Theory of Computation Excercise for TTF (4)

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Problem 4.1:

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Problem 4.2:

The equivalent grammar in Chomsky normal form would be $(\{S, A, B, U, V, X, Y\}, \{0, 1\}, P, S)$ with productions

$$\begin{array}{ll} S \to XA, & S \to YB, \\ A \to YS, & B \to XS, \\ A \to 0, & B \to 1, \\ A \to XU, & B \to YV, \\ U \to AA, & V \to BB, \\ X \to 1, & Y \to 0. \end{array}$$

Problem 4.3:

(1) Let $G_1 = (N_1, T_1, P_1, S_1)$ be a grammar with $L(G_1) = L$ and $G_2 = (N_2, T_2, P_2, S_2)$ be a grammar with $L(G_2) = L'$. The combined grammar

$$G = (N_1 \cup N_2 \cup \{\xi\}, \quad T_1 \cup T_2, \quad P_1 \cup P_2 \cup \{(\xi, S_1), (\xi, S_2)\}, \quad \xi)$$

would have $L(G) = L \cup L'$.

(2) Let $M = \{\mathtt{a}^n \mathtt{b}^n \mathtt{c}^k : n, k \in \mathbb{N}_0\}, N = \{\mathtt{a}^k \mathtt{b}^n \mathtt{c}^n : n, k \in \mathbb{N}_0\}$ and $L = \overline{M} \cup \overline{N}$. Assume that the complement of a context free language is also context free. Then $\overline{M}$ and $\overline{N}$ are context-free, therefore L will be context-free aswell. By our assumption $\overline{L}$ should be context free but we see that

$$\overline{L} = \overline{\left(\overline{N} \cup \overline{M}\right)} = N \cap M = \underbrace{\{\mathtt{a}^n \mathtt{b}^n \mathtt{c}^n : n \in \mathbb{N}_0\}}_{\text{not context free}}.$$

We have a contradiction — complement of a context free grammar is not always context free.

Problem 4.4:

Let $M = (\mathrm{Z}, \Sigma, \Gamma, \delta, z_0, \mathrm{Z}_A)$

(1) We can set $M' = (\mathrm{Z}, \Sigma, \Gamma, \delta', z_0, \mathrm{Z}_A)$ and $\delta'(\alpha, \varepsilon, \beta) = \begin{cases} \{(\alpha, \text{pop})\} & \text{if } \alpha \in \mathrm{Z}_A \wedge \beta \neq \varepsilon \\ \{\} & \text{otherwise} \end{cases}$

(2) We can set $M' = (\mathrm{Z}, \Sigma, \Gamma, \delta, z_0, \mathrm{Z})$.

Problem 4.5:

Proof: We can employ the construction:
If a state transition wants to push a sequence of symbols on the store

$$(\omega, \text{push } ABCD) \in \delta(\alpha, \beta, \gamma)$$

it would first transition through a sequence of intermediate states

$$\begin{aligned}
\left\{\left(\langle(\alpha,\omega,ABCD)_1\rangle, \text{push } A\right)\right\} &= \delta(\alpha,\beta,\gamma) \\
\left\{\left(\langle(\alpha,\omega,ABCD)_2\rangle, \text{push } B\right)\right\} &= \delta\left(\langle(\alpha,\omega,ABCD)_1\rangle, \varepsilon, \varepsilon\right) \\
\left\{\left(\langle(\alpha,\omega,ABCD)_3\rangle, \text{push } C\right)\right\} &= \delta\left(\langle(\alpha,\omega,ABCD)_2\rangle, \varepsilon, \varepsilon\right) \\
\{(\omega, \text{push } D)\} &= \delta\left(\langle(\alpha,\omega,ABCD)_3\rangle, \varepsilon, \varepsilon\right).
\end{aligned}$$

This way, the sequence $ABCD$ will get pushed on the store and the state will become α. □