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Nonlinear average stochastic resonance for image enhancement

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Abstract

How to extract useful information from noised image is always an important issue for image processing. Many methods have been proposed in image enhancement field. However, in these methods the noise is usually considered as harmful and should be removed as much as possible. Stochastic resonance is a very different method, in which the noise is regarded as a driver to push the stochastic resonance system to output enhanced image. In this paper, the cumulative gain is introduced and the sequence average is used to enhance the original image information which hidden in a noised image sequence produced by bistable stochastic resonance. We present the one-dimensional and two-dimensional stochastic resonance methods and discuss their performance in this paper. Experiments illustrate that the one-dimensional average stochastic resonance has the best performance considering the indicator PSNR and SSIM. Compared with traditional filters such as median and Wiener filters, the proposed methods have significant advantages.

Keywords

Stochastic resonance, average, image enhancement, noise

Introduction

Image is not only an important information carrier, but also a crucial way for human to acquire knowledge and perceive the world. However, due to the unbalanced sensitivity of optoelectronic transform device, circuit noise, quantization

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noise produced by digital process and external environment factors, the acquired images contain many kinds of noises.[1] And then these noises affect the quality of images and even submerge the image features. It will bring lots of difficulties to the subsequent image processing. Therefore, image enhancement is a necessary step for the acquired noised image.

To enhance a noised image, two ways can be considered. One widely accepted and adapted way is noise suppression. Its main idea is to reduce noise so as to outstand original image information. Another is to enhance the original image information from noise polluted to improve the quality of images, for example, stochastic resonance technique.

Currently, many methods can be used to suppress noise so as to improve image quality. All these methods are broadly classified into spatial filters and frequency domain filters. Traditional filters such as many kinds of spatial filters usually blur the images when filtering the noise, since the current pixel value is related to its neighbor's values. The frequency domain filter's function is to reduce the out-band noise. However, since much out-band signal is also filtered, some of the image information is lost.

Another method to enhance a noised image is to improve the original image information to obtain better Peak Signal-to-Noise Ratio (PSNR), Structure Similarity (SSIM) and some other indicators which are the objective evaluation of an image. With this type of method the hidden image information will be outstood from the noised image. The noise of images in the stochastic resonance systems is no longer seen as harmful but a driving force to drive a nonlinear stochastic resonance system. It can efficiently outstand the original image information and improve the quality of images.

In this paper the one-dimensional average stochastic resonance (ODASR) algorithm based on sequence average is presented. First of all, given a noised image, an image sequence was obtained by performing the bistable stochastic resonance several times with independent stochastic noise, and then we achieved larger cumulative gain using the sequence average method and obtained higher objective indicators: PSNR and SSIM. On this basis, we discuss the application of sequence average method in two-dimensional stochastic resonance, propose the two-dimensional average stochastic resonance algorithm (TDASR) and analyze its performance. Experiments illustrate that the indicators PSNR and SSIM of ODASR exhibit better than that of TDASR and the stochastic resonance algorithms without using sequence average. Furthermore, the TDASR is better than the stochastic resonance algorithms without using sequence average, though its high frequency component is lost, that means the output image becomes blurring.

The structure of the rest of the paper is as follows. The next section enumerates some works on image denoising and enhancement. The stochastic resonance method is presented in the Non-linear resonate average method section and the ODASR method is described in the One-dimensional average stochastic resonance enhancement section. Then, the Performance analysis section which is given below the One-dimensional average stochastic resonance enhancement section describes the performance analysis of the ODASR. The TDASR is described in the Two-dimensional average stochastic resonance enhancement section and the performance analysis is described in the Performance analysis section which is given below the Two-dimensional average stochastic resonance enhancement section. The penultimate section illustrates the comparison results of average stochastic resonance to some classical filters. The Conclusion section summarizes the paper.

