Homework 1
Part 1

1. Find the vector $u$ that is perpendicular to the vector $v = (3, 4)$ and the size of which is 15.

2. Verify Lagrange's identity $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|^2\|\vec{v}\|^2 - (\vec{u} \cdot \vec{v})^2$ for vectors $\vec{u} = (-1, 1, -2)$ and $\vec{v} = (2, -1, 0)$.

3. If we locate vectors $\vec{u}$ and $\vec{v}$ such that they form adjacent sides of a parallelogram, then the area of the parallelogram is given by $\|\vec{u} \times \vec{v}\|$. Consider points $A(2, -3, 4), B(0, 1, 2)$ and $C(-1, 2, 0)$.

   (a) Find the area of parallelogram $ABCD$ with adjacent sides $\vec{AB}$ and $\vec{AC}$;

   (b) Find the area of triangle $ABC$.

4. Nonzero vector $\vec{u}$ and $\vec{v}$ are called collinear if there exists a nonzero scalar $\alpha$ such that $\vec{v} = \alpha\vec{u}$. Show that vectors $\vec{AB}$ and $\vec{AC}$ are collinear, where $A(4, 1, 0), B(6, 5, -2)$ and $C(5, 3, -1)$.

5. Consider points $P(3, 7, -2)$ and $Q(1, 1, -3)$. Determine the angle between vectors $\vec{OP}$ and $\vec{OQ}$. ($O$ represents the origin).

6. Find $A^T A - 2A$, if $A = \begin{pmatrix} 3 & -1 \\ 0 & 2 \end{pmatrix}$.

7. Solve the matrix equation: $XA = B$, if $A = \begin{pmatrix} 2 & 1 \\ -4 & -3 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & 2 \\ 6 & 4 \end{pmatrix}$

8. To find the inverse of $n \times n$ matrix $A$, you can use formula:

$$A^{-1} = \frac{1}{\det(A)} \cdot adj(A); \det A \neq 0,$$

   where

$$adj(A) = \begin{pmatrix} C_{11} & C_{21} & ... & C_{n1} \\ C_{12} & C_{22} & ... & C_{n2} \\ ... & & & \\ C_{1n} & C_{2n} & ... & C_{nn} \end{pmatrix}$$

   (This is transpose of cofactor matrix).

   Find the inverse of the following matrices:

(a) $\begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \\ 0 & -1 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix}$