Homework 9

1. Solve $Ax = b$ for the following A and $b = [1, 0, 0]^T$, using GMRES with $x_0 = [0, 0, 0]^T$. Report all approximations x_k up to and including the correct solution.
$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

2. Find QR-factorization of A.

   (a) $A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix}$

   (b) $A = \begin{pmatrix} 2 & 3 \\ -2 & -6 \\ 1 & 0 \end{pmatrix}$

   (c) $A = \begin{pmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{pmatrix}$

3. Use the QR factorization to solve the least squares problem:

   (a) $\begin{pmatrix} 2 & 3 \\ -2 & -6 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 4 \\ -1 & 1 \\ 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 1 \\ -3 \end{pmatrix}$

Optional tasks:

1. Let $A = \begin{pmatrix} 1 & 0 & a_{13} \\ 0 & 1 & a_{23} \\ 0 & 0 & 1 \end{pmatrix}$. Prove that for any x_0 and b, GMRES converges to the exact solution after two steps.

2. If R is upper triangular and invertible,show that there exists a diagonal matrix D with diagonal entries ± 1 such that $R_1 = DR$ is invertible, upper triangular, and has positive diagonal entries.