Exercises below are your homework; after submission, they will also be discussed during exercise classes.

Week two

1. Show: The language $L = \{a^n b^n \ : \ n \in \mathbb{N}\}$ is not regular.

2.
   (1) Describe the nondeterministic automaton $M = (Z, A, \delta, z_0, Z_A)$ on Figure 1 by identifying Z, A, δ and Z_A.

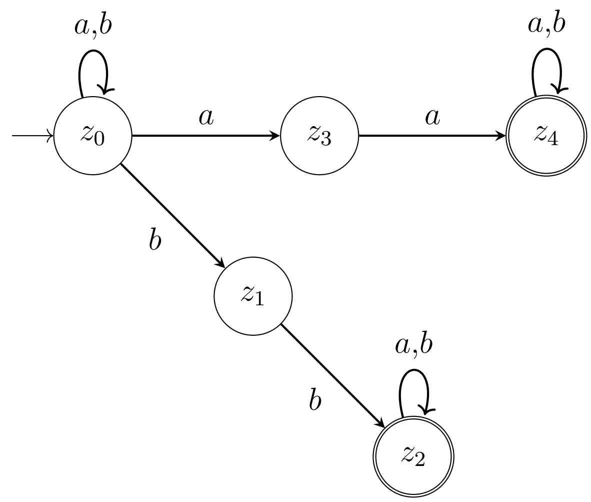

Figure 1

   (2) Describe a language accepted by nondeterministic automaton on Figure 1.

3. Draw a nondeterministic automaton M, which accepts the language

$$L = \{1^i \ : \ i \equiv 0 \bmod 3\} \cup \{1^i \ : \ i \equiv 0 \bmod 5\}.$$

4. Give a counterexample to show that the following construction fails to prove the closure of the class of languages accepted by NFA under the star operation.

Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1. Construct $N = (Q_1, \Sigma, \delta, q_1, F)$ as follows. N is supposed to recognize A_1^*.

a. The states of N are the states of N_1.

b. The start state of N is the same as the start state of N_1.

c. $F = \{q_1\} \cup F_1$.
The accept states F are the old accept states plus its start state.

d. Define δ so that for any $q \in Q_1$ and any $a \in \Sigma_\varepsilon$,

$$\delta(q,a) = \begin{cases} \delta_1(q,a) & q \notin F_1 \text{ or } a \neq \varepsilon \\ \delta_1(q,a) \cup \{q_1\} & q \in F_1 \text{ and } a = \varepsilon \end{cases}$$

5. Complete the proof of Lemma 4 from the lecture on NFA i.e. show that $L(M'') = L^*$.