A Review of Parameter-Induced Stochastic Resonance and Current Applications in Two-Dimensional Image Processing

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Abstract. Stochastic resonance (SR) is a phenomenon that nonlinear system synchronizes with noise to boost a resonant-like behavior. Parameter-induced stochastic resonance (PSR) proposed in this paper is realized by optimally tuning system parameters without adding any noise. It has been proved the performance of PSR is better than traditional SR technique which is in fact a particular case in PSR region. The applications of PSR to signal processing and target detection in shallow water reverberation have been reviewed. A new concept of two-dimensional parameter-induced stochastic resonance (2D-PSR) and its applications in the restoration of degraded image and pattern recognition of remote sensing image are developed.

Keywords: Stochastic resonance; Nonlinear system; Image processing.

1 Review of Parameter-Induced Stochastic Resonance

Stochastic Resonance (SR) is a phenomenon manifested in nonlinear systems in which generally feeble input information, such as a weak signal, can be amplified and optimized by the presence of noise. This concept was first proposed by Benzi in 1981 [1], addressing the problem of the periodically recurrent ice ages. The effect requires three basic ingredients:

1. An energetic activation barrier or, more generally, a form of threshold;
2. A weak coherent input, such as a periodic signal;
3. A source of noise that is inherent in the system or that adds to the coherent input.

Given these features, the response of the system undergoes resonant-like behavior as a function of the noise level, hence the name Stochastic Resonance. Over the last few decades, Stochastic Resonance has continuously attracted considerable attention, and has been applied to large variety of fields, including physics, chemistry, biomedical sciences and engineering systems.

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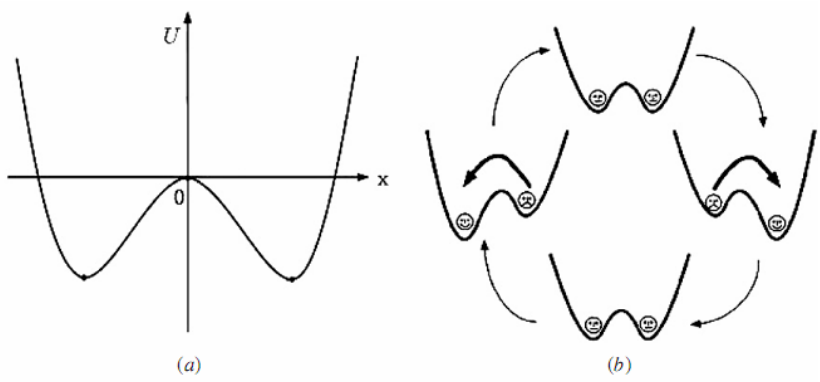

Fig. 1 (a) Symmetric double-well potential. (b) Double-well potential under periodic weak forcing.

In order to introduce the mechanism of stochastic resonance, let us consider a heavily damped particle of mass m and viscous friction γ moving in a symmetric double-well potential $U(x)$ (see Fig. 1(a)). If we apply a weak periodic forcing to the particle, the double-well potential would be titled asymmetrically up and down, periodically raising and lowering the potential barrier, as shown in Fig 1(b). Generally, the weak force itself cannot cause the particle to shift between the two wells. However, if the system is interfered by noise with certain intensity which is synchronized with the periodic force, the particle will roll between the two wells in accordance with the periodic force. This synchronization is called the stochastic resonance.

Conventional SR is realized by adding an optimal amount of noise into the system. However, this method has some shortcomings. For example, in most situations, the input signal is corrupted by noise with a given quantity. So it is easy to increase the input noise but impossible to decrease it. If the input signal is already suprathreshold, Conventional SR can do nothing with it. Recently, a new approach called Parameter-induced Stochastic Resonance (PSR) is proposed to realize the SR effect by optimally tuning the system parameters. Unlike Conventional SR, PSR which in fact tunes the height of the barrier to affect the system's response speed can occur both in subthreshold case and in suprothreshold case. Moreover, it has been proved that SR realized by conventional method (adding noise) is an optimization problem in a subregion of the parameter space [2]. Parameter-induced Stochastic Resonance has been successfully applied to many fields including one-dimensional (1D) digital signal and analog signal processing, target detection in shallow-water reverberation, two-dimensional (2D) image processing and etc.

