1. **Proof.**
   Given

$$\begin{aligned} d(1) &= d(\text{FA}) \\ d(n) &= d(n/2) + d(\text{MUX}) \end{aligned}$$

assume

$$d(n) = d(\text{FA}) + d(\text{MUX}) \cdot \log_2 n$$

base case:

$$\begin{aligned} d(1) &= d(\text{FA}) + d(\text{MUX}) \cdot \log_2(1) \\ &= d(\text{FA}) + 3 \cdot 0 \\ &= d(\text{FA}) \end{aligned}$$

induction step:

$$\begin{aligned} d(n) &= d(n/2) + d(\text{MUX}) \\ &= d(\text{FA}) + d(\text{MUX}) \cdot \log_2(n/2) + d(\text{MUX}) \\ &= d(\text{FA}) + d(\text{MUX}) \cdot (\log_2 n - 1) + d(\text{MUX}) \\ &= d(\text{FA}) + d(\text{MUX}) \cdot \log_2 n \\ &= d(\text{FA}) + 3 \cdot \log_2(n) \end{aligned}$$

proven.

2. Given

$$\begin{aligned} d(2) &= 1, \\ d(n) &= d(n/2) + 2, \\ c(2) &= 1, \\ c(n) &\leq c(n/2) + n \end{aligned}$$

Proof.
assume $d(n) = 2 \cdot \log_2 n - 1$
base case:

$$\begin{aligned} d(2) &= 2 \cdot \log_2 2 - 1 \\ &= 2 \cdot 1 - 1 \\ &= 1 \end{aligned}$$

induction step:

$$\begin{aligned} d(n) &= d(n/2) + 2 \\ &= 2 \cdot \log_2(n/2) - 1 + 2 \\ &= 2 \cdot (\log_2 n - 1) + 1 \\ &= 2 \cdot \log_2 n - 2 + 1 \\ &= 2 \cdot \log_2 n - 1 \end{aligned}$$

$$d(n) = 2 \cdot \log_2 n - 1 = O(\log n)$$

proven.

Proof.
assume $c(n) \leq 2n$
base case:

$$c(2) = 1 \quad \leq \quad 4$$

induction step:

$$\begin{aligned} c(n) &\leq c(n/2) + n \\ &\leq (n/2) \cdot 2 + n \\ &\leq 2n \end{aligned}$$

$$c(n) \leq 2n = O(n)$$

proven.

3. :(

4.

$$\begin{aligned} \text{ovf}u &= (\text{sub} \wedge (\langle a \rangle < \langle b \rangle)) \vee (\overline{\text{sub}} \wedge C_n) \wedge u \\ &= ((\text{sub} \wedge \overline{C_n}) \vee (\overline{\text{sub}} \wedge C_n)) \wedge u \\ &= (\text{sub} \oplus C_n) \wedge u \end{aligned}$$

5. Given $a_{n-1} \oplus b_{n-1} \oplus c_{n-1} = S_{n-1}$ we can simply say that $c_{n-1} = a_{n-1} \oplus b_{n-1} \oplus S_{n-1}$

6. :(