

$$\int_{-\infty}^{\infty} \cos 2t \, dt = \sum_{k \in \mathbb{Z}} \int_{\pi k}^{\pi(k+1)} \cos 2t \, dt = \sum_{k \in \mathbb{Z}} 0 = 0$$

8.1.25

$$\begin{aligned}
I &= \int_{-1}^1 \sqrt{1+x^2} \, dx \\
&= 2 \int_0^1 \sqrt{1+x^2} \, dx \\
&= 2 \int_0^{\frac{\pi}{2}} \sec \theta \sec^2 \theta \, d\theta && [x = \tan \theta, dx = \sec^2 \theta d\theta] \\
&= 2 \left( [\sec \theta \tan \theta]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sec \theta \tan^2 \theta \, d\theta \right) \\
&= 2 \left( [\sec \theta \tan \theta]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sec \theta (\sec^2 \theta - 1) \, d\theta \right) \\
&= 2 \left( [\sec \theta \tan \theta]_0^{\frac{\pi}{2}} - \frac{I}{2} + \int_0^{\frac{\pi}{2}} \sec \theta \, d\theta \right) \\
&= 2 \left( [\sec \theta \tan \theta]_0^{\frac{\pi}{2}} - \frac{I}{2} + [\ln |\sec \theta + \tan \theta|]_0^{\frac{\pi}{2}} \right) \implies \\
2I &= 2 [\sec \theta \tan \theta + \ln |\sec \theta + \tan \theta|]_0^{\frac{\pi}{2}} \implies \\
I &= \left[ x\sqrt{1+x^2} + \ln \left| \sqrt{1+x^2} + x \right| \right]_0^1 \\
&= \left[ \sqrt{2} + \ln |\sqrt{2} + 1| - \ln |1| \right] \\
&= \ln(\sqrt{2} + 1) + \sqrt{2}
\end{aligned}$$

$$\lim_{t \rightarrow \frac{\pi}{2}} \tan t$$