Numerical Analysis
Homework 3

1. The nonlinear system

$$x_1^2 - 10x_1 + x_2^2 + 8 = 0$$
$$x_1x_2^2 + x_1 - 10x_2 + 8 = 0$$

can be transformed into the fixed-point problem

$$x_1 = g_1(x_1, x_2) = \frac{x_1^2+x_2^2+8}{10}$$
$$x_2 = g_1(x_1, x_2) = \frac{x_1x_2^2+x_1+8}{10}$$

(a) Show that $\mathbf{G}= (g_1, g_2)^t$ mapping $D \subset \mathbb{R}^2$ into $\mathbb{R}^2$ has a unique fixed point in $D = \{(x_1, x_2)^t | 0 \leq x_1, x_2 \leq 1.5\}$

(b) Apply functional iteration to approximate the solution.

(c) Does the Gauss-Seidel method accelerate convergence?

2. Use functional iteration to find solutions to the following nonlinear system, accurate to within 10^{-5}, using the l_∞ norm:

$$3x_1^2 - x^2 = 0$$
$$3x_1x_2^2 - x_1^3 - 1 = 0$$

3. Use Newton's method with $\mathbf{x}^{(0)} = \mathbf{0}$ to compute $\mathbf{x}^{(2)}$ for the following nonlinear system:

(a)

$$4x_1^2 - 20x_1 + \frac{1}{4}x_2^2 + 8 = 0$$
$$\frac{1}{2}x_1x_2^2 + 2x_1 - 5x_2 + 8 = 0$$

(b)

$$3x_1 - cos(x_2x_3) - \frac{1}{2} = 0$$
$$4x_1^2 - 625x_2^2 + 2x_2 - 1 = 0$$
$$e^{-x_1x_2} + 20x_3 + \frac{10\pi-3}{3} = 0$$