To prove that

$$\texttt{mul c (sum l 0) 0 = c * summa l}$$

we first proove that

$$\texttt{sum l a = summa l + a}$$

by induction on length $x$ of `t`.

- Base case: $x = 0 \implies$ `l = []`

$$\begin{aligned}
\texttt{sum l a} &\overset{\texttt{sum}}{=} \texttt{match l with [] -> a | h::t -> sum t (h+a)} \\
&\overset{\texttt{def l}}{=} \texttt{match [] with [] -> a | h::t -> sum t (h+a)} \\
&\overset{\texttt{match}}{=} \texttt{a} \\
&\overset{\texttt{math}}{=} \texttt{0 + a} \\
&\overset{\texttt{match}}{=} \texttt{match [] with [] -> 0 | h::t -> h + summa t + a} \\
&\overset{\texttt{summa}}{=} \texttt{summa [] + a} \\
&\overset{\texttt{def l}}{=} \texttt{summa l + a}
\end{aligned}$$

- Induction step: $x > 0$ `t = h::t`

$$\begin{aligned}
\texttt{sum l a} &\overset{\texttt{sum}}{=} \texttt{match l with [] -> a | h::t -> sum t (h+a)} \\
&\overset{\texttt{def l}}{=} \texttt{match h::t with [] -> a | h::t -> sum t (h+a)} \\
&\overset{\texttt{match}}{=} \texttt{sum t (h+a)} \\
&\overset{\texttt{I.H.}}{=} \texttt{summa t + h+a} \\
&\overset{\texttt{comm.}}{=} \texttt{h + summa t + a} \\
&\overset{\texttt{match}}{=} \texttt{(match h::t with [] -> 0 | h::t -> h + summa t) + a} \\
&\overset{\texttt{def l}}{=} \texttt{(match l with [] -> 0 | h::t -> h + summa t) + a} \\
&\overset{\texttt{summa}}{=} \texttt{summa l + a}.
\end{aligned}$$

Now we proove that

$$\texttt{mul c n a = c * n + a}$$

by induction on $\texttt{c} \geq 0$

- Base case: $c = 0$

$$\begin{aligned}
\texttt{mul c n a} &\overset{\texttt{mul}}{=} \texttt{if c <= 0 then a else mul (c-1) n (n+a)} \\
&\overset{\texttt{def c}}{=} \texttt{if 0 <= 0 then a else mul (0-1) n (n+a)} \\
&\overset{\texttt{if}}{=} \texttt{a} \\
&\overset{\texttt{math}}{=} \texttt{0 * n + a} \\
&\overset{\texttt{def c}}{=} \texttt{c * n + a}
\end{aligned}$$

- Induction step: `c` $> 0$

$$
\begin{aligned}
\texttt{mul c n a} &\overset{\texttt{mul}}{=} \texttt{if c <= 0 then a else mul (c-1) n (n+a)} \\
&\overset{\texttt{if}}{=} \texttt{mul (c-1) n (n+a)} \\
&\overset{\texttt{I.H.}}{=} \texttt{(c-1) * n + (n+a)} \\
&\overset{\texttt{math}}{=} \texttt{c * n - n + n + a} \\
&\overset{\texttt{math}}{=} \texttt{c * n + a}
\end{aligned}
$$

Prooven.

To prove that

```
renroh p (horner p y s) list = horner p (rev y list) s
```

holds for all `int` values `p` and `s` and lists `y, list`, we first need to prove that

```
renroh p n (h::t) = horner p (n::t) h
```

By induction on length $x$ of `list`

- Base case: $x = 0 \implies$ `list = []`

$$\texttt{renroh p (horner p y s) list} \overset{\texttt{renroh}}{=} \texttt{match list with [] -> s | ...}$$
$$\overset{\texttt{def\_list}}{=} \texttt{match [] with [] -> s | ...}$$
$$\overset{\texttt{match}}{=} \texttt{s}$$
$$\overset{\texttt{match}}{=} \texttt{s}$$