ENGINEERING TRIPOS PART IIB

IIB Project Weekly Progress

Name: Daniil Slavin

College: Downing

crsID: ds779

Week 1, 2 and 3

Reading

GA

In order to understand Geometric Algebra, [2] was read as well as [4] to understand its extension for five dimensions.

NURBS

Read [5] as an introduction to NURBS. Was captivated by the simple representation of a circle and refered to [3] to understand how a smaller arc can be created.

Week 4

Target:

Make some shapes using GA and explore how these shapes would be made with NURBS.

1st Goal: Making a donut

Attempt 1: Formed two circles on top of one another and tried to exploit opposite handedness to form a 360°torus. Did not work as I would assume it would try to use the shortest path.
Attempt 2: Formed two circles that were reflected in the y-z plane to form a 180°torus. Did not work as the transformation stayed within the x-y plane.
Attempt 3: Used a 120°rotor to create two circles. Used the $rotor\_generator\_function$, worked better than attempt 2 but an inflated torus was created.
Attempt 3.1: Used a 120°rotor to create two circles. Instead of using the rotor generator, I simply cut the previous rotor into 20 and was finally able to make one third of a torus. Now will it work with a 360°or a 180°rotor?
Success attempt 4: Formed a rotor which would rotor through $\frac{2\pi}{N}$ and then applied it N times to form the donut. Parameterised using N and saved under Donut Success.

2nd Goal: Making an enclosed circular surface

Circular Arc: Used the same rotor as before but added an extra parameter, f to allow the user to choose the fraction of the circle they want drawn. Rotated a point around the origin, then formed point pairs between all points to linearly interpolate a circular arc.
Circular Surface: Used the same rotor as before on a point pair to form a surface.
Enclosed Circular Surface: Rotated the above point pair and applied a projector

to each point pair to find the edge point of every pair. Connected the edge points using interpolated point pairs.

Summary of discussions with supervisors

JL: Target is to now create surfaces that also combine translation and scaling (week 4 only included rotations) and to work on a quadratic interpolation between three surfaces. I also suggested working on a combination of two rotors and parameterising their significance in order to create a range of possible surfaces between two point pairs.
HH: Explained the limitations of $rotor\_between\_objects$ and suggested I attempt to use $TRS\_between\_rounds$ to remake the donut.

Week 5

Target:

Redo the donut using TRS. Get to grips with scaling and translation. Experiment with surfaces that combine the two.

1st Goal: TRS donut

Completed and parametrised using N and f as before, only worked for circles $\frac{2\pi}{3}$ apart.

2nd Goal: Understanding coordinates

Turns out the coordinates make a left handed set. Explaining why my donuts went in a clockwise fashion.

3rd Goal: Get to grips with translation

Bivector in the rotor is made up of the direction of translation and the infinity vector. Normalised by a factor of a half and multiplied by the magnitude of translation.

4th Goal: Set up basis surface

Created two line which are a translation and a rotation apart. Used TRS to form a basis surface between the two.

5th Goal: Creating the variable surface

Change of approach: Previously I have constructed a rotor list which starts from 1 and finishes at the complete rotor, each rotor is multiplied against the first line. Instead I will now just use one small rotor and iteratively apply it to the most recent

line to create an object list.
Approach to order of transformations: Since I am only dealing with one translation and one rotation. I will create a list of numbers and feed it through a loop which will apply a translation for odd numbers and the rotation for even numbers.
Error in order of transformations: The order of transformation matters, therefore the loop was adjusted such that the translation occurs along the vector rather than being inline with e2.
Variable surface complete: Order of odd and even numbers determines the shape of the surface. End and start points are the same because the translation issue has been addressed by translating along the vector.

Summary of discussions with supervisors

JL: To move onto using scaling. Also to try out surfaces where the order of transformation does not matter, for instance one where the translation vector is perpendicular to the plane of rotation.
HH: Discussed imaging issues and using the clean object.

Week 6, 7 and 8

Target:

Sort out imaging issues. Get the hang of scaling and apply some TS rotors.

1st Goal: Imaging issues

Replace objects using the following code:

```
sc.add_objects([o(2).normal().clean(1E-4) for o in object_list])
```

The '2' references the grade of the object, in this case a point pair, which is then normalised and cleaned up to the nearest 0.0001 on all values. Would need to be changed to '3' for circles and so on.

2nd Goal: Scale Practise

Uses the bivector of e_4 and e_5. Works relative to the origin only, therefore adjustment required if something is to be shortened off the origin. By adjustment I mean translation.