In one-dimensional case, the theory of Parameter-induced Stochastic Resonance and the principle of parameter optimization have been founded. It is proved that the low SNR baseband binary pulse amplitude modulated (PAM) signals can get a high SNR gain after being processed by PSR technique with optimally tuned parameters. Besides, a marked enhancement of the channel capacity for binary PAM signal is achieved [3]. Since the PSR system is double-well and nonlinear, if we process analog signal directly by PSR technique, strong distortion may occur in outputs. To overcome this problem, a simple but effective nonlinear inversion method is developed to remove the distortion, which makes it possible to tackle with analog signal using PSR technique [4]. The target detection with active sonar is often limited by the presence of reverberation which is highly correlated with the transmitted signal. Using PSR-based technique, the detection performance can be optimized by tuning system parameters. Theoretical and experimental results have shown that PSR technique performs well under the condition of weak signal-to-reverberation-noise ratio [5]. Image processing is another important field to which SR technique can be applied [6]. Based on PSR theory, we propose a two-dimensional parameter-induced stochastic resonance (2D-PSR) system with parameters a, b to be adjustable. It will be proved in this paper that the SNR gain of 2D-PSR system can surpass one which is impossible for linear filtering [7].

This paper is organized as follows. In Section 2, we describe the 2D-PSR system and its output probability density function which is derived by solving corresponding Fokker-Planck Equation. In Section 3, we will introduce how to use theories discussed in Section 2 to improve SNR gain. Experimental results of degraded image processing and an application to pattern recognition of remote sensing image by our proposed technique are presented in Section 4 and Section 5. Finally, we draw the conclusions of 2D-PSR technique and its prospect in Section 6.

2 2D-PSR System and the Corresponding Fokker-Planck Equation

Similar to one-dimensional case, we propose the following two-dimensional parameter-induced stochastic resonance system

$$\frac{\partial w}{\partial x}+\frac{\partial w}{\partial y}=a\cdot w-b\cdot w^{3}+h+\Gamma(x,y) \tag{1}$$

where $w=w(x,y)$ is the state variable (also taken as the system output), a, b are system parameters to be adjustable, h is the original signal and $\Gamma(x,y)$ is the Gaussian white noise with $\langle \Gamma(x,y),\Gamma(x_1,y_1)\rangle = 2D\cdot\delta(x-x_1,y-y_1)$. Here D is the noise intensity, which is related to noise variance σ^2 by $\sigma^2=\dfrac{2\cdot D}{\Delta x\cdot\Delta y}$ in two-dimensional case, where $\Delta x, \Delta y$ are the sampling intervals along horizontal and vertical directions. According to the characteristic method in Partial Differential Equation (PDE) theory [8], Equation (1) is equivalent to a set of Ordinary Differential Equations (ODE)

$$\frac{dx}{1}=\frac{dy}{1}=\frac{dw}{aw-bw^3+h+\Gamma} \tag{2}$$

The characteristic line is $\frac{dy}{dx}=1$ or $y=x+C$ with C being the constant, which indicates that in any arbitrarily small region, the solution of Eq. (1) is symmetric along the diagonal direction. Thus we can solve Eq. (1) by independently dealing with Eq. (2)

$$\frac{dw}{dx}=aw-bw^3+h(x)+\Gamma(x) \tag{3-1}$$

$$\frac{dw}{dy}=aw-bw^3+h(y)+\Gamma(y) \tag{3-2}$$

The corresponding Fokker-Planck Equation (FPE) for Eq. (3-1) is

$$\frac{\partial\rho(w,x)}{\partial x}=-\frac{\partial}{\partial x}\left[f(w)\cdot\rho(w,x)\right]+\frac{D}{\Delta x}\cdot\frac{\partial^2\rho(w,x)}{\partial w^2} \tag{4}$$

where $\rho(w,x)$ is the output probability density function (PDF). When $x\to\infty$ we can obtain the static PDF of Eq. (4)

