Numerical Analysis
Homework 1

1. Find the error term and order for the approximation formula
$$f'(x) = \frac{4f(x+h) - 3f(x) - f(x-2h)}{6h}.$$

2. Develop a second-order method for approximating $f'(x)$ that uses the data $f(x-h), f(x)$ and $f(x+3h)$ only.

3. Develop a first-order method for approximating $f''(x)$ that uses the data $f(x-h), f(x)$ and $f(x+3h)$ only.

4. Prove the second-order formula for the third derivative
$$f'''(x) = \frac{-f(x-2h) + 2f(x-h) - 2f(x+h) + f(x+2h)}{2h^3} + \mathcal{O}(h^2)$$

5. Let $f'(x)$ be a six-times continuously differentiable function.
   Prove that if $f(x) = f'(x) = 0$, then
$$f^{(IV)}(x+h) - \frac{16f(x+h) - 9f(x+2h) + \frac{8}{3}f(x+3h) - \frac{1}{4}f(x+4h)}{h^4} = \mathcal{O}(h^2)$$
   Hint:
   Use formula: $f^{(iv)}(x) = \frac{f(x-2h)-4f(x-h)+6f(x)-4f(x+h)+f(x+2h)}{h^4} + \mathcal{O}(h^2)$;
   Show that if $f(x) = f'(x) = 0$, then $f(x-h) - 10f(x+h) + 5f(x+2h) - \frac{5}{3}f(x+3h) + \frac{1}{4}f(x+4h) = \mathcal{O}(h^6)$