$$10! \sum_{i=1}^{10} i^{10} \tag{1}$$

1. (a)
$$\begin{aligned} Z &= ABC\overline{D} + AB\overline{C}D + A\overline{B}CD + \overline{A}BCD + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}C\overline{D} + \overline{A}B\overline{C}\,\overline{D} + A\overline{B}\,\overline{C}\,\overline{D}. \\ Y &= AB\overline{C}\,\overline{D} + A\overline{B}C\overline{D} + \overline{A}BC\overline{D} + A\overline{B}\,\overline{C}D + \overline{A}B\overline{C}D + \\ &\quad \overline{A}\,\overline{B}CD + A\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}\,\overline{D} + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D. \\ X &= \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}. \end{aligned}$$

   (b)
$$\begin{aligned} Z &= (A+B+C+D)(A+B+\overline{C}+\overline{D})(A+\overline{B}+C+\overline{D})(\overline{A}+B+C+\overline{D}) \cdot \\ &\quad (A+\overline{B}+\overline{C}+D)(\overline{A}+B+\overline{C}+D)(\overline{A}+\overline{B}+C+D)(\overline{A}+\overline{B}+\overline{C}+\overline{D}). \\ Y &= (A+B+C+\overline{D})(A+B+\overline{C}+D)(A+\overline{B}+C+D)(\overline{A}+B+C+D) \cdot \\ &\quad (A+\overline{B}+\overline{C}+\overline{D})(\overline{A}+B+\overline{C}+\overline{D})(\overline{A}+\overline{B}+C+\overline{D})(\overline{A}+\overline{B}+\overline{C}+D). \end{aligned}$$

2. (a) $X = \overline{A}\,\overline{B}\,\overline{C}DE\overline{F}G$

   (b) $Y = \overline{A}\,\overline{B}\,\overline{C}D\overline{E}F\overline{G}$

   (c) $Z = A + B$, numbers 0-31 (or rather 0000000-0011111) are ignored, so we only care about the numbers, one of two most significant bits of which are 1 (11XXXXX)

3. $F_1 = \sum m(0,4,5,6)$, $F_2 = \sum m(0,3,6,7)$

$$\begin{aligned} F_1 + F_2 &= \sum m(0,4,5,6) + \sum m(0,3,6,7) \\ &= \sum m(0,3,4,5,6,7) && \text{since } (a \lor b) \lor (b \lor c) = a \lor b \lor c \end{aligned}$$

Proof.

let $F_1 = \sum_{i=0}^{2^n-1} a_i m_i, \;\; F_2 = \sum_{j=0}^{2^n-1} b_j m_j.$

$$\begin{aligned} F_1 + F_2 &= \sum_{i=0}^{2^n-1} a_i m_i + \sum_{j=0}^{2^n-1} b_j m_j \\ &= a_0 m_0 + a_1 m_1 + \cdots + b_0 m_0 + b_1 m_1 + b_2 m_2 + \ldots \\ &= (a_0 + b_0) m_0 + (a_1 + b_1) m_1 + (a_2 + b_2) m_2 + \ldots \\ &= \sum_{i=0}^{2^n-1} (a_i + b_i) m_i \end{aligned}$$

4. (a) $F = A \oplus \overline{B}, \;\; X_1 = 1, X_2 = 0$

   (b) $G = C, \;\; X_1 = 0, X_2 = 0$