Electrical engineering students will benefit from this paper about how to enhance the image. In this paper, we have used a standard image. The image can be any electrical engineering related image for enhancement. The post graduates can take into the same algorithms discussed via coding and implement the same for any motor speed analysis via noncontact image processing methods.[2] Thus this study is most relevant to the electrical engineering research groups.

Related works

Until now, many researchers have dedicated to design specific filters[3–8] to eliminate noise for improving the quality of images. However, as we have mentioned above, the stochastic resonance is stemmed from different idea and improves a noised image by enhancing the useful signal via stochastic resonance system.

Benzi et al.[9] first presented the stochastic resonance phenomenon when they studied paleoclimate and glacier. From then on, the stochastic resonance has attracted much attention.[10–12] This interesting technique has been gradually used in image processing.

Leng et al.[13] used bitable stochastic resonance to stretch the histogram distribution of a given noised image and then obtained an enhanced image by tuning the parameter of the resonance system. Their work exhibits the impact of bistable stochastic resonance system on histogram. Experiments also demonstrated that their method is effective.

Xu et al.[14] introduced the two-dimensional SR system into image processing and its function is just as a nonlinear filter. To illustrate its efficiency the authors used it to process a binary image polluted by additive white Gaussian noise. The results show it is an effective method and can significantly improve the quality of image.

Yang et al.[15] developed a theory of 2D parameter-induced stochastic resonance (PSR) for nonlinear image processing. The authors used this theory to establish a Fokker-Planck equation (FPE) based framework performed using rigorous analysis to study some applications in which images were processed by 2D-PSR. Besides, many other researchers have also made important contributions to this filed.[16–18]

In general, how to apply the stochastic resonance technique into image processing to enhance the noised images is still an interesting issue which needs to be further studied.

Non-linear resonate average method

On accumulation

In the field of Radar signal processing, the accumulation is always used to improve the Signal-to-Noise Ratio (SNR). This method can be further divided into coherent accumulation and incoherent accumulation. The former is to accumulate complex data, including amplitude and phase, and the later only accumulates amplitude.

Assume that the measured signal contains complex data $Ae^{j\phi}$ and the independent and random additive noise w with power σ^2. Therefore, the SNR of single signal is denoted as follows.

$$\chi_1 = \frac{P_s}{P_n} = \frac{A^2}{\sigma^2} \tag{1}$$

Here, P_s is the signal power and P_n is the noise power. Suppose that we have measured the signal N-1 times independently, the signal responses were unchanged and the noise examples were independent for each measurement. We accumulate these measured values and obtain a new result z. The value z can be expressed as follows.

$$\begin{aligned} z &= \sum_{n=0}^{N-1} \left\{ Ae^{j\phi} + w[n] \right\} \\ &= NAe^{j\phi} + \sum_{n=0}^{N-1} w[n] \end{aligned} \tag{2}$$

Therefore, the accumulated signal power is N^2A^2 and the accumulated noise power is the sum of the independent noise examples with mean value 0. As we have assumed above, each noise example has power σ^2, then the total of noise power is $N\sigma^2$. With the accumulation we obtain the new SNR:

$$\chi_N = \frac{N^2A^2}{N\sigma^2} = N\chi_1 \tag{3}$$

From above description, the accumulation of the multiple measurements can efficiently improve the SNR and make it easy for signal extraction.

Average stochastic resonated method

In the field of stochastic resonance research, the potential function in non-linear bistable system model is usually expressed as following:

$$U(x) = -\frac{1}{2}ax^2 + \frac{1}{4}bx^4 \tag{4}$$

Here a and b are the system parameters. Let $f(x) = -dU(x)/dt$ and $u = a/b$ ($b \neq 0$), the determinate dynamic equation of one-dimension nonlinear bistable system without considering the noise is as follows.

$$\dot{x} = f(x) = ux - x^3 \tag{5}$$

Assume that the noised input image is I_{noise}, in order to produce stochastic resonance we introduce a random noise $w(t)$, and then the stochastic resonance model is following.