$$\rho(w,x\to\infty)=\rho_s(w)=N\cdot\exp[\varphi_0(w)]=N\cdot\exp\left[\int_{-\infty}^{+\infty}\Delta x\cdot\frac{aw-bw^3+h}{D}dw\right] \tag{5}$$

with N to be the normalized factor. The asymptotic dynamic PDF of Eq. (4) is [3]

$$\rho(w,x)=\sum_{i=0}^{n-1}N_i\cdot\Phi_i(w)\cdot\exp(-\lambda_i\cdot x)+\left[\rho_s(w)-\sum_{i=0}^{n-1}N_i\cdot\Phi_i(w)\right]\cdot\exp(-\lambda_n\cdot x) \tag{6}$$

where $0=\lambda_0<\lambda_1<...<\lambda_n$ and $\Phi_0=\exp[\varphi_0(w)],\Phi_1,...,\Phi_n$ are the n orders eigenvalues and eigenfunctions of Eq. (3-1), N_i is the constant to be determined by orthodonal condition of eigenfunctions [3]. λ_1 which is the dominant factor of system's settling down to steady state is regarded as the system response speed.

The static and dynamic PDF of Eq. (3-2) can be derived similarly.

3 Theory of 2D-PSR for Degraded Image Processing

Previously we set λ_1 to be around 3 so that the error between dynamic and static PDF is within $e^{-3} \approx 5\%$. However this constrained condition will restrict our choice for parameters a, b. Therefore we introduce the concept of dynamic signal-to-noise ratio (DSNR)

$$DSNR(a,b) = \frac{E[w]}{\sqrt{Var[w]}} \tag{7}$$

where $E[\bullet]$ is an expectation operator and $Var[\bullet] = E[\bullet^2] - E^2[\bullet]$. Without the restriction of $\lambda_1 \approx 3$, the system response speed can either be higher or lower. Consequently the valid sample points of the output will vary following the value of λ_1. Previously, when processing a noisy image through 2D-PSR system, we just directly pick up the last sample point of each sample period [6], which is assumed to have the best statistic performance in the means of probability density. According to the dynamic solution, when the system response speed grows faster, the output performance will decrease, however, there will be more sample points valid to be considered. If these available sample points are averaged, we will get a better result. The statistic characteristics of averaged outputs can be calculated by the theory of local average random field [9]. Assume $w(x)$ is a random field with expectation m and variance σ^2. $W_X(x)$ is length average of $w(x)$ over a period X. Here $W_X(x)$ is called the local average random field, which has expectation and variance

$$\begin{aligned} E[W_X(x)] &= m = \int w \cdot \rho(w,x) dw \\ Var[W_X(x)] &= \Omega(X) \cdot \sigma^2 \end{aligned} \tag{8}$$

where $\Omega(X)$ is called the variance function of $W_X(x)$. Let $\rho(\xi) = Cov(\xi) / \sigma^2$ be the normalized correlation function of $w(x)$. The relationship between $\Omega(X)$ and $\rho(\xi)$ can be described as follows

$$\Omega(X) = \frac{1}{X^2} \int_0^X \int_0^X \rho(x_1 - x_2) dx_1 dx_2 = \frac{2}{X} \int_0^X \left(1 - \frac{\xi}{X}\right) \rho(\xi) d\xi \tag{9}$$

In order to obtain variance function $\Omega(X)$ from Eq. (9), we have to figure out the second-order statistic characteristics of $w(x)$ .Thus we rewrite Eq. (4) as

$$\frac{\partial \rho(w,x|w_0,x_0)}{\partial x} = -\frac{\partial}{\partial x}\left[f(w) \cdot \rho(w,x|w_0,x_0)\right] + \frac{D}{\Delta x} \cdot \frac{\partial^2 \rho(w,x|w_0,x_0)}{\partial w^2} \tag{10}$$

where $\rho(w,x|w_0,x_0)$ is the conditional PDF with $\rho(w,x_0|w_0,x_0)=\delta(w-w_0)$ and $\lim_{x\to\infty}\rho(w,x|w_0,x_0)=\rho(w,x)$. The first-order approximate solution of Eq. (10) is

$$\rho(w,x|w_0,x_0)=\rho(w,x)+[\delta(w-w_0)-\rho(w,x)]\cdot\exp[-\lambda_1\cdot(x-x_0)] \tag{11}$$

with λ_1 the system response speed. Thus

$$\Omega(X)=\frac{2}{X}\int_0^X\left(1-\frac{\xi}{X}\right)\exp(-\lambda_1\xi)d\xi \tag{12}$$

