Homework for week 1

Dimitri Tabatadze

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2.1

Problem 5:

(a) (i)

$$\begin{aligned}\frac{(80-4.9\cdot 4.1^2)-(80-4.9\cdot 4^2)}{0.1} &= 4.9\frac{4^2-4.1^2}{0.1}\\ &= 4.9\frac{(4+4.1)(4-4.1)}{0.1}\\ &= 49\cdot 8.1\cdot(-0.1)\\ &= 8.1\cdot -4.9\\ &= -39.69\end{aligned}$$

(ii)

$$\begin{aligned}\frac{(80-4.9\cdot 4.05^2)-(80-4.9\cdot 4^2)}{0.05} &= 4.9\frac{4^2-4.05^2}{0.05}\\ &= 4.9\frac{(4+4.05)(4-4.05)}{0.05}\\ &= 98\cdot 8.05\cdot(-0.05)\\ &= 8.05\cdot -4.9\\ &= -39.445\end{aligned}$$

(iii)

$$\begin{aligned}v_{[4,4.01]} = \frac{(80-4.9\cdot 4.01^2)-(80-4.9\cdot 4^2)}{0.01} &= 4.9\frac{4^2-4.01^2}{0.01}\\ &= 4.9\frac{(4+4.01)(4-4.01)}{0.01}\\ &= 490\cdot 8.01\cdot(-0.01)\\ &= 8.01\cdot -4.9\\ &= -39.249\end{aligned}$$

(b)

$$\begin{aligned}v_4 = \lim_{x\to 0}\frac{(80-4.9\cdot(4+x)^2)-(80-4.9\cdot 4^2)}{x} &= \lim_{x\to 0}\frac{4.9\cdot 4^2-4.9\cdot(4+x)^2}{x}\\ &= 4.9\cdot\lim_{x\to 0}\frac{4^2-(4+x)^2}{x}\\ &= 4.9\cdot\lim_{x\to 0}\frac{16-16-8x-x^2}{x}\\ &= 4.9\cdot\lim_{x\to 0}-(8+x)\\ &= 4.9\cdot(-8)\\ &= -39.2\end{aligned}$$

2.2

Problem 5:

(a)

$$\lim_{x\to 1} f(x) = 2$$

(b)

$$\lim_{x\to 3^-} f(x) = 1$$

(c)

$$\lim_{x\to 3^+} f(x) = 4$$

(d)

$$\lim_{x\to 3} f(x) \text{ doesn't exist}$$

(e)

$$f(3) = 3$$

Problem 7:

(a)

$$a = 4$$

(b)

$$a = 5$$

(c)

$$a = 2, 4$$

(d)

$$a = 4$$

Problem 9:

(a)

$$\lim_{x\to -7} f(x) = -\infty$$

(b)

$$\lim_{x\to -3} f(x) = \infty$$

(c)

$$\lim_{x\to 0} f(x) = \infty$$

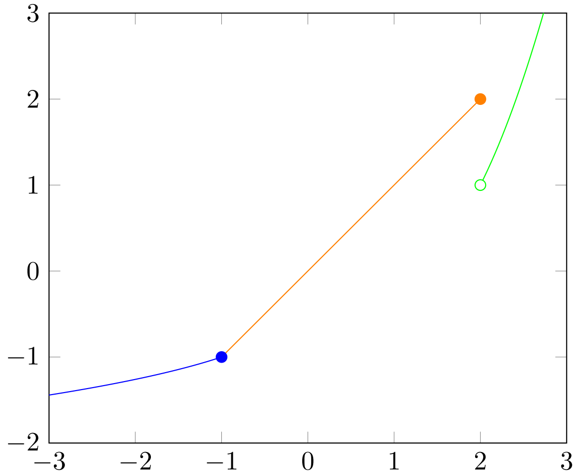
(d)

$$\lim_{x\to 6^-} f(x) = -\infty$$

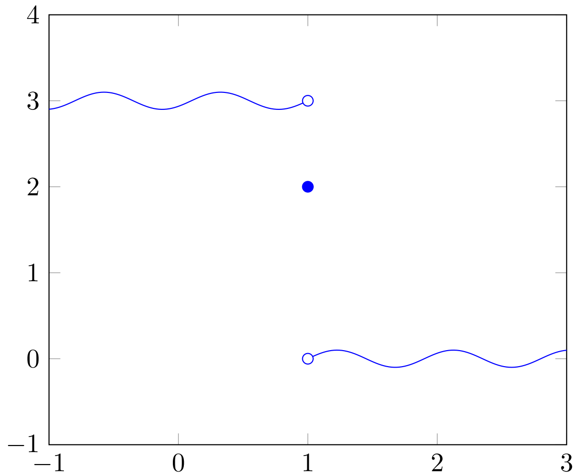
(e)