$$\dot{x} = ux - x^3 + I_{noise} + w(t) \tag{6}$$

Suppose that the image I_{noise} is composed of unpolluted image I and noise w_I, and has following equation.

$$I_{noise} = I + w_I \tag{7}$$

Then we have following expression.

$$\dot{x} = ux - x^3 + I + (w_I + w(t)) \tag{8}$$

It indicates that the noise $(w_I + w(t))$ is still independent and random since w_I is determinate. The Nth resonated image $I_{resonate-n}$ contains the noise and part of the input image information. Furthermore, we assume that the noise in Nth resonance is w_n and the image intensity is A_n, the accumulation of the total N resonated images is denoted as follows.

$$z = \frac{1}{N}\sum_{n=0}^{N-1}\{A + \mathrm{w[n]}\} \tag{9}$$

Therefore, the SNR will be improved significantly according to the On accumulation section.

One-dimensional average stochastic resonance enhancement

Suppose that the input two-dimensional noised image I_{noise} is of size $\mathrm{m} \times \mathrm{n}$, and a nonlinear bistable stochastic resonance system $U(x)$ can be expressed as equation (4). The One-dimensional Average Stochastic Resonance method (**ODASR**)is composed of following steps.

Step 1. Transform the noised image I_{noise} into one-dimensional signal $I_{noise}^{one-\mathrm{dim}}$ with size $1 \times \mathrm{mn}$.

Step 2. Put the one-dimensional noised signal into the stochastic resonated system $U(x)$ and add proper noise w_i to drive the resonant system to output resonated signal $I^{one-\dim}$.
Step 3. Repeat step 2 for n times and obtain a set of one-dimensional signals resonated with different independent random noise, that is $I^{one-\dim} = \{I_1^{one-\dim}, I_2^{one-\dim}, \ldots, I_n^{one-\dim}\}$. Here we suppose that the n noise constitute a set $W = \{w_i, w_2, \ldots, w_n\}$.
Step 4. Transform one-dimensional signal sequence $I^{one-\dim} = \{I_1^{one-\dim}, I_2^{one-\dim}, \ldots, I_n^{one-\dim}\}$ into two-dimensional images $I_{IMAGES} = \{I_1^{two-\dim}, I_2^{two-\dim}, \ldots, I_n^{two-\dim}\}$.

Step 5. Compute $I_{output} = \frac{1}{n}\sum_{i=1}^{n} I_i^{two-\dim}$ and process the I_{output} with adaptive equalization algorithm as the final output image.

In this algorithm, the number of repeats n is a crucial parameter. We will discuss it in the later section. The Gaussian white noise whose mean value is 0 and variance is σ is added to drive the nonlinear bistable stochastic resonance system. According to the algorithm described above, we process the classical image 'lina.jpg' and obtain the one-dimensional average stochastic resonated (ODASR) image. Furthermore, we compare the image processed using ODASR with that processed using ODSR, and illustrate the results in Figure 1. Intuitively, Figure 1(d) is better than (c) in the situation that the noise density is 0.4.

Performance analysis

The number of repeat resonances n is a crucial parameter. Increasing n will not output continuous and efficient performance improvement, but lead to more time-consuming. Therefore, an appropriate parameter n is necessary and worth discussing.

In this section we will mainly discuss the performance of the ODASR to ODSR. Appropriate and efficient evaluation indicators are very crucial for performance discuss. Currently, many image evaluation indicators have been used in different researcher's works, such as Peak Signal-to Noise Ratio (PSNR), Structural

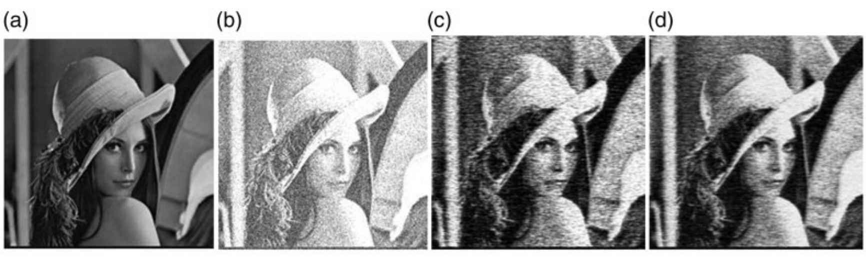

Figure 1. Image processing results: (a) original image, (b) noised image (noise density 0.4), (c) processed using ODSR, and (d) processed using ODASR.