Replacing $E[w],Var[w]$ in Eq. (7) with Eqs. (8) and (12) we obtain

$$DSNR(a,b)=\frac{\int w\cdot\rho(w,x)dw}{\frac{2\cdot\sigma^2}{X}\int_0^X\left(1-\frac{\xi}{X}\right)\exp(-\lambda_1\xi)d\xi} \tag{13}$$

Here X is determined as follows. Define an allowance error: $\exp(-\lambda_1\cdot\xi)=err$ $\Rightarrow\xi=-\frac{1}{\lambda_1}\ln err$. If $\xi\ge X_b$ (here X_b means each length period), we directly use the last sample point to calculate the DSNR. If $\xi<X_b$, we take $X=X_b-\xi$. Our goal is to maximize DSNR by optimizing system parameters a,b , which can be solved by gradient descent algorithm very efficiently.

It has been proved that the SNR gain of linear systems cannot surpass one [7]. We will prove that the SNR gain of 2D-PSR system can outstrip one. The input SNR in our case is $SNR_{input}=h/\sqrt{2\cdot D}$ with h the original image and D the noise intensity. The SNR gain can be written as

$$SNR_{gain}=\frac{DSNR(a,b)}{SNR_{input}}=\frac{\sqrt{2\cdot D}\cdot\int w\cdot\rho(w,x)dw}{2h\cdot\sigma^2/X\int_0^X\left(1-\frac{\xi}{X}\right)\exp(-\lambda_1\xi)d\xi} \tag{14}$$

Fig. 2 shows the SNR gain as a function of parameter b with $a=1.5$. It indicates when system parameters a,b are properly set, the SNR gain will surpass one ,which is impossible for linear systems. In addition, the SNR gain remains high and descending slowly as b grows larger, which means 2D-PSR technique performs strong robustness to system parameters variations. If the parameters are not best optimized or even biased seriously, we can still obtain high SNR gain.

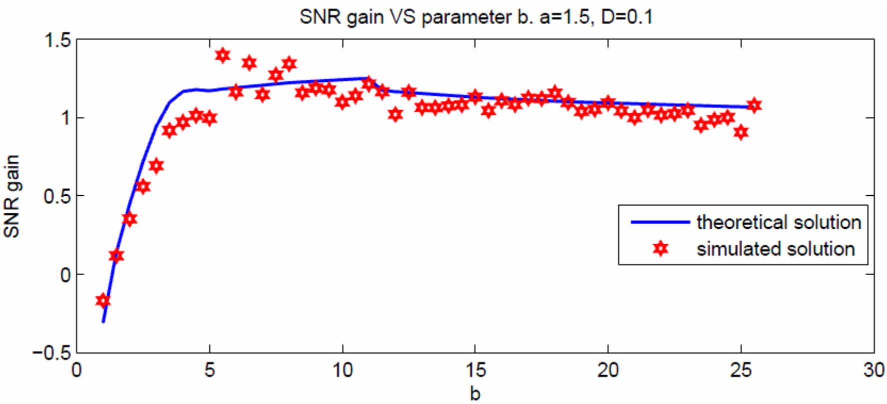

Fig. 2 SNR gain as a function of parameter b

4 Example of 2D-PSR Image Restoration

The simulation equation equivalent to Equation (3) is

$$\begin{cases} w_{m,n} = \left[aw_{m,n-1} - bw_{m,n-1}^3 + h_{m,n-1} + \Gamma_{m,n-1}\right]\cdot \Delta x + w_{m,n-1} \\ w_{m,n} = \left[aw_{m-1,n} - bw_{m-1,n}^3 + h_{m-1,n} + \Gamma_{m-1,n}\right]\cdot \Delta y + w_{m-1,n} \end{cases} \tag{15}$$

where the subscripts represent the locations of sample points, and $\Delta x, \Delta y$ are the sampling intervals along horizontal and vertical directions.

