University of Moratuwa
Faculty of Engineering
Department of Electronic & Telecommunication Engineering

Modelling and Analysis of Physiological Systems

Properties of the Hodgkin-Huxley equations

Bandara D.M.D.V.
Undergraduate (Biomedical engineering)
Department Electronic and Telecommunications
Faculty of Engineering
University of Moratuwa

July 14, 2023

Contents

Question 1

sub-threshold $\mu \mathrm{A} cm^{-2}$	super-threshold $\mu \mathrm{A} cm^{-2}$	Action potential
6.5	7	-
6.75	7	-
6.87	7	-
6.9375	7	-
6.9688	7	Found
6.9844	7	Found

Table 1

I then checked for 6.94,6.95,6.96 to find the threshold to the second decimal place. From the graphs it was found to be 6.96 $\mu \mathrm{A} cm^{-2}$.

Refer **Appendix: live transcript** for the derivation.

Question 2

$\int_{t_0}^{t_f} \sum_k J_k dt$	$\int_{t_0}^{t_f} J_{ei} dt$
4	3.9999
5	4.9998
6.96	6.9620
9	8.984
10	9.9984

Table 2

Hence we can say that

$$\int_{t_0}^{t_f} \sum_k J_k dt \approx \int_{t_0}^{t_f} J_{ei} dt$$

Question 3 & 4

Delay	I˙2th $\mu A cm^{-2}$
25	13.7
20	11.6
18	11.3
16	12.7
14	17.0
12	25.5
10	40.8
8	70.1
6	145.2

Table 3

Now by plotting I_{2th}/I_{1th} vs. delay we get the following graph.

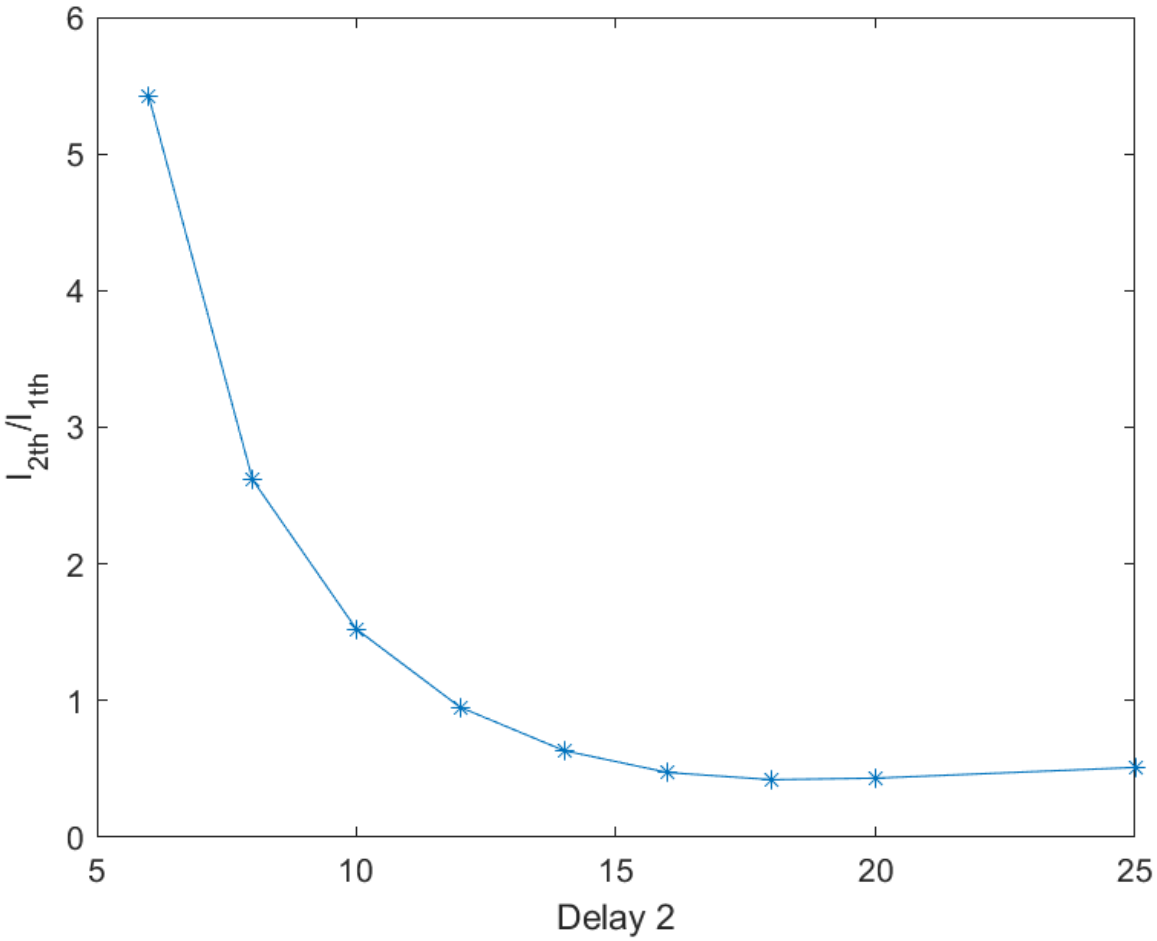


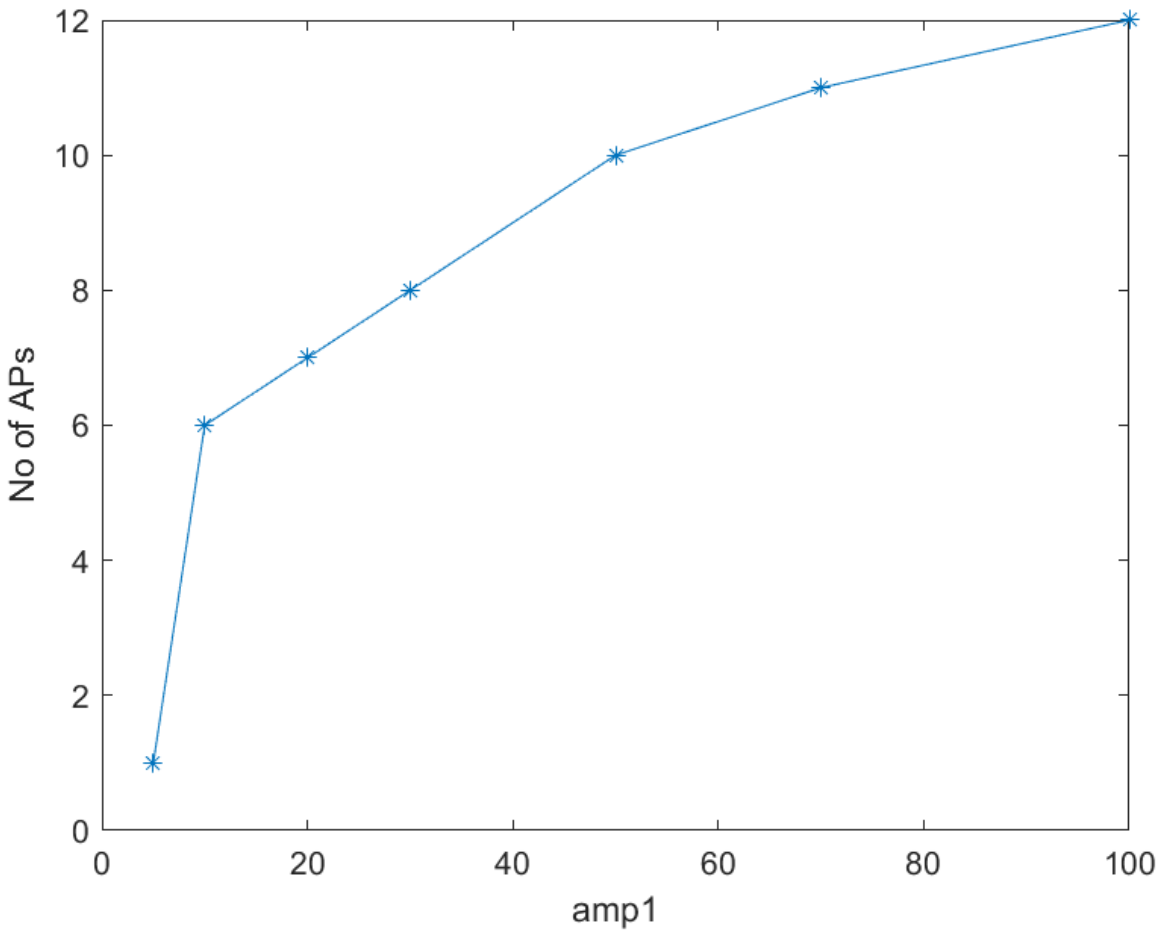
Figure 1: I_{2th}/I_{1th} vs. delay

The Current increases exponentialy with decreasing time delay. At 6ms, time delay value is more than 5 times the current. Hence, comparatively large. So the absolute refractory period is 6ms. Thereafter, its clear that is grater than current at 10ms. Therefore relative refractory period is 10ms.

Question 5

amp1	Action Potential (AP)
5	1
10	6
20	7
30	8
50	10
70	11
100	12

Table 4



amplitude of the action potentials decreases as the amplitude increases. This can be observed from the graphs in the **Appendix: live transcript**.

Question 6

When the amp1 is very large instead of generating APs, it reaches a constant value(saturates).

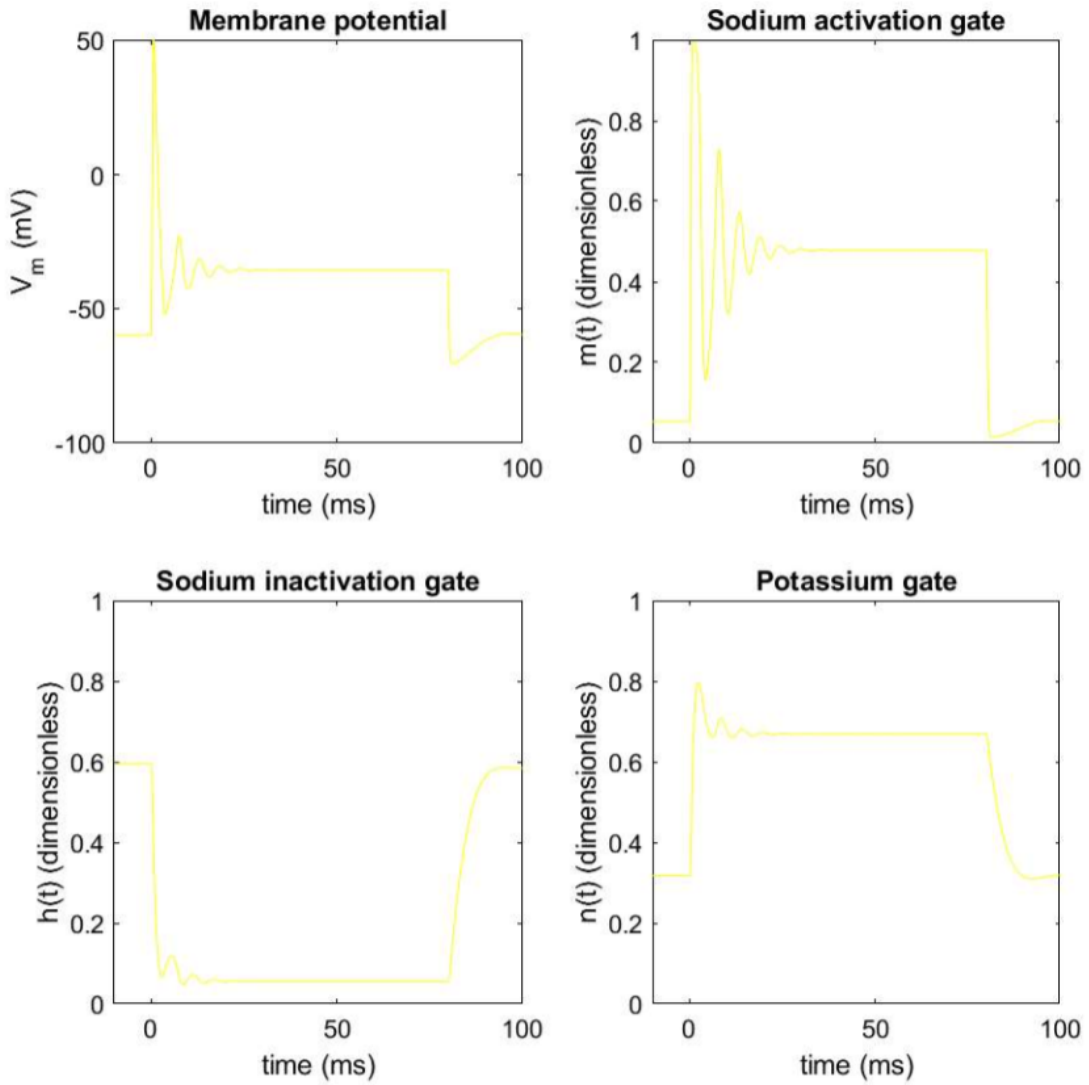


Figure 2: Caption

As the stimulating current increases membrane potential of neurons depolarises rapidly. This causes the voltage-gated sodium channel to activate (m factor). Activation of the Sodium channel(h factor) increases causing the inactivation of voltage-gated Na channels. hence less number of sodium channels will be available for the rising phase. which results in the drop of the amplitude we see in the graphs of **Question 5** (see **Appendix: live transcript**.

n factor(activation of potassium channels) decreases with high depolarization. Eventually, the number of potassium channels will be not enough to repolarize. Hence we see a constant depolarized state.

Question 7

From the graphs in the **Appendix: live transcript** we can see the Amplitude and Duration of AP decreases with temperature .

Appendix: live transcript

Assignment 3 : Properties of the Hodgkin-Huxley equations

200061N - Bandara D.M.D.V.

```
hhconst;
```

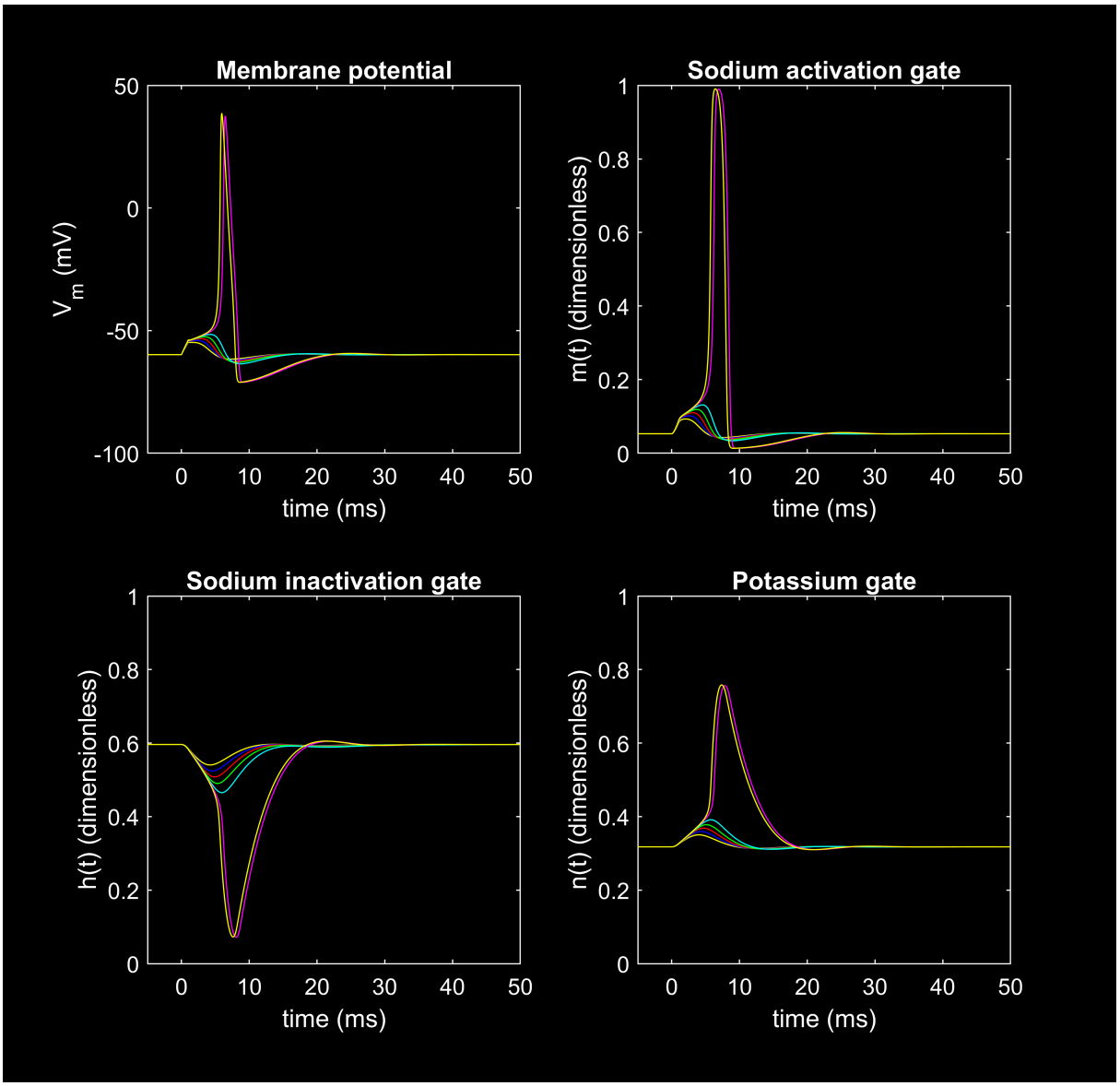
Threshold

Question 1

```
amp1=6;
width1=1;
hhmplot(0,50,0);
%amp1=7;
%hhmplot(0,50,1);
n=1;
while true
    amp1=(amp1+7)/2;
    disp(amp1);
    hhmplot(0,50,1);
    if n>5
        break
    end
    n=n+1;
end
```