Similarity (SSIM), Information Entropy and Definition etc. General speaking, one indicator cannot efficiently evaluate an image quality, so we place emphasis on PSNR and SSIM in our work.[8] Of course, subjective evaluation is also very important and should be valued.

On the number of repeats n

According to the description in the Non-linear resonate average method section, the average of independent stochastic resonated images is an efficiently method to enhance a noised image. Figure 2 is the PSNR trend as the noise density increases.

Figure 2 indicates that the PSNR change in the low noise density region (<0.4) is not significant when the number of the resonated images is larger than 5. However, more images are needed when the noise density is greater than 0.6. Similarly, the indicator SSIM has same trend as PSNR. According to the experiment results $n = 10$ is appropriate in most cases.

The performance on PSNR

To analyze the performance of the proposed ODASR quantitatively, we perform the algorithm in different noise density from 0.1 to 0.9 and compare the results processed using ODSR, see Table 1.

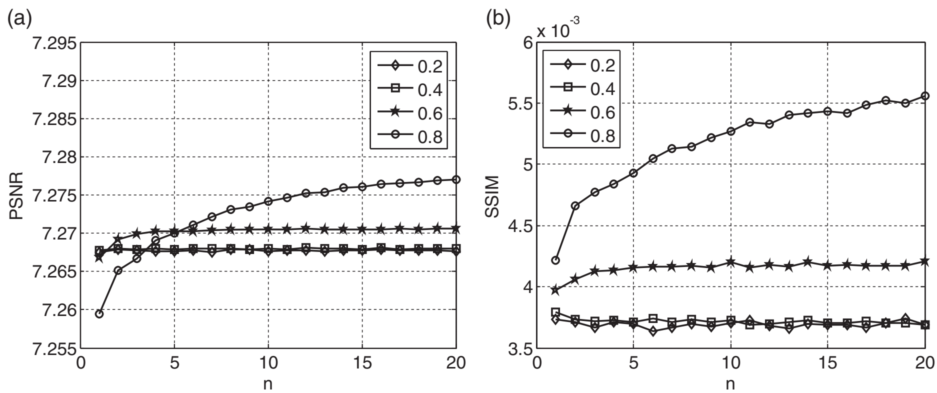

Figure 2. Different noise density impact on PSNR and SSIM: (a) PSNR and (b) SSIM.

Table 1. Comparison results.

Method \ Noise	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
ODSR	7.267130	7.267624	7.267951	7.267993	**7.268084**	**7.265582**	**7.263823**	**7.263544**	**7.254852**
ODASR	7.267381	7.267567	7.267715	7.267951	**7.268471**	**7.270404**	**7.273919**	**7.274968**	**7.265114**

From Table 1 we can see that the PSNRs of ODSR and ODASR are very similar when the noise density is lower than 0.5. However, when the noise density exceeds 0.5, the advantage of ODASR is significant.

To further illustrate the trend of PSNR as the noise density increases, following Figure 3 is given. Generally speaking, when the noise density is higher than 0.5, the PSNR change trend of ODSR is decreasing but that of ODASR exhibits rising and then falling. Anyway, the performance of ODASR is better than ODSR in indicator PSNR.

The performance on SSIM

The SSIM is the structural similarity. The value tends to 1 means the two images are becoming more and more like each other. The Table 2 is the comparison results of ODSR and ODASR in indictor SSIM. It demonstrates that when the noise density is higher than 0.5, the ODASR has obvious advantage. Below this value, the two have similar performance.

