Numerical Analysis
Homework 2

1. Find a second-order formula for approximating $f'(x)$ by applying extrapolation to the two-point forward-difference formula.

2. (a) Develop a first-order method for approximating $f''(x)$ that uses the data $f(x-h)$, $f(x)$ and $f(x+3h)$ only; (task from the previous homework)

   (b) Apply extrapolation to the formula developed in the first part to get a second-order formula for $f''(x)$;

   (c) Demonstrate the order of the new formula by approximating $f''(0)$, where $f(x) = \cos(x)$, with $h = 0.1$ and $h = 0.01$.

   (d) Programming: Start with $h = 0.5$. Reduce $h$ in each step by 0.025 until $h = 0.01$. Output approximating values of $f(x)$ using the first and the second methods; Output errors of each method on each step.

3. Get partial derivative's approximation formulas, find the error term and order:

   (a) $\frac{\partial f}{\partial x} \approx \frac{f(x+h,y)-f(x-h,y)}{2h}$;

   (b) $\frac{\partial^2 f}{\partial y^2} \approx \frac{f(x,y+h)-2f(x,y)+f(x,y-h)}{h^2}$