EQUATIONS OF STRAIGHT LINES ON VARIOUS GRAPH PAPERS

STRAIGHT LINE ON ARITHMETIC GRAPH PAPER   (LINEAR FUNCTION)

EQUATION: $y = mx + b$

$m = \text{slope of line} = \dfrac{\Delta y}{\Delta x}$

b = y-intercept: value of y where the line crosses the x-axis, i.e., value of y when x = 0

The equation of this line is readily gotten from the plot. As shown in the graph to the right, find two points on the line (x_1, y_1) and (x_2, y_2) where you can easily determine the values of x and y.
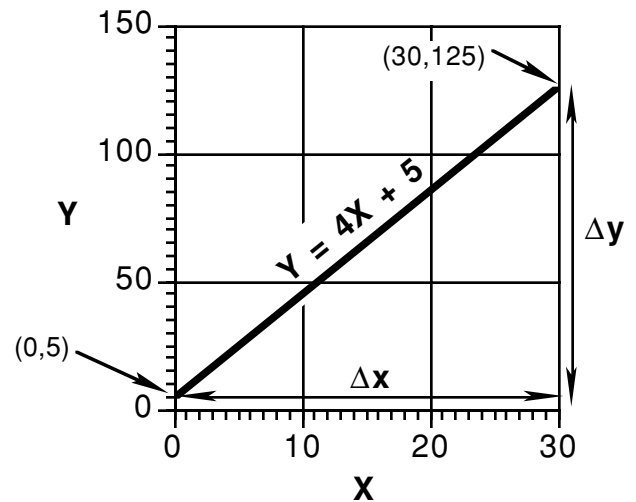
Then

$$\Delta x = x_2 - x_1$$
$$\Delta y = y_2 - y_1$$

Calculate

$$m = \frac{\Delta y}{\Delta x}$$

Read b directly from the graph. It's the value of y where the line crosses the x-axis.

$$\Delta x = x_2 - x_1 = 30 - 0 = 30$$

$$\Delta y = y_2 - y_1 = 125 - 5 = 120$$

$$m = \frac{\Delta y}{\Delta x} = \frac{120}{30} = 4$$

$$b = \text{y-intercept (at x = 0)} = 5$$

BRIEF REVIEW OF LOGARITHMS:

If you have forgotten about logarithms, here is a short refresher:

if $A = 10^c$ then $\log_{10} A = c$

that is, the *logarithm* (to the base 10) of a number is the power to which you must raise 10 to equal that number.

for example: $100 = 10^2$ so $\log 100 = 2$

The *antilog* ($\log^{-1}$) of a number is 10 raised to that power.

for example: $\log^{-1}(3) = 10^3 = 1000$

STRAIGHT LINE ON LOGARITHMIC GRAPH PAPER (POWER FUNCTION)

EQUATION: $\mathbf{y = kx^m}$

or

$\mathbf{\log\ y = m\ \log\ x + \log\ k}$

On logarithmic graph paper:

$m = \text{slope of line} = \dfrac{\Delta(\log y)}{\Delta(\log x)}$

k = value of y where line crosses the x = 1 axis

Power functions have the form $y = kx^m$, where m is any positive or negative constant. If a power function is plotted on arithmetic graph paper, the result is a curved line; that is, the the relationship between x and y is not linear (see graph to right). It is difficult to determine the equation of the line from such a plot.

If however, we take the logarithms of both sides of the equation, we get :

$\log y = m \log x + \log k$

Note that this is the equation of a straight line! That is, the *logarithms* of x and y have a linear relationship. If we wanted to, we could take the logs of all our data points, plot them on arithmetic graph paper, fit a straight line, and determine m and a just as we did before. In this case the y-intercept a = log k, so $k = 10^a$.

An easier method, described on the next page, is to use logarithmic graph paper.

