Homework 7

1. Prove Sherman-Morrison-Woodbury formula:

$$(A+UCV)^{-1} = A^{-1} - A^{-1}U(C^{-1} + VA^{-1}U)VA^{-1}$$

   where $A \in \mathbb{R}^{n\times n}, U \in \mathbb{R}^{n\times k}, C \in \mathbb{R}^{k\times k}, V \in \mathbb{R}^{k\times n}$,

2. Use the Cholesky factorization procedure to express the following matrices in the form $A = R^T R$

   (a) $\begin{pmatrix} 1 & 2 \\ 2 & 8 \end{pmatrix}$

   (b) $\begin{pmatrix} 4 & -2 & 0 \\ -2 & 2 & -3 \\ 0 & -3 & 10 \end{pmatrix}$

3. Show that the Cholesky factorization procedure fails for the matrices:

   (a) $\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

4. $$A = \begin{pmatrix} 10 & 3 & 0 \\ 4 & 9 & 4 \\ 0 & 3 & 5 \end{pmatrix}$$

   For the linear system $Ax = b$

   (a) write Jacobi method in:
      i. componentwise form;
      ii. matrix form.

   (b) write Gaus-Seidel method in:
      i. componentwise form;
      ii. matrix form.

   (c) check the convergence of the methods using
      i. sufficient condition;
      ii. necessary and sufficient condition

(d) How many iterations is needed to reduce initial error 10-times?

5. Apply two steps of SOR to the system $\begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix} \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 6 \\ 3 \\ 5 \end{pmatrix}$

   Use starting vector $(0, ..., 0)$ and $\omega = 1.5$

6. $$A = \begin{pmatrix} 5 & 3 \\ 1 & 4 \end{pmatrix}, \; P = \begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}$$

   For the linear system $Ax = b$ write Richardson'si method in componentwise and matrix forms using pre-conditioner matrix P.

Useful material in'Numerical analysis', Sauer