaMP	$O(MN)$
SAMP	$O((MN + MK^2)N_I)$
IOMP	$O(2K(M - L)N)$

4 Simulation results and performance analysis

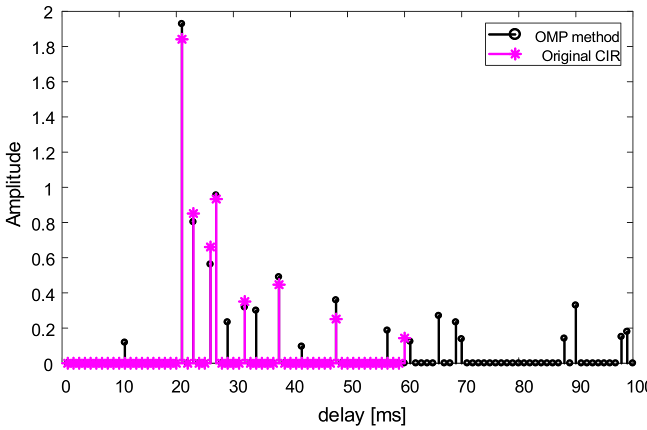
In this section, the effectiveness of the intersection process of the IOMP algorithm and the noise immunity of sparse channel estimation based on various recovery algorithms mentioned above are verified by MATLAB. We use two different multipath channel models, the Extended Vehicular A (EVA) and Extended Pedestrian A (EPA), to test the effectiveness of these algorithms. Besides, we consider a practical OFDM system in which some OFDM subcarriers are set to null subcarriers. The parameters of the CP-OFDM system are specified in Table 2, and the data modulation scheme with QPSK is employed.

Figure 3 illustrates the effectiveness of the intersection process of the IOMP algorithm. This simulation is investigated from the recovery procedure of channel impulsive response (CIR) by superimposing AWGN noise under the EVA channel model. We set AWGN noise with relative power as Eb/No = 10 dB, the number of iterations to 20. In Fig. 3, the legend mentioned as "Original CIR" represents the CIR of theoretical channel impulsive response based on known EVA channel parameters. Specifically, the black line with "OMP method" in Fig. 3a represents the estimated CIR of the OMP algorithm based on the pilot sequence 1 with the number of 54, and Fig. 3b represents the one based on the pilot sequence 2 with the number of 44. The positions where the red line and the black line overlap in Fig. 3a–c indicate the correctly discovered delay paths; otherwise, the found paths are false, which we call ghost paths. Also, as shown in the three subgraphs of Fig. 3, the estimated path supports are more than the theoretical paths of EVA due to ghost paths. These ghost paths are induced by channel noise, causing the leakage of channel energy and the decline of the synthesis accuracy of channel response in frequency domain.

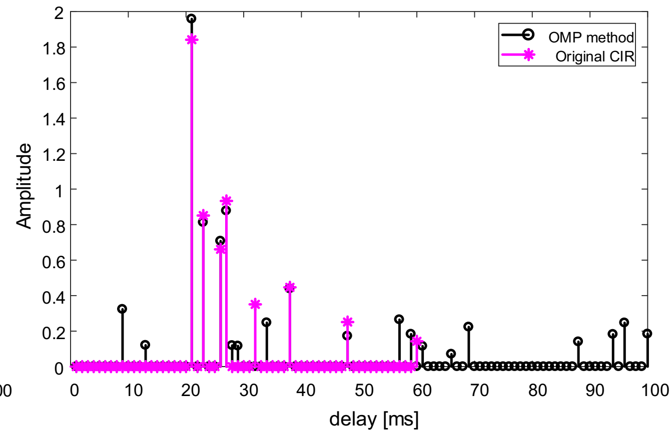
Our proposed IOMP algorithm aims to reduce the ghost paths, to reduce the noise's influence. Compared Fig. 3a with Fig. 3b, we can notice that the low-energy ghost paths appear at different positions due to the uncertainty of noise. Figure 3c is the CIR estimation of the proposed IOMP

