Discrete Probability Theory — Homwework 1

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Variant 1

1. 1.1. Assuming that there are 26 letters and 10 digits, the number of possible license plates is
$$26^5 \cdot 10^2 = 1188137600. \text{ (not my id number)}$$

   1.2. We need to count the number of permutations insde the groups and then multiply that by the total permutations of all groups
$$\underbrace{2! \cdot 3! \cdot 4! \cdot 1!}_{\text{inside the groups}} \cdot 4! = 6912.$$

2. 2.1. We should multiply the number of possible choices of 3 women out of 5 and 5 men out of 8
$$\binom{5}{3} \cdot \binom{8}{5} = 10 \cdot 56 = 560.$$

   2.2. We can consider placing the functional antenas and then looking at the inbetweens as availbale spots for defective ones. That would give us $6 + 1$ spots for 4 antenas which is
$$\binom{6+1}{4} = 35.$$

3. 3.1. There are $7! = 5040$ permutations of 7 gifts but since the order for each child doesn't matter, we must divide that by number of permutations inside each group
$$\frac{7!}{3! \cdot 2! \cdot 2!} = 210.$$

   3.2. We can consider all possible partitions of 7 answered questions
$$\binom{5}{3}\binom{5}{4} + \binom{5}{4}\binom{5}{3} + \binom{5}{5}\binom{5}{2} = 110.$$

4. 4.1. Since there is no restriction on prefixed 0s, I will consider 000…000 to be a n-digit number. The first digit doesn't matter so it has 10 possible values. However, all consecutive digits have only 9 possible since they can not be equal to the previous digit
$$10 \cdot \underbrace{9 \cdot 9 \cdot \ldots \cdot 9}_{n-1} = 10 \cdot 9^{n-1}.$$
   If we were to be restricted to numbers starting with a non-zero digit, we would have only 9 possibilities for the first digit as well as the consecutive ones and we would get
$$\underbrace{9 \cdot 9 \cdot \ldots \cdot 9}_{n} = 9^n.$$

   4.2. If we have to pick one student from each group, the answer would be 3^n but if we can pick any 3 students, the answer would very straightforwardly be $\binom{3n}{3}$.