Exercises for week 1

1 A simple grammar

Consider the grammar G with:

$$\begin{aligned} N &= \{S\} \\ T &= \{a, e, m\} \\ S &\to m \mid aSee \end{aligned}$$

1. Draw derivation trees for all words in $L(G)$ which can be derived with at most 3 steps (derivation trees with at most 3 inner nodes)? (20 credit points)
2. Prove:
$$a^n m e^{2n} \in L(G) \quad \text{for all} \quad n \geq 0.$$
(10 credit points)
3. Prove:
$$L(G) = \left\{ a^n m e^{2n} \middle| n \geq 0 \right\}.$$
(20 bonus credit points)

2 Getting familiar with the C0 grammar

This follows the pattern of the first and second week's slides (they are uploaded). Draw derivation trees—if there is one—showing derivations.

1. Constants: from $\langle C \rangle$ to 324.
2. Names: from $\langle Na \rangle$ to $G680$.
3. Expressions: from $\langle E \rangle$ to $8/X - 989$.
   Compress subtrees with roots $\langle C \rangle$ and $\langle N \rangle$.
4. Statements: from $\langle StS \rangle$ to $Y \ = \ Y - 1; P \ = \ \texttt{new}\ OLD*$.
   Compress subtrees with roots $\langle Na \rangle$ and $\langle E \rangle$. Do not worry about the effect of the new-statement. This is only about syntax.
5. Function calls: from $\langle St \rangle$ to $v = ge(60,\ t * 8)$.

6. Type declarations: from $\langle TyD \rangle$ to `typedef uint` $[0]$ $empty\_arr$.
   Compress subtrees with roots $\langle Na \rangle$.

7. Type declarations: from $\langle TyD \rangle$ to `typedef bool` $[N]$ $parametrized\_arr$.
   Compress subtrees with roots $\langle Na \rangle$.

8. Function declarations: from $\langle FuD \rangle$ to `char` $main(\ )\{t = t/8; \texttt{return}\ A\}$.
   Compress subtrees with roots $\langle Na \rangle$ or $\langle E \rangle$.

Do not show pointed brackets in the trees. If there is no tree, explain why. (Each item 10 credit points)