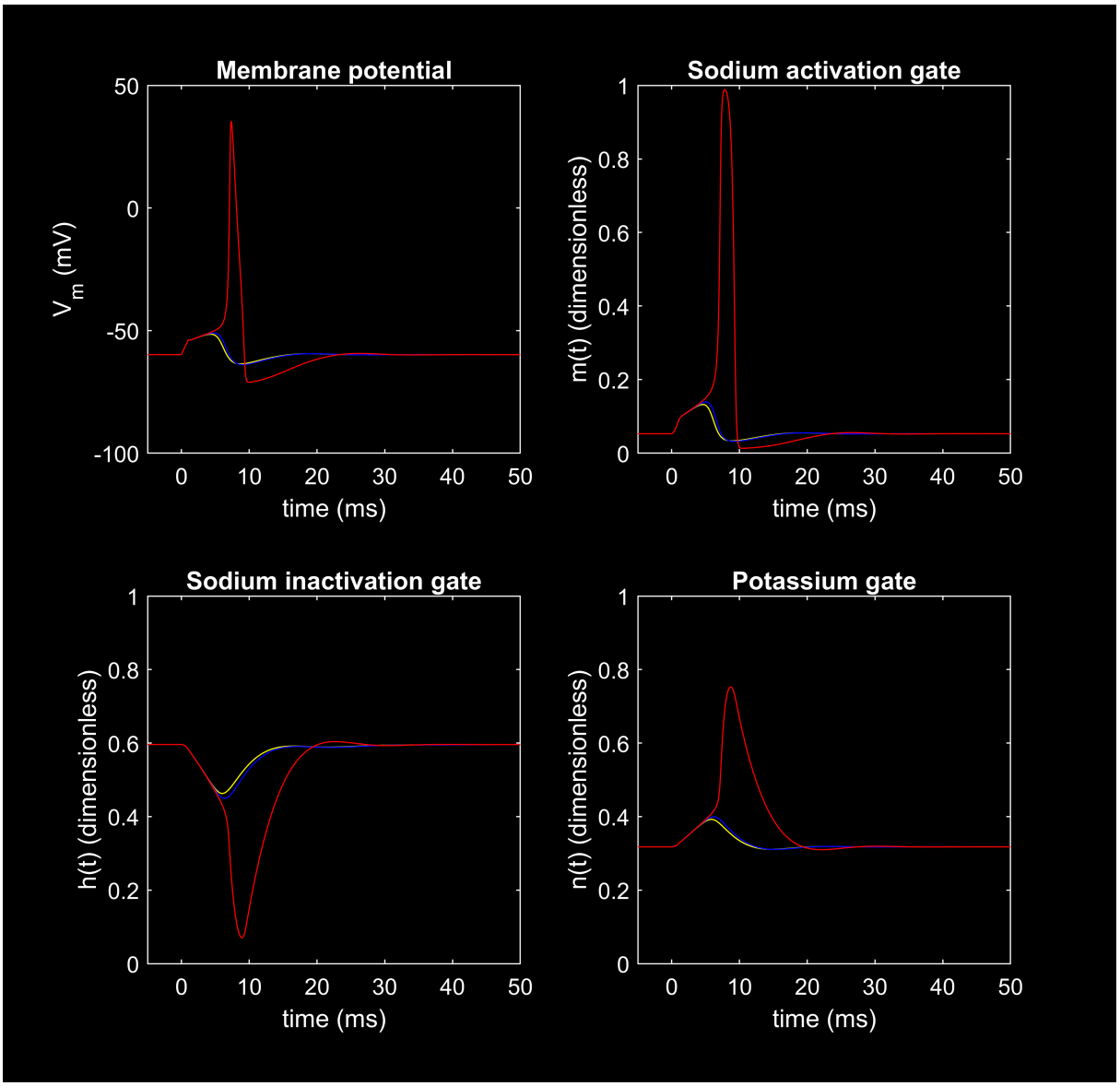
```
    6.5000

    6.7500

    6.8750

    6.9375

    6.9688

    6.9844
```

1



From the graphs we can say that the threhold lie above 0.9375 and below 6.9688 To find the closest threshold value to second decimal place following samples are used.

```
amp1=6.94;
hhmplot(0,50,0);
amp1=6.95;
hhmplot(0,50,1);
amp1=6.96;
hhmplot(0,50,1);
```



From the graphs we can estimate that the threshold is at 6.96 $\mu\mathrm{Acm}^{-2}$.

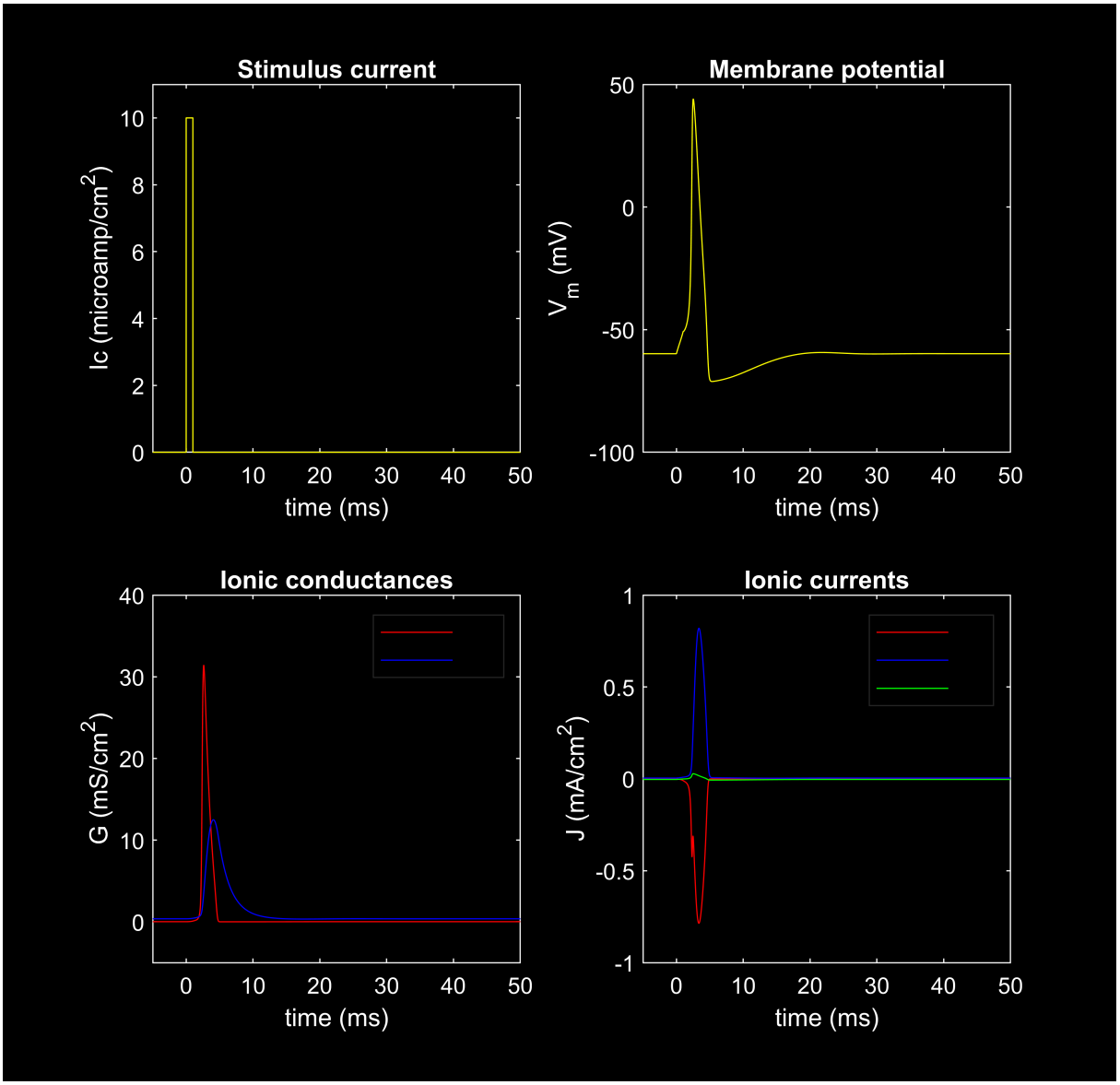
(Different approach is described in the explanations.)

Question 2

```
for amp1 = [4 5 6.96 9 10]
    [qna,qk,ql]=hhsplot(0,50);
    total_Current_densities=qna+qk+ql
    Sum_Jei=width1*amp1
end
```

```
total_Current_densities = 3.9999
Sum_Jei = 4
total_Current_densities = 4.9998
Sum_Jei = 5
total_Current_densities = 6.9620
```

```
Sum_Jei = 6.9600
total_Current_densities = 8.9984
Sum_Jei = 9
```




```
total_Current_densities = 9.9984
Sum_Jei = 10
```

Therefore $\int_{t_0}^{t_f} \sum_k J_k dt \approx \int_{t_0}^{t_f} J_{ei} dt$

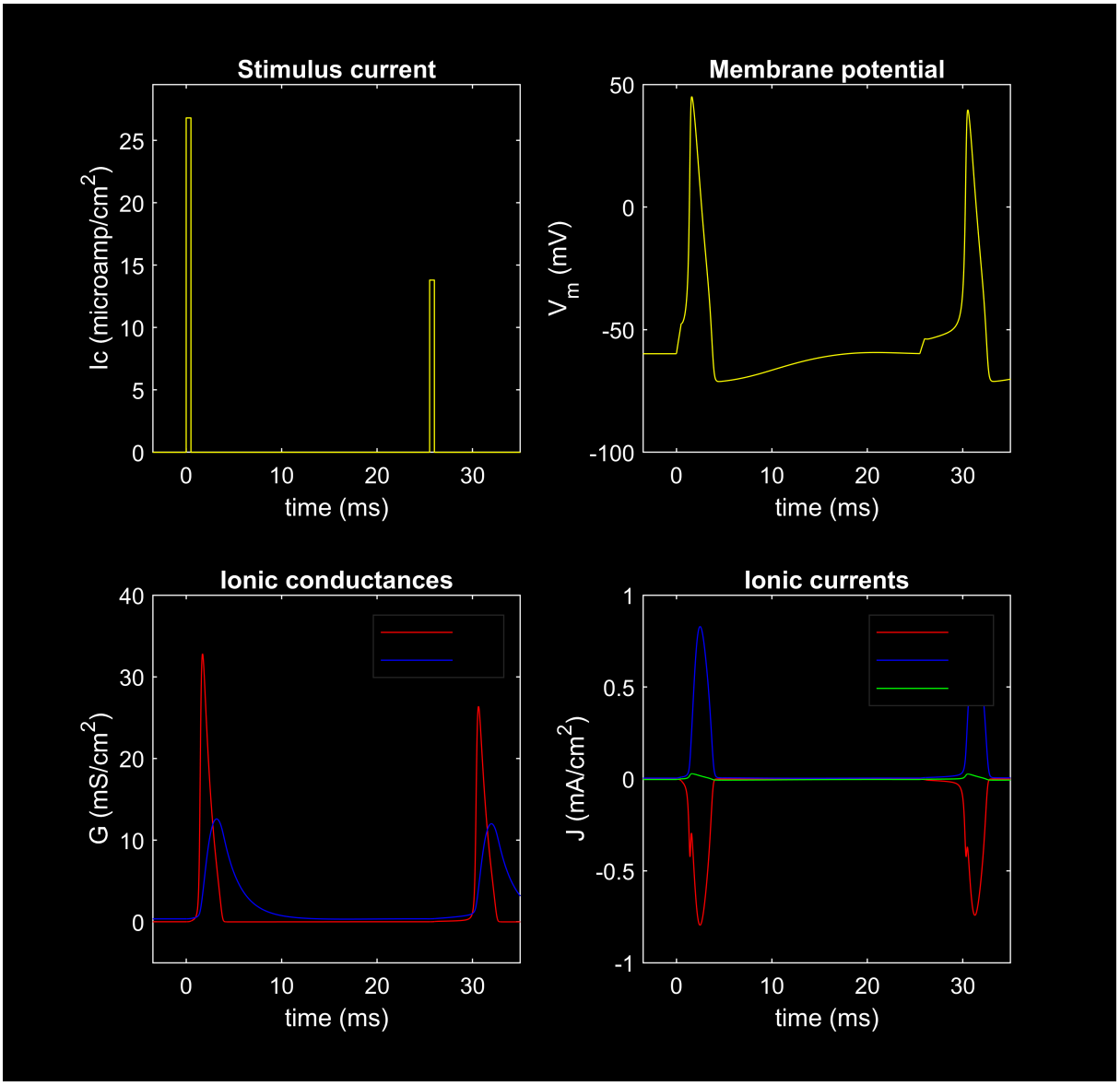
Refractoriness

Question 3

Delay 2=25 / amp2=13.7

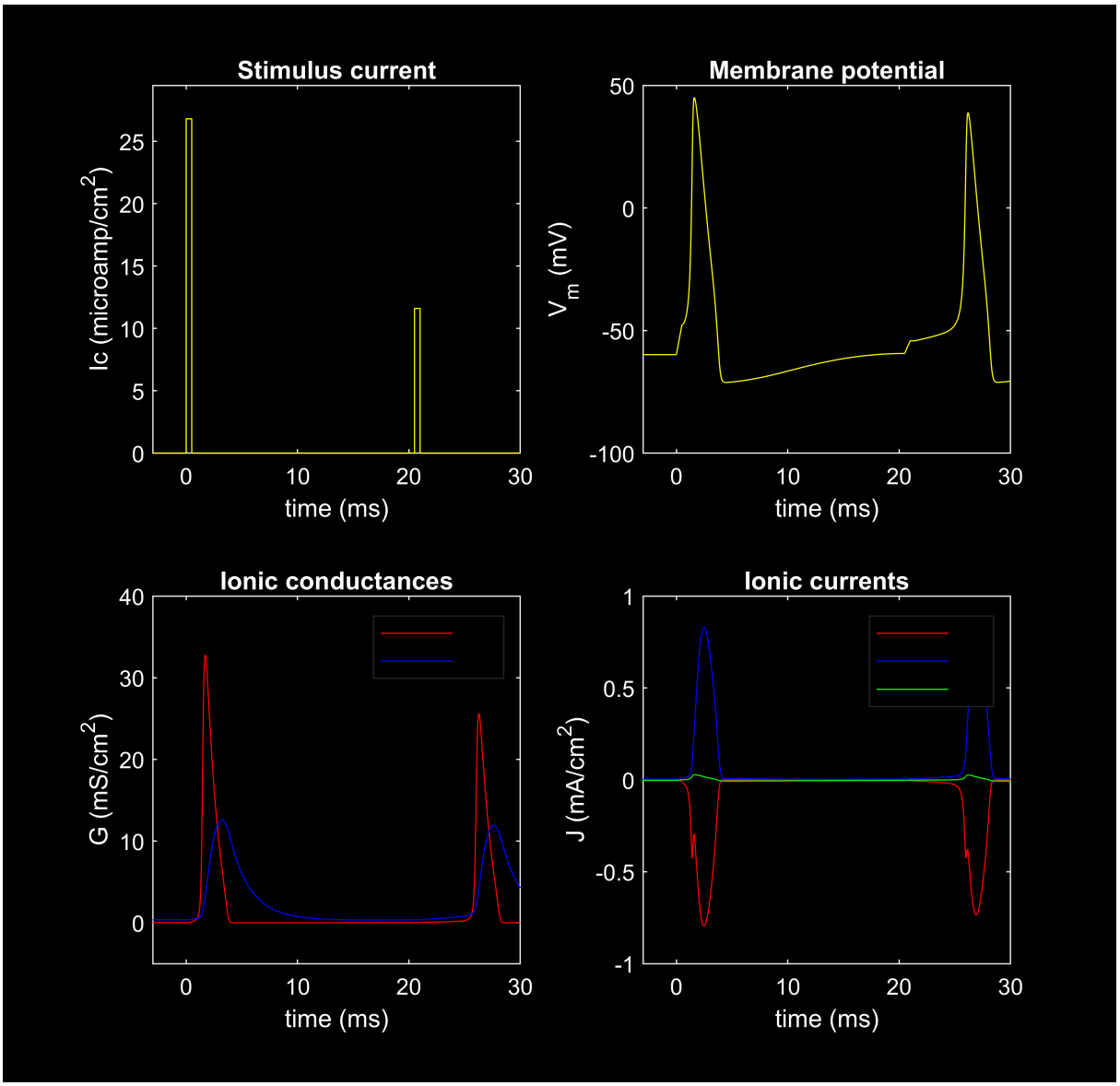
```
amp1 = 26.8;
width1 = 0.5;
delay2 = 25;
```

```
amp2 = 13.8;
width2 = 0.5;
hhsplot(0,35);
```



