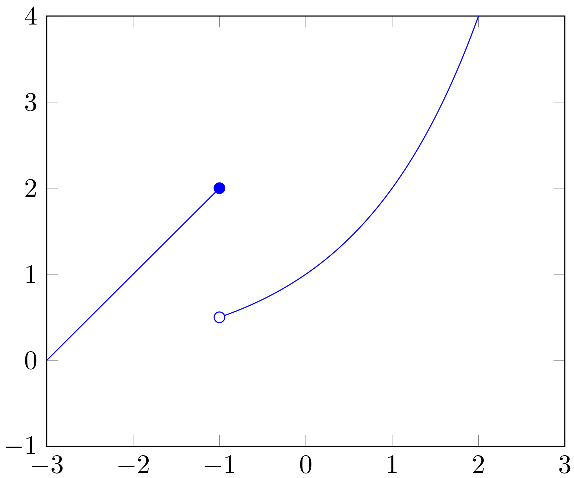
2.5

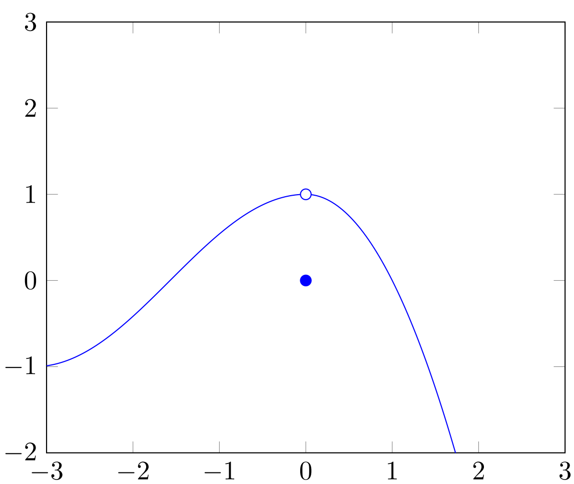
Problem 21.

$$\begin{aligned}
\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x) \quad &\implies \\
\lim_{x\to 2^-} (x+3) \neq \lim_{x\to 2^+} 2^x \quad &\implies \\
\lim_{x\to 2^-} x + \lim_{x\to 2^-} 3 \neq \lim_{x\to 2^+} 2^x \quad &\implies \\
2+3 \neq 2^2 &
\end{aligned} \tag{1}$$



Problem 23.

$$\begin{aligned}
\lim_{x\to 0} f(x) \neq f(0) \quad &\implies \\
\lim_{x\to 0} (1-x^2) \neq 0 \quad &\implies \\
\lim_{x\to 0} 1 + \lim_{x\to 0} x^2 \neq 0 \quad &\implies \\
1+0 \neq 0 &
\end{aligned} \tag{2}$$



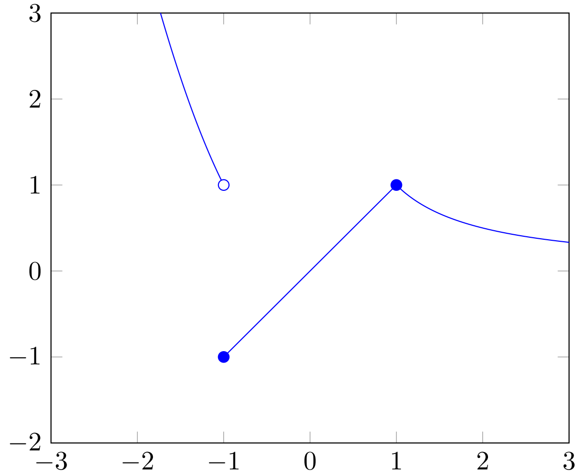
Problem 25.

(a) since $\lim_{x\to 3} f(x)$ exists and is equal to $\frac{1}{6}$ but $f(3)$ doesn't exist, the discontinuity is removable.

(b)

$$f(x) = \begin{cases} \frac{x-3}{x^2-9} & \text{if } x \neq 3 \\ \frac{1}{6} & \text{if } x = 3 \end{cases}$$

Problem 43. $f(x)$ is discontinuous but continuous from the right at $x = -1$.



