KUTAISI INTERNATIONAL UNIVERSITY

Exercises for TTF
Introduction to Theory of Computation
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Summer semester 2024

Exercises below are your homework; they will be discussed during exercise classes. Problems marked with a (*) are more challenging.

Week 4

1. **Pumping Lemma for context free grammars.** Fix the argument in Lemma 6 from the lecture on Chomsky hierarchy so that $vx \neq \epsilon$ and $|vwx| \leq n_0$ follow.

2. **Chomsky normal form.** Consider the grammar $(\{S, A, B\}, \{0, 1\}, P, S)$ with productions

$$S \to 1A, \qquad S \to 0B,$$
$$A \to 0S, \qquad B \to 1S,$$
$$A \to 0, \qquad B \to 1,$$
$$A \to 1AA, \qquad B \to 0BB.$$

Find an equivalent grammar in Chomsky normal form.

3. **Clusure operations for context free languages.** Show
   (1) if L and L' are context free, then $L \cup L'$ is context free.
   (2) If L is context free, then it is possible that $\overline{L}$ is not context free. *Hint:* try to write a non context free language as the intersection of context free languages.

4. **Variants of the npda model.** We have defined that an npda accepts if both the pushdown store is empty and the state is accepting. There are obvious variants of this definition:
   - accept by (reaching an) accepting state.
   - Accept by (reaching an) empty pushdown store.

Show that all these are equivalent, that is
   (1) if an npda M accepts L by accepting state, then there is an npda M' which accepts L by accepting state and empty pushdown store.
   (2) If an npda M accepts L by empty pushdown store, then there is an npda M' which accepts L by accepting state and empty pushdown store.

5. **Extended npda.** Call a push down automaton *extended* if it can push more than one symobl on the stack simultaneously. Prove that notions of extended and (regular) push down automatons are equivalent.