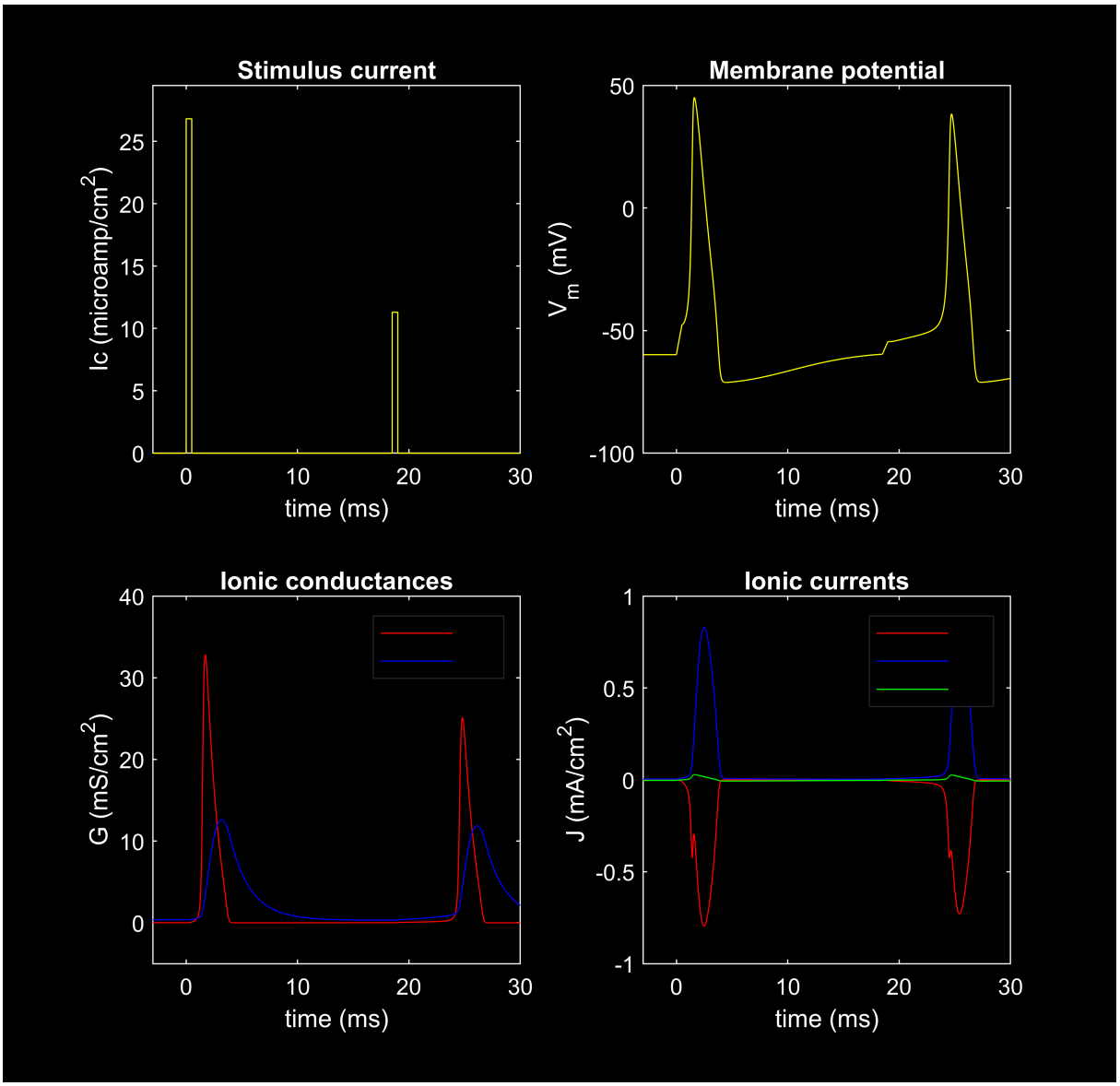
Delay 2=20 / amp2=11.6

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 20;
amp2 = 11.6;
width2 = 0.5;
hhsplot(0,30);
```



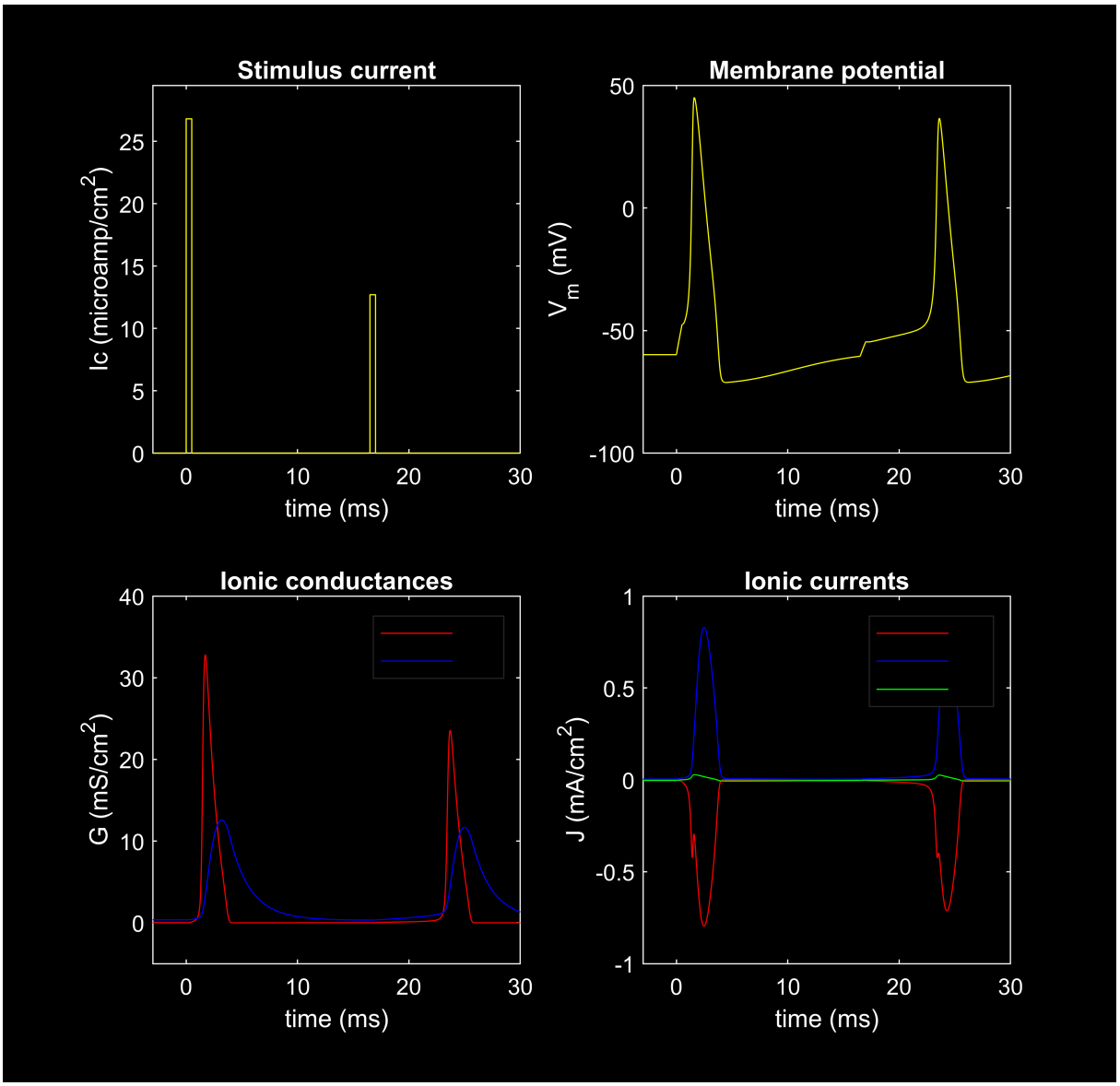
Delay 2=18 / amp2=11.3

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 18;
amp2 = 11.3;
width2 = 0.5;
hhsplot(0,30);
```



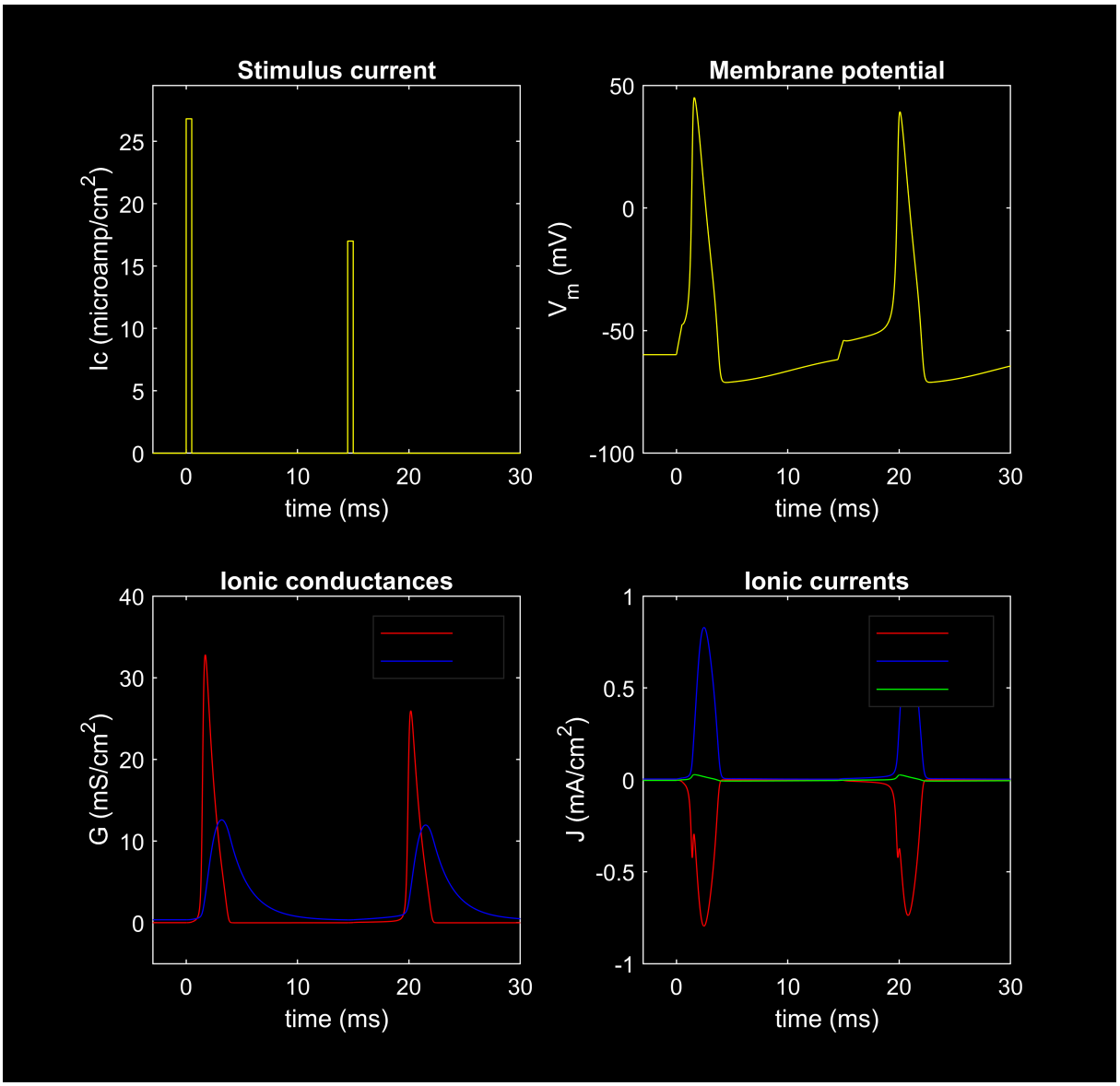
Delay 2=16 / amp2=12.7

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 16;
amp2 = 12.7;
width2 = 0.5;
hhsplot(0,30);
```



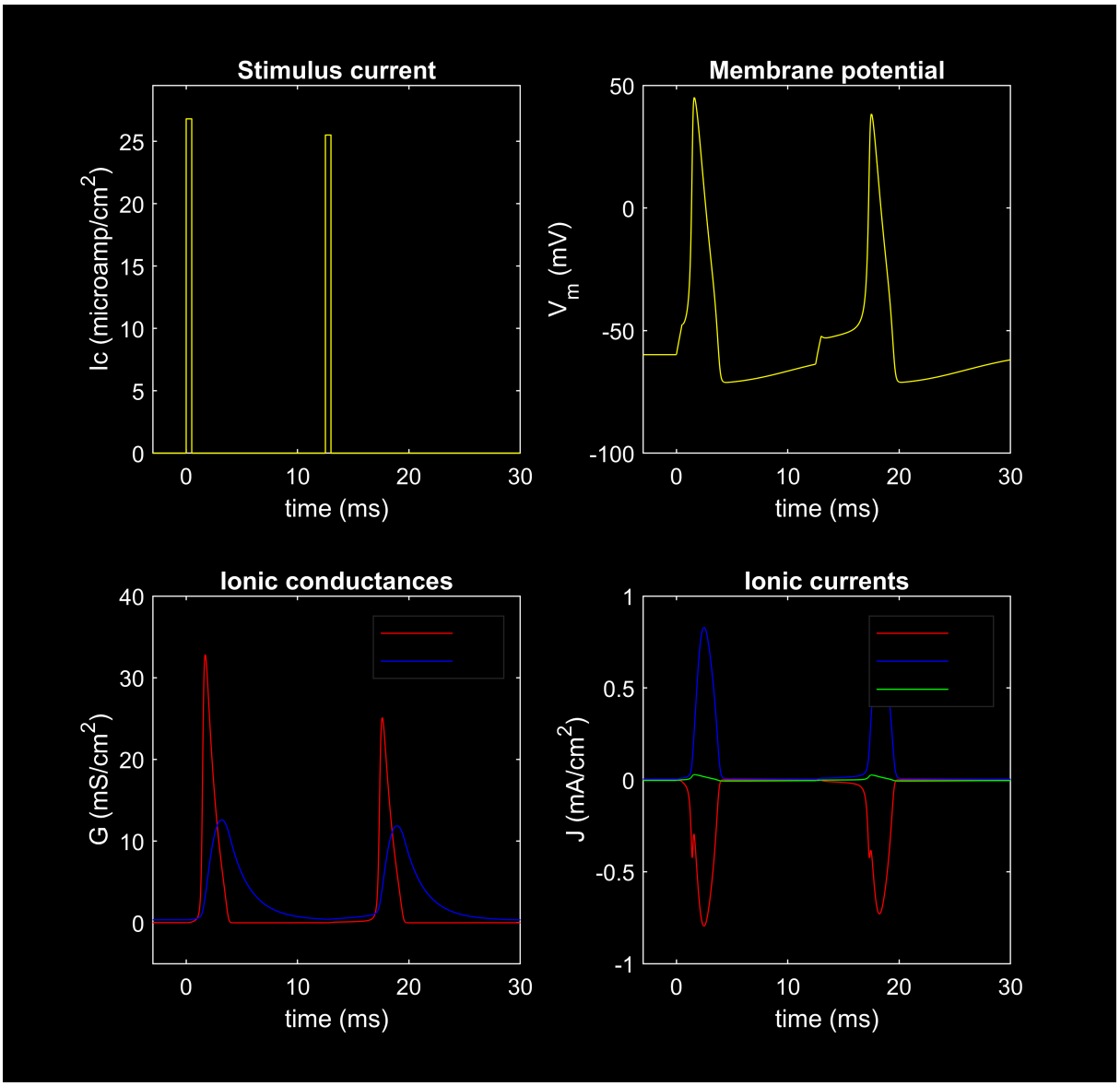
Delay 2=14 / amp2=17

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 14;
amp2 = 17;
width2 = 0.5;
hhsplot(0,30);
```



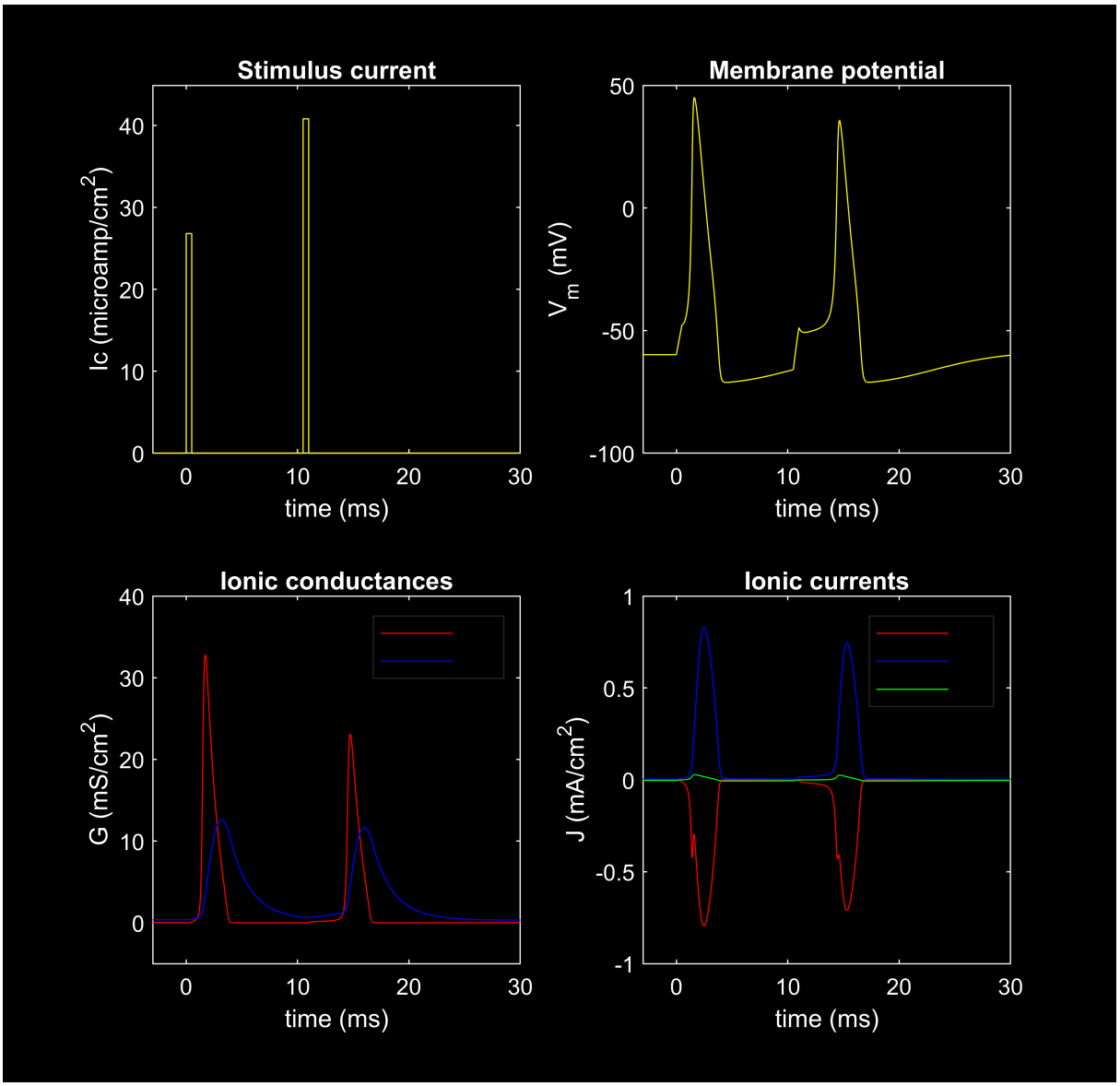
Delay 2=12 / amp2=25.5

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 12;
amp2 = 25.5;
width2 = 0.5;
hhsplot(0,30);
```



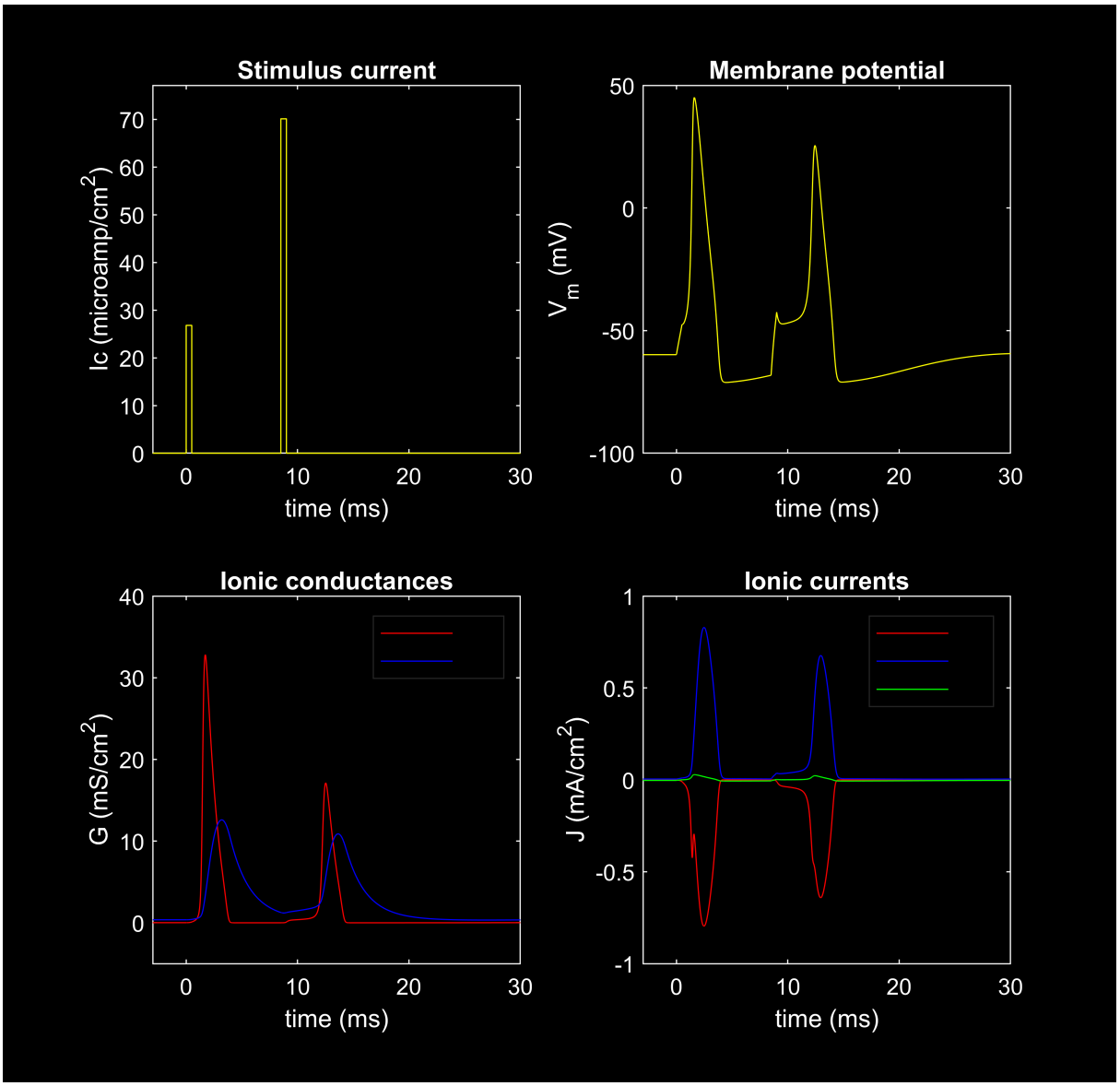
Delay 2=10 / amp2=40.8

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 10;
amp2 = 40.8;
width2 = 0.5;
hhsplot(0,30);
```