Problem 48.

$$f(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \leq x < 3 \\ 2x - a + b & \text{if } 3 \leq x \end{cases}$$

we know that $\lim_{x \to 2^+} f(2) = f(2)$ and $\lim_{x \to 3^+} f(x) = f(3)$, so all that's left to align are $\lim_{x \to 2^-} f(x) = f(2)$ and $\lim_{x \to 3^-} f(x) = f(3)$

$$\begin{aligned} \lim_{x \to 2^-} f(x) &= \lim_{x \to 2^-} \frac{x^2-4}{x-2} \\ &= \lim_{x \to 2^-} \frac{(x-2)(x+2)}{x-2} \\ &= \lim_{x \to 2^-} (x+2) \\ &= 4 \end{aligned} \tag{3}$$

$$\begin{aligned} f(2) &= a \cdot 2^2 - b \cdot 2 + 3 \\ &= 4a - 2b + 3 \end{aligned} \tag{4}$$

$$\lim_{x \to 2^-} f(x) = f(2) \implies 4 = 4a - 2b + 3 \implies 4a - 2b = 1 \tag{5}$$

and

$$\begin{aligned} \lim_{x \to 3^-} f(x) &= \lim_{x \to 3^-} (ax^2 - bx + 3) \\ &= 9a - 3b + 3 \end{aligned} \tag{6}$$

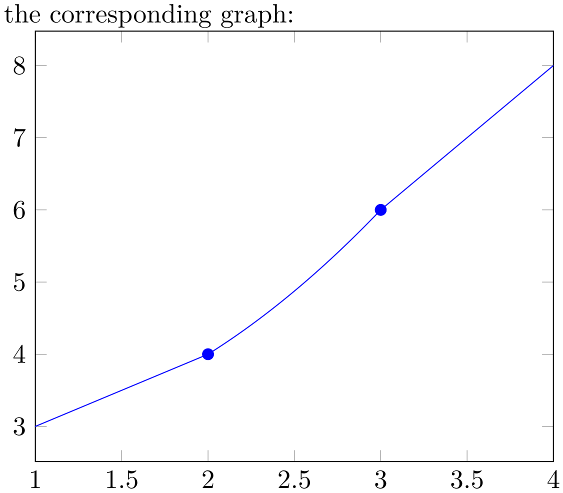
$$\begin{aligned} f(3) &= 2 \cdot 3 - a + b \\ &= 6 - a + b \end{aligned} \tag{7}$$

$$\lim_{x \to 3^-} f(x) = f(3) \implies 9a - 3b + 3 = 6 - a + b \implies 10a - 4b = 3 \tag{8}$$

then, we simply have the following system to solve:

$$
\begin{aligned}
&\begin{cases} 10a - 4b = 3 & (5) \\ 4a - 2b = 1 & (8) \end{cases} \\
\implies &\begin{cases} 10a - 4b = 3 \\ -8a + 4b = -2 \end{cases} \\
\implies &\begin{cases} 2a = 1 \\ 4(\frac{1}{2}) - 2b = 3 \end{cases} \\
\implies &\begin{cases} 2a = 1 \\ 2b = 1 \end{cases} \\
\implies &\begin{cases} a = \frac{1}{2} \\ b = \frac{1}{2} \end{cases}
\end{aligned} \tag{9}
$$

the corresponding graph:



Problem 55. Given that $f(x) = -x^3 + 4x + 1$ is continuous and the range $(-1, 0)$ maps to $(-2, 1)$, by the Intermediate Value Theorem we know that if $f(a) = 0$ then $a \in (-1, 0)$.

Problem 72. $f(x)$ is never continuous.

2.6

Problem 3.

