Homework 8

1. $$A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix},\ P = I$$

   For the linear system $Ax = b$ write Richardson's method in componentwise or matrix form using the given preconditioner matrix and find optimal step(parameter).

2. Use SOR method with $\omega = 1.2$ to solve the given linear system with a tolerance $TOL = 10^{-3}$ in the l_∞ norm.

$$\begin{cases} 10x_1 - x_2 = 9 \\ -x_1 + 10x_2 - 2x_3 = 7 \\ -2x_2 + 10x_3 = 6 \end{cases}$$

3. Write a program to solve the linear system $Ax = b$ using SOR method.

   INPUT

   - the number of equations and unknowns n;
   - the entries $a_{ij},\ 1 \le i, j \le n$ of the matrix A;
   - the entries $b_i,\ 1 \le i \le n$ of b;
   - the entries $x_i^{(0)},\ 1 \le i \le n$ of $x^{(0)}$
   - the parameter ω;
   - tolerance TOL
   - maximum number of iterations N

   OUTPUT

   - the approximate solution $x_1, ..., x_n$ or a message that the number of iterations was exceeded.

4. Use the SOR method to solve the linear system $Ax = b$ to within 10^{-5} in the l_∞ norm, where the entries of A are

$$a_{i,j} = \begin{cases} 2i,\ when\ j = i\ and\ i = 1, 2, ..., 80 \\ 0.5i,\ when \begin{cases} j = i + 2\ and\ i = 1, 2, ..., 78, \\ j = i - 2\ and\ i = 3, 4, ..., 80 \end{cases} \\ 0.25i\ when \begin{cases} j = i + 4\ and\ i = 1, 2, ..., 76, \\ j = i - 4\ and\ i = 5, 6, ..., 80 \end{cases} \\ 0,\ otherwise \end{cases}$$

   and those of b are $b_i = \pi$, for each $i = 1, 2, ..., 80$.

(Useful material in 'Central Exercises' → 'SOR')