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Jan Švihlík, Karel Fliegel, Stanislav Vítek, Jaromír Kukal, Zuzana Krbcová, "Estimation of Poisson noise in spatial domain," Proc. SPIE 10396, Applications of Digital Image Processing XL, 103962X (19 September 2017); doi: 10.1117/12.2274149

Event: SPIE Optical Engineering + Applications, 2017, San Diego, California, United States

Estimation of Poisson noise in spatial domain

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ABSTRACT

This paper deals with modeling of astronomical images in the spatial domain. We consider astronomical light images contaminated by the dark current which is modeled by Poisson random process. Dark frame image maps the thermally generated charge of the CCD sensor. In this paper, we solve the problem of an addition of two Poisson random variables. At first, the noise analysis of images obtained from the astronomical camera is performed. It allows estimating parameters of the Poisson probability mass functions in every pixel of the acquired dark frame. Then the resulting distributions of the light image can be found. If the distributions of the light image pixels are identified, then the denoising algorithm can be applied. The performance of the Bayesian approach in the spatial domain is compared with the direct approach based on the method of moments and the dark frame subtraction.

Keywords: Poisson noise, Poisson distribution, dark current, minimum mean square estimator, method of moments

1. INTRODUCTION

Astronomical cameras used for light image acquisition are usually cooled to suppress dark current. Dark current presents thermally generated charge depending on the temperature of the CCD sensor. Cooling systems in commonly used cameras allow decreasing the sensor temperature by about 30 K. The cooling performance also depends on ambient temperature. Hence, the astronomical images acquired by such cameras must be corrected for dark current. In practice, a set of so-called dark frames is taken at the same temperature and exposure time as for the light image. To overcome the dark current fluctuation, the set of dark frames are averaged in time to create the master dark frame. Master dark frame is then subtracted from the light image.

If we consider one cell of the darkened CCD sensor in particular time interval, then we could observe that a number of generated electrons approximately match the Poisson distribution $Po(\lambda)$.[1] Hence, the creation of master dark frame could be seen as the estimation of λ in a pixel-wise manner. However, some authors stated that even in common CCD sensors the dark current could be modeled by another distribution. They recommend using a Log-Normal, Gamma and Inverse Gamma distributions, which were found empirically.[2] In[3] authors modeled the sensor noise by Skellam distribution. The Skellam distribution has random variable generated as the difference of two Poissonian random variables. The dark current could also be modeled in the transform domain. In our previous paper,[4] we modeled the dark current in the wavelet domain by using Gaussian mixture model, and the model parameters were estimated using the method of moments. The relation between the dark frame moments in the spatial and in the wavelet domain is derived in this paper.[5]

The light images acquired by CCD sensor are usually modeled by a combination of Poisson and Gaussian distribution.[6] The Poisson distribution came from the fact that incoming light is determined by a random number of photons counted in the specified time interval. The number of photons is given by Poisson distribution. The Gaussian part of the image model presents so-called read-out noise of the sensor.[7]

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Applications of Digital Image Processing XL, edited by Andrew G. Tescher, Proc. of SPIE
Vol. 10396, 103962X · © 2017 SPIE · CCC code: 0277-786X/17/$18 · doi: 10.1117/12.2274149

As we mentioned before, the dark current could be eliminated by subtracting the master dark frame. Also a special chip configurations were developed to suppress the effect of the dark current.[8] For the big professional telescopes, the CCD sensor is usually cooled by liquid nitrogen. For very low temperatures of the CCD sensor (approximately below 173 K), the dark current is negligible.[9]

The paper is organized as follows. Section 2 discusses modeling of the light images and the real dark frames using Poisson distribution. Then an MMSE (minimum mean square error) estimator is described for the case of addition of two Gaussian random variables. We consider utilization of the heteroscedastic normal approximation of Poisson distribution. At the end of this section, the estimation of the model parameters based on the method of moments is described. Section 3 is dedicated to acquisition and analysis of real data using BART (Burst Alert Robotic Telescope) system.[10] This section also presents results of performed simulations.

2. METHOD

In this section, we first introduce a model of useful image data and noise. Then the MMSE estimator in the spatial domain is described, and direct method for model parameters estimation is discussed.

2.1 Model of image and noise

We assume the acquired image y consisting of clean image x contaminated by additive dark current υ

$$y(a,b) = x(a,b) + \upsilon(a,b). \tag{1}$$

All the mentioned variables in equation (1) are considered to be the image matrixes with spatial coordinates (a,b). We model the image $x(a,b)$ as the Poisson random variable $x(a,b) \sim Po(\lambda_x(a,b))$. It means that every pixel $x(a,b)$ presents independent random variable. The Poissonian model is also used for the dark current $\upsilon(a,b) \sim Po(\lambda_\upsilon(a,b))$.