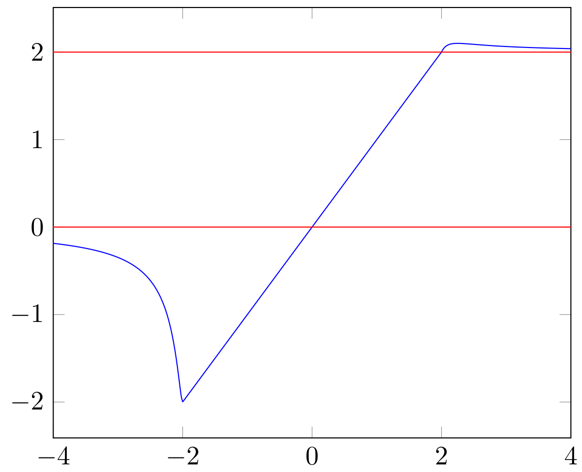
(a) $\lim_{x\to\infty} f(x) = -2$

(b) $\lim_{x\to-\infty} f(x) = 2$

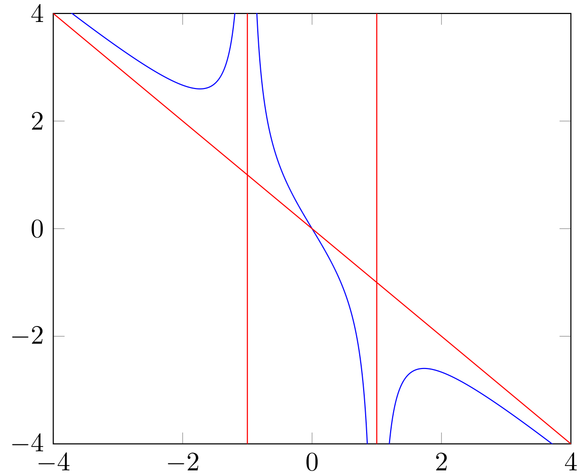
(c) $\lim_{x\to 1} f(x) = \infty$

(d) $\lim_{x\to 3} f(x) = -\infty$

(e)
- $y = 2$
- $y = -2$
- $x = 1$
- $x = 3$



Problem 5.



Problem 9.

Problem 13.

$$
\begin{aligned}
\lim_{x\to\infty} \frac{2x^2-7}{5x^2+x-3} &= \lim_{x\to\infty} \frac{\frac{2x^2-7}{x^2}}{\frac{5x^2+x-3}{x^2}} = \lim_{x\to\infty} \frac{2-\frac{7}{x^2}}{5+\frac{1}{x}-\frac{3}{x^2}} \\
&= \frac{\lim\limits_{x\to\infty}\left(2-\frac{7}{x^2}\right)}{\lim\limits_{x\to\infty}\left(5+\frac{1}{x}-\frac{3}{x^2}\right)} && \text{(by Limit Law 5)} \\
&= \frac{\lim\limits_{x\to\infty} 2 - \lim\limits_{x\to\infty} \frac{7}{x^2}}{\lim\limits_{x\to\infty} 5 + \lim\limits_{x\to\infty} \frac{1}{x} - \lim\limits_{x\to\infty} \frac{3}{x^2}} && \text{(by 1, 2 and 3)} \\
&= \frac{2-0}{5+0-0} && \text{(by 8 and Theorem 5)} \\
&= \frac{2}{5}
\end{aligned}
$$

Problem 17.

$$\begin{aligned}
\lim_{t\to-\infty}\frac{3t^2+t}{t^2-4t+1} &= \lim_{t\to-\infty}\frac{\frac{3t^2+t}{t^2}}{\frac{t^2-4t+1}{t^2}} = \lim_{t\to-\infty}\frac{3+\frac{1}{t}}{1-\frac{4}{t}+\frac{1}{t^2}}\\
&= \frac{\lim\limits_{t\to-\infty}\left(3+\frac{1}{t}\right)}{\lim\limits_{t\to-\infty}\left(1-\frac{4}{t}+\frac{1}{t^2}\right)}\\
&= \frac{\lim\limits_{t\to-\infty}3+\lim\limits_{t\to-\infty}\frac{1}{t}}{\lim\limits_{t\to-\infty}1-\lim\limits_{t\to-\infty}\frac{4}{t}+\lim\limits_{t\to-\infty}\frac{1}{t^2}}\\
&= \frac{3+0}{1-0+0}\\
&= 3
\end{aligned}$$

Problem 23.

$$\begin{aligned}
\lim_{x\to\infty}\frac{\sqrt{x+3x^2}}{4x-1} &= \lim_{x\to\infty}\frac{\sqrt{\frac{1}{x}+3}}{4-\frac{1}{x}}\\
&= \frac{\lim\limits_{x\to\infty}\sqrt{\frac{1}{x}+3}}{\lim\limits_{x\to\infty}\left(4-\frac{1}{x}\right)}\\
&= \frac{\sqrt{\lim\limits_{x\to\infty}\frac{1}{x}+\lim\limits_{x\to\infty}3}}{\lim\limits_{x\to\infty}4-\lim\limits_{x\to\infty}\frac{1}{x}}\\
&= \frac{\sqrt{0+3}}{4-0}\\
&= \frac{\sqrt{3}}{4}
\end{aligned}$$

Problem 29.

$$\begin{aligned}
\lim_{t\to\infty}\left(\sqrt{25t^2+2}-5t\right) &= \lim_{t\to\infty}t\frac{\sqrt{25t^2+2}-5t}{t}\\
&= \lim_{t\to\infty}t\left(\sqrt{25+\frac{2}{t}}-5\right)\\
&= (\lim_{t\to\infty}t)\cdot\left(\lim_{t\to\infty}\sqrt{25+\frac{2}{t}}-\lim_{t\to\infty}5\right)\\
&= (\lim_{t\to\infty}t)\cdot\left(\sqrt{\lim_{t\to\infty}25+\lim_{t\to\infty}\frac{2}{t}}-\lim_{t\to\infty}5\right)\\
&= \infty\cdot\left(\sqrt{25+0}-5\right)\\
&= 0
\end{aligned}$$