3rd Goal: Making a cone

Basic Cone: Took a circle of fixed radius. Created a set of scalar transformations of $\frac{1}{1}, \frac{1}{2}, ..., \frac{1}{10}$ (placing a list of numbers as the denominators). Results in an odd looking

cone.
Better Cone: Took a circle of fixed radius. Created a set of transformations of $\frac{1}{10}, \frac{2}{10}, ..., \frac{10}{10}$ to apply to the circle. Results in a cone with a constant gradient but with an imaginary pointy top.
Advanced Cone: Parameterised the slop and height of the cone. Made sure the final translation is performed on a point at the origin to ensure a pointy top.
The reason why the point forms the perfect top is because the first scalar is 1 with a translaiton of 0. Therefore by adding the extra origin and extra max translation, it reaches the desired level as if it were the circle being collapsed to nothing

4th Goal: Making a piece of 'Holy Macaroni'

Unlike the *variable surface* from before, this surface is constructed out of a translation and rotation where the axis of translation and rotation line up with one another and therefore sequencing does not matter.
Parameters included the number of full rotations of the object and the length of the object. One translation followed one rotation to produce a macaroni like object from a point pair centered on the origin.

Clean up

Since sequencing did not matter, the individual translation and rotation could be multiplied together to produce a small TR rotor and produced a much smoother surface.

Week L123

Work following TMR: NURBS Arc

From what I wrote in my TMR is was clear that I did not understand how to draw a circular arc of any angle using NURBS. Therefore I found a NURBS package in Python [1] and set about the algebraic work to find the relevant control points and weights.
Successfully created a function which draws a NURBS arc using the fraction of the circle one needs creating.

Work following TMR: NURBS Torus

After revisiting [5] I understood that I could replicated the CGA Torus using NURBS by taking the control points of a circle and mapping them onto a circular arc centered at the origin.
Plan is to attempt this once 2D arcs are fully understood as well how to construct surfaces using the NURBS package.

Work following TMR: Dual

After discussing the Dual with HH, I have progress in its uses and plan to use this for interacting surfaces.

Discussion with HH and JL

After finishing the NURBS torus, the plan to next draw a curve using both NUBRS and CGA. The curve will have the same start point and end point as well as the same tangent at the start and end point. For CGA, the work in [6] will be used to interpolate between the start and end point.

Conformal Point Imaging

After discussion with HH, the following code is needed to display conformal points

```
[normalise_n_minus_1(o(1)).clean(1E-6) for o in object_list]
```

Week L45678

Target: Work on conversion L45

Spent the majority of this time working on conversions between hermite curves to various options of conformal interpolation. So far all resulting curves produced look absolutely nothing like that of a NURBS curve.

Target: Conversion Breakthrough L6

By accidentally setting all the orientations of the circle the same, a NURBS curve is produced.

Discussion with HH and JL

In hindsight the 3 of us realise that is simply because the centre of the circles is interpolated with orientation having no effect. The goal going forward is to investigate further.

Target: Angle Exploration L78

Different combinations of misalignment of circle planes were attempted, making it clear that symmetrical misalignment produces symmetrical centre point curves. Whilst anti-symmetric misalignment produces the opposite.
Another thing to note is that full conformal surface is not unique, different initial

orientations of all 4 circles produce different rules. This is despite them all sharing the same orientation.

Week E1

After discussion with HH and JL, a new form of surface was introduced. I was successfully able to produce these surfaces but not using rotors as JL and HH suggested.

Discussion with HH and JL

After the discussion it became a lot clearer what was required and subsequently managed by forming rotors as functions of the definitions of Canal Surfaces.

References

[1] Onur Rauf Bingol and Adarsh Krishnamurthy. NURBS-Python: An open-source object-oriented NURBS modeling framework in Python. *SoftwareX*, 9:85–94, 2019.

[2] Chris Doran and Anthony Lasenby. *Geometric Algebra for Physicists.* Cambridge University Press, 2003.

[3] D. Eberly. Representing a circle or a sphere with nurbs. 2003.

[4] A Lasenny, Joan Lasenby, and Rj Wareham. A covariant approach to geometry using geometric algebra. 2004.

[5] L. Piegl. On nurbs: a survey. *IEEE Computer Graphics and Applications*, 11(1):55–71, Jan 1991.

[6] Rich Wareham and Joan Lasenby. Mesh vertex pose and position interpolation using geometric algebra. In Francisco J. Perales and Robert B. Fisher, editors, *Articulated Motion and Deformable Objects*, pages 122–131, Berlin, Heidelberg, 2008. Springer Berlin Heidelberg.