Problem 3.1:

I. Since $\{E, F, G, H\}$ are not reachable, we can remove them

II.

III.
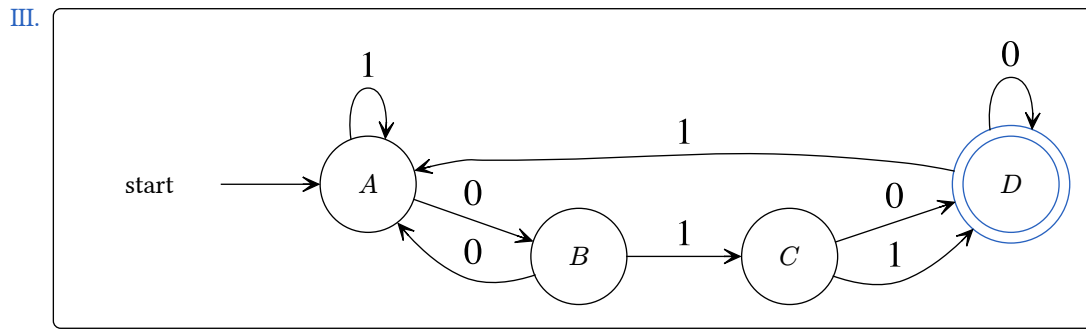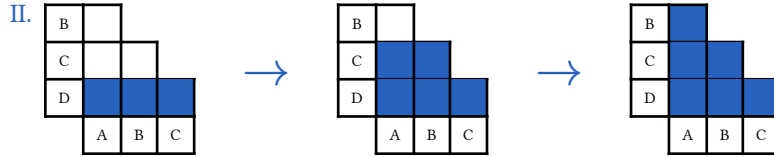
Problem 3.2:

Proof:

- $B \xrightarrow[G]{*} w \implies$ there exists a derivation tree with root B and border word w.

To go from a derivation to a derivation tree, we to find a construction:
Let $B \Rightarrow w_1 \Rightarrow w_2 \Rightarrow ... \Rightarrow w_n \Rightarrow w$ be intermediate yieldings of productions to B that result in w.
1. Start with the initial symbol B as the root.
2. For every intermediate yielding, attach new nodes (right hand side of the production) as children to the respective node.
3. Repeat step 2 until you reach the desired border word.

- $B \xRightarrow[G]{*} w \impliedby$ there exists a derivation tree with root B and border word w.

In a derivation tree, the immediate children of every inner node is specified by a production, therefore given a root node B, there exist a sequence of direct derivations that result in the given borderword, i.e. $B \xRightarrow[G]{*} w$.

$\square$

Problem 3.3:

Problem 3.4:

Proof: We use pumping lemma and show that there is no combination $uvxyz$ such that you can pump on v and y at the same time ($uv^i xy^i z \in L(G) \forall i \geq 0$). For that we can consider all combinations:

- Consider that the number of v consists only of as or bs and similarly y consists only of bs or cs (but v and y dont consist of bs at the same time). In this case, pumpint on v and y increases the amount of corresponding symbols while the amoun of the excluded one stays the same — the decomposition is not valid.

- v consists of k as and k bs and y consists of k cs. Pumping will not result in imbalanced amounts, but it will cause the order of as and bs to change (eg. aaabbbccc $\rightarrow$ aaababbbcccc) — the decomposition is not valid.

- The case where either v or y contains all three symbols at the same tims violates the pumping length restriction. No matter what we pick p (pumping length) to be, a string that looks like $a^p b^p c^p$ will violate the length restriction.

These are all sensible decompositions, for others it's very clear that they fail. ☐