

Homework 4

October 2023

Problem 1

Give examples of a matrix A such that:

- (a) $\|A\|_1 < \|A\|_\infty$,
- (b) $\|A\|_1 = \|A\|_\infty$, and
- (c) $\|A\|_1 > \|A\|_\infty$.

Problem 2

Consider a sequence of 2×2 matrices $\{A_n\}$ where each A_n is defined as:

$$A_n = \begin{bmatrix} \frac{n}{n^2+1} & 0 \\ 0 & \frac{1}{n^2+1} \end{bmatrix}.$$

Determine whether this sequence converges, and if it does, find the limit matrix.

Problem 3

Calculate the condition number of a Hilbert matrix H_3 , a 3×3 matrix with entries $H_{ij} = \frac{1}{i+j-1}$.

Problem 4

Let $A = (a_1, a_2, \dots, a_n)$, where a_j is the j -th column of A . Then prove that $\|A\|_F^2 = \sum_{i=1}^n \|a_i\|_2^2$, where F stands for a Frobenius norm.