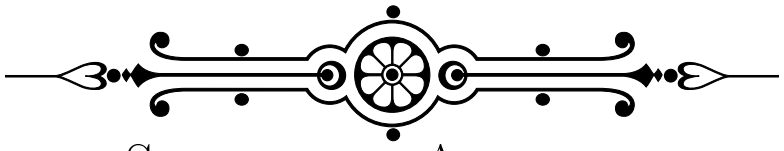


SURVEY OF ALGEBRA

MATH 320

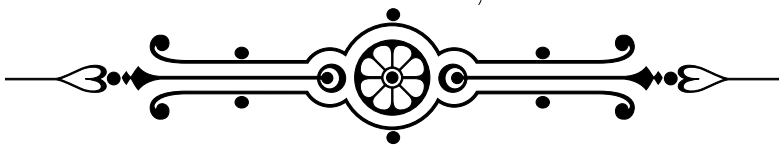
Dr. Alia Hamieh

# Assignment 6

*Deepak Jassal*

**Due Date:**

November 20$^{\text{th}}$, 2025



# Question 1 [2 marks]

What is the order of the element $14 + \langle 8 \rangle$ in the factor group $\mathbb{Z}_{24}/\langle 8 \rangle$?

# Question 2 [2 marks]

Explain why the correspondence $x \mapsto 3x$ from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{10}$ is not a homomorphism.

# Question 3 [2 marks]

Let $H$ be a normal subgroup of a finite group $G$, and let $a$ belong to $G$. If the element $aH$ has order 3 in the group $G/H$ and $|H| = 10$, what are the possibilities for the order of $a$ in $G$?

# Question 4 [3 marks]

Prove that a factor group of a cyclic group is cyclic.

# Question 5 [3 marks]

Let $H$ and $K$ be normal subgroups of a group $G$. Prove that $HK$ is also a normal subgroup of $G$.

# Question 6 [3 marks]

Let $G$ be a group acting on a set $X$. Suppose that the stabilizer $G_x$ of a certain point $x \in X$ is a proper normal subgroup of $G$. Prove that every element of $G_x$ fixes every element $y \in G_x$.

# Question 7 [5 marks]

In what follows, you prove the third isomorphism theorem. Let $M, N$ be normal sub-groups of a group $G$ such that $N$ is a subgroup of $M$.

(a) Show that $N$ is a normal subgroup of $M$.

(b) Show that $(G/N)/(M/N) \cong G/M$.