Table 2 OFDM system parameters

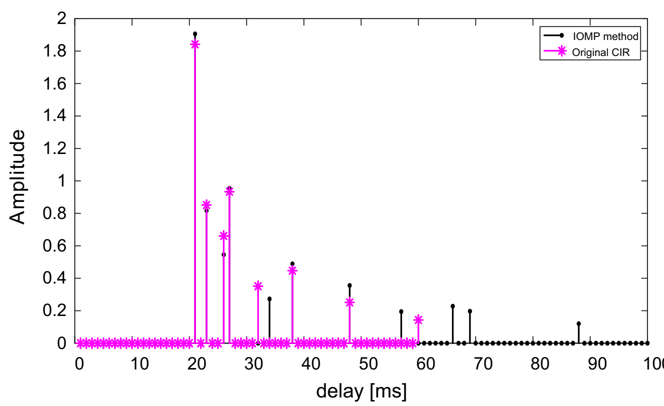
Symbol	Quantity	
f_c	Carrier frequency	15 KHz
N_f	Length of FFT	1024
N	Number of active subcarriers	800
T	Symbol duration	66.7 us
T_{cp}	Guard interval	6.51 us
q_kp	Number of pilots	44/54



(a) The estimated CIR (K_p=54)



(b) The estimated CIR (K_p=44)



(c) The estimated CIR by intersecting (a) and (b)

Fig. 3 The effect of pilot number on CIR results

obtained by intersecting the black line of Fig. 3a, b. It is observed that the intersect operation significantly reduces the ghost paths.

Figure 4 uses the noiseless EVA channel model to verify the effectiveness of the proposed IOMP algorithm. The pilot sequences used are the same as those in Fig. 3. It demonstrates that the proposed IOMP algorithm is as successful as the OMP algorithm in estimating all 8 channel paths without energy leakage.

The noise immunity of the "IOMP" algorithm, as shown in Figs. 5, 6, 7 and 8 is investigated from bit error rate (BER) and normalized mean square error (NMSE) by

