I2DS25 exercise 1

Wolfgang J. Paul

March 4, 2025

All exercises are about distributed systems with asynchronous communication. Use direct or indirect addressing as you wish. Solutions should be strictly in the formalism, in which systems were defined in the lecture. We consider $0 \in \mathbb{N}$.

1 Sending numbers to oneself and increasing them (20 Pt)

This exercise is very important. You are *not* supposed to learn the definition of distributed systems by examples. You are suppose to construct examples yourself in order to understand the definitions. You have seen many formal models of computations, so you should be able to do it.

- Define a system with a single process, wich can send messages to itself.
- in a set S, which initially contains only 0, it can store a finite set of natural numbers. In a 'register' R it can store a single natural number. Initially $R = 0$.
- In an internal step the process can pick an element from S and put it into register R
- in a send step it can send the number in R (to itself).
- in a receive step it increments the number received and puts it into S

Hints: use as your set of states

$$Z = \{S \subset \mathbb{N} \mid S \text{ finite}\} \times \mathbb{N}$$

For the internal transition relation you have to define when

$$S \times R \rightarrow^i S' \times R'$$

2 Constructing example computations (20 Pt)

Show for the system of the previous exercise

1. there is an infinite computation which puts every natural number into S.
2. there is an infinite computation in which set S stays finite.

3 Invariants (20 P)

Show: if $\{P\} \rightarrow \{P\}$ holds for a system, then P holds in every state of every computation of the system. Hint: the proof in 'Tel' has a typo. Do not copy it blindly.

4 Lexicographic order (20 Pt)

Let $<$ be an order on an alphabet A. Extend it to $<_{lex}$ on A^+ in the following way. Number strings from left to right

$$a = a[1:n] \in A^+$$

For two strings $a, b \in A^+$ let $fd(a,b)$ be the first index (from the left), where the strings differ if it exists. For

$$a, b \in A^+ \ , \ a \neq b$$

define $a <_{lex} b$ if

- $a_{f(a,b)} < b_{f(a,b)}$ if $f(a,b)$ exists or
- a is prefix of b

Prove or disprove:

1. $<_{lex}$ is well founded on A^n for every n
2. $<_{lex}$ is well founded on A^+.

5 Reasons for limiting the power of transitions (20 Pt)

We have restricted transition functions to be computable. Imagine we would not have done this. Constract (well, better: define) a process, which on inputs x received sends non computable answers $f(x)$.