To reveal the change trend of the SSIM as the noise density increases, we draw Figure 4. It is obvious that no matter ODSR or MOASR, the change trend of SSIM is increasing as the noise density increases. The ODASR has obvious advantage when the density is larger than 0.5.

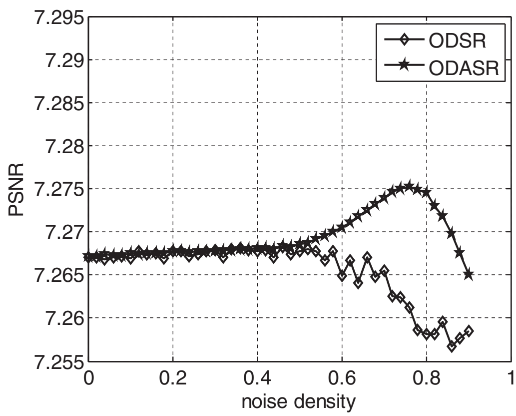

Figure 3. PSNR change trend as the noise increases.

Table 2. Comparison results.

Method \ Noise	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
ODSR	0.003730	0.003699	0.003736	0.003796	**0.003841**	**0.004032**	**0.004035**	**0.004408**	**0.004557**
ODASR	0.003691	0.003715	0.003672	0.003733	**0.003847**	**0.004184**	**0.004802**	**0.005324**	**0.005096**

Two-dimensional average stochastic resonance enhancement

In this section we will firstly describe the two-dimensional stochastic resonance method which has been proposed in other researcher's works. It can be considered as an extension of ODSR. The symbols are defined as that in the One-dimensional average stochastic resonance enhancement section.

Suppose that the input two-dimensional noised image I_{noise} is of size $\mathrm{m} \times \mathrm{n}$, and a nonlinear bistable stochastic resonated system $U(x)$ can be expressed as equation (4). The traditional two-dimensional stochastic resonance enhancement method (TDSR) is composed of following steps.

Step 1. Transform the noised image I_{noise} into one-dimensional signal $I_{noise}^{one-\dim}$ in row with size $1 \times \mathrm{mn}$.
Step 2. Put the one-dimensional noised signal into the resonated system $U(x)$ and add proper noise w_i to drive the resonant system to output one-dimensional resonated signal $I_r^{one-\dim}$.
Step 3. Transform the output one-dimensional signal $I_r^{one-\dim}$ into two-dimensional image.
Step 4. Transform the recovered image into one-dimensional signal in column with size $\mathrm{mn} \times 1$.
Step 5. Put the one-dimensional noised signal into the resonated system $U(x)$ and add proper noise to drive the resonant system again to output resonated signal $I_{rc}^{two-\dim}$.
Step 6. Transform the signal $I_{rc}^{two-\dim}$ into a two-dimensional image as the final output I_{output}.

Above description indicates that in fact the traditional two-dimensional stochastic resonance method firstly processes the noised image in row and then in

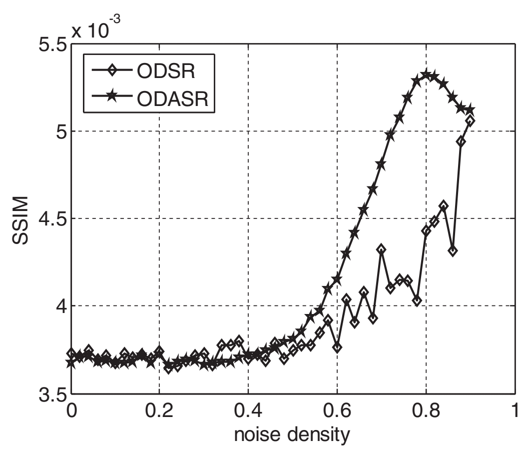

Figure 4. SSIM change trend.

column. Experiments illustrate that this method is effective in enhancing a noised image. Next, based on traditional two-dimensional stochastic resonance, a two-dimensional average stochastic resonance (TDASR) method is proposed as follows.