We consider the light image $y(a,b)$ taken in more realizations in time $\{y(a,b)\}_t$, where $t = 1 \dots T$. Hence, we have T samples for every pixel of the light image. The same number of realizations are also acquired in the case of the dark frame.

2.2 MMSE estimator in the spatial domain

In this paragraph the minimum mean square error estimator (MMSE) of the clean image x is derived. For the sake of the simplicity, we employed so-called heteroscedastic normal approximation[6] of the Poisson distribution $Po(\lambda) \approx \mathcal{N}(\lambda, \lambda)$. The conditional mean of the posterior PDF (probability density function) $p_{x|y}(x|y)$ provides the least square estimation of x. Due to used normal approximation, the MMSE estimator[11] has the closed form[12]

$$\begin{aligned} \widehat{x}(y) &= \int_{-\infty}^{+\infty} p_{x|y}(x|y)x \; dx \\ &= \frac{\int_{-\infty}^{+\infty} p_{y|x}(y|x)p_x(x)x \; dx}{\int_{-\infty}^{+\infty} p_{y|x}(y|x)p_x(x) \; dx}, \\ &= \frac{\int_{-\infty}^{+\infty} p_\upsilon(y-x)p_x(x)x \; dx}{\int_{-\infty}^{+\infty} p_\upsilon(y-x)p_x(x) \; dx} \\ &= E(x) + \frac{\sigma_x^2}{\sigma_\upsilon^2 + \sigma_x^2}(y - E(y)) \end{aligned} \tag{2}$$

where $p_{y|x}(y|x)$ denotes the likelihood function, $p_x(x)$ represents the a priori model, and $p_\upsilon(x)$ stands for the noise model. In equation (2), we consider $\upsilon(a,b) \sim \mathcal{N}(E(\upsilon), \sigma_\upsilon^2)$ and $x(a,b) \sim \mathcal{N}(E(x), \sigma_x^2)$. If the heteroscedastic normal approximation is taken into account, then equation (2) could be written as follows

$$\widehat{x}(y) = \lambda_x + \frac{\lambda_x}{\lambda_\upsilon + \lambda_x}(y - E(y)), \tag{3}$$

where $\lambda_x = E(x)$.

2.3 Model parameters estimation (direct method)

The direct method is used for model parameter estimation. The development is presented in the two following paragraphs while using the first or the second moments approach.

2.3.1 Using the first moments

Parameters of the models in equation (2) must be estimated. In the case of Poisson distribution, the simplest approach is to use the first moments for the estimation of the parameters. The estimate of $\lambda(\upsilon)$ denoted as $\widehat{\lambda_\upsilon}$ could be simply obtained by averaging the dark current samples in the time

$$\widehat{\lambda_\upsilon} = \frac{1}{T}\sum_{t=1}^{T}\{\upsilon(a,b)\}_t. \tag{4}$$

The estimation of the $\lambda(\upsilon)$ based on the first moment is unbiased and consistent. The final estimate of λ_x is then given by

$$\widehat{\lambda_x} = \widehat{\lambda_y} - \widehat{\lambda_\upsilon}. \tag{5}$$

Equation (5) denotes $\widehat{\lambda_\upsilon}$ the estimate derived by the method of moments. However, if we use the first moments for the λ_x estimation, equation (2) degenerates as follows

$$\widehat{x}\{(y(a,b))\}_t = \widehat{\lambda_x} + \frac{\widehat{\lambda_x}}{\widehat{\lambda_\upsilon}+\widehat{\lambda_x}}(\{y(a,b)\}_t - E(y)). \tag{6}$$

Hence, we obtained estimate for each realization of the light image at given pixel (a,b). Mean value through the realizations then runs as

$$\frac{1}{T}\sum_{t=1}^{T}\{\widehat{x}(y(a,b))\}_t = \frac{1}{T}\sum_{t=1}^{T}\left(\widehat{\lambda_x} - \frac{\widehat{\lambda_x}}{\widehat{\lambda_\upsilon}+\widehat{\lambda_x}}E(y) + \frac{\widehat{\lambda_x}}{\widehat{\lambda_\upsilon}+\widehat{\lambda_x}}\{y(a,b)\}_t\right), \tag{7}$$

hence

$$\frac{1}{T}\sum_{t=1}^{T}\{\widehat{x}(y(a,b))\}_t = \widehat{\lambda_x} - \frac{\widehat{\lambda_x}}{\widehat{\lambda_\upsilon}+\widehat{\lambda_x}}E(y) + \frac{\widehat{\lambda_x}}{\widehat{\lambda_\upsilon}+\widehat{\lambda_x}}\frac{1}{T}\sum_{t=1}^{T}\{y(a,b)\}_t, \tag{8}$$

where $\frac{1}{T}\sum_{t=1}^{T}\{y(a,b)\}_t = E(y)$, so that we obtained the same results as in the case of equation (5).