The intensity value of an image usually distributes in the region of [0, 255]. However, the double-well potential of 2D-PSR system is symmetric to zero. Thus we should first subtract the mean value of an image before processing with 2D-PSR technique and add it back later. Fig. 3(a) shows a computed tomography (CT) image corrupted by additive Gaussian white noise $N(0,57)$ (Fig. 3(b)). We sample the degraded image 5×5 times per pixel (five by row and five by column), and operate it on 2D-PSR system. After optimizing parameters a, b according to Eq. (13) we come up with the recovered image Fig. 3(c). Fig. 3(d) shows the result obtained by linear mean filtering, which takes the average value of every 5×5 blocks of Fig. 3(b). As a comparison, we have further processed Fig. 3(b) with total variation method [10], wavelet de-noising [11] and adaptive Wiener filtering [12]. The results are shown in Figs. 3(e), (f) and (g) respectively.

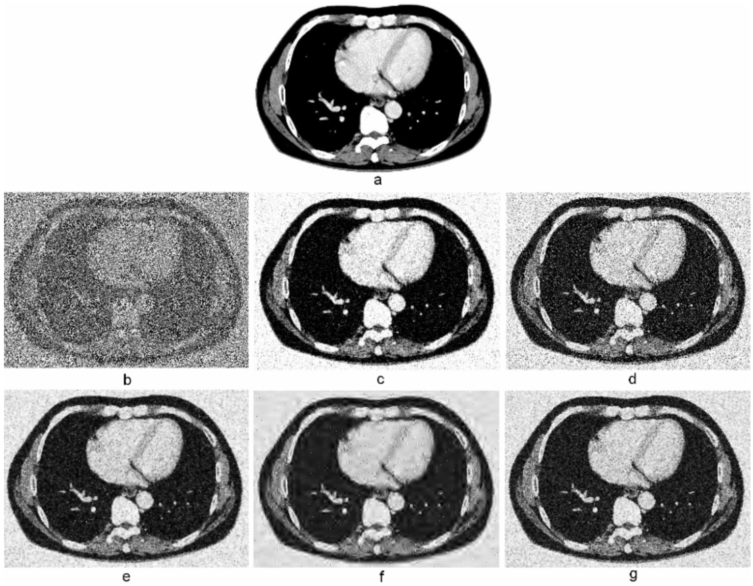

Fig. 3 (a) Original CT image. (b) Sampled CT image degraded by additive Gaussian white noise. (c) Image processed by 2D-PSR technique. (d)Image processed by mean filtering. (e) Image processed by total variation method. (f) Image processed by wavelet de-noising. (g) Image processed by adaptive Wiener filtering.

5 Example of 2D-PSR Pattern Recognition of Remote Sensing Image

The remote sensing multi-spectral image is a set of registered images with different bands of wavelength ranging from visible light to infrared. Fig. 4 shows a set of multi-spectral images of bands blue, green, red and infrared. Our goal is to classify rivers, buildings and forests from the multi-spectral images using 2D-PSR technique. To this end, we rewrite 2D-PSR equation in vector's form

$$\frac{\partial \mathbf{w}}{\partial \mathrm{x}}+\frac{\partial \mathbf{w}}{\partial y}=a\mathbf{w}-b\mathbf{w}^{3}+\mathrm{h}+\Gamma(x,y) \tag{16}$$

where $\mathtt{w}=[w_1,w_2,...,w_N]^T$, $\mathrm{h}=[h_1,h_2,...,h_N]^T$, $\Gamma=[\Gamma_1,\Gamma_2,...,\Gamma_N]^T$, N is the number of registered images. The static PDF of corresponding FPE in the horizontal direction is

$$\rho(\mathtt{w})=\mathtt{N}\cdot\exp\left(\frac{1/2\cdot a\mathtt{w}^2-1/4\cdot b\mathtt{w}^4+[h_1w_1,h_2w_2,...,h_Nw_N]^T}{D}\right) \quad , \tag{17}$$

