**Test quiz**
Numerical Linear Algebra
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# 1 True or False

(a) True

(b) True

(c) True

(d) False

(e) False

(f) True

(g) False

(h) True

(i) False

(j) True

(k) False

# 2 Condition Numbers

1. $\mathrm{Cond}_2(A) = \frac{1}{\mathrm{Cond}_2(A^{-1})}$
2. $\mathrm{Cond}_2(cA) = \|cA\| \cdot \|A^{-1}c^{-1}\| = c\|A\| \cdot c^{-1}\|A^{-1}\| = \|A\| \cdot \|A^{-1}\| = \mathrm{Cond}_2(A)$

# 3 Othogonal Matrices

since $\mathrm{Cond}_2(cA) = \|cA\| \cdot \|A^{-1}c^{-1}\| = c\|A\| \cdot c^{-1}\|A^{-1}\| = \|A\| \cdot \|A^{-1}\| = \mathrm{Cond}_2(A)$ and $\mathrm{Cond}_2(Q) = 1$ where $Q$ is an orthogonal matrix, we can say that $\mathrm{Cond}_2(A) = \mathrm{Cond}_2(cQ) = \mathrm{Cond}_2(Q)$

# 4 Properties of Matrix Norms

By the definition, the norm $\|\cdot\|$ is an induced norm.

(a)

$$\|Ay\| \leq \|A\| \cdot \|y\| \implies \|Ay\| \leq \max_{\|x\|=1} \|Ax\| \cdot \|y\|$$

# 5 Matrix Products

$$\mathrm{Cond}(AB) = \underbrace{\|AB\| \cdot \|B^{-1}A^{-1}\| \leq \|A\| \cdot \|A^{-1}\| \cdot \|B\| \cdot \|B^{-1}\|}_{\text{by sub-multiplicativity}} = \mathrm{Cond}(A) \cdot \mathrm{Cond}(B)$$

Although, for this, the norm must exhibit sub-multiplicativity.