Homework — Algorithms and Data Structures

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Worksheet 4

1. *Proof.* By induction on i. Base case: $i = 1$

$$p_b(b(j), c) = 1 = 1!$$

Induction step:

$$p_b(b(j), c) = \frac{1}{i} \cdot p_{b-b(j)}(c) = \frac{1}{i} \cdot \frac{1}{(i-1)!} = \frac{1}{i!}$$

$\square$

2. They probably used the fact that $(a+b)^2 = a^2 + b^2 + 2ab$. They could calculate the sum of a and b and the squares $a^2, b^2, (a+b)^2$ fairly easily, which in (also fairly easy) combination $(a+b)^2 - a^2 - b^2$ would give them the double of the product they were looking for $2ab$ — which they would also be able to halve easily to obtain ab.

3.
```
let p = random(0, size(A));
let L = [];
let R = [];
for i in 0..size(A) {
    if i != p {
        if A[i] < A[p] { L.push(A[i]) }
        else { R.push(A[i]) }
    }
}
```

4. First of all, this definition is not full. The case where either $n = 0$ or $m = 0$ is not handeled. I can not make an assumption fot what sould be done, since it is very important at least for the 2-nd task, so I will just assume that $n = m$

   (1) We can write the number of comparisons as $t'(n, m) = 1 + t'(n-1, m-1)$ which can be shortened to $t'(n, m) = n = m$. Whereas, for the original merge $t(n, m) = n + m - 1$. This modified merge has less comparisons.

   (2) This can be disproven with a counterexample: Let $A = [1, 2, 3], B = [4, 5, 6]$, then $\text{merge}'(A, B) = [1, 4, 2, 5, 3, 6]$ which is not sorted.

5. (1) We can say that $S_n = [1 : 365]^n$. Thereby $p_n(x) = 365^{-n}$

   (2)

$$\#E_n = 365 \cdot 364 \cdot \ldots \cdot (365 - n + 1) = \frac{365!}{(365-n)!}$$

   This also introduces a constraint $n \leq 365$ which is quite logical – you can't have more people with distinct birthdays than the number of possible birthdays.

   (3)

$$p(E_n) = \frac{\#E_n}{365^n} = \frac{365!}{365^n (365-n)!}$$

(4)

$$p(E_{23}) = \frac{365!}{(365-23)!}\frac{1}{365^{23}} = \frac{342 \cdot 343 \cdot \ldots \cdot 364}{365^{22}} \approx \frac{3699}{7509} \approx 0.4927...$$

(5) $p(E_n)$is the probability of there being no pair in n people that share a birthday. What was probably ment to be written in the worksheet was the statement $1 - p(E_{23}) > 1/2$, which means that given 23 people, the chance of there being a pair sharing a birthday is more than 50%.