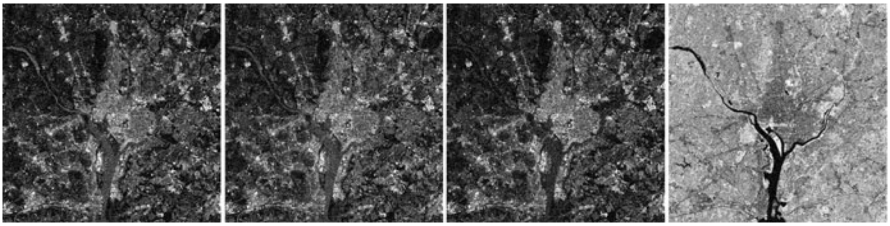

Fig. 4 Multi-spectral images with blue, green, red and infrared bands

Assume that the noise in the registered images is uncorrelated, thus $\rho(\mathtt{w}) = \prod_{i=1}^{N} \rho(w_i)$ where $\rho(w_i) = N_i \cdot \exp\left[\left(1/2 \cdot a w_i^2 - 1/4 \cdot b w_i^4 + h_i w_i\right)/D\right]$. The dynamic PDF in the horizontal direction is

$$\rho(\mathtt{w}, x) = \prod_{i=1}^{N} \rho(w_i, x) \tag{18}$$

where $\rho(w_i, x) = \sum_{j=0}^{K-1} N_j \Phi_j(w) \exp(-\lambda_j x) + \left[\rho(w_i) - \sum_{j=0}^{K-1} N_j \Phi_j(w_i)\right] \exp(-\lambda_K x)$.

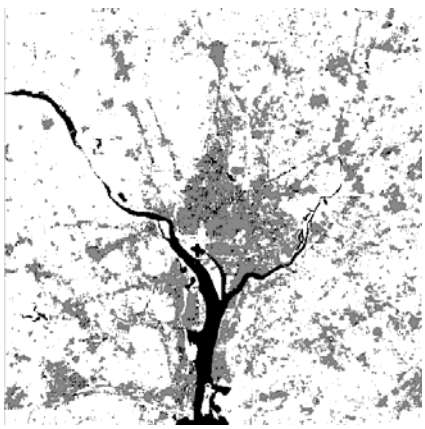

Fig. 5 Classified image by 2D-PSR technique with black, grey and white representing rivers buildings and forests respectively

The decision function of 2D-PSR based classifier is

$$D_k(\mathtt{w}) = \ln(d_k(\mathtt{w})) = \ln\left[\rho(\mathtt{w}, x | \omega_k) \cdot P(\omega_k)\right] = \ln \rho(\mathtt{w}, x | \omega_k) + \ln P(\omega_k) \tag{19}$$

where ω_k is the number of classes (such as rivers, buildings and forests) and $P(\omega_k)$ is the probability of ω_k. In pattern recognition of remote sensing image, each class ω_k is determined by the input value h_k (for example the class of rivers buildings and forests can be classified by their grey levels). Thus we can replace ω_k in Eq. (19) by h_k. The final decision is

$$D(\mathtt{w}_{i,j}) = \left\{ k \middle| \max[D_k(\mathtt{w}_{i,j})], k = 1,2,...M \right\} \tag{20}$$

Fig 5 shows the classified result by 2D-PSR based pattern recognition.

6 Conclusions

In this paper, the researches on PSR technique and its applications in signal processing and target detection in shallow water reverberation have been reviewed. A current developed 2D-PSR technique is proposed. The corresponding static and dynamic PDF of the FPE is derived. A concept of dynamic signal-to-noise ratio (DSNR) is introduced to utilize more valid sample points to upgrade the output performance. It has been proved 2D-PSR technique can have SNR gain greater than one which is impossible in linear cases. Examples of 2D-PSR based image restoration and pattern recognition of remote sensing image have shown 2D-PSR technique to be promising in the field of image processing.

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