Theory of Computation — Week two

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1. To show that a language is not regular, we can use the pumping lemma and show that a word of the form $a^n b^n$ can not be decomposed into uvx such that $uv^i x \in L$. We must consider all decompositions:
   - $uxv = (a^m)(a^k)(b^n), n = m + k$. We can see that if we pump x, the number of as will be $m + ik$ which is not equal to $m + k = n$ thereby it's not in L.
   - $uxv = (a^m)(a^k b^l)(b^p), n = m + k = l + p$ If we pump x, we will get a word with $k(i - 1)$ number of as after the first b for all $i \geq 2$ a thereby it's not in L.
   - $uxv = (a^n)(b^k)(b^m), n = m + k$ If we pump x, the number of as will be $m + ik$ which is not equal to $m + k = n$ thereby it's not in L.

2. (1) The NFA would be described as

$$\begin{aligned} M &= (Z, A, \delta, z_0, Z_A) \\ Z &= \{z_0, z_1, z_2, z_3, z_4\} \\ A &= \{a, b\} \\ Z_A &= \{z_2, z_4\} \end{aligned} \qquad \begin{aligned} \delta(z_0, x) &= \begin{cases} \{z_0, z_3\} & \text{if } x = a \\ \{z_0, z_1\} & \text{if } x = b \end{cases} \\ \delta(z_1, x) &= \begin{cases} \{\} & \text{if } x = a \\ \{z_2\} & \text{if } x = b \end{cases} \end{aligned} \qquad \begin{aligned} \delta(z_2, x) &= \{z_2\} \\ \delta(z_3, x) &= \begin{cases} \{z_4\} & \text{if } x = a \\ \{\} & \text{if } x = b \end{cases} \\ \delta(z_4, x) &= \{z_4\}. \end{aligned}$$

   (2) M accepts all words that contain at least one a followed by another a or a b followed by another b.

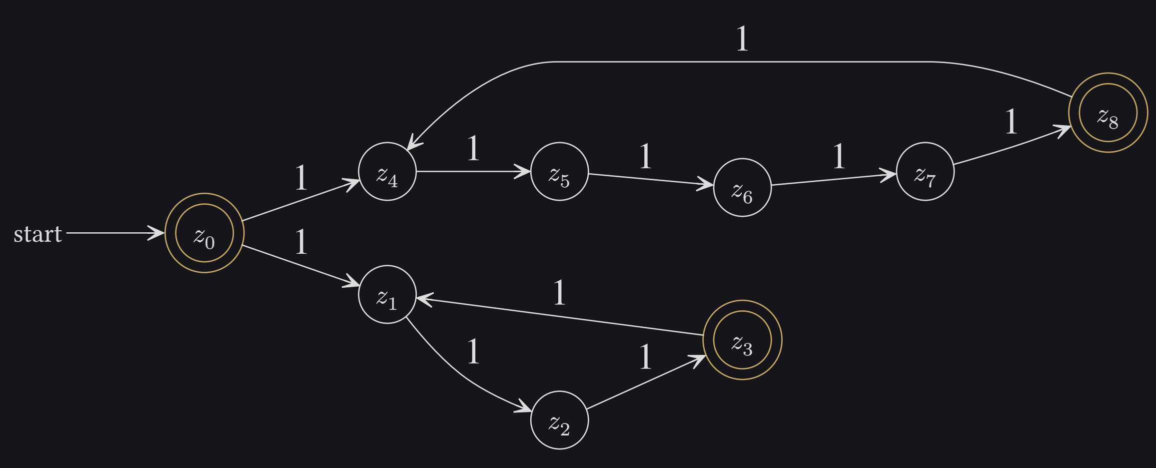
3. 

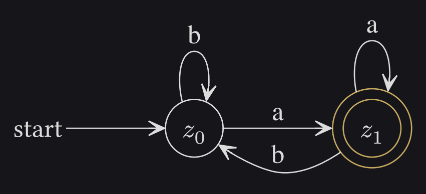
   Figure 1: NFA that accepts $L = \{1^i : i \equiv 0 \bmod 3\} \cup \{1^i : i \equiv 0 \bmod 5\}$

4. A counterexample would be an NFA that accepts all strings from alphabet $\{a, b\}$ which end with a.

$$Q_1 = \{z_0, z_1\}, \quad \Sigma_1 = \{a, b\}, \quad F_1 = \{z_1\}$$

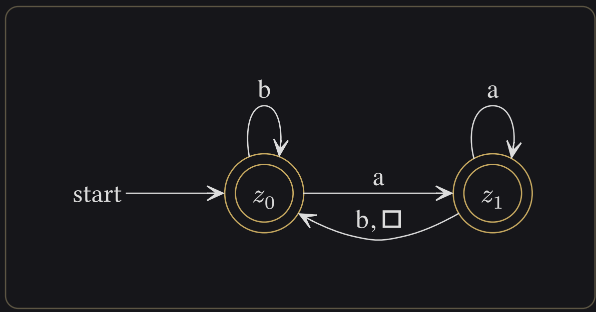
$$\delta_1(z_0, x) = \begin{cases} \{z_1\} \text{ if } x = a \\ \{z_0\} \text{ if } x = b \end{cases}$$

$$\delta_1(z_1, x) = \begin{cases} \{z_1\} \text{ if } x = a \\ \{z_0\} \text{ if } x = b \end{cases}$$



while accepting $L(N_1) = \{a, b\}^* \circ \{a\}$. If we apply the given construction, we get

$$N = (Q, \Sigma, \delta, q, F), \quad Q = Q_1,$$
$$q = q_1, \quad F = F_1 \cup \{q_1\}, \quad \Sigma_\varepsilon = \Sigma_1 \cup \{\varepsilon\}$$
$$\delta_1(z_0, x) = \begin{cases} \{z_1\} \text{ if } x = a \\ \{z_0\} \text{ if } x = b \end{cases}$$
$$\delta_1(z_1, x) = \begin{cases} \{z_1\} \text{ if } x = a \\ \{z_0\} \text{ if } x = b \\ \{z_0\} \text{ if } x = \varepsilon \end{cases}$$



If we look closely, we can see that N would accept something that's not in $L(N_1)^*$ such as bbb.

5. We modify the construction from the previous problem a little bit, and get the solution
$$M = (Z, A, \delta, z_0, Z_A), \quad M'' = (Z'', A, \delta'', z_{\text{start}}, Z''_A)$$
$$Z'' = Z \cup \{z_{\text{start}}\}$$
$$Z''_A = Z_A \cup \{z_{\text{start}}\}$$
$$\delta''(z, x) = \begin{cases} \delta(z, x) & z \notin Z_A \lor x \neq \varepsilon \\ \delta(z, x) \cup \{z_{\text{start}}\} & z \in Z_A \land x = \varepsilon. \end{cases}$$
$$\delta''(z_{\text{start}}, \varepsilon) = \{z_0\}.$$