2.3.2 Using the second moments

In this section we employed the second moments. The second moment of y is given by

$$m_2(y) = m_2(x) + m_2(\upsilon), \tag{9}$$

where all the moments in equation (9) are the central theoretical moments of the given distribution. Firstly, we need to estimate $m_2(\upsilon)$. The estimation of $m_2(y)$ is done by second central sample moment as follows

$$M_2(y) = \frac{1}{T}\sum_{t=1}^{T}(\{y(a,b)\}_t - E(y))^2, \tag{10}$$

where $E(y) = \frac{1}{T}\sum_{t=1}^{T}\{y(a,b)\}_t$. From equation (9) we can express $M_2(x) = M_2(y) - M_2(\upsilon)$, which can be seen as the estimate of λ_x

$$\widehat{\lambda_x} = M_2(x). \tag{11}$$

Figure 1. BART telescope, photo taken by Jan Štrobl.

3. RESULTS

3.1 Simulated data

We generated 100 realizations for the one image pixel. We consider the constant signal $\lambda_x = 300$. The noise parameter is changing in the following range $\lambda_v = 1 : 500$.

In the first experiment we use the first moments for the estimation of λ_x (equation (5)). The results can be seen in Fig. 2 (a). This approach is similar to the dark frame subtraction. However, in contrast with dark frame subtraction, we also use more than one realization of the same light image to determine λ_v more precisely.

The equation (2) applied to the noisy observation y results in 100 estimates of $\widehat{\lambda_x}$. Hence, the final estimation of $\widehat{\lambda_x}$ for each λ_v is an average of 100 estimations. As we expected the results of MMSE were the same as for the first moments (see section 2.3.1), see Fig. 2 (c).

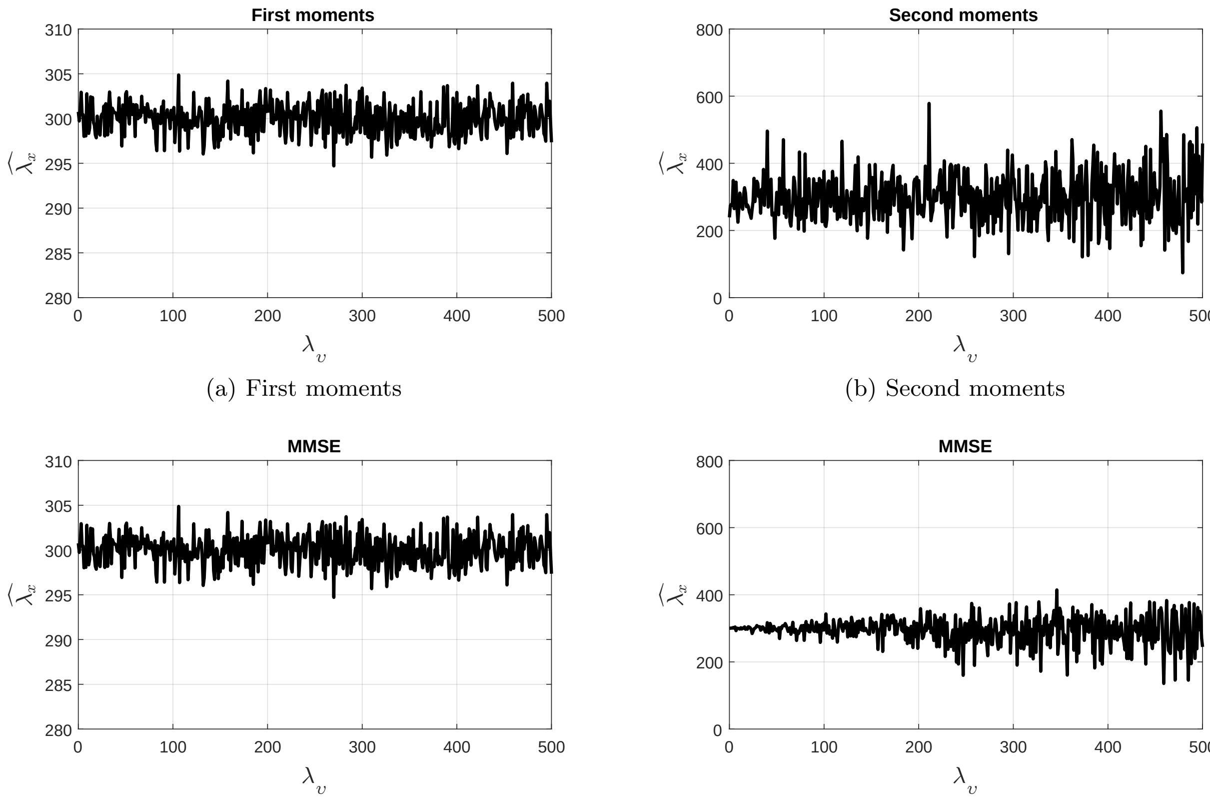
If we consider a special case, where we replace the parameters in equation (2) by the second moments, the variance of the $\widehat{\lambda_x}$ is decreased in comparison with estimation of λ_x by directly using the second moments, see Fig. 2 (b) and (d).