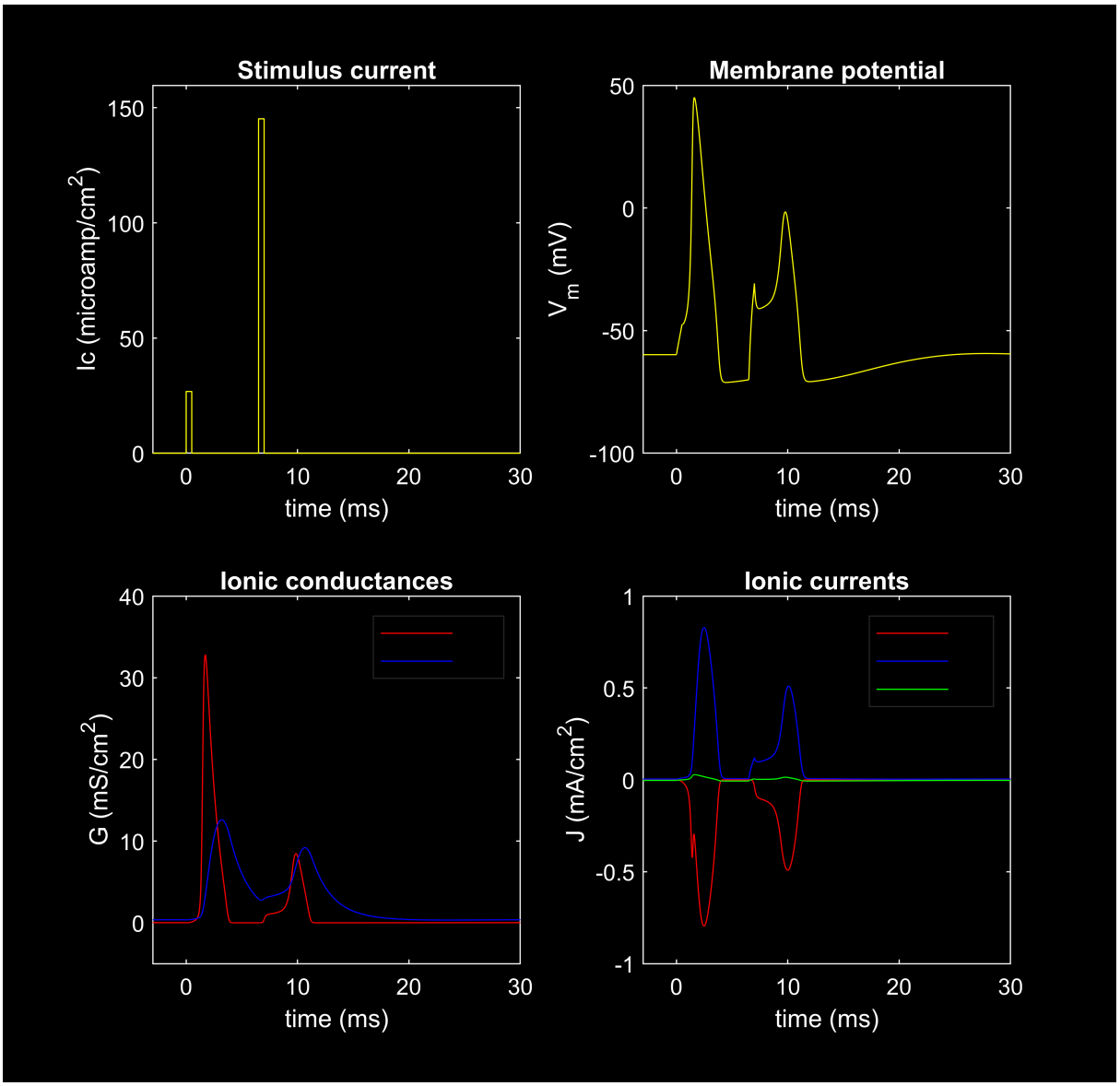
Delay 2=8 / amp2=70.1

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 8;
amp2 = 70.1;
width2 = 0.5;
hhsplot(0,30);
```



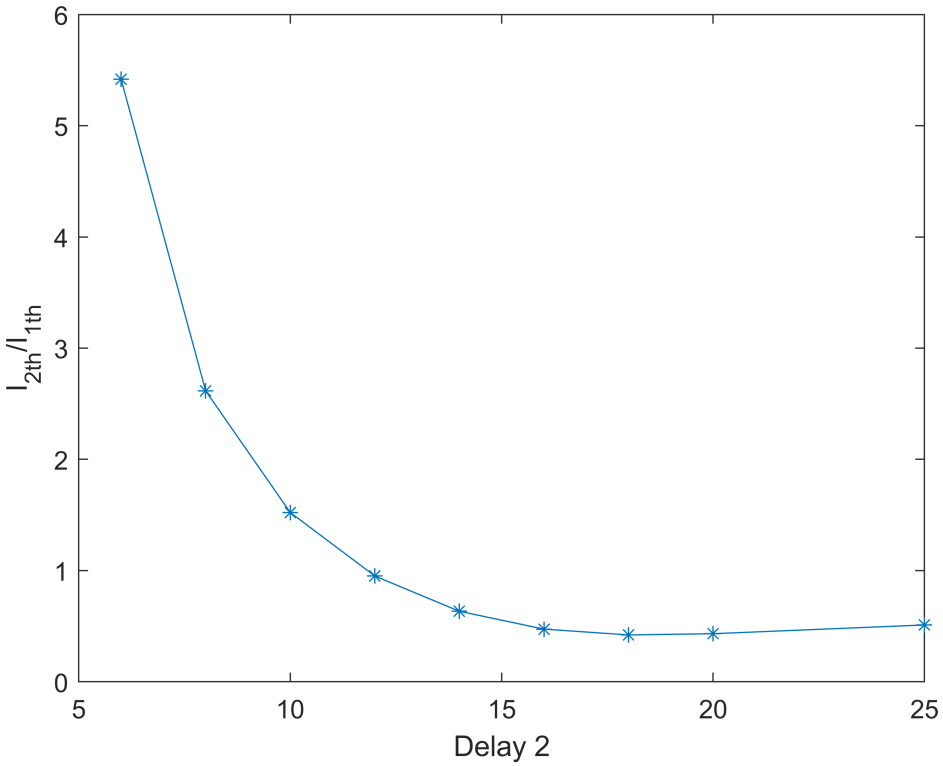
Delay 2=6 / amp2=145.2

```
amp1 = 26.8;
width1 = 0.5;
delay2 = 6;
amp2 = 145.2;
width2 = 0.5;
hhsplot(0,30);
```



With the results,

```
delay_2=[6 8 10 12 14 16 18 20 25];
I2th=[145.2 70.1 40.8 25.5 17 12.7 11.3 11.6 13.7]./26.8;
figure;
plot(delay_2,I2th,'*-');
xlabel("Delay 2");
ylabel('I_{2th}/I_{1th}');
```



I_{2th} current increases exponentialy with decreasing time delay.

At 6ms, time delay I_{2th} value is more than 5 times the I_{1th} current. Hence, comparatively large. So the absolute refractory period is 6ms.

Thereafter, its clear that I_{2th} is grater than I_{1th} current at 10ms. Therefore relative refractory period is 10ms.

Repetitive Activity

Question 5

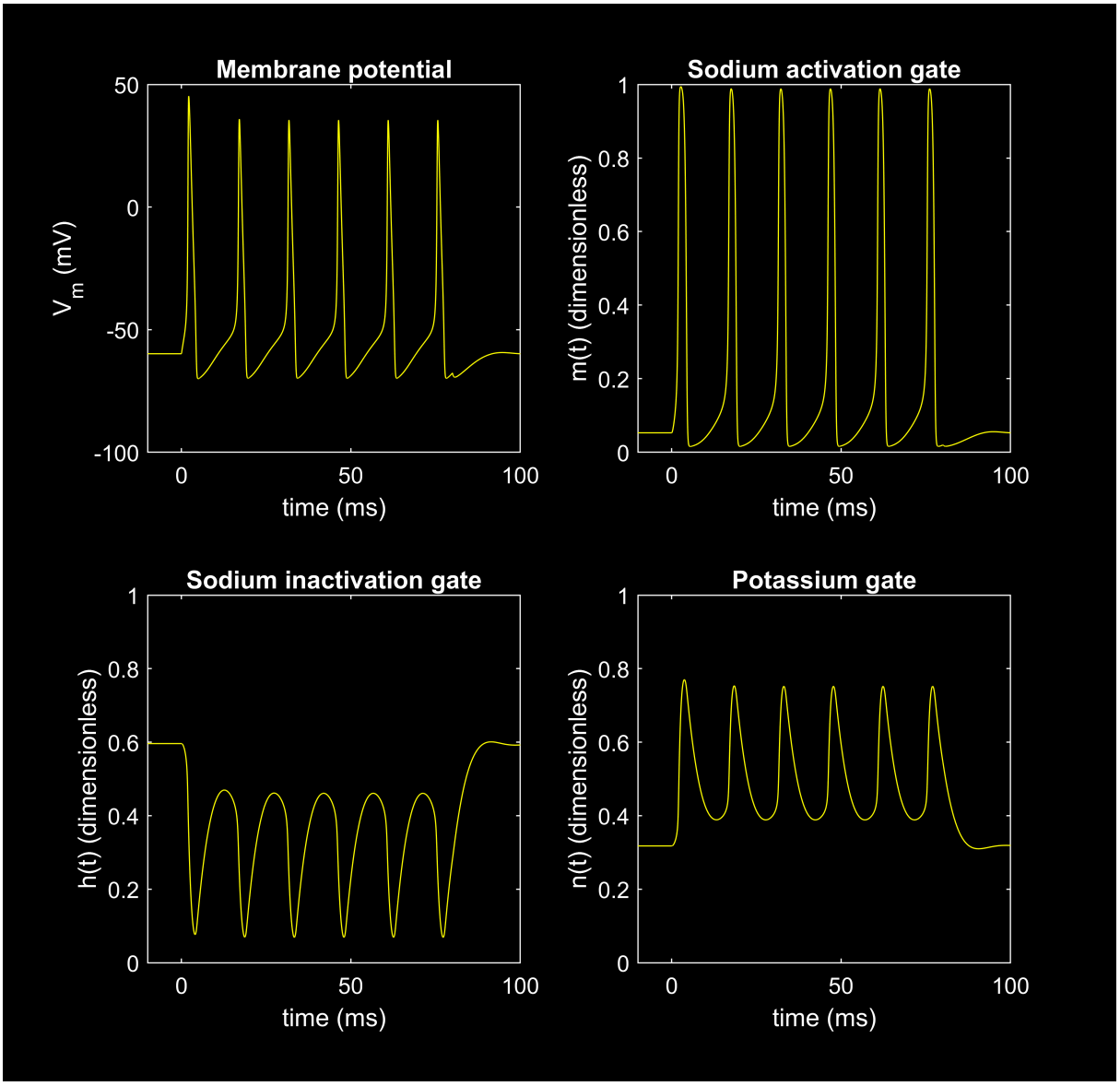
amp1=5

```
amp1=5;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
%AP=1
```

amp1=10

```
amp1=10;
width1 = 80;
delay2 = 0;
```

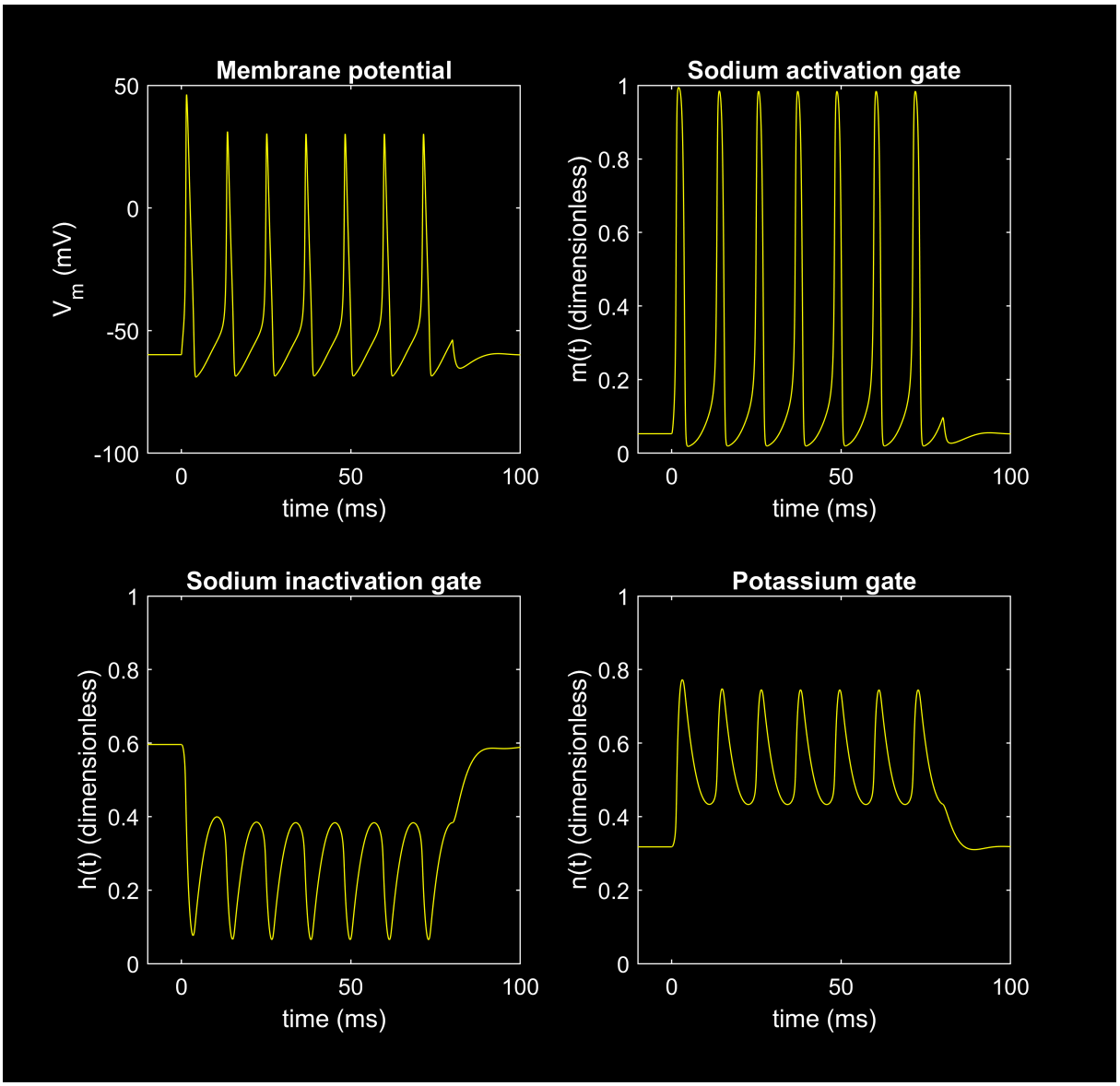
```
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```




```
% AP=6
```

amp1=20

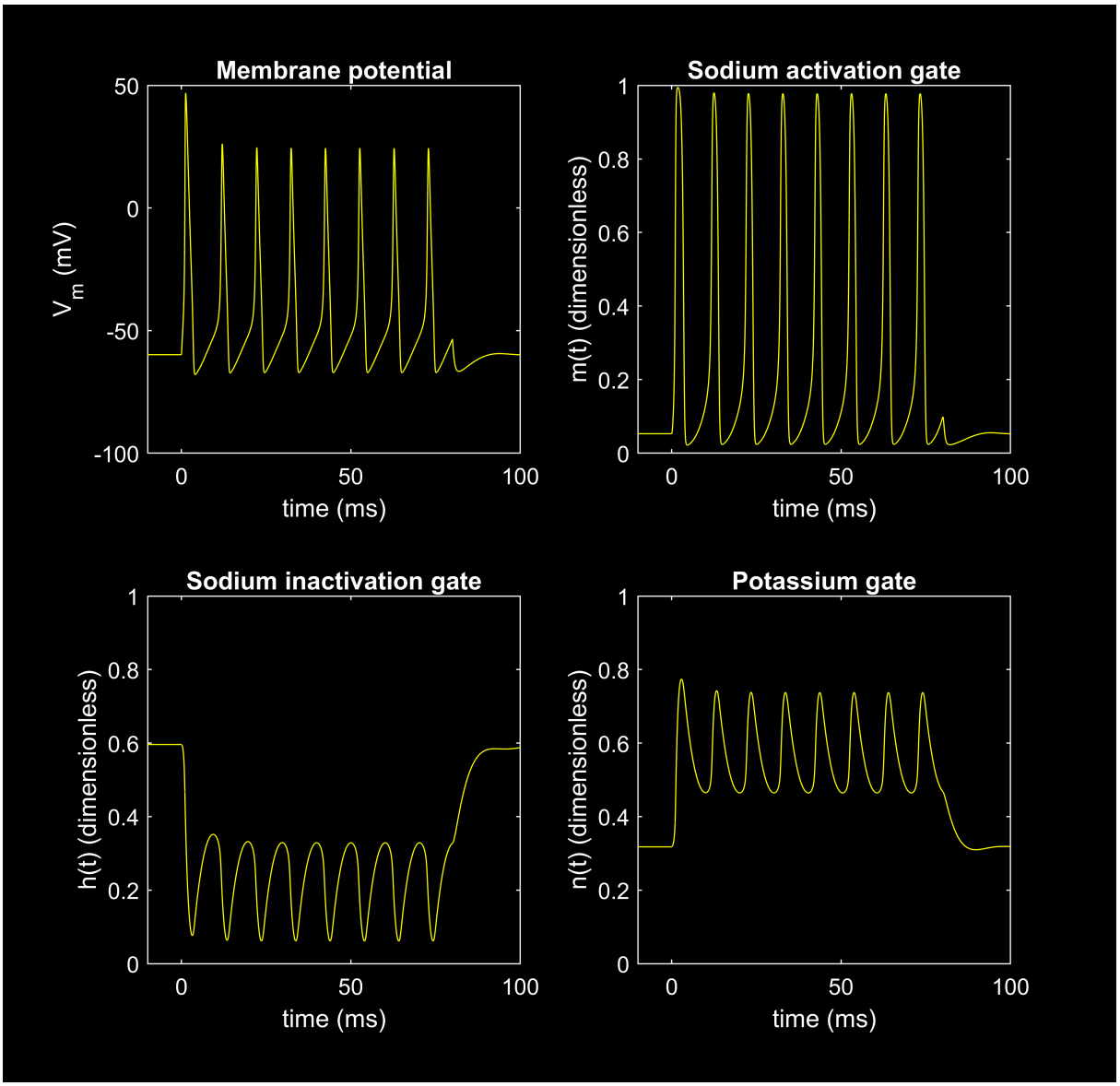
```
amp1=20;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```




