

Numerical Analysis  
Homework 5

1. Use Lagrange interpolation to find a polynomial that passes through the points:  
  - (a)  $(0, 1), (2, 3), (3, 0)$
  - (b)  $(-1, 0), (2, 1), (3, 1), (5, 2)$ .
2. Use Newton's divided differences to find the interpolating polynomials of the points in Exercise 1.
3. Find  $P(0)$ , where  $P(x)$  is the degree 10 polynomial that is zero at  $x = 1, \dots, 10$  and satisfies  $P(12) = 44$ .
4. Can a degree 3 polynomial intersect a degree 4 polynomial in exactly five points? Explain.
5. Write down the degree 25 polynomial that passes through the points  $(1, -1), (2, -2), \dots, (25, -25)$  and has constant term equal to 25.
6. Prove that the characteristic polynomials  $l_i \in \mathbb{P}_n$  defined as  $l_i(x) = \prod_{j=0, j \neq i}^n \frac{x-x_j}{x_i-x_j}$ ,  $i = 0, \dots, n$  form a basis for  $\mathbb{P}_n$
7. Prove the recursive relation  $f[x_0, \dots, x_n] = \frac{f[x_1, \dots, x_n] - f[x_0, \dots, x_{n-1}]}{x_n - x_0}$ ,  $n \geq 1$ , for Newton divided differences.