In the last experiment, we consider known $E(y)$. We applied MMSE estimator given by equation (2) to the simulated noisy observation y, where λ_v is s changing in the following range $\lambda_v = 1 : 500$. We compared the results obtained by MMSE estimator with the simplest estimation given by $\{\widehat{x}(a,b)\}_t = \{y(a,b)\}_t - \widehat{\lambda_v}$. The final averaging $\forall\ t$ is omitted. This experiment could be seen as the typical dark frame subtraction (mean master dark frame is considered). The resulting variance of the described two approaches can be seen in Fig. 3. As it could be expected the dark frame subtraction cannot decrease the variance of noisy observation. In contrats, MMSE estimator can decrease the variance through the realizations of each pixel. Each point of the depicted dependency of the variance was obtained as an average of variances estimated for 100 realizations.

In practice, a number of realizations of the same light images are quite often acquired. If we want to eliminate the dark current in each realization and also decrease the variance of light image pixels, we could estimate the model parameters and then apply the MMSE estimator to the noisy observation in pixel-wise manner.

3.2 Real data

In our experiment, we used images of M33 galaxy obtained with the wide-field camera G2 1600 equipped with 1536×1024 full-frame CCD image sensor Kodak KAF1603ME. This camera, made by Moravian Instruments, is the main camera of the robotic telescope BART[13] (an acronym for Burst Alert Robotic Telescope). This telescope, placed in Ondřejov, Czech Republic, is one of the oldest fully autonomous robotic telescopes in the



(a) First moments (b) Second moments

(c) MMSE using the first moments (d) MMSE using the second moments

Figure 2. The results of MMSE estimator utilizing method of moments for the parameters estimation and the method of moments in the spatial domain for simulated data and 100 samples, (a) the first moments used for λ_x estimation, (b) the second moments used for λ_x estimation, (c) MMSE, parameters estimated by using the second moments, (d) MMSE, parameters estimated by using the first moments.

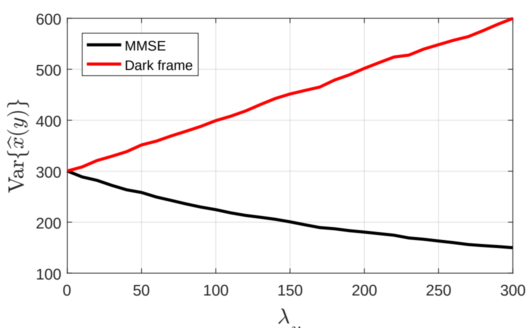
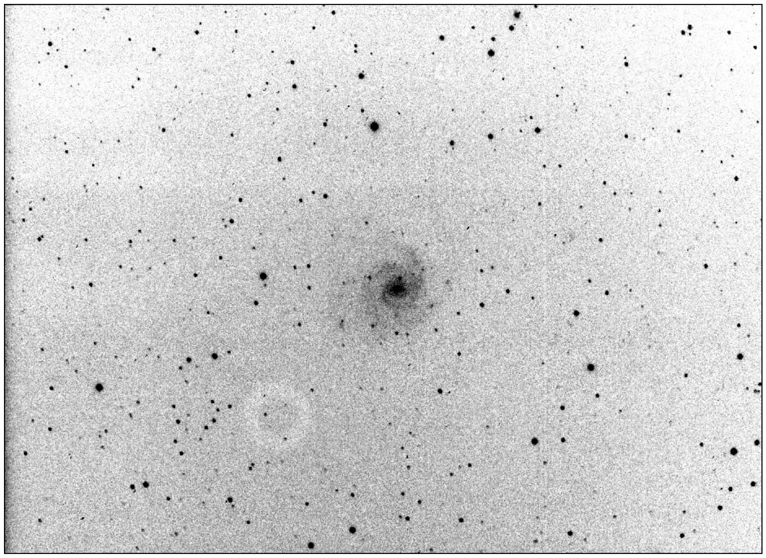