```
%AP=7
```

amp1=30

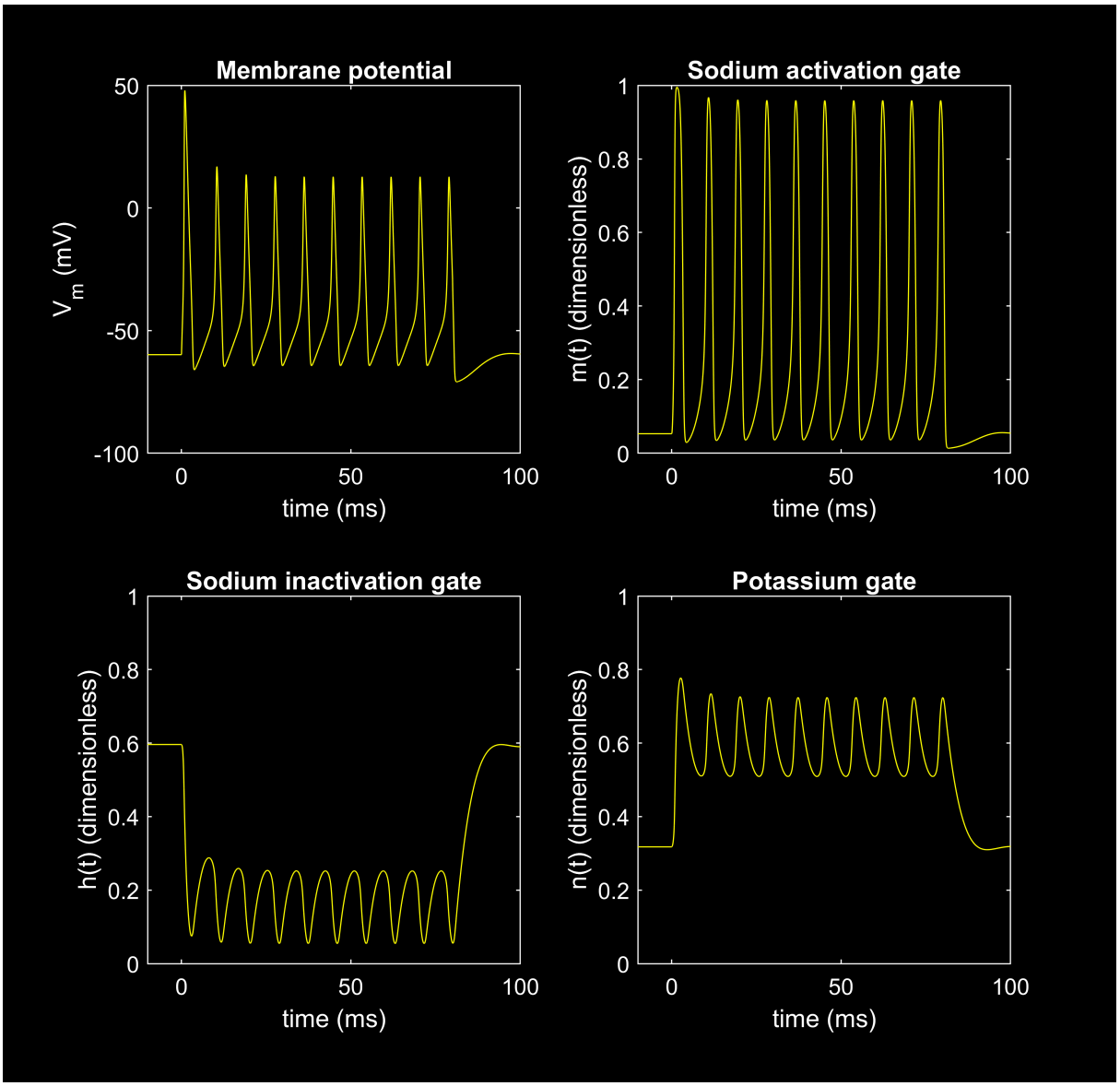
```
amp1=30;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```




```
%AP=8
```

amp1=50

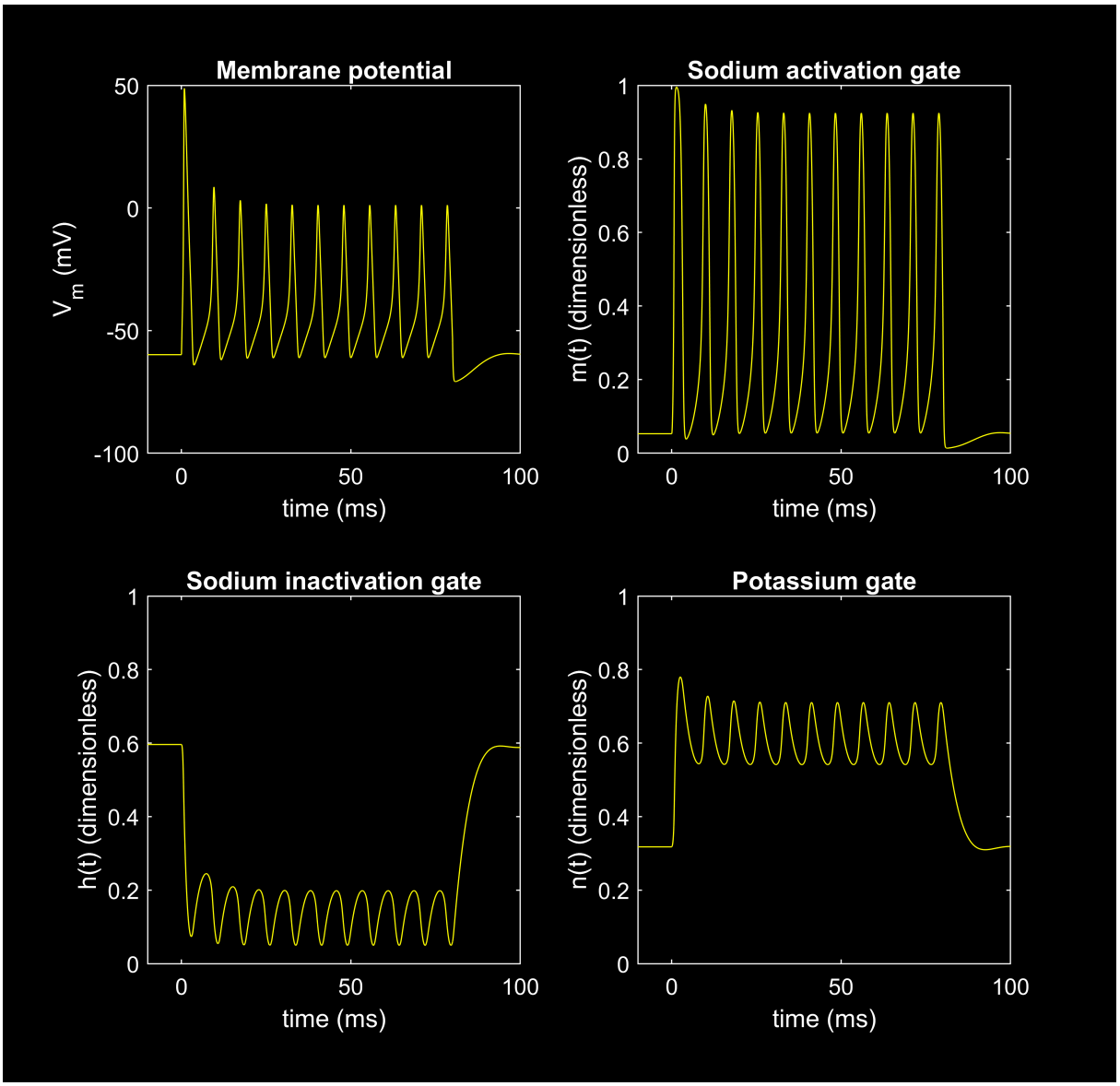
```
amp1=50;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```




```
%AP10
```

amp1=70

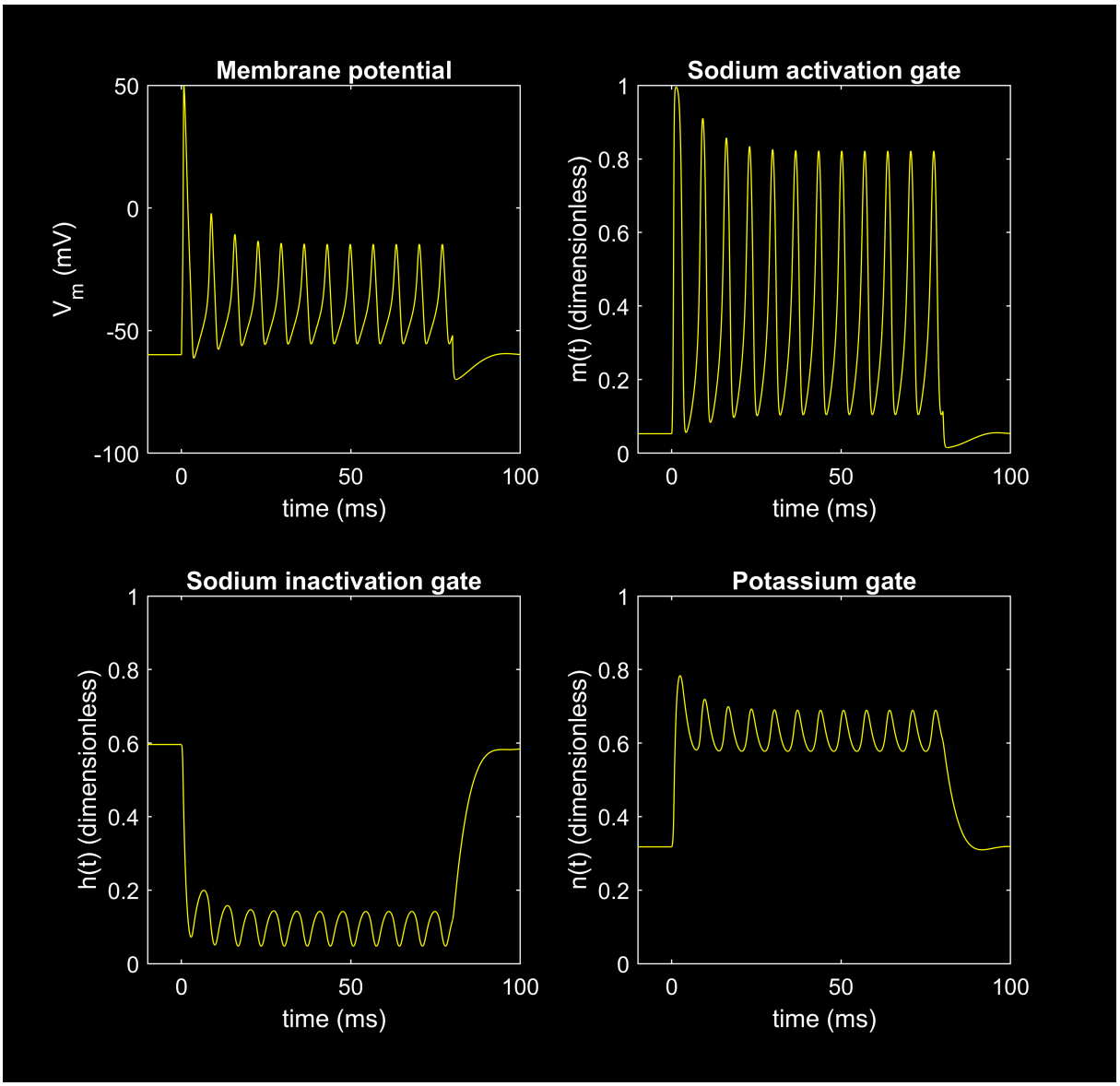
```
amp1=70;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```




```
%AP = 11
```

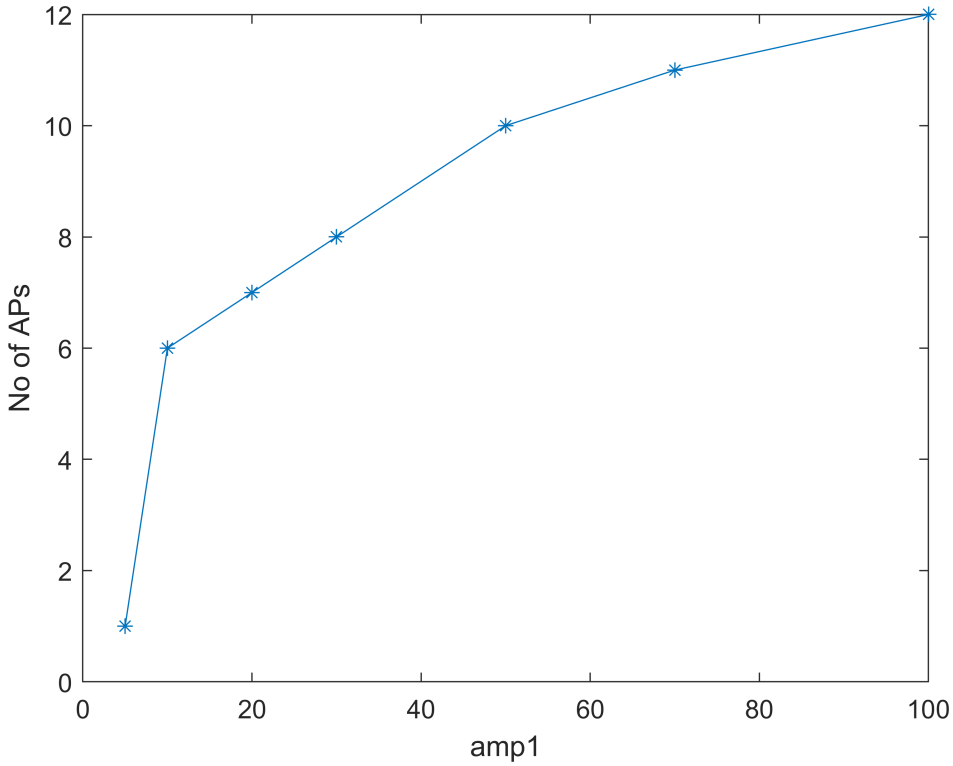
amp1 = 100

```
amp1=100;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```




```
%AP = 12
```

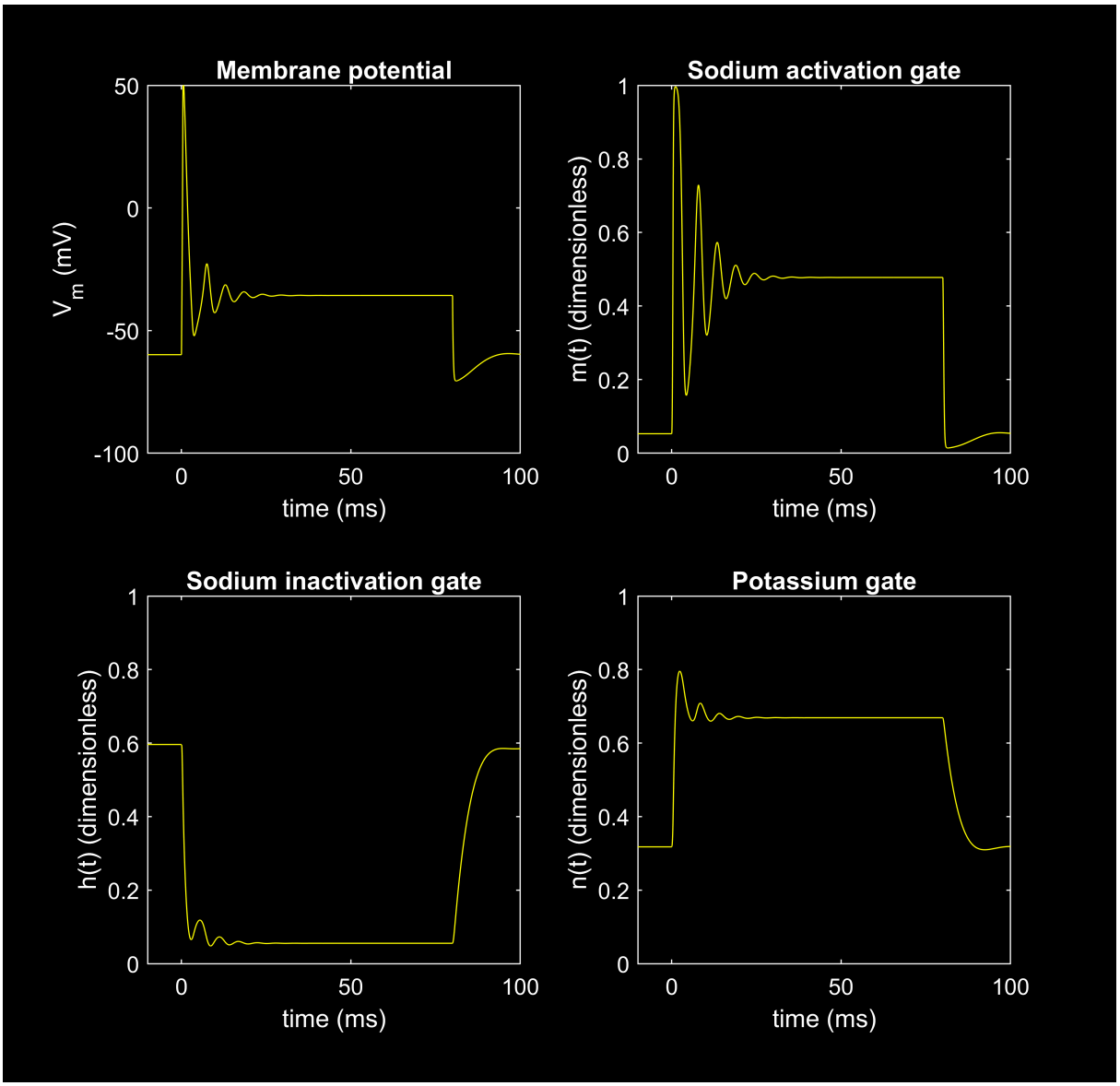
```
amp1=[5 10 20 30 50 70 100];
no_of_AP=[1 6 7 8 10 11 12];
figure;
plot(amp1,no_of_AP,'*-');
xlabel("amp1");
ylabel('No of APs');
```



Amplitude of action potential is decreasing as the AMP1 value is increasing

Question 6