Problem 33.

$$\begin{aligned}
\lim_{x\to-\infty}\left(x^2+2x^7\right) &= \lim_{x\to-\infty}x^7\left(\frac{1}{x^5}+2\right)\\
&= \lim_{x\to-\infty}x^7\cdot\lim_{x\to-\infty}\left(\frac{1}{x^5}+2\right)\\
&= -\infty
\end{aligned}$$

Problem 37.

$$\begin{aligned}\lim_{x\to\infty}\frac{1-e^x}{1+2e^x} &= \lim_{x\to\infty}\frac{\frac{1-e^x}{e^x}}{\frac{1+2e^x}{e^x}}\\ &= \lim_{x\to\infty}\frac{\frac{1}{e^x}-1}{\frac{1}{e^x}+2}\\ &= \frac{\lim\limits_{x\to\infty}\frac{1}{e^x}-\lim\limits_{x\to\infty}1}{\lim\limits_{x\to\infty}\frac{1}{e^x}+\lim\limits_{x\to\infty}2}\\ &= \frac{0-1}{0+2}\\ &= -\frac{1}{2}\end{aligned}$$

Problem 41.

$$\begin{aligned}\lim_{x\to\infty}\left[\ln(1+x^2)-\ln(1+x)\right] &= \lim_{x\to\infty}\ln\left(\frac{1+x^2}{1+x}\right)\\ &= \ln\left(\lim_{x\to\infty}\frac{1+x^2}{1+x}\right)\\ &= \ln\left(\lim_{x\to\infty}\frac{\frac{1}{x^2}+1}{\frac{1}{x^2}+\frac{1}{x}}\right)\\ &= \ln(\infty)\\ &= \infty\end{aligned}$$

Problem 47. asymptotes: $x=-3, y=4$

Problem 49. asymptotes: $x=-2, x=1, y=2$

Problem 61.

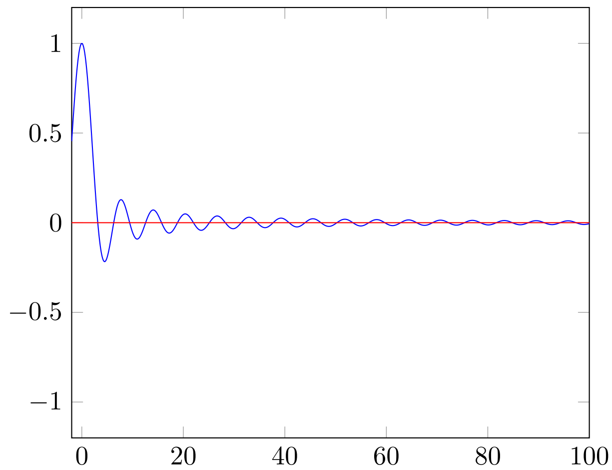
$$\begin{aligned}\lim_{x\to\infty}\left(x^4-x^6\right) &= \lim_{x\to\infty}x^6\left(\frac{1}{x^2}-1\right)\\ &= (\lim_{x\to\infty}x^6)\cdot\lim_{x\to\infty}\left(\frac{1}{x^2}-1\right)\\ &= (\lim_{x\to\infty}x^6)\cdot\left(\lim_{x\to\infty}\frac{1}{x^2}-\lim_{x\to\infty}1\right)\\ &= \infty\cdot(0-1)\\ &= -\infty\end{aligned} \tag{1}$$

Problem 65.

(a) $f(x)=\frac{\sin x}{x}$. Let $g(x)=-\frac{1}{|x|}$ and $h(x)=\frac{1}{|x|}$. It's clear that $g(x)\le f(x)\le h(x)$. Since $\lim\limits_{x\to\infty}g(x)=\lim\limits_{x\to\infty}h(x)$, by the Squeeze Theorem, we can say that $\lim\limits_{x\to\infty}f(x)=\lim\limits_{x\to\infty}g(x)=\lim\limits_{x\to\infty}h(x)$.

$$\begin{aligned}\lim_{x\to\infty}f(x) &= \lim_{x\to\infty}h(x)\\ &= \lim_{x\to\infty}\frac{1}{|x|}\\ &= 0\end{aligned} \tag{2}$$

(b)



the function crosses it's asymptote infinitely many times.