The **step 1** and **step 2** are the same as TDSR.

Step 3. Repeat step 2 for n times and obtain a set of one-dimensional signals resonated with different independent random noise i.e. $I_r^{one-\dim} = \{I_{r1}^{one-\dim}, I_{r2}^{one-\dim}, \ldots, I_{rn}^{one-\dim}\}$. Here we suppose the n noise constitute a set $W = \{w_i, w_2, \ldots, w_n\}$.
Step 4. Transform each of the one-dimensional signals $I_r^{one-\dim} = \{I_{r1}^{one-\dim}, I_{r2}^{one-\dim}, \ldots, I_{rn}^{one-\dim}\}$ into image $I = \{I_1, I_2 \ldots I_n\}$. And then transform sequence I into one-dimensional signals in column, and put it into the bistable stochastic resonance system $U(x)$. The outputs are denoted as $I_{rc}^{one-\dim} = \{I_{rc1}^{one-\dim}, I_{rc2}^{one-\dim}, \ldots, I_{rcn}^{one-\dim}\}$.
Step 5. Transform one-dimensional signal sequence $I_{rc}^{one-\dim} = \{I_{rc1}^{one-\dim}, I_{rc2}^{one-\dim}, \ldots, I_{rcn}^{one-\dim}\}$ into two-dimensional images $I_{IMAGES} = \{I_1^{two-\dim}, I_2^{two-\dim}, \ldots, I_n^{two-\dim}\}$.
Step 6. Compute $I_{output} = \frac{1}{n}\sum_{i=1}^{n} I_i^{two-\dim}$ and process the I_{output} with adaptive equalization algorithm as the final enhancement image.

To exhibit the effect of TDASR, we process the noised 'lina.jpg' by TDSR and TDASR respectively and the results are in Figure 5.

Figure 5 shows that the methods TDAR and TDASR both blur the image. Of cause, we intuitively feel that the TDASR i.e. Figure 5(d) is better than (c) not matter to PSNR or SSIM.

Performance analysis

In this section, we will mainly discuss the repeat number n and two indicators PSNR and SSIM of two-dimensional stochastic resonance.

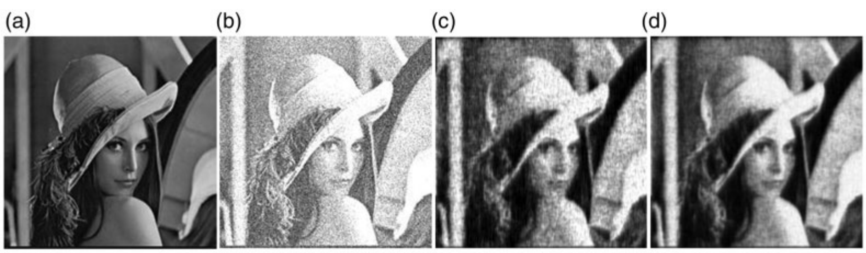

Figure 5. Resonated results: (a) original image, (b) noise added image (noise density 0.4), (c) TDSR, and (d) TDASR.

Number of repeats

Figure 6 indicates that the PSNR is stable as the repeat number n increases when the noise density is lower than 0.7. When the noise density is larger than 0.7, for example density is 0.8 as shown in Figure 6, the PNSR is improved as the repeat number n increases. The change trend of SSIM is similar to that of PSNR. Without loss of generality, we take $n = 10$.

Performance on PSNR

To better exhibit the performance of the TDSR and TDASR, we generate following Table 3. It demonstrates that the PSNR of TDASR is higher than that of TDSR when the noise density is above 0.6.

Figure 7 shows the PSNR change trend of TDASR and TDSR. It illustrates that the PSNR of TDASR is higher than that of TDSR, especially when the noise density is larger than 0.6. Furthermore, the PSNR value of TDASR first rise and then fall, and that of RDSR is steadily dropping when the noise density is above 0.6.

