

denominator²: $[\omega_1(1-u)^2 + 2\omega_2u(1-u) + \omega_3u^2]^2$

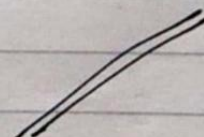
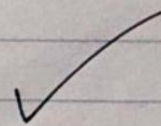
- ① $\omega_1^2(1-u)^4$
- ② $2\omega_1\omega_2u(1-u)^3$
- ③ $\omega_1\omega_3u^2(1-u)^2$
- ④ $2\omega_1\omega_2u(1-u)^3$
- ⑤ $4\omega_2^2u^2(1-u)^2$
- ⑥ $2\omega_2\omega_3u^3(1-u)$
- ⑦ $\omega_1\omega_3u^2(1-u)^2$
- ⑧ $2\omega_2\omega_3u^3(1-u)$
- ⑨ $\omega_3^2u^4$

Expansion

(Test $\omega_1 = \omega_3 = \sqrt{2}$ $\omega_2 = 1$)

$$\begin{aligned} (1-u)^4 &= \omega_1^2 \\ u(1-u)^3 &= 4\omega_1\omega_2 \\ u^2(1-u)^2 &= 4\omega_2^2 + 2\omega_1\omega_3 \\ u^3(1-u) &= 4\omega_2\omega_3 \\ u^4 &= \omega_3^2 \end{aligned}$$

$$\begin{aligned} &= 2 \\ &= \cancel{8\sqrt{2}} = 4\sqrt{2} \\ &= 8 \\ &= 4\sqrt{2} \\ &= 2 \end{aligned}$$



Control Points & Weights

$$1 : \quad \omega_0 = 1 \quad ; \quad (1, 0)$$

$$2 : \quad \omega_1 = ? \quad ; \quad (s \cos \theta, s \sin \theta)$$

$$3 : \quad \omega_2 = 1 \quad ; \quad (\cos 2\theta, \sin 2\theta)$$

$$\text{num}(x) = (1-u)^2 + 2s \cos \theta \omega_1 u(1-u) + u^2 \cos 2\theta$$

$$\text{num}(y) = 2s \sin \theta \omega_1 \underbrace{u(1-u)}^*$$

$$\text{denom} = (1-u)^2 + 2u(1-u)\omega_1 + u^2$$

$$\therefore \text{denom}^2 : \quad (1-u)^4 = 1$$

$$u(1-u)^3 = 4\omega_1$$

$$u^2(1-u)^2 = 4\omega_1^2 + 2$$

$$u^3(1-u) = 4\omega_1$$

$$u^4 = 1$$

$$x^2 = \left[(1-u)^2 + 2s \cdot \cos \theta \omega_1 \cdot u(1-u) + u^2 \cos 2\theta \right]^2$$

$$(1-u)^4 = 1$$

$$u(1-u)^3 = 4[s \cdot \cos \theta \omega_1]$$

$$u^2(1-u)^2 = 4[s \cdot \cos \theta \omega_1]^2 + 2 \cos 2\theta$$

$$u^2(1-u)^2 = 4s^2 \cos^2 \theta \omega_1 + 2 \cos 2\theta$$

$$u^3(1-u) = 4[s \cos \theta \omega_1][\cos 2\theta]$$

$$u^3(1-u) = 4s \cos \theta \cos 2\theta \omega_1$$

$$u^4 = \cos^2 2\theta$$

$$y^2 = [2s \cdot \sin \theta \omega_1 \cdot u(1-u) + u^2 \sin 2\theta]$$

$$u^2(1-u)^2 = 4s^2 \sin^2 \theta \omega_1^2$$

$$u^3(1-u) = 4s \sin \theta \sin 2\theta \omega_1$$

$$u^4 = \sin^2 2\theta$$

$$x^2 + y^2 \Rightarrow$$

$$(1-u)^4 = 1$$

$$u(1-u)^3 = 4s \cos \theta \omega_1$$

$$u^2(1-u)^2 = 4s^2 \cos^2 \theta \omega_1 + 2 \cos 2\theta + 4s^2 \sin^2 \theta \omega_1^2$$

$$u^3(1-u) = 4s \omega_1 [\cos \theta \cos 2\theta + \sin \theta \sin 2\theta]$$

$$u^4 = 1$$

$$U(1-u)^3 : 4s \cos \theta \omega_1 = 4\omega_1$$

$$s \cos \theta = 1$$

$$U^2(1-u)^2 : 4s^2 \cos^2 \theta \omega_1 + 4s^2 \sin^2 \theta \omega_1 + 2 \cos 2\theta = 4\omega_1^2 + 2$$

$$4s^2 \omega_1^2 + 2 \cos 2\theta = 4\omega_1^2 + 2$$

$$\frac{4\omega_1^2}{\cos^2 \theta} + 2 \cos 2\theta = 4\omega_1^2 + 2$$

$$4\omega_1^2 + 4\cos^4 \theta - 2\cos^2 \theta = 4\omega_1^2 \cos^2 \theta + 2\cos^2 \theta$$

$$4\cos^4 \theta + (4\omega_1^2 + 4)\cos^2 \theta + 4\omega_1^2 = 0$$

$$\cancel{4\cos^4 \theta}$$

$$\cos^4 \theta + (-\omega_1^2 - 1)\cos^2 \theta + \omega_1^2 = 0$$

$$\therefore \cos^2 \theta = \frac{\omega_1^2 + 1 \pm \sqrt{\omega_1^4 + 2\omega_1^2 + 1 + \omega_1^2}}{2}$$

$$= \frac{\omega_1^2 + 1 \pm \sqrt{\omega_1^4 + 6\omega_1^2 + 1}}{2}$$

$$= \frac{\omega_1^2 + 1 \pm \sqrt{\omega_1^4 - 2\omega_1 + 1}}{2}$$

$$\cos^2 \theta = \frac{\omega_1^2 + 1 \pm \sqrt{(\omega_1^2 - 1)^2}}{2}$$

$$\cos^2 \theta = \frac{\omega_1^2 + 1 \pm (\omega_1^2 - 1)}{2}$$

$$= \omega_1^2$$

$$\boxed{\cos \theta = \omega_1}$$

OR

$$\cos^2 \theta = \frac{\omega_1^2 + 1 - \omega_1 + 1}{2}$$

$$\cos^2 \theta = 1$$

$$\cos \theta = \omega_1$$

$$\omega_1 = \cos \theta$$

$$\sin \theta = 1$$