```
amp1=200;
width1 = 80;
delay2 = 0;
amp2 = 0;
width2 = 0;
hhmplot(0,100,0);
```



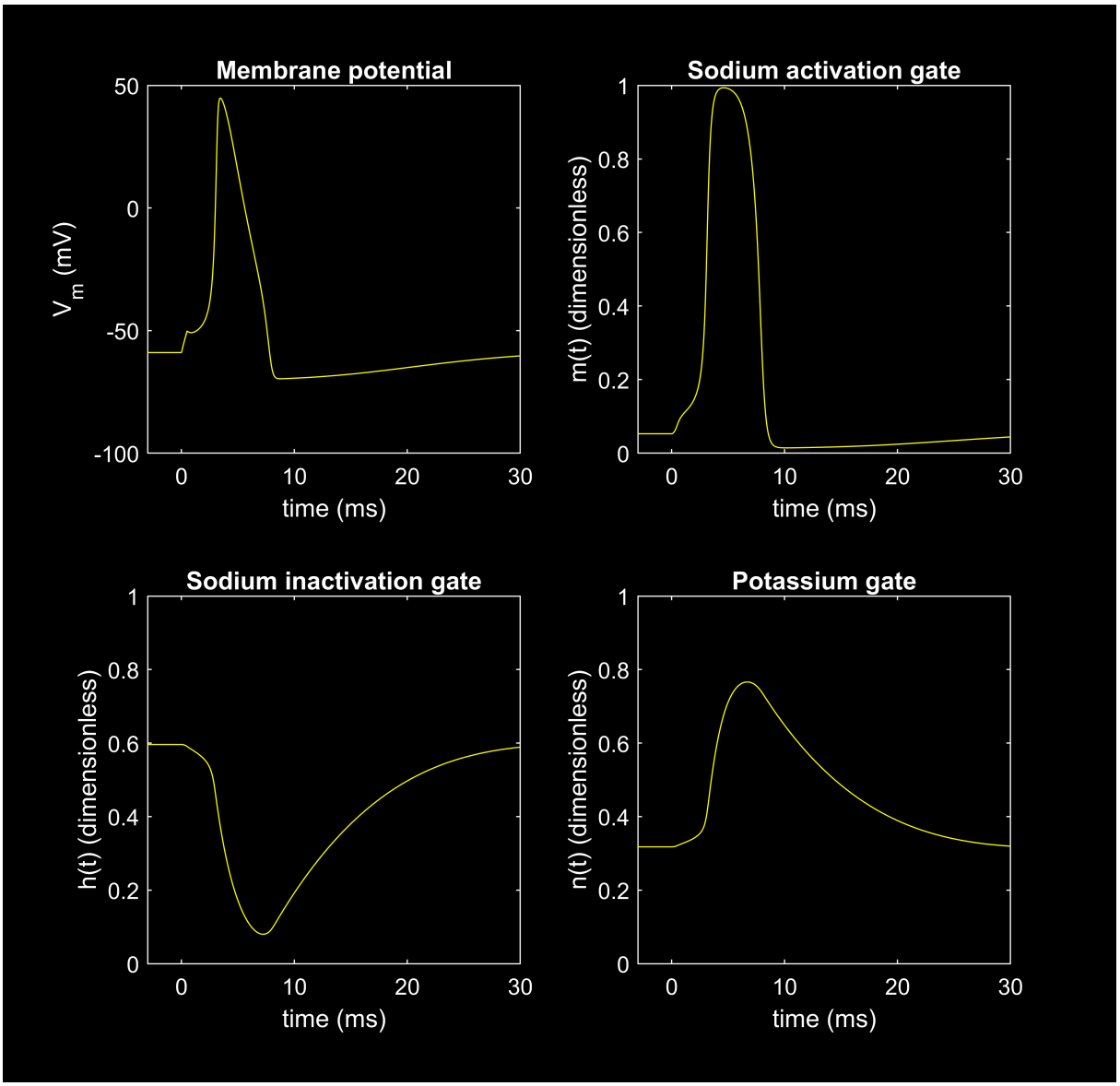
When the amp1 is very large instead of generating APs, it reaches a constant value\saturates.

Temperature Dependence

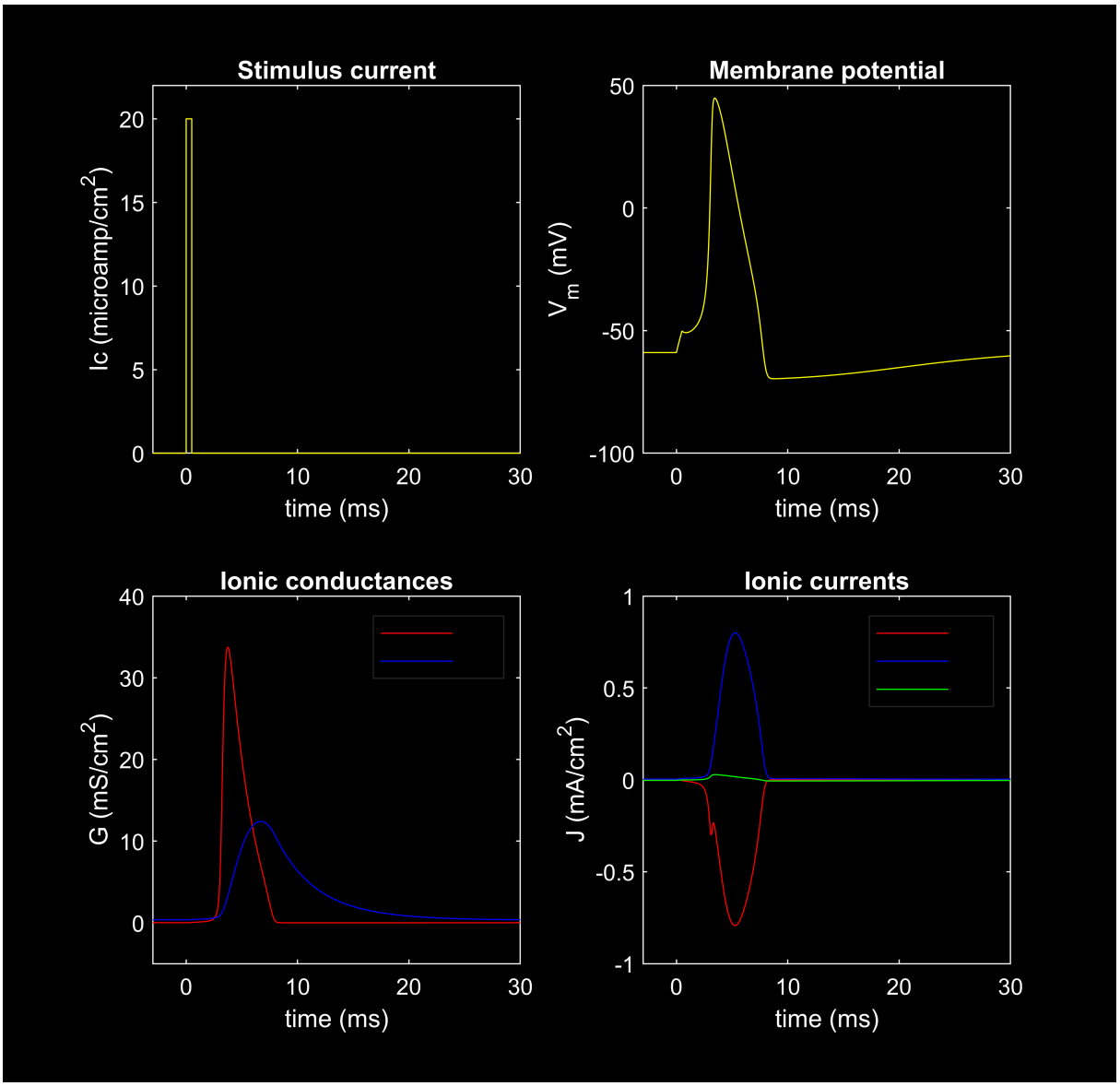
Question 7

plotting by increasing the temperature with 5 degrees incriment to see changes

```
vclamp = 0;
amp1 = 20;
width1 = 0.5;
tempc = 0;
hhmplot(0,30,0);
```

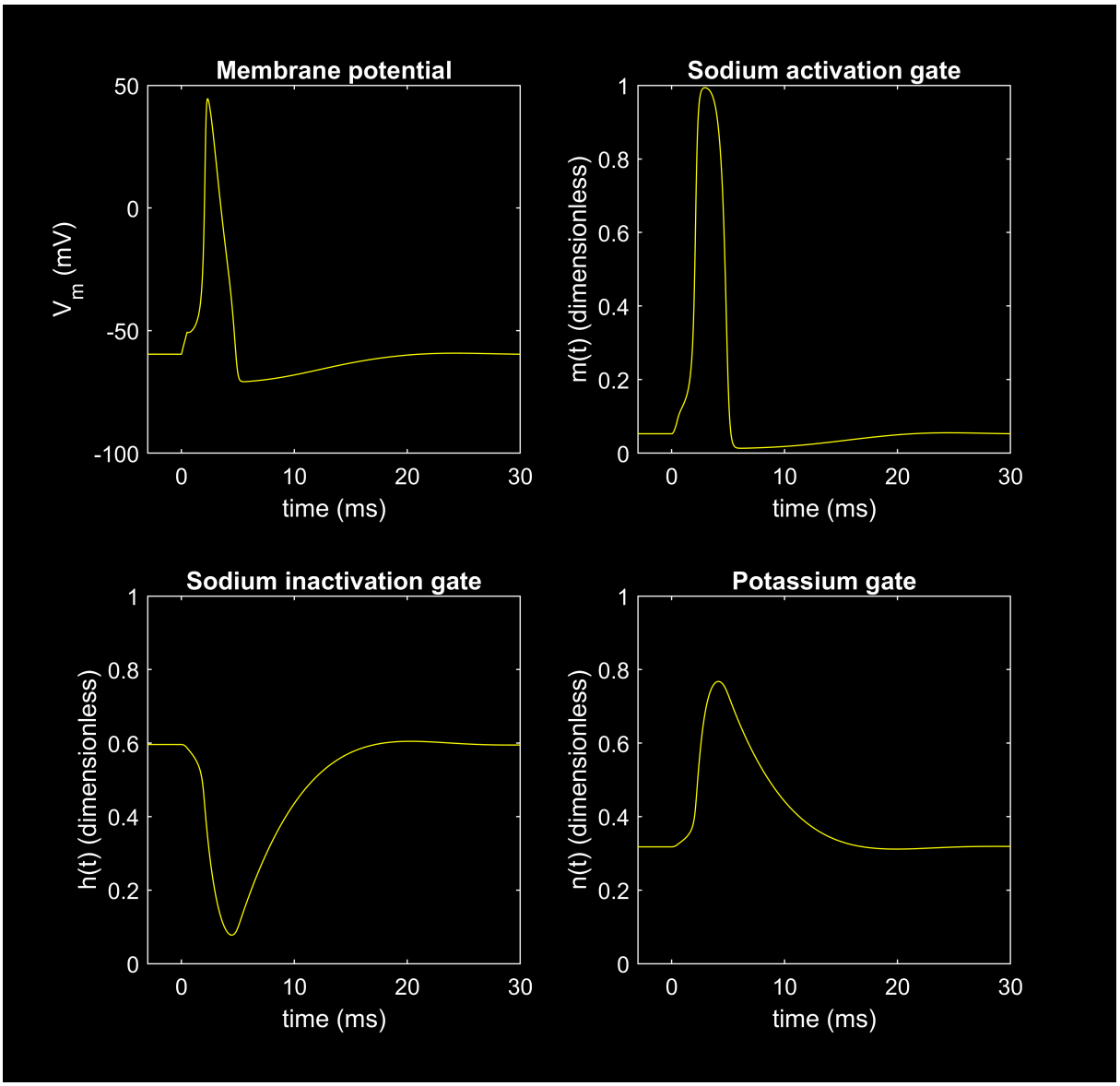

Membrane potential
50
0
-50
-100
V m (mV)
0
10
20
30
time (ms)
Sodium activation gate
1
0.8
0.6
0.4
0.2
0
m(t) (dimensionless)
0
10
20
30
time (ms)
Sodium inactivation gate
1
0.8
0.6
0.4
0.2
0
h(t) (dimensionless)
0
10
20
30
time (ms)
Potassium gate
1
0.8
0.6
0.4
0.2
0
n(t) (dimensionless)
0
10
20
30
time (ms)

```
hhsplot(0,30)
```

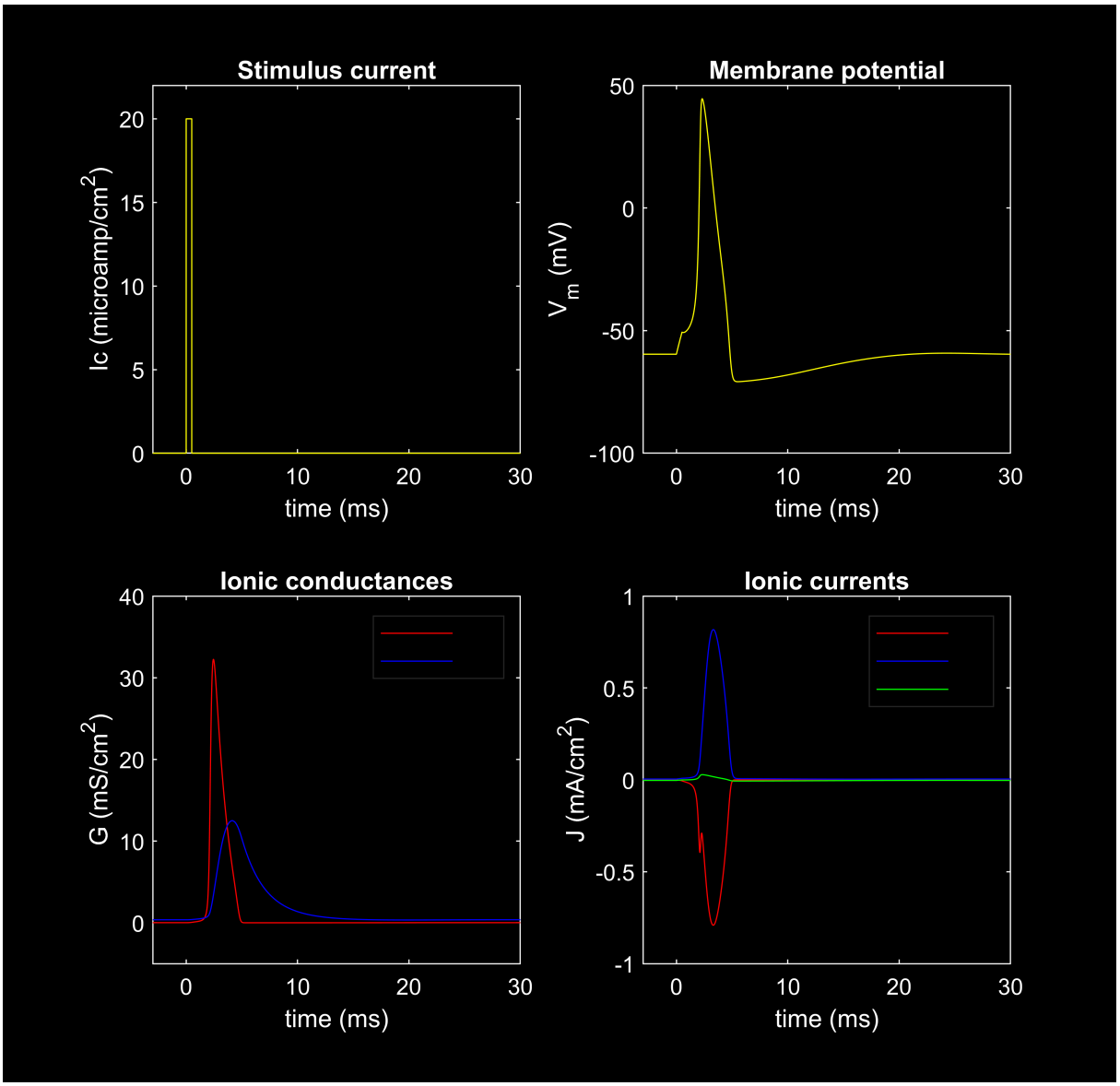



```
ans = -2.7016e+03
```

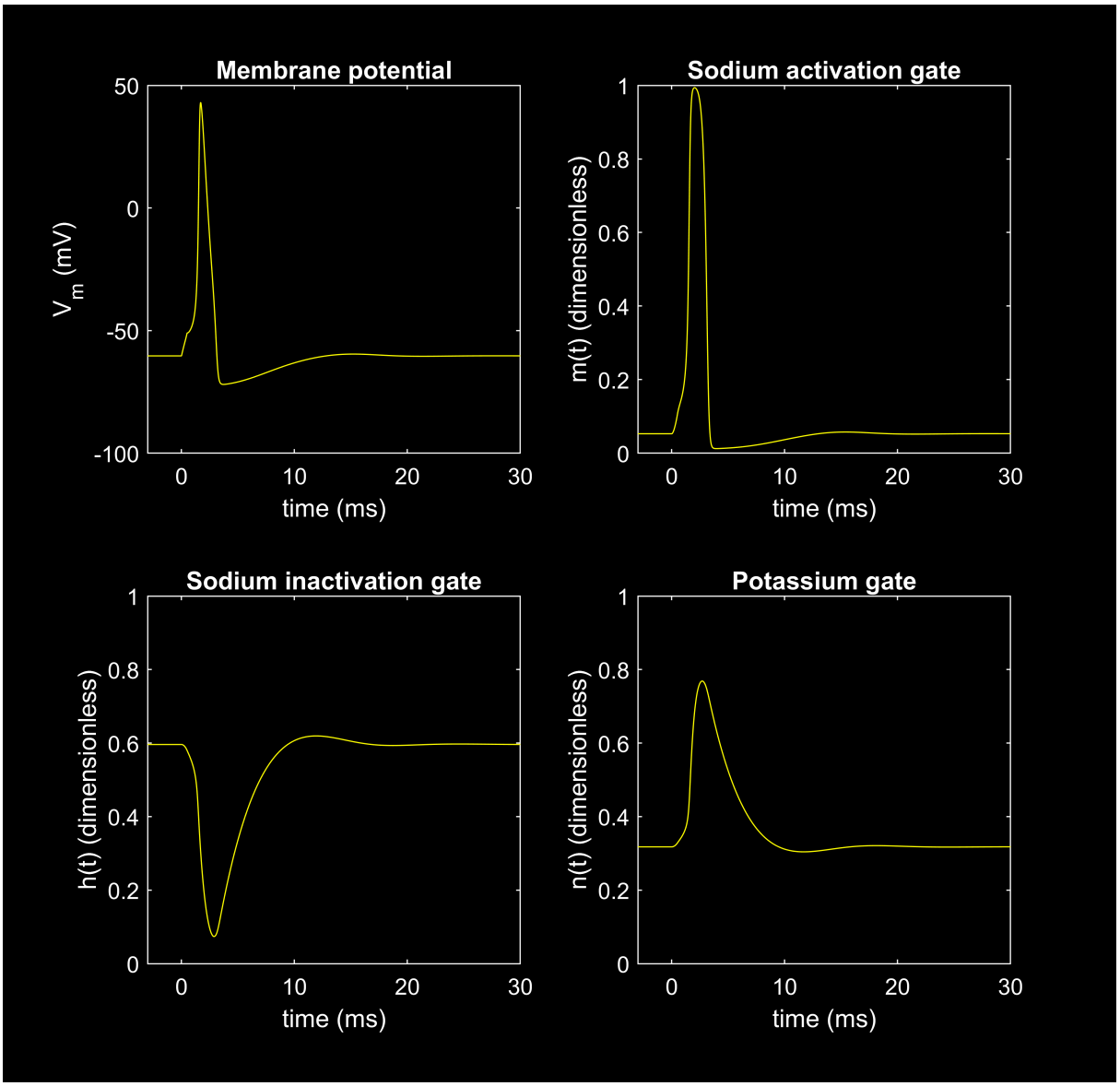
```
vclamp = 0;
amp1 = 20;
width1 = 0.5;
tempc = 5;
hhmplot(0,30,0);
```


Membrane potential
50
0
-50
-100
V m (mV)
0
10
20
30
time (ms)
Sodium activation gate
1
0.8
0.6
0.4
0.2
0
m(t) (dimensionless)
0
10
20
30
time (ms)
Sodium inactivation gate
1
0.8
0.6
0.4
0.2
0
h(t) (dimensionless)
0
10
20
30
time (ms)
Potassium gate
1
0.8
0.6
0.4
0.2
0
n(t) (dimensionless)
0
10
20
30
time (ms)

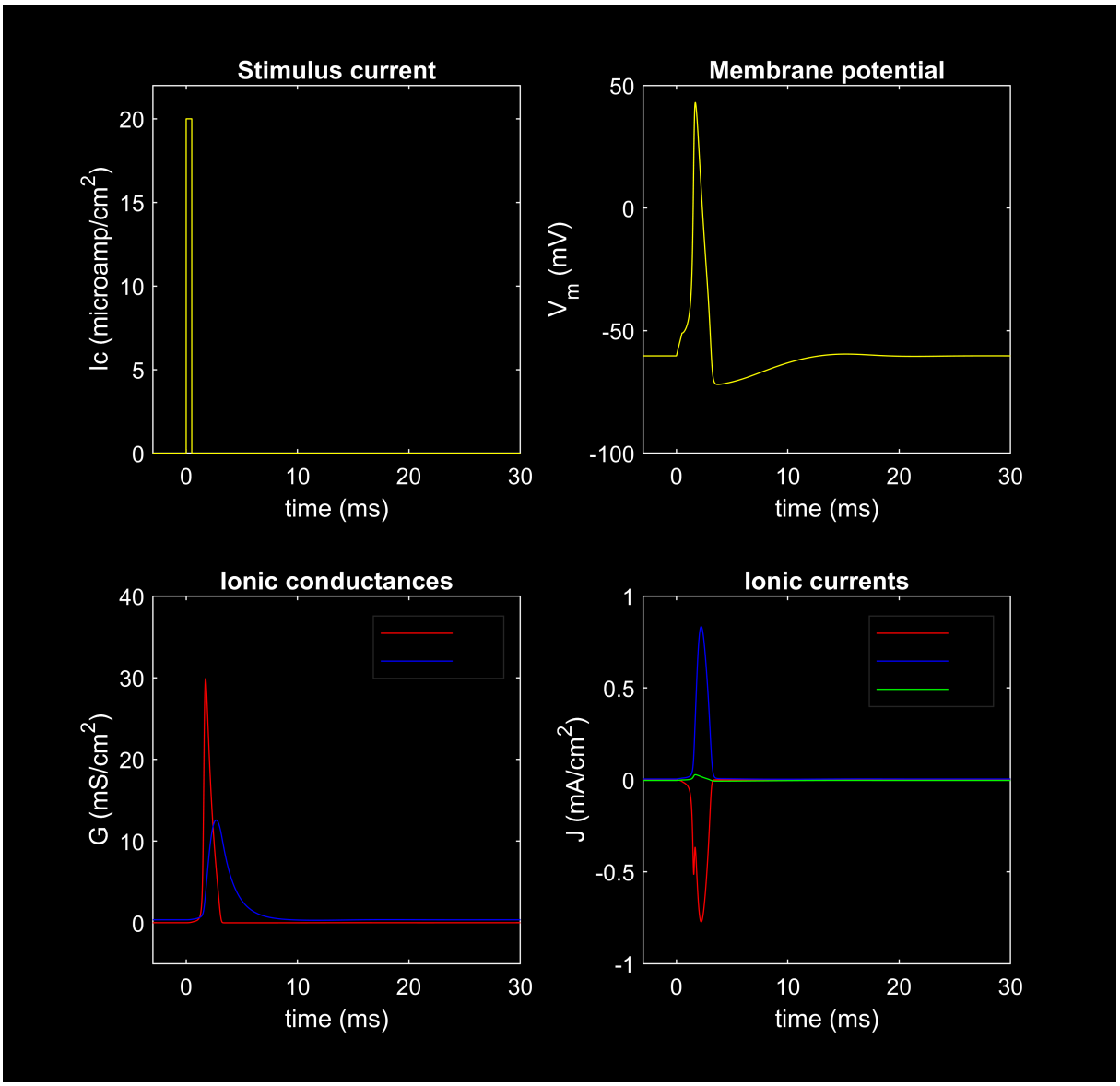
```
hhsplot(0,30);
```



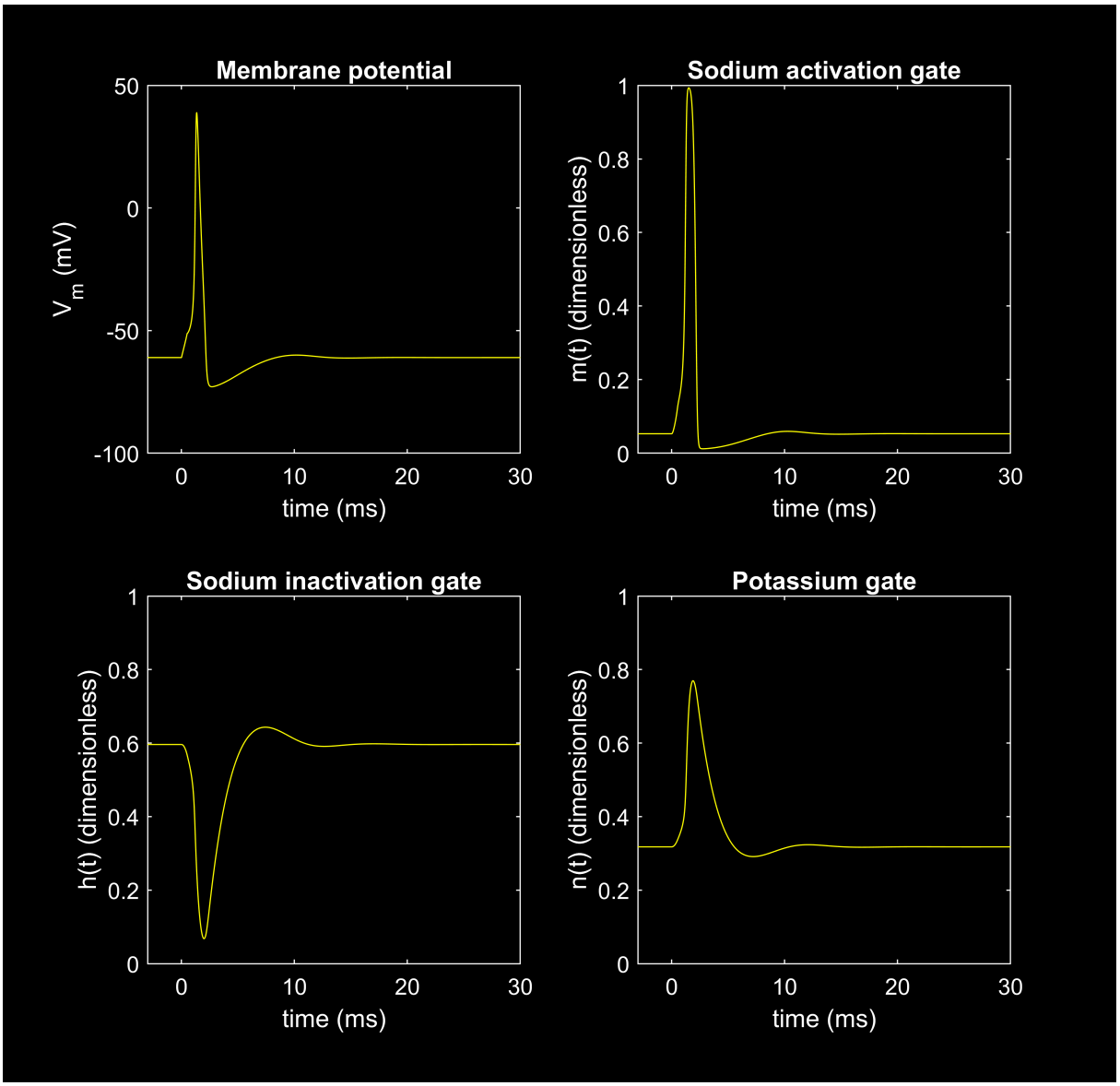

```
vclamp = 0;
amp1 = 20;
width1 = 0.5;
tempc = 10;
hhmplot(0,30,0);
```


Membrane potential
50
0
-50
-100
V m (mV)
0
10
20
30
time (ms)
Sodium activation gate
1
0.8
0.6
0.4
0.2
0
m(t) (dimensionless)
0
10
20
30
time (ms)
Sodium inactivation gate
1
0.8
0.6
0.4
0.2
0
h(t) (dimensionless)
0
10
20
30
time (ms)
Potassium gate
1
0.8
0.6
0.4
0.2
0
n(t) (dimensionless)
0
10
20
30
time (ms)

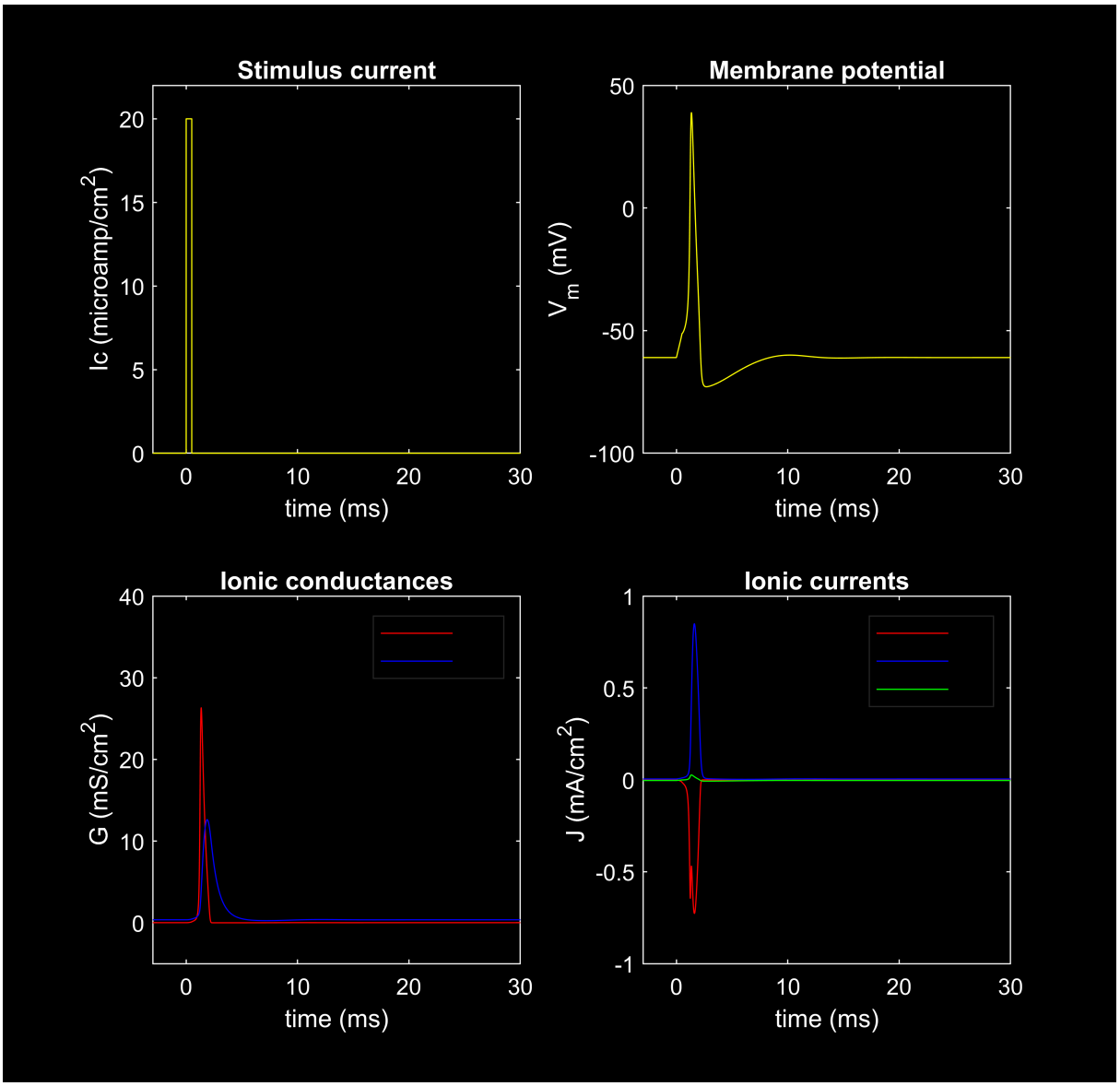
```
hhsplot(0,30);
```



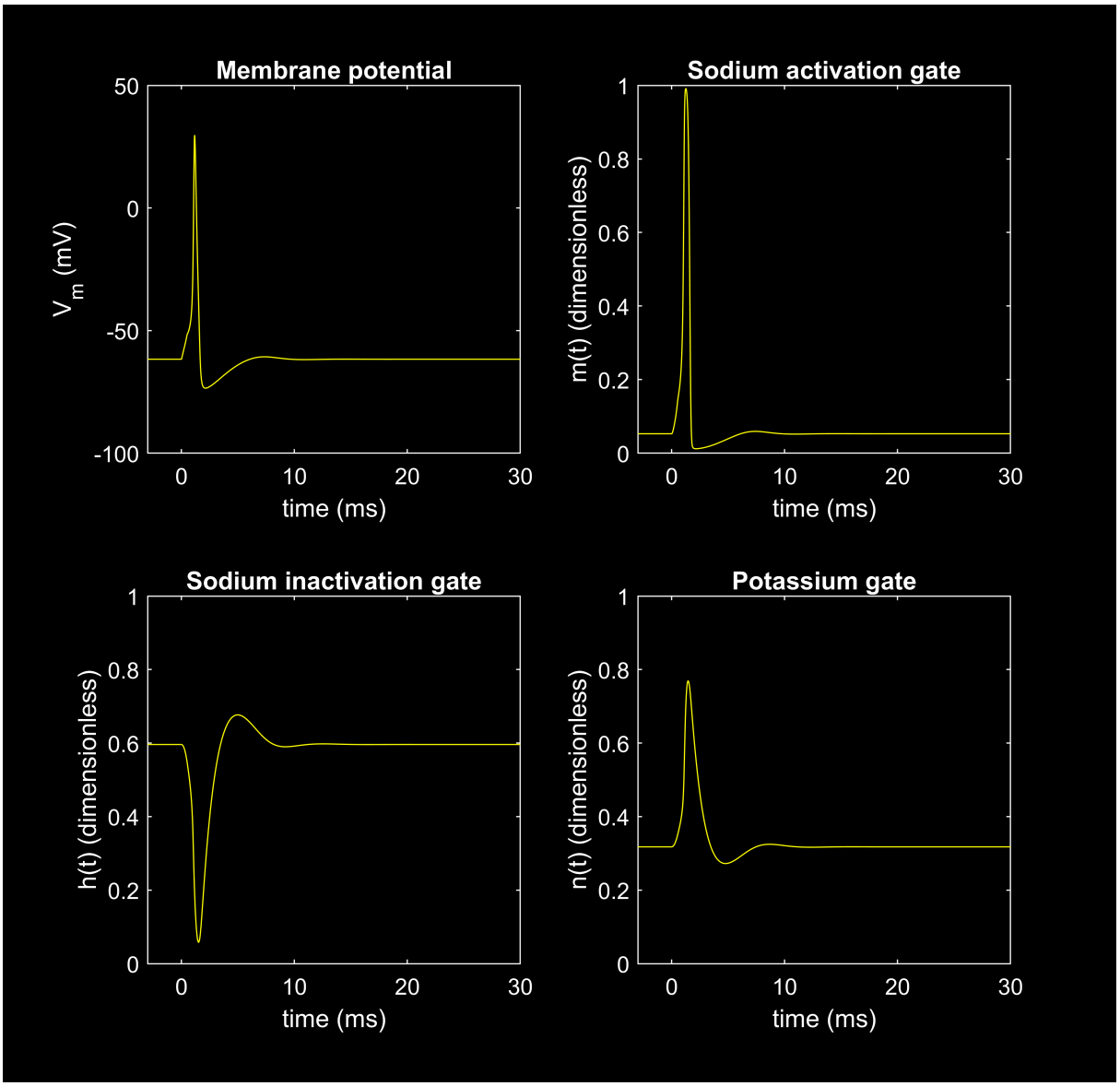

```
vclamp = 0;
amp1 = 20;
width1 = 0.5;
tempc = 15;
hhmplot(0,30,0);
```


Membrane potential
50
0
-50
-100
V m (mV)
0
10
20
30
time (ms)
Sodium activation gate
1
0.8
0.6
0.4
0.2
0
m(t) (dimensionless)
0
10
20
30
time (ms)
Sodium inactivation gate
1
0.8
0.6
0.4
0.2
0
h(t) (dimensionless)
0
10
20
30
time (ms)
Potassium gate
1
0.8
0.6
0.4
0.2
0
n(t) (dimensionless)
0
10
20
30
time (ms)

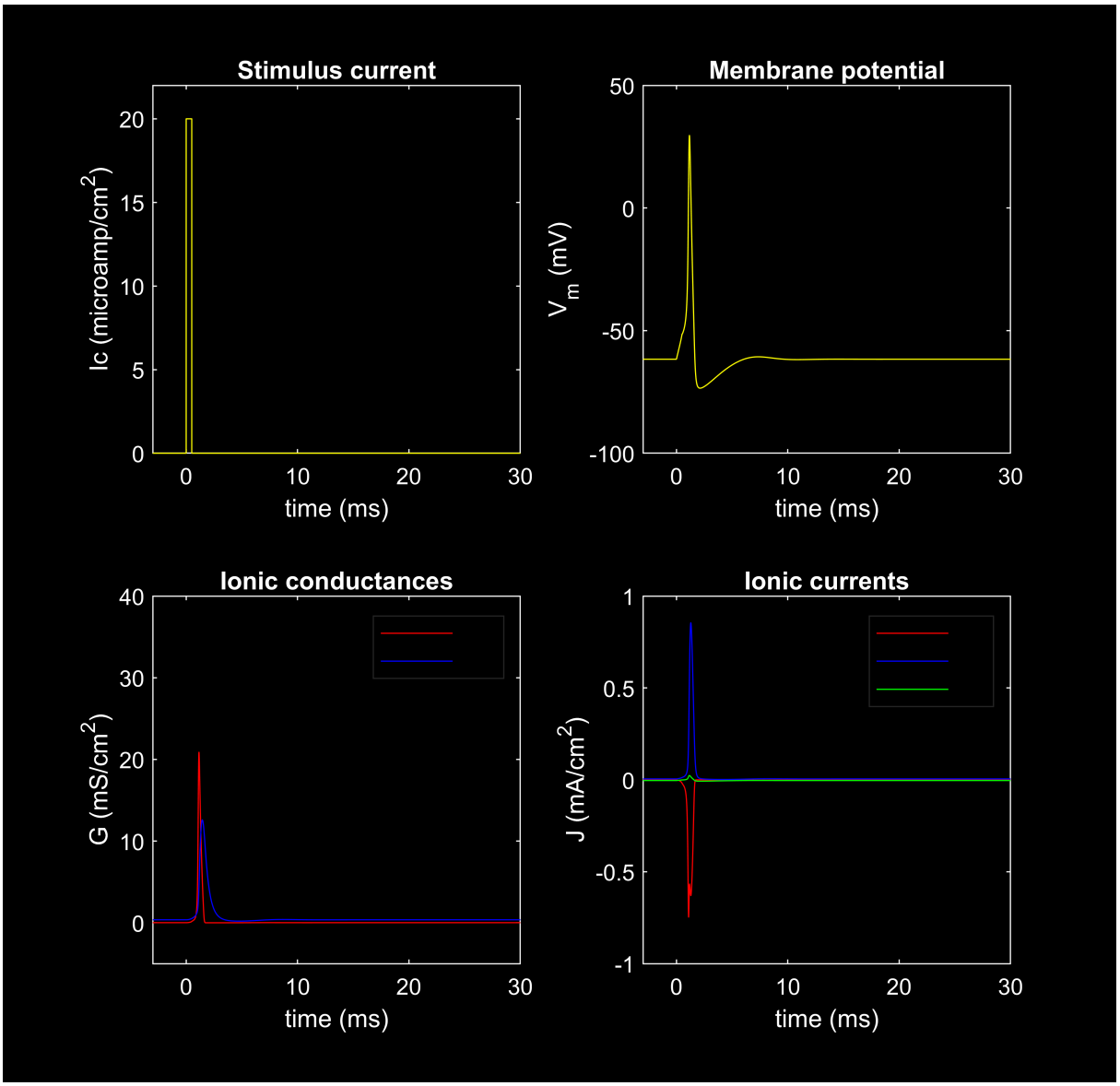
```
hhsplot(0,30);
```



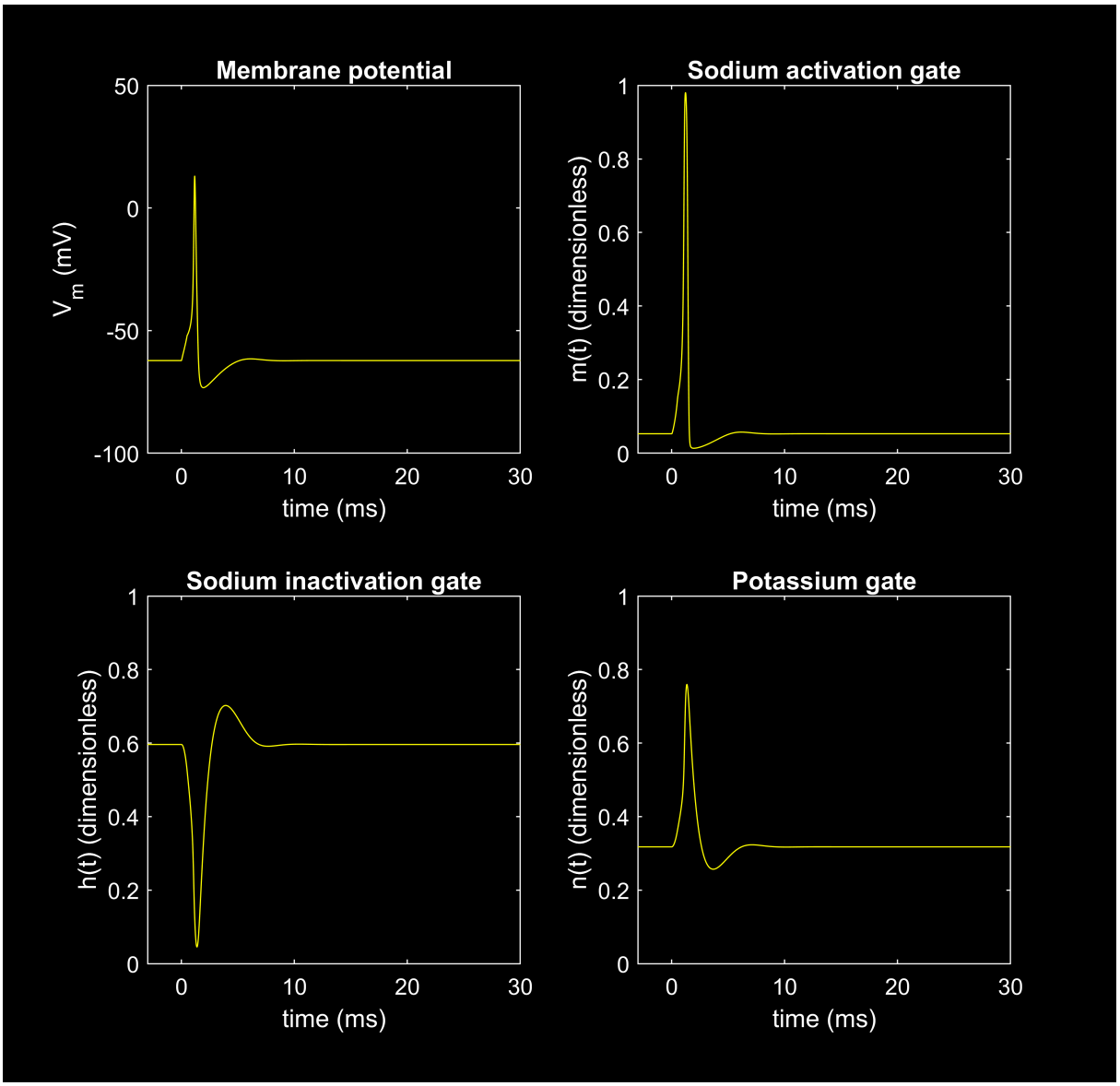

```
vclamp = 0;
amp1 = 20;
width1 = 0.5;
tempc = 20;
hhmplot(0,30,0);
```


Membrane potential
50
0
-50
-100
V m (mV)
0
10
20
30
time (ms)
Sodium activation gate
1
0.8
0.6
0.4
0.2
0
m(t) (dimensionless)
0
10
20
30
time (ms)
Sodium inactivation gate
1
0.8
0.6
0.4
0.2
0
h(t) (dimensionless)
0
10
20
30
time (ms)
Potassium gate
1
0.8
0.6
0.4
0.2
0
n(t) (dimensionless)
0
10
20
30
time (ms)

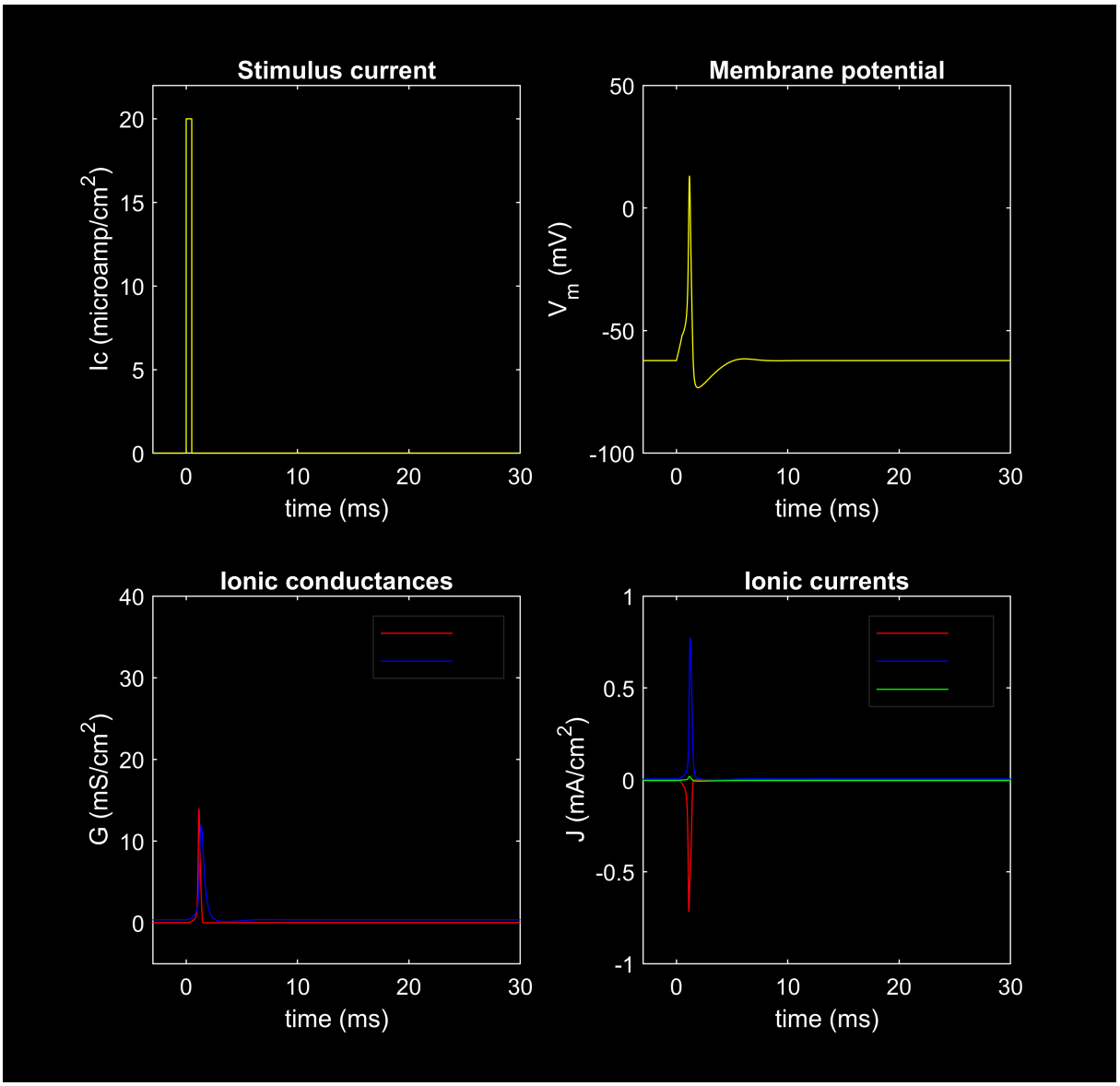
```
hhsplot(0,30);
```



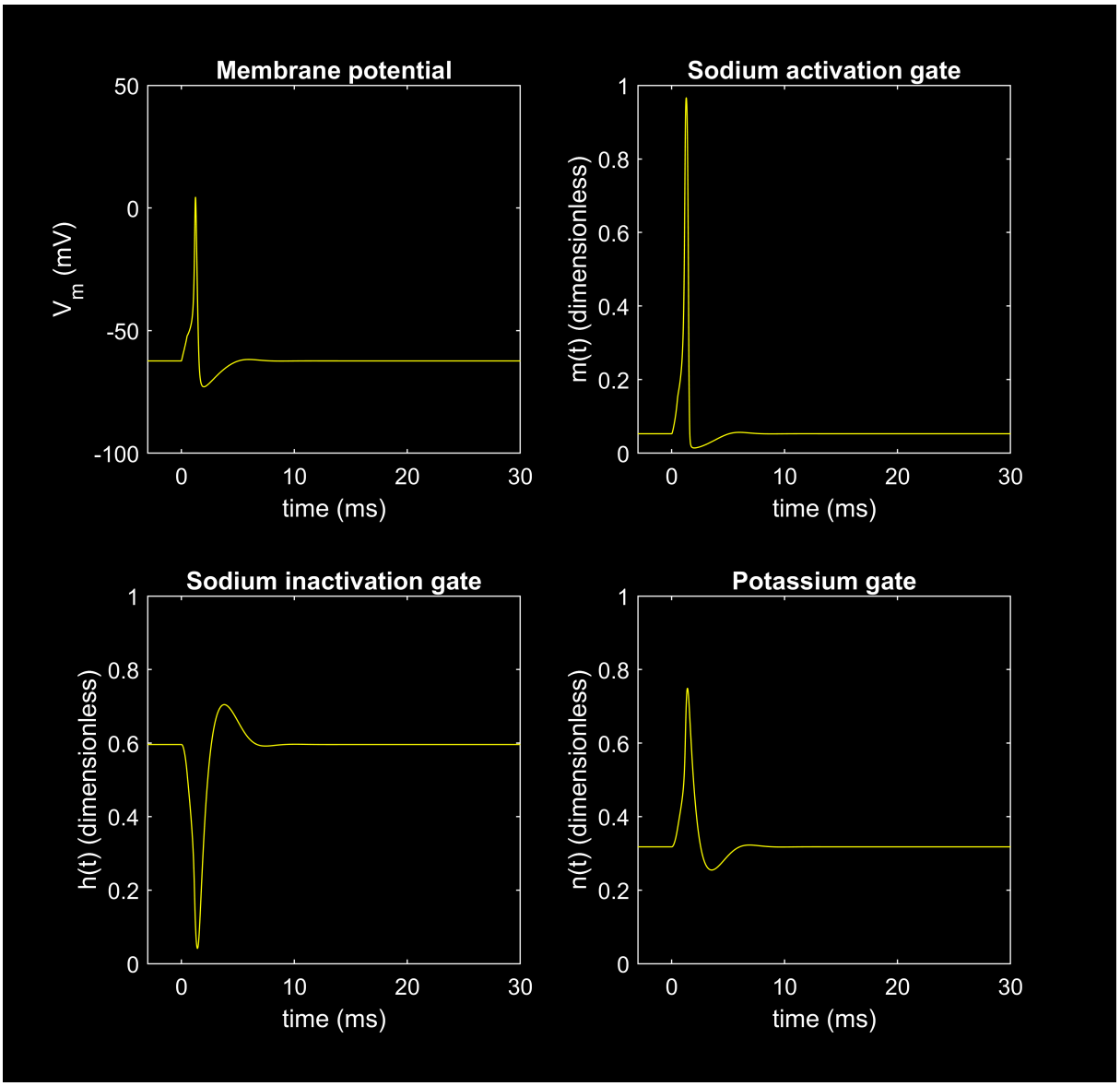

```
vclamp = 0;
amp1 = 20;
width1 = 0.5;
tempc = 24;
hhmplot(0,30,0);
```


Membrane potential
50
0
-50
-100
V m (mV)
0
10
20
30
time (ms)
Sodium activation gate
1
0.8
0.6
0.4
0.2
0
m(t) (dimensionless)
0
10
20
30
time (ms)
Sodium inactivation gate
1
0.8
0.6
0.4
0.2
0
h(t) (dimensionless)
0
10
20
30
time (ms)
Potassium gate
1
0.8
0.6
0.4
0.2
0
n(t) (dimensionless)
0
10
20
30
time (ms)

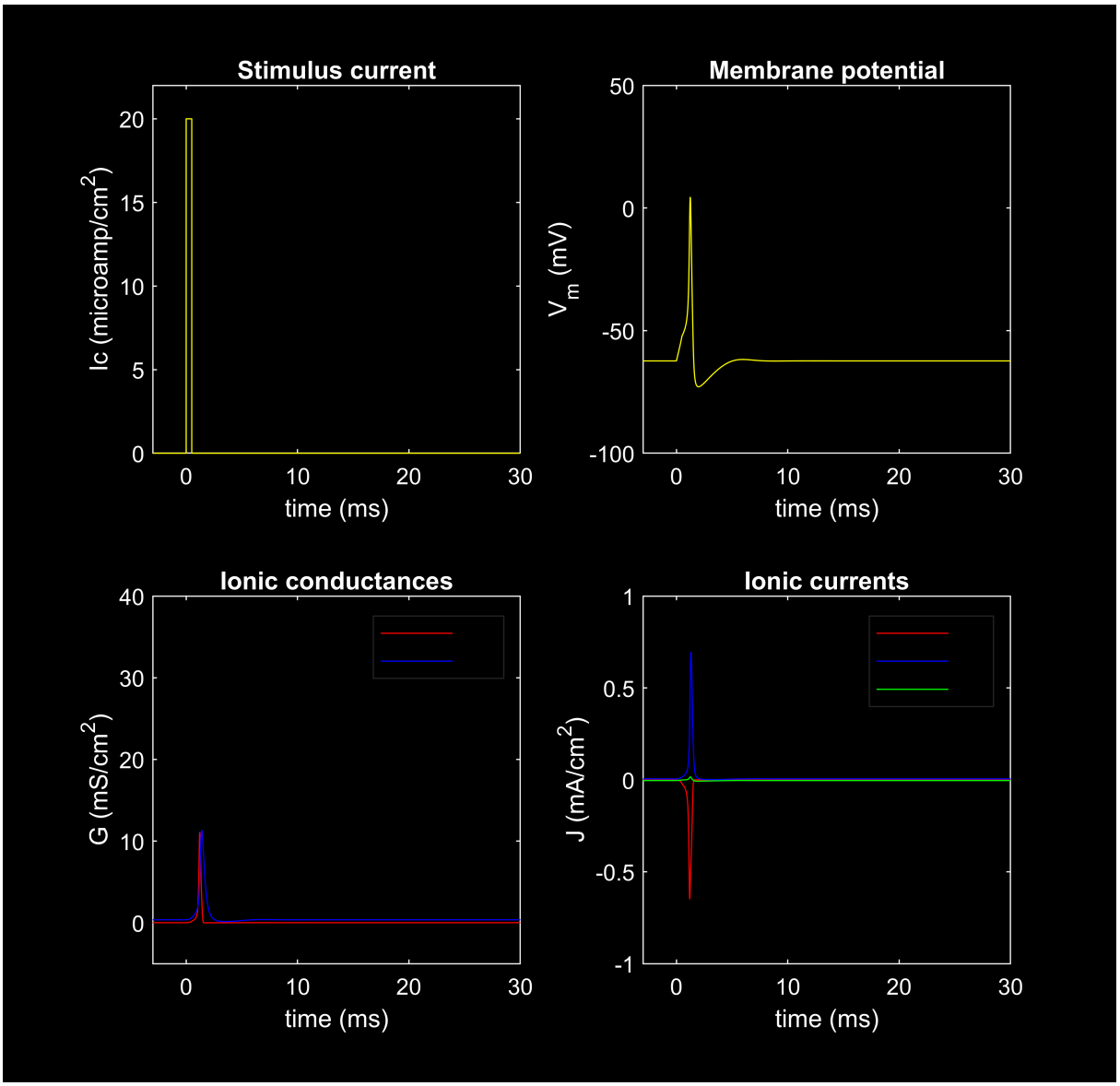
```
hhsplot(0,30);
```



