Homework — Numerical Linear Algebra

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1. The steps:

   (1)

$$r_0 = b - Ax_0 = \begin{pmatrix}1\\0\\0\end{pmatrix} - \begin{pmatrix}0\\0\\0\end{pmatrix} = \begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$\alpha_0 = \frac{\langle Ar_0, r_0\rangle}{\langle Ar_0, Ar_0\rangle} = \frac{\left\langle \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}1\\0\\0\end{pmatrix} \right\rangle}{\left\langle \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix} \right\rangle} = \frac{1}{2}$$

$$x_1 = x_0 + \alpha_0 r_0 = \begin{pmatrix}1/2\\0\\0\end{pmatrix}$$

   (2)

$$r_1 = b - Ax_1 = \begin{pmatrix}1\\0\\0\end{pmatrix} - \begin{pmatrix}1/2\\0\\1/2\end{pmatrix} = \begin{pmatrix}1/2\\0\\-1/2\end{pmatrix}$$

$$\alpha_1 = \frac{\langle Ar_1, r_1\rangle}{\langle Ar_1, Ar_1\rangle} = \frac{\left\langle \begin{pmatrix}1/2\\0\\0\end{pmatrix}, \begin{pmatrix}1/2\\0\\-1/2\end{pmatrix} \right\rangle}{\left\langle \begin{pmatrix}1/2\\0\\0\end{pmatrix}, \begin{pmatrix}1/2\\0\\0\end{pmatrix} \right\rangle} = 1$$

$$x_2 = x_1 + \alpha_1 r_1 = \begin{pmatrix}1\\0\\-1/2\end{pmatrix}$$

   (3)

$$r_2 = b - Ax_2 = \begin{pmatrix}1\\0\\0\end{pmatrix} - \begin{pmatrix}1\\0\\-1/2\end{pmatrix} = \begin{pmatrix}0\\0\\1/2\end{pmatrix}$$

$$\alpha_2 = \frac{\langle Ar_2, r_2\rangle}{\langle Ar_2, Ar_2\rangle} = \frac{\left\langle \begin{pmatrix}0\\0\\1/2\end{pmatrix}, \begin{pmatrix}0\\0\\1/2\end{pmatrix} \right\rangle}{\left\langle \begin{pmatrix}0\\0\\1/2\end{pmatrix}, \begin{pmatrix}0\\0\\1/2\end{pmatrix} \right\rangle} = 1$$

$$x_3 = x_2 + \alpha_2 r_2 = \begin{pmatrix}1\\0\\-1\end{pmatrix}$$

(4)

$$r_2 = b - Ax_3 = \begin{pmatrix}1\\0\\0\end{pmatrix} - \begin{pmatrix}1\\0\\-1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

stop.

2. (a)

$$A = \begin{pmatrix}1 & 2\\1 & 1\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{2\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{2\sqrt{2}}\end{pmatrix}\begin{pmatrix}\sqrt{2} & \sqrt{2}\\0 & \sqrt{2}\end{pmatrix} = QR$$

(b)

$$A = \begin{pmatrix}2 & 3\\-2 & -6\\1 & 0\end{pmatrix} = \begin{pmatrix}\frac{2}{9} & \frac{23}{3\sqrt{337}}\\ -\frac{2}{9} & -\frac{50}{3\sqrt{337}}\\ \frac{1}{9} & -\frac{2}{3\sqrt{337}}\end{pmatrix}\begin{pmatrix}\sqrt{2} & 2\\0 & \frac{\sqrt{337}}{3}\end{pmatrix} = QR$$

3. (a)

$$\begin{pmatrix}2 & 3\\-2 & -6\\1 & 0\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}3\\-3\\6\end{pmatrix}$$

$$R\begin{pmatrix}x_1\\x_2\end{pmatrix} = Q^T\begin{pmatrix}3\\-3\\6\end{pmatrix}$$

$$\begin{pmatrix}\sqrt{2} & 2\\0 & \frac{\sqrt{337}}{3}\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}\frac{2}{9} & -\frac{2}{9} & \frac{1}{9}\\ \frac{23}{3\sqrt{337}} & -\frac{50}{3\sqrt{337}} & -\frac{2}{3\sqrt{337}}\end{pmatrix}\begin{pmatrix}3\\-3\\6\end{pmatrix}$$

$$= \begin{pmatrix}2\\ \frac{77}{\sqrt{337}}\end{pmatrix}$$

$$\begin{pmatrix}x_1\\x_2\end{pmatrix} = \begin{pmatrix}\frac{229}{\sqrt{2}}\\231\end{pmatrix}$$