Problem 57.

$$f(x) = \frac{1}{-|x|} + \frac{1}{3-x} - \frac{1}{(x-2)^2+2}$$

2.7

Problem 3.

(a) (i)

$$\begin{aligned} m &= \lim_{x\to-1} \frac{f(x)-f(-1)}{x+1} \\ &= \lim_{x\to-1} \frac{(x^2+3x)-((-1)^2+3(-1))}{x+1} \\ &= \lim_{x\to-1} \frac{x^2+3x+2}{x+1} \\ &= \lim_{x\to-1} \frac{(x+1)(x+2)}{x+1} \\ &= \lim_{x\to-1} (x+2) \\ &= 1 \end{aligned}$$

(ii)

$$\begin{aligned} m &= \lim_{h\to0} \frac{f(-1+h)-f(-1)}{h} \\ &= \lim_{h\to0} \frac{((-1+h)^2+3(-1+h))-((-1)^2+3(-1))}{h} \\ &= \lim_{h\to0} \frac{(1-2h+h^2-3+3h)-(1-3)}{h} \\ &= \lim_{h\to0} \frac{1-2h+h^2-3+3h+2}{h} \\ &= \lim_{h\to0} \frac{h+h^2}{h} \\ &= \lim_{h\to0} 1+h \\ &= 1 \end{aligned}$$

(b) $y = x - 1$

Problem 7.

$$\begin{aligned} m &= \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} \\ &= \lim_{h\to 0} \frac{\frac{2+h+2}{2+h-3} - \frac{2+2}{2-3}}{h} \\ &= \lim_{h\to 0} \frac{\frac{4+h}{h-1} + 4}{h} \\ &= \lim_{h\to 0} \frac{4+h+4h-4}{h(h-1)} \\ &= \lim_{h\to 0} \frac{5h}{h(h-1)} \\ &= \lim_{h\to 0} \frac{5}{(h-1)} \\ &= \frac{5}{(0-1)} \\ &= -5 \end{aligned}$$

Problem 11.

(a)

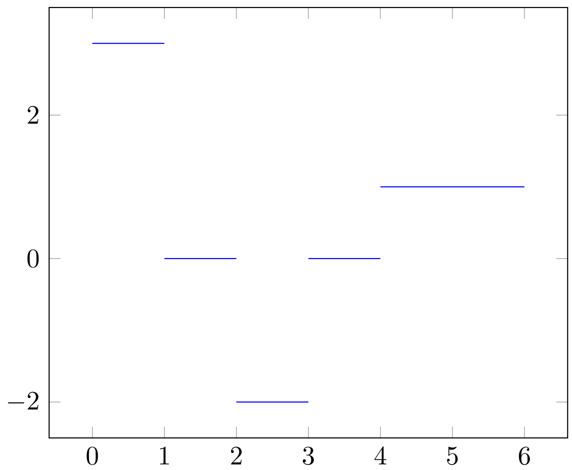
$$\begin{aligned} 4.9t^2 &= 30 && \implies \\ t^2 &= \frac{30}{4.9} && \implies \\ t &= \sqrt{\frac{30}{4.9}} = \frac{10\sqrt{3}}{7} \end{aligned}$$

(b)

$$\begin{aligned} v &= \lim_{h\to 0} \frac{4.9\left(\frac{10\sqrt{3}}{7} + h\right)^2 - 4.9\left(\frac{10\sqrt{3}}{7}\right)^2}{h} \\ &= \lim_{h\to 0} \frac{4.9\left(\frac{300}{49} + 2h\frac{10\sqrt{3}}{7} + h^2\right) - 4.9\left(\frac{300}{49}\right)}{h} \\ &= \lim_{h\to 0} \frac{30 + 4.9 \cdot 2h\frac{10\sqrt{3}}{7} + 4.9h^2 - 30}{h} \\ &= 4.9 \cdot \lim_{h\to 0} \frac{2h\frac{10\sqrt{3}}{7} + h^2}{h} \\ &= 4.9 \cdot \lim_{h\to 0} \left(2\frac{10\sqrt{3}}{7} + h\right) \\ &= 14\sqrt{3} \end{aligned}$$

Problem 15.

(a) The particle is moving to the right during time intervals $[0, 1]$ and $[4, 6]$. It's moving to the left during time interval $[2, 3]$. It's standing still during $[1, 2]$ and $[3, 4]$.



(b)