$Y = 2.5\ X^{0.67}$
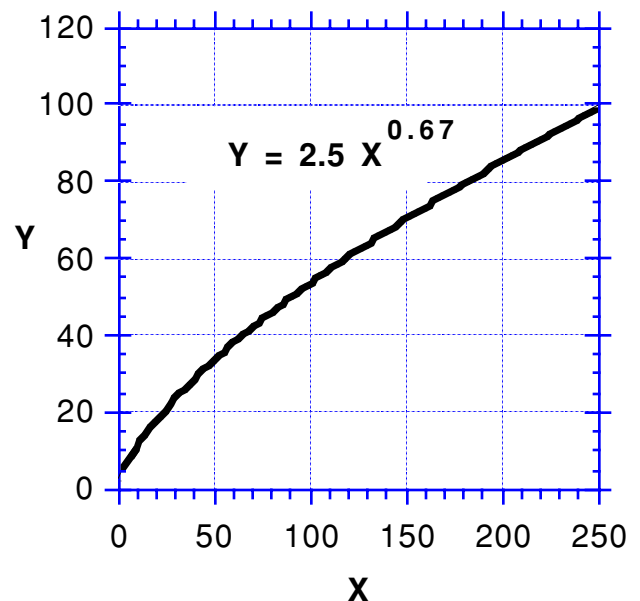
Logarithmic graph paper has both X and Y axes calibrated in log cycles. When you plot a number on one of the axes, you are actually positioning it according to its logarithm.(i.e., the paper takes the log for you; the numbers on the paper are antilogs.) The logarithms themselves are not shown, but are spaced arithmetically along the axes. For example, log 1 = 0; log 10 =1; log 100 =2; in the graph at right, note that there is the same distance between 1 and 10 as there is between 10 and 100. *Note that you can never have 0 on a log axis because log 0 doesn't exist.*

Power functions plot as straight lines on logarithmic graph paper. The slope of the line (m) gives the exponent in the equation, while the value of y where the line crosses the x =1 axis gives us k.

If the log cycles in both x and y directions are the same size (this is usually, but not always the case), the easiest way to determine the slope is to extend the line so it crosses one complete log cycle (i.e., one power of 10). Then simply measure with a ruler the distance between these crossing-points in both X and Y directions, and divide Δy by Δx (see example at right.) To do this properly, you will need a ruler with fairly fine calibrations, such as mm or twentieths of an inch.

Alternatively, you can:

a) find two points (x_1,y_1) and (x_2,y_2) on the line where you can easily determine the values of x and y;

b) take the logs of these numbers and compute $\Delta \log y$ and $\Delta \log x$;

c) divide $\Delta \log y$ by $\Delta \log x$ to get m.

This approach is also shown in the graph at right.

You *must* use this second technique if the log cycles are not the same width on both axes.

$\Delta \textbf{log Y} = 24$ mm
$\Delta \textbf{log X} = 35.5$ mm
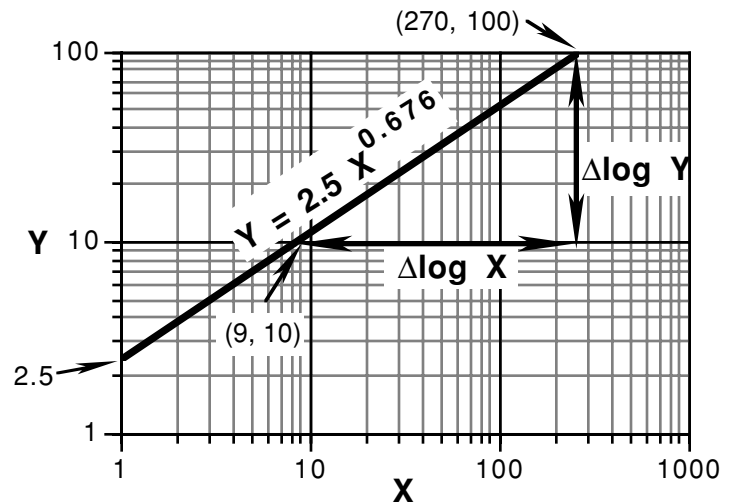
$$m = \frac{\Delta \log Y}{\Delta \log X} = 0.676$$

alternatively:

$\Delta \textbf{log Y} = \log y_2 - \log y_1 = \log 100 - \log 10$
$\qquad = (2 - 1) = 1$

$\Delta \textbf{log X} = \log x_2 - \log x_1 = \log 270 - \log 9$
$\qquad = (2.43 - 0.95) = 1.48$

$$m = \frac{\Delta \log Y}{\Delta \log X} = \frac{1}{1.48} = 0.675$$

y-intercept (at x =1) = 2.5

STRAIGHT LINE ON SEMI-LOGARITHMIC GRAPH PAPER (EXPONENTIAL FUNCTION)

