Homework 6

1. Use Crout factorization for tridiagonal systems to solve the following linear system:
$$\begin{cases} 2x_1 + x_2 = 3 \\ x_1 + 2x_2 + x_3 = -2 \\ 2x_2 + 3x_3 = 0 \end{cases}$$

2. Find LU factorization of the given matrices using Gaussian elimination:

   (a) $\begin{pmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & -1 & 0 \\ 2 & 2 & 3 \\ -1 & 3 & 2 \end{pmatrix}$

3. Consider the following matrices. Find the permutation matrix P so that PA can be factored into the product LU, where L is lower triangular with 1s on its diagonal and U is upper triangular for these matrices.

   (a) $\begin{pmatrix} 0 & 2 & -1 \\ 1 & -1 & 2 \\ 1 & -1 & 4 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 1 & -1 & 2 \\ -1 & -1 & 1 & 5 \\ 2 & 2 & 3 & 7 \\ 2 & 3 & 4 & 5 \end{pmatrix}$

4. Find LDU decomposition of the matrix:
$$\begin{pmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{pmatrix}$$

Remark: You can find useful material in the folder 'Central Exercise' → 'Material for Homework 6' and on the lecture slides