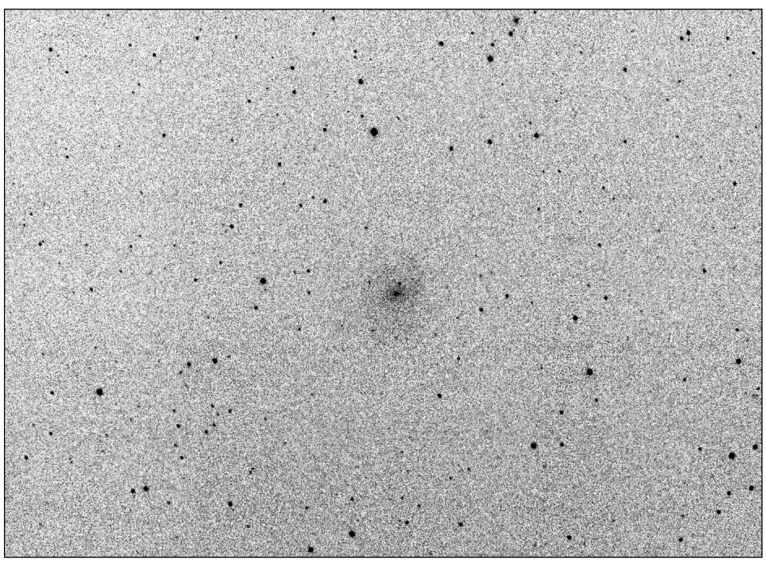
Figure 3. Variance estimated for the two approaches, MMSE estimator and the simulation of dark frame subtraction. The master dark frame is subtracted from each light image in the subsequently acquired sequence.

world - it exists since early 2000, see Fig. 1. Nowadays it uses German-type mount Losmandy Titan, which is holding two optical telescopes: the narrow-field (NF), which is 0.25 m Schmidt-Cassegrain from Meade (f = 1600 mm), and wide-field (WF), which is 0.1 m Maksutov-Cassegrain from Rubinar (f = 500 mm). The testing dataset consists of astrometrically aligned frames with the exposure times of 32 s. The frames were acquired consecutively, so we can expect the same condition during observation, in terms of stellar objects visibility and seeing. Dark frame subtraction was applied to each of the images in the dataset.

In this experiment 35 light images were available. We applied MMSE estimator to all the images in the sequence. Hence, the final estimation of the correct pixels for each image is based on the averaged astrometrically registered image realizations. A comparison of the selected light image corrected by MMSE estimator can be seen in Fig. 4 (a) and the image corrected by the master dark frame can be seen in Fig. 4 (b). However, only the master dark frame computed as the median of the particular dark frames was available. For the sake of the simplicity, we consider that median is approximately equal to mean value. We evaluated the variance of each pixel in time. These variances were then averaged to get mean variance through the image sequence. The mean variance for MMSE estimator is equal to 71 and for dark frame subtraction is equal to 496.



(a)



(b)

Figure 4. Light image selected from the acquired sequence of 35 astrometrically registered images, (a) corrected by MMSE estimator in the spatial domain, mean $\mathrm{Var}\{\widehat{x}(y)\} = 71$, (b) corrected by dark frame subtraction, mean $\mathrm{Var}\{\widehat{x}(y)\} = 496$.

The considerable reduction of the mean variance is given by the fact that MMSE estimation uses averaged image sequence (λ_x estimation). This approach becomes useful if we want to get one smooth light image computed from the image sequence. In this case, the final estimate of the corrected light image is given by the first moments, see equation (5). The second case, where the described algorithm could be useful is if we want to reduce the variance of the acquired sequence.

4. CONCLUSION

We have presented a method for dark current suppression based on pixel-wise MMSE estimator. We used heteroscedastic normal approximation. Hence, both noise and image are modeled by Gaussian PDFs. The performed simulations illustrated that this method could be useful for the denoising of the light image sequences. There are two basic algorithm usages. In the first case, the algorithm application results in one smooth light image estimated from the acquired sequence. In the second case, the algorithm application results in the corrected image sequence with the considerably reduced pixel variance.

ACKNOWLEDGMENTS

This work has been supported by the grant GA17-05840S "Multicriteria Optimization of Shift-Variant Imaging System Models" of the Czech Science Foundation.

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