Discrete Probability Theory Homwework (3)

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Problem 3.1:

We can think of this problem as picking one ball from urn B and then measuring the probability of a random ball picked form urn A matching the one from B. That for any ball picked from B there is a $\frac{1}{2}$ probability that a ball from A matches.

Problem 3.2:

We can count all the possible combinations of a committee consisting of one student from each class and then divide that by the total number of combinations

$$\frac{\overbrace{3}^{\text{freshmen}} \cdot \overbrace{4}^{\text{sophomores}} \cdot \overbrace{4}^{\text{juniors}} \cdot \overbrace{3}^{\text{seniors}}}{\binom{14}{4}} = \frac{144}{1001}$$

Problem 3.3:

It's been proven in class that

a) $P(M) = 1 - P(M^c)$.

c) $\left(\bigcap_{i=1}^{\infty} A_i\right)^c = \bigcup_{i=1}^{\infty} A_i^c$.

d) $P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$ for mutually independent sequence $\{A_i\}$.

Now we just write

$$\begin{aligned} P\left(\bigcap_{i=1}^{\infty} A_i\right) &= P\left(\left(\bigcup_{i=1}^{\infty} A_i^c\right)^c\right) && (3) \\ &= 1 - P\left(\bigcup_{i=1}^{\infty} A_i^c\right) && (1) \\ &= 1 - \sum_{i=1}^{\infty} P(A_i^c) && (4) \\ &= 1 - \sum_{i=1}^{\infty} (1 - P(A_i)) && (1) \\ &= 1 - \sum_{i=1}^{\infty} (1 - 1) = 1 \end{aligned}$$

Problem 3.4:

Since the probability of picking a blue ball out of $4 + 8 + 5$ balls is $\frac{8}{4+8+5}$. Then picking another blue ball would have the probability $\frac{8-1}{4+8+5-1}$ and so on. Thereby the probability of the first 5 balls picked would be blue is

$$\prod_{i=0}^{4} \frac{8-i}{4+8+5-i} = \frac{8}{17} * \frac{7}{16} * \frac{6}{15} * \frac{5}{14} * \frac{4}{13} = \frac{2}{221}$$