

Numerical Linear Algebra Dimitri Tabatadze 01:30 Thursday 9<sup>th</sup> November, 2023

- 1. • raw = 3.1415926...
  - chopped at 5 digits = 3.1415
  - rounded at 5 digits = 3.1416
- absolute error =  $|\hat{x} x| = 0.1 \cdot 10^{-4}$ 
  - relative error =  $\frac{|\hat{x}-x|}{|x|} = \frac{0.1 \cdot 10^{-4}}{0.3 \cdot 10^{-3}} = \frac{1}{30}$
  - (b) absolute error =  $|\hat{x} x| = |40320 39900| = 420$ 
    - relative error =  $\frac{|\hat{x}-x|}{|x|} = \frac{420}{39900} = \frac{1}{95}$
- $x + y = \frac{5}{7} + \frac{1}{3} = \frac{22}{21} = 1.047619...$ 3.
  - Chopped = 1.0476.
  - Absolute error =  $\frac{22-21\cdot1.0476}{21} = \frac{22-21.9996}{21} = \frac{0.0004}{21} = \frac{4}{210000}$
  - Relative error =  $\frac{|\frac{4}{210000}|}{|\frac{22}{21}|} = \frac{2}{11000}$
  - $-x-y=\frac{5}{7}-\frac{1}{3}=\frac{8}{21}=0.380952...$ 
    - Chopped = 0.38095.
    - Absolute error =  $\frac{8-21\cdot0.38095}{21} = \frac{8-7.99995}{21} = \frac{0.00005}{21} = \frac{1}{4200000}$
    - Relative error =  $\frac{\left|\frac{1}{4200000}\right|}{\left|\frac{8}{21}\right|} = \frac{1}{1600000}$
  - $-x \cdot y = \frac{5}{7} \cdot \frac{1}{3} = \frac{5}{21} = 0.2380952...$ 
    - Chopped = 0.23809.
    - Absolute error =  $\frac{5-0.23809\cdot21}{21} = \frac{5-4.99989}{21} = \frac{0.00011}{21} = \frac{11}{2100000}$
    - Relative error =  $\frac{\left|\frac{1}{2100000}\right|}{\left|\frac{5}{27}\right|} = \frac{11}{500000}$
  - $-x/y = \left(\frac{5}{7}\right)/\left(\frac{1}{3}\right) = \frac{15}{7} = 2.1428571...$ 
    - Chopped = 2.1428.
    - Absolute error =  $\frac{15-2.1428\cdot7}{7} = \frac{15-14.9996}{7} = \frac{0.0004}{7} = \frac{1}{17500}$
    - Relative error =  $\frac{\left|\frac{1}{17500}\right|}{\left|\frac{15}{2}\right|} = \frac{1}{37500}$
- 4. 0.54617 + 0.54611 = 1.09228. Absolute error = 0.00028 and relative error =  $\frac{0.00028}{1.09228}$ . The result has 4 significant digits.
- 5. (a)  $\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{-x\sin x \cos x}{-\cos x} = 1$ 
  - (b)  $f(0.1) = \frac{0.1\cos 0.1 \sin 0.1}{0.1 \sin 0.1} = \frac{0.1 \cdot 0.9950 0.09983}{0.1 0.09983} = -\frac{0.00033}{0.00017} = -1.941176...$
  - (c)  $f(0.1) = \frac{0.1(1 \frac{(0.1)^2}{2!}) (0.1 \frac{(0.1)^3}{3!})}{0.1 (0.1 \frac{(0.1)^3}{2!})} = \frac{\frac{(0.1)^3 \cdot (2! 3!)}{3! \cdot 2!}}{\frac{(0.1)^3}{2!}} = \frac{2! 3!}{2!} = \frac{-4}{2} = -2.0$
  - (d) For (b) the relative error =  $\frac{|-1.9989999 (-1.9411)|}{|-1.9989999|} = \frac{1.9989999 1.9411}{1.9989999} = \frac{0.0578999}{1.9989999}$  For (c) the relative error =  $\frac{|-1.9989999 (-2)|}{|-1.9989999|} = \frac{2 1.9989999}{1.9989999} = \frac{0.0010001}{1.9989999}$
- 6. (a)  $13.0_{10} = 1101.0_2$ 
  - (b)  $\frac{3}{8} = 0.375_{10} = 0.011_2$
  - (c)  $1.32_{10} = 1.0101000111101..._2$
- 7. (a) 1+4+16+64=85
  - (b) 11.625

(c) 
$$39.0 + \sum_{i=1}^{\infty} \frac{1}{4^i} = 39.\overline{3}$$

8. (a) 
$$-5_{10} = 11111011_2$$

(b) 
$$-17_{10} = 11101111_2$$

9. 
$$100111111 = 31 - 128 = -97$$