1

denominator^2: $\left[\omega_1(1-u)^2 + 2\cdot\omega_2\cdot u(1-u) + \omega_3 u^2\right]^2$

① $\omega_1^2(1-u)^4$
② $2\omega_1\omega_2\, u\,(1-u)^3$
③ $\omega_1\omega_3\, u^2(1-u)^2$
④ $2\omega_1\omega_2\, u\,(1-u)^3$
⑤ $4\omega_2^2 u^2(1-u)^2$
⑥ $2\omega_2\omega_3\, u^3(1-u)$
⑦ $\omega_1\omega_3\, u^2(1-u)^2$
⑧ $2\omega_2\omega_3\, u^3(1-u)$
⑨ $\omega_3^2\, u^4$

Expansion

$$
\begin{aligned}
(1-u)^4 &= \omega_1^2 \\
u(1-u)^3 &= 4\omega_1\omega_2 \\
u^2(1-u)^2 &= 4\omega_2^2 + 2\omega_1\omega_3 \\
u^3(1-u) &= 4\omega_2\omega_3 \\
u^4 &= \omega_3^2
\end{aligned}
$$

(Test $\omega_1=\omega_3=\sqrt{2}$ $\omega_2=1$

$= 2$
~~$=8\sqrt{2}$~~ $= 4\sqrt{2}$
$= 8$
$= 4\sqrt{2}$
$= 2$

✓

Control Points & Weights

1 : $\omega_0 = 1$; $(1, 0)$

2 : $\omega_1 = ?$; $(s\cos\theta, s\sin\theta)$

3 : $\omega_2 = 1$; $(\cos 2\theta, \sin 2\theta)$

num(x) = $(1-u^2) + 2s\cos\theta\,\omega_1 u(1-u) + u^2\cos 2\theta$

num(y) = $2s\sin\theta\,\omega_1 \, {}^{*}(u(1-u)) + u^2\sin 2\theta$

denom: $(1-u)^2 + 2u(1-u)\omega_1 + u^2$

$\therefore$ denom^2 :

$$(1-u)^4 = 1$$
$$u(1-u)^3 = 4\omega_1$$
$$u^2(1-u)^2 = 4\omega_1^2 + 2$$
$$u^3(1-u)^2 = 4\omega_1$$
$$u^4 = 1$$

$$x^2 = \left[(1-u)^2 + 2s\cdot\cos\theta\omega_1\, u(1-u) + u^2\cos 2\theta\right]^2$$

$$(1-u)^4 = 1$$

$$u(1-u)^3 = 4\left[s\cdot\cos\theta\,\omega_1\right]$$

$$u^2(1-u)^2 = 4\left[s\cdot\cos\theta\omega_1\right]^2 + 2\cos 2\theta$$

$$u^2(1-u)^2 = 4s^2\cos^2\theta\omega_1 + 2\cos 2\theta$$

$$u^3(1-u) = 4\left[s\cos\theta\omega_1\right]\left[\cos 2\theta\right]$$

$$u^3(1-u) = 4s\cos\theta\cos 2\theta\,\omega_1$$

$$u^4 = \cos^2 2\theta$$

$$y^2 = \left[2s\cdot\sin\theta\omega_1\cdot u(1-u) + u^2\cdot\sin 2\theta\right]$$

$$u^2(1-u)^2 = 4s^2\sin^2\theta\omega_1^2$$

$$u^3(1-u) = 4s\sin\theta\sin 2\theta\omega_1$$

$$u^4 = \sin^2 2\theta$$

$$x^2+y^2 \Longrightarrow$$

$$(1-u)^4 = 1$$

$$u(1-u)^3 = 4s\cos\theta\omega_1$$

$$u^2(1-u)^2 = 4s^2\cos^2\theta\omega_1 + 2\cos 2\theta + 4s^2\sin^2\theta\omega_1^2$$

$$u^3(1-u) = 4s\omega_1\left[\cos\theta\cos 2\theta + \sin\theta\sin 2\theta\right]$$

$$u^4 = 1$$

$u(1-u)^3$: $\quad 4s\cos\theta\,\omega_1 = 4\omega_1$

$$s\cos\theta = 1$$

$u^2(1-u)^2 \quad 4s^2\cos^2\theta\omega_1 + 4s^2\sin^2\theta\omega_1 + 2\cos2\theta = 4\omega_1^2+2$

$$4s^2\omega_1^2 + 2\cos2\theta = 4\omega_1^2+2$$

$$\frac{4\omega_1^2}{\cos^2\theta} + 2\cos2\theta = 4\omega_1^2+2$$

$$4\omega_1^2 + 4\cos^4\theta - 2\cos^2\theta = 4\omega_1^2\cos^2\theta + 2\cos^2\theta$$

$$4\cos^4\theta - (4\omega_1^2+4)\cos^2\theta + 4\omega_1^2 = 0$$

$\cancel{4\cos^4\theta}$

$$\cos^4\theta + (-\omega_1^2-1)\cos^2\theta + \omega_1^2 = 0$$

$$\therefore \cos^2\theta = \frac{\omega_1^2+1 \pm\sqrt{\omega_1^4+2\omega_1^2+1+4\omega_1^2}}{2}$$

$$\cancel{= \frac{\omega_1^2+1 \pm\sqrt{\omega_1^4+6\omega_1^2+1}}{2}}$$

$$= \frac{\omega_1^2+1 \pm\sqrt{\omega_1^4-2\omega_1+1}}{2}$$

$$\cos^2\theta = \frac{\omega_1^2+1 \pm\sqrt{(\omega_1^2-1)^2}}{2}$$

$$\cos^2\theta = \frac{\omega_1^2+1 \pm(\omega_1^2-1)}{2}$$

2

$= \omega_1^2$

$$\cos^2\theta = \frac{\omega_1^2 + 1 - \omega_1 + 1}{2}$$

$\boxed{\cos\theta = \omega_1}$ OR $\cos^2\theta = 1$

$\cos\theta = \omega_1$ $\qquad \omega_1 = \cos\theta$

$S\cos\theta = 1$