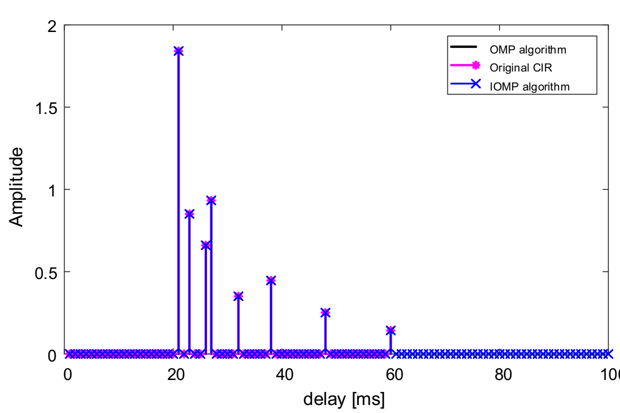


Fig. 4 The estimated CIR under noiseless

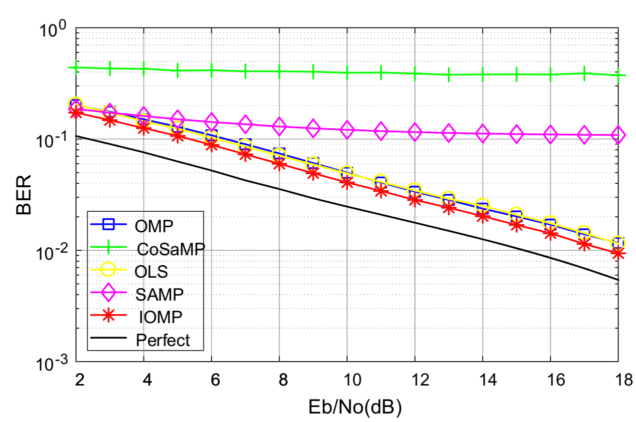


Fig. 5 The comparison of BER performances in EVA channel model

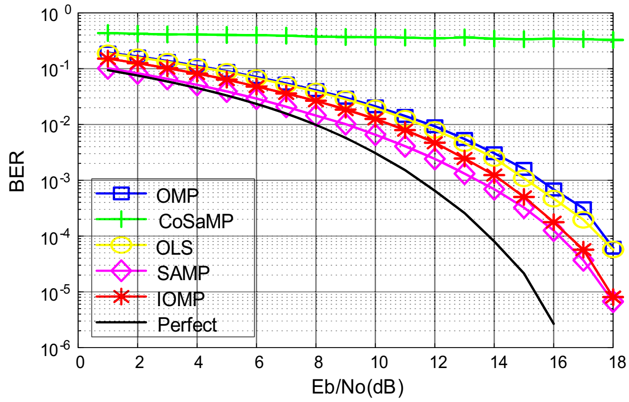


Fig. 6 The comparison of BER performances in EPA channel model

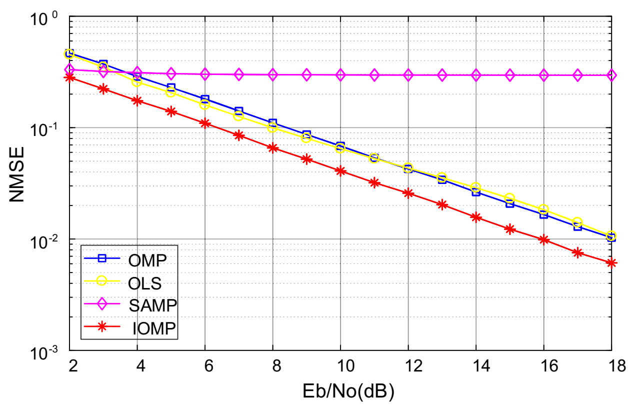


Fig. 7 The comparison of NMSE performances in EVA channel model

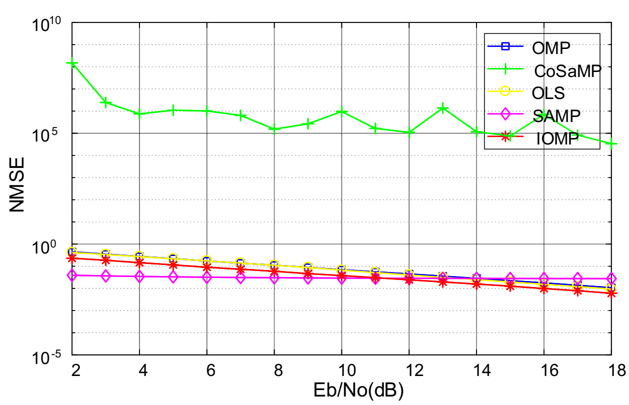


Fig. 8 The comparison of NMSE performances in EPA channel model

superimposing AWGN noise under EVA and EPA channel model. For all algorithms applied here, the channel sparsity is assumed to be unknown as a precondition. The initial step size of SAMP is set to 5, while the iterations of other reconstruction algorithms are 20.

In Figs. 5 and 6, OMP, SAMP, CoSaMP, and OLS algorithm are implemented to compare BER performance. Figure 5 shows that the anti-noise ability of our proposed IOMP algorithm under the EVA channel model is better than all other commonly used BP algorithms. Meanwhile, we can note that the BER performance of the conventional SAMP algorithm is inferior under this channel model. Figure 6 illustrates that although the noise immunity of the IOMP algorithm under the EPA channel model is slightly weaker than that of traditional SAMP, it still maintains good performance.

Figures 7 and 8 respectively verify that the IOMP algorithm has good channel estimation accuracy under EVA and EPA channel models with NMSE. The NMSE is calculated as follows.

$$NMSE = \frac{\sum \left|H(f_k) - \hat{H}(f_k)\right|^2}{\sum \left|H(f_k)\right|^2} \tag{16}$$

where H and $\hat{H}$ represent the actual and estimated value, respectively. As can be seen from the illustration above, the proposed IOMP algorithm yields better performance compared to the OMP, SAMP, CoSaMP, and OLS algorithm. It should be noted that the OLS and OMP algorithms show similar NMSE performance and are superior to that of the SAMP algorithm as Eb/No increases. Moreover, the performance of the CoSaMP algorithm is relatively poor and instable, which can be interpreted that the input channel sparsity deviates from the actual situation.

5 Conclusion

This paper proposes a modified OMP sparse channel estimation algorithm called the IOMP algorithm based on the intersect operation for OFDM systems. Compared with the traditional OMP algorithm, one of the advantages of the IOMP algorithm is that it does not need a priori knowledge of channel sparsity. Meanwhile, to improve the performance of the IOMP for channel estimation, both the DSA and IES schemas are exploited to search for optimal pilot allocation. Besides, the proposed IOMP algorithm can offer a better tradeoff between the accuracy and complexity of the algorithm against the traditional ones. Simulation results demonstrate that our proposed IOMP algorithm achieves excellent BER and normalized mean square error performances among various greedy pursuit and adaptive algorithms.

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