KUTAISI INTERNATIONAL UNIVERSITY

Homework 4
Numerical Linear Algebra
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1. (a)

$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$$

   (b)

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

   (c)

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$$

2. Since all of the elements of the matrix are convergent

$$\lim_{n\to\infty} \frac{n}{n^2+1} = \lim_{n\to\infty} \frac{1}{2n} = 0$$
$$\lim_{n\to\infty} \frac{1}{n^2+1} = 0$$

   the matrix will also be convergent

$$\lim_{n\to\infty} \begin{pmatrix} \frac{n}{n^2+1} & 0 \\ 0 & \frac{1}{n^2+1} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

3.

$$H_3 = \begin{pmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 1/3 & 1/4 \\ 1/3 & 1/4 & 1/5 \end{pmatrix}, H_3^{-1} = \begin{pmatrix} 9 & -36 & 30 \\ -36 & 192 & -180 \\ 30 & -180 & 180 \end{pmatrix}$$

   now the condition number:

$$\|H_3\|\|H_3^{-1}\| = \frac{11}{6} \cdot 30 = 55$$

4. This is literaly the definition of the frobenius norm, but sure:

$$\|A\|_F = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2 \right)^{\frac{1}{2}} \qquad \Longrightarrow$$

$$\begin{aligned} \|A\|_F^2 &= \sum_{i=1}^{m} \sum_{j=1}^{n} |a_{ij}|^2 \\ &= \sum_{j=1}^{n} \left(\sum_{i=1}^{m} |a_{ij}|^2 \right) \\ &= \sum_{j=1}^{n} \left(\left(\sum_{i=1}^{m} |a_{ij}|^2 \right)^{\frac{1}{2}} \right)^2 \\ &= \sum_{j=1}^{n} (|a_j|_2)^2 \end{aligned}$$