In the performance analysis section which is given below the One-dimensional average stochastic resonance enhancement section, we have discussed the performance of ODSR and ODASR. And then we will put the four methods together (i.e. ODSR, ODASR, TDSR and TDASR) to compare their performance and

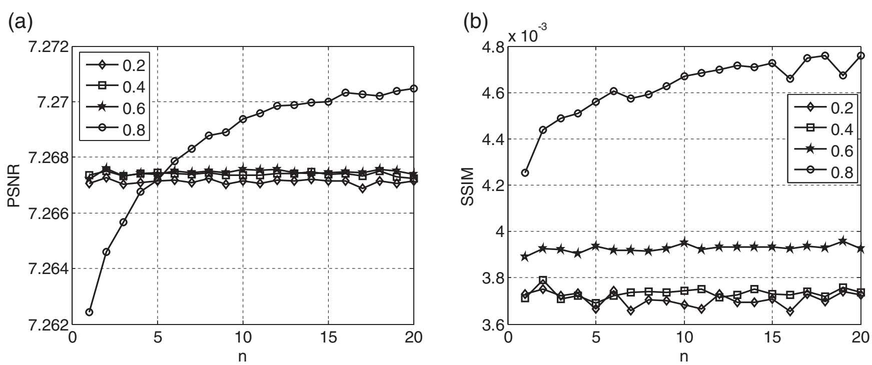

Figure 6. Different noise density impact on PSNR and SSIM: (a) PSNR and (b) SSIM.

Table 3. Comparison result.

Method \ Noise	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0 9
TDSR	7.266696	7.267098	7.267084	7.267267	7.267255	**7.266972**	**7.265928**	**7.262277**	**7.258609**
TDASR	7.266929	7.267031	7.267256	7.267476	7.267409	**7.267467**	**7.267914**	**7.269256**	**7.265954**

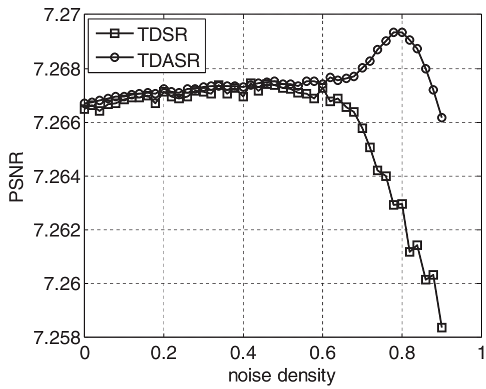

Figure 7. PSNR change trend of TDSR and TDASR.

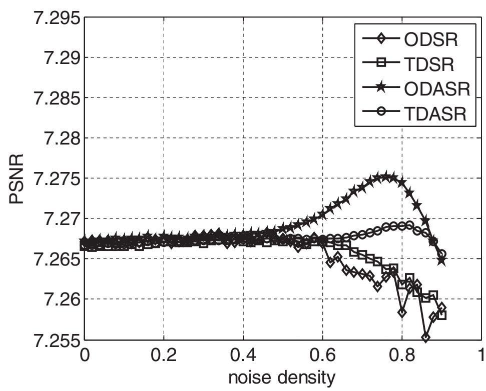

Figure 8. PSNR of the four methods.

illustrate their PNSR change trend. Figure 8 illustrates that the ODASR outperforms the other three and the TDASR is better than TDSR and ODSR.

Performance on SSIM

Table 4 exhibits the comparison results of TDSR and TDASR. It illustrates that the TDASR has obvious advantage when the noise density is above 0.5.

Figure 9 is the SSIM change trend as the noise density increases. From this figure, we can see that the general trend is rising and the TDASR is better than TDSR when the noise density is larger than 0.5 as we have discussed above.