$$\lim_{x\to 6^+} f(x) = \infty$$

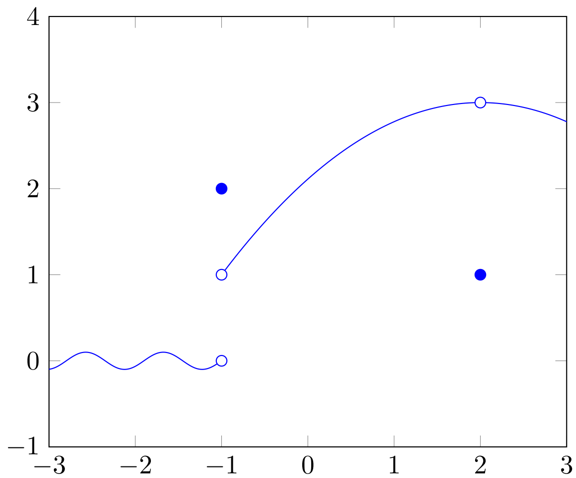
Problem 12:



Problem 15:



Problem 17:



Problem 29:

$$\lim_{x \to 5^+} \frac{x+1}{x-5} = \infty$$

Because when x approaches 5 from the right we have that $x - 5 > 0$ ($x - 5$ approaches 0 from the right) and $x + 2 > 0$.

Problem 31:

$$\lim_{x \to 2} \frac{x^2}{(x-2)^2} = \infty$$

Because when x approaches 2 we have that $(x+2)^2$ approaches 0. and $x^2 > 0$, $(x-2)^2 > 0$.

Problem 33:

$$\lim_{x \to 1^+} \ln(\sqrt{x} - 1) = -\infty$$

Because $\ln(\lim_{x \to 1^+} \sqrt{x} - 1) = -\infty$, $\lim_{x \to 1^+} \sqrt{x} - 1 = 0$ since $\lim_{x \to 1^+} \sqrt{x} = 1$ and $1 - 1 = 0$ and $\lim_{a \to 1^0} \ln(a) = -\infty$.

Problem 35:

$$\lim_{x \to \frac{\pi}{2}^+} \frac{1}{x} \sec x = -\infty$$

Because when x approaches $\frac{\pi}{2}$ from the right, we have that $\cos x$ approaches 0 and $cosx < 0$ and $\frac{1}{x \cdot \cos x} < 0$.

Problem 37:

$$\lim_{x \to 1} \frac{x^2 + 2x}{x^2 - 2x + 1} = \lim_{x \to 1} \frac{(x-1)()}{(x-1)^2} = \infty$$

Because $(x-1)^2$ approaches 0 and $x(x+2)$ approaches 3.

Problem 39:

$$\lim_{x \to 0} (\ln x^2 - x^{-2}) = -\infty$$

Problem 41:

$x = -2$

2.3

Problem 5:

$$\begin{aligned}
\lim_{v\to 2}(v^2+2v)(2v^3-5) &= (\lim_{v\to 2}(v^2+2v))\cdot(\lim_{v\to 2}(2v^2-5)) && \text{Product law}\\
&= (\lim_{v\to 2} v^2 + \lim_{v\to 2} 2v)\cdot(\lim_{v\to 2} 2v^3 - 5) && \text{Sum \& Difference law}\\
&= (\lim_{v\to 2} v^2 + 2\cdot \lim_{v\to 2} v)\cdot(2\cdot \lim_{v\to 2} v^3 - 5) && \text{Constant multiple law}\\
&= (4+2\cdot 2)(2\cdot 8-5)\\
&= 8\cdot 11\\
&= 88
\end{aligned}$$

Problem 7:

$$\begin{aligned}
\lim_{u\to -2}\sqrt{9-u^3+2u^2} &= \sqrt{\lim_{u\to -2}(9-u^3+2u^2)} && \text{Root law}\\
&= \sqrt{\lim_{u\to -2} 9 - \lim_{u\to -2} u^3 + \lim_{u\to -2} 2u^2} && \text{Sum \& Difference laws}\\
&= \sqrt{9 - \lim_{u\to -2} u^3 + 2\cdot \lim_{u\to -2} u^2} && \text{Constant Multiple law}\\
&= \sqrt{9+8+2\cdot 4}\\
&= 5
\end{aligned}$$

Problem 9:

$$\begin{aligned}
\lim_{t\to -1}(\frac{2t^5-t^4}{5t^2+4})^3 &= \left(\frac{\lim_{t\to -1}(2t^5-t^4)}{\lim_{t\to -1}(5t^2+4)}\right)^3 && \text{Quotient law}\\
&= \left(\frac{\lim_{t\to -1} 2t^5 - \lim_{t\to -1} t^4}{\lim_{t\to -1} 5t^2 + \lim_{t\to -1} 4}\right)^3 && \text{Sum \& Differnece laws}\\
&= \left(\frac{2\cdot \lim_{t\to -1} t^5 - \lim_{t\to -1} t^4}{5\cdot \lim_{t\to -1} t^2 + 4}\right)
\end{aligned}$$