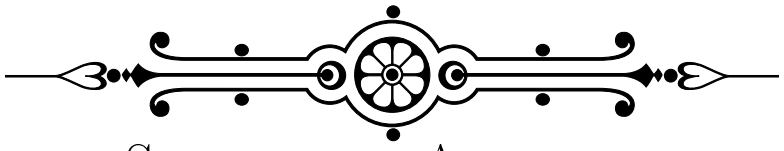


SURVEY OF ALGEBRA

MATH 320

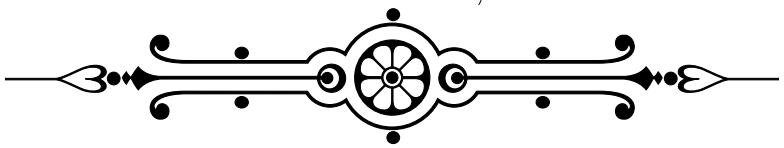
Dr. Alia Hamieh

Assignment 6

Deepak Jassal

Due Date:

November 20$^{\text{th}}$, 2025



Question 1 [2 marks]

What is the order of the element $14 + \langle 8 \rangle$ in the factor group $\mathbb{Z}_{24}/\langle 8 \rangle$?

Question 2 [2 marks]

Explain why the correspondence $x \mapsto 3x$ from $\mathbb{Z}_{12}$ to $\mathbb{Z}_{10}$ is not a homomorphism.

Question 3 [2 marks]

Let H be a normal subgroup of a finite group G, and let a belong to G. If the element aH has order 3 in the group G/H and $|H| = 10$, what are the possibilities for the order of a in G?

Question 4 [3 marks]

Prove that a factor group of a cyclic group is cyclic.

Question 5 [3 marks]

Let H and K be normal subgroups of a group G. Prove that HK is also a normal subgroup of G.

Question 6 [3 marks]

Let G be a group acting on a set X. Suppose that the stabilizer G_x of a certain point $x \in X$ is a proper normal subgroup of G. Prove that every element of G_x fixes every element $y \in G_x$.

Question 7 [5 marks]

In what follows, you prove the third isomorphism theorem. Let M, N be normal sub-groups of a group G such that N is a subgroup of M.

(a) Show that N is a normal subgroup of M.

(b) Show that $(G/N)/(M/N) \cong G/M$.