EQUATION: $y = k\ 10^{mx}$ (or $y = ke^{mx}$ note that $e^{2.303} = 10$)

or

$\log y = mx + \log k$ (or $\ln y = mx + \ln k$)

On semi-logarithmic graph paper:

m = slope of line $= \dfrac{\Delta(\log y)}{\Delta x}$

k = y-intercept: value of y where line crosses the $x = 0$ axis

Exponential functions have the form $y = k\ 10^{mx}$, or $y = ke^{mx}$ where m is any positive or negative constant. If an exponential function is plotted on arithmetic graph paper, the result is a curved line; that is, the the relationship between x and y is not linear (see graph to right). It is difficult to determine the equation of the line from such a plot.

If however, we take the logarithms of both sides of the equation, we get :

$\log y = mx + \log k$

Note that this is the equation of a straight line! That is, x and the *logarithms* of y have a linear relationship.
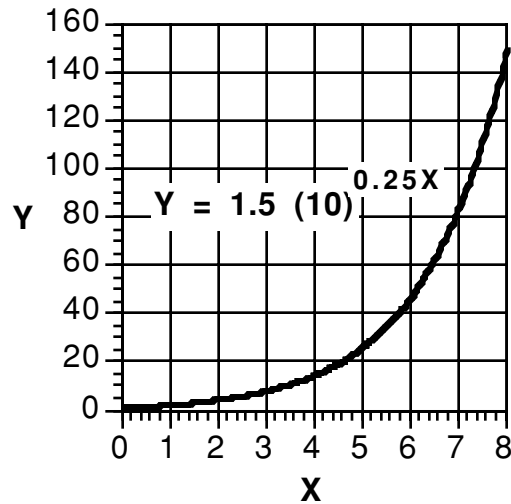

Exponential functions plot on semilog paper as straight lines. Semi-log paper has one arithmetic and one logarithmic axis. The slope of the line (m) gives the exponential constant in the equation, while the value of y where the line crosses the $x = 0$ axis gives us k.

To determine the slope of the line:

a) extend the line so it crosses one complete log cycle (i.e., one power of 10)

b) find the points (x_1, y_1) and (x_2, y_2) on the line where it crosses the top and bottom of a log cycle. $\Delta(\log y)$ will thus $= 1$.

c) determine Δx as $x_2 - x_1$, where x_2 is the x-value corresponding to the top of the log cycle

d) divide Δx into 1 to get m.

This procedure is shown in the graph at right.
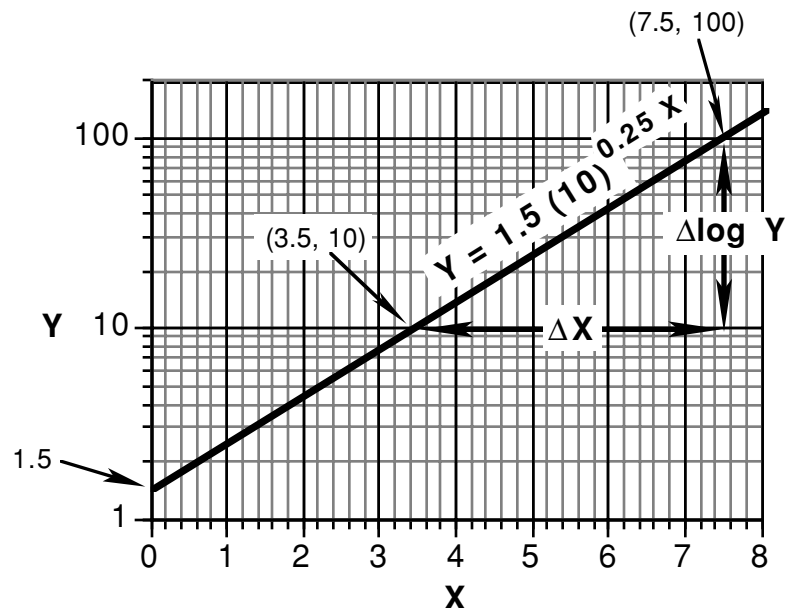
$\log y_2 = \log 100 = 2$ $x_2 = 7.5$

$\log y_1 = \log 10 = 1$ $x_1 = 3.5$

$$m = \frac{\Delta \log y}{\Delta x} = \frac{1}{4} = 0.25$$

k = y-intercept (at $x = 0$) = 1.5