Homework 2

1. For which real values of a does the matrix $A = \begin{pmatrix} 2 & a \\ -1 & 1 \end{pmatrix}$ have real eigenvalues?

2. Solve the systems of linear equations using Gaussian elimination:

   (a) $\begin{cases} 3x - 4y = -7 \\ -6x + 8y = 14 \end{cases}$

   (b) $\begin{cases} -x + 2y - 4z = 8 \\ 3y + 8z = -4 \\ -7x + y + 2z = 1 \end{cases}$

3. In a triangle, the smallest angle measures 10° more than one-half of the largest angle. The middle angle measures 12° more than the smallest angle. Find the measure of each angle.

4. Find the $\|.\|_1$, $\|.\|_2$, $\|.\|_3$, $\|.\|_\infty$, $\|.\|_A$ norms for the vector

$$v = (-5, 4, 5)^T \text{ if } A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

5. Check if the following expression is a norm of a vector $x = (x_1, ..., x_n)$:

$$\sum_{k=1}^{n} \left| \sum_{i=1}^{k} x_i \right|$$

6.
$$A = \begin{pmatrix} 5 & 2 \\ 2 & 5 \end{pmatrix}, \|x\|_* = 5|x_1| + |x_2|$$

   Find the set of points on the plane where $\|x\|_* = 1$.

7. Find the $\|.\|_1$, $\|.\|_2$ $\|.\|_\infty$ norms for the matrix $\begin{pmatrix} 0 & 1 \\ 9 & 0 \end{pmatrix}$

8. Prove that $\|A\|_1, \|A\|_2$ are matrix norms.

9. The condition number of an invertible matrix A is defined to be $K(A) = \|A\|\|A^{-1}\|$

(a) Find the condition numbers of a matrix $A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$ in $\|.\|_1$ norm

(b) Find the condition numbers of a matrix $A = \begin{pmatrix} -3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ in infinity-norm.