The SSIM change trend of the four stochastic resonance methods (i.e. ODSR, ODASR, TDSR and TDASR) is illustrated as following Figure 10. The SSIM general trend of the four methods are all go up and the performance of two average

Table 4. Comparison results.

Method \ Noise	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0 9
TDSR	0.003707	0.003674	0.003710	0.003734	**0.003788**	**0.003891**	**0.004091**	**0.004265**	**0.004470**
TDASR	0.003728	0.003717	0.003723	0.003705	**0.003786**	**0.003941**	**0.004148**	**0.004659**	**0.004790**

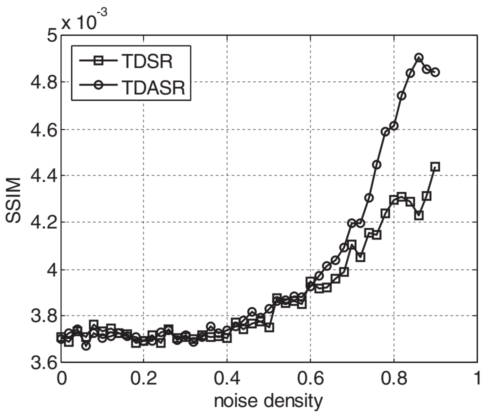

Figure 9. SSIM trend as the noise increases.

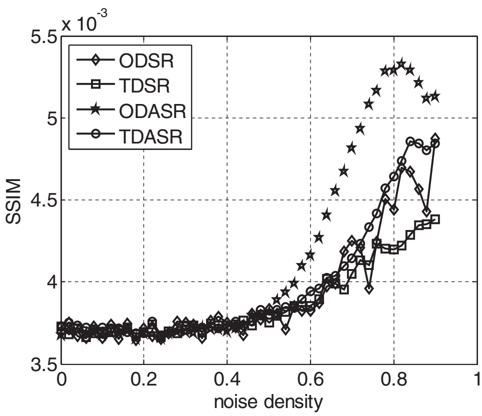

Figure 10. SSIM trend of four resonance methods.

based methods, that is ODASR and TDASR, is better than the others. As we have observed in above Tables, when the noise density is lower than 0.6, the performance of the four methods are similar, but when the density is higher than 0.6, the advantage of OTASR is outstanding.

Figure 11. Four enhancement methods: (a) noise added image (noise density 0.4) (b) median filter, (c) Wiener filter, (d) ODASR and (e) TDASR.

Table 5. PSNR of the four processing methods.

Indicator \ Method	medfilter	wienerfilter	ODASR	TDASR	Noise density
PSNR	8.224836	8.335276	14.698261	14.101312	0.4
PSNR	4.079466	4.208081	13.648746	12.688589	0.8

Comparison results

This part will compare the ODASR and TDASR with other filters, such as median filtering and Wiener filtering. Figure 11 is the processing results and the 'lina.jpg' is added Gaussian noise with density 0.4. From the subjective point of view, the ODASR is the best. Although the output image processed by TDASR looks well, it is obviously blurry.

To demonstrate the efficiency of the proposed methods ODASR and TDASR, we use the four methods to process the image 'lina.jpg' and generate the following Table 5. It exhibits that no matter the noise density is 0.4 or 0.8 the ODASR is the best among the four methods.

Conclusions

Different from traditional image enhancement methods, the stochastic resonance uses noise as power to drive a stochastic resonance system to enhance the image information. Since the image information is enhanced, the quality of the image is improved. From the objective point of view, some indicators of image such as PSNR and SSIM are improved. In this paper, we introduce the cumulative gain, and generate an image sequence from the one noised image using bistable stochastic resonance system. The one-dimensional average stochastic resonance (ODASR) and two-dimensional average stochastic resonance (TDASR) methods are discussed from the subjective point of view. Experiments illustrate that these two methods outperform some traditional stochastic resonance filters, such as median and wiener filters. Furthermore, the ODASR is better than TDASR.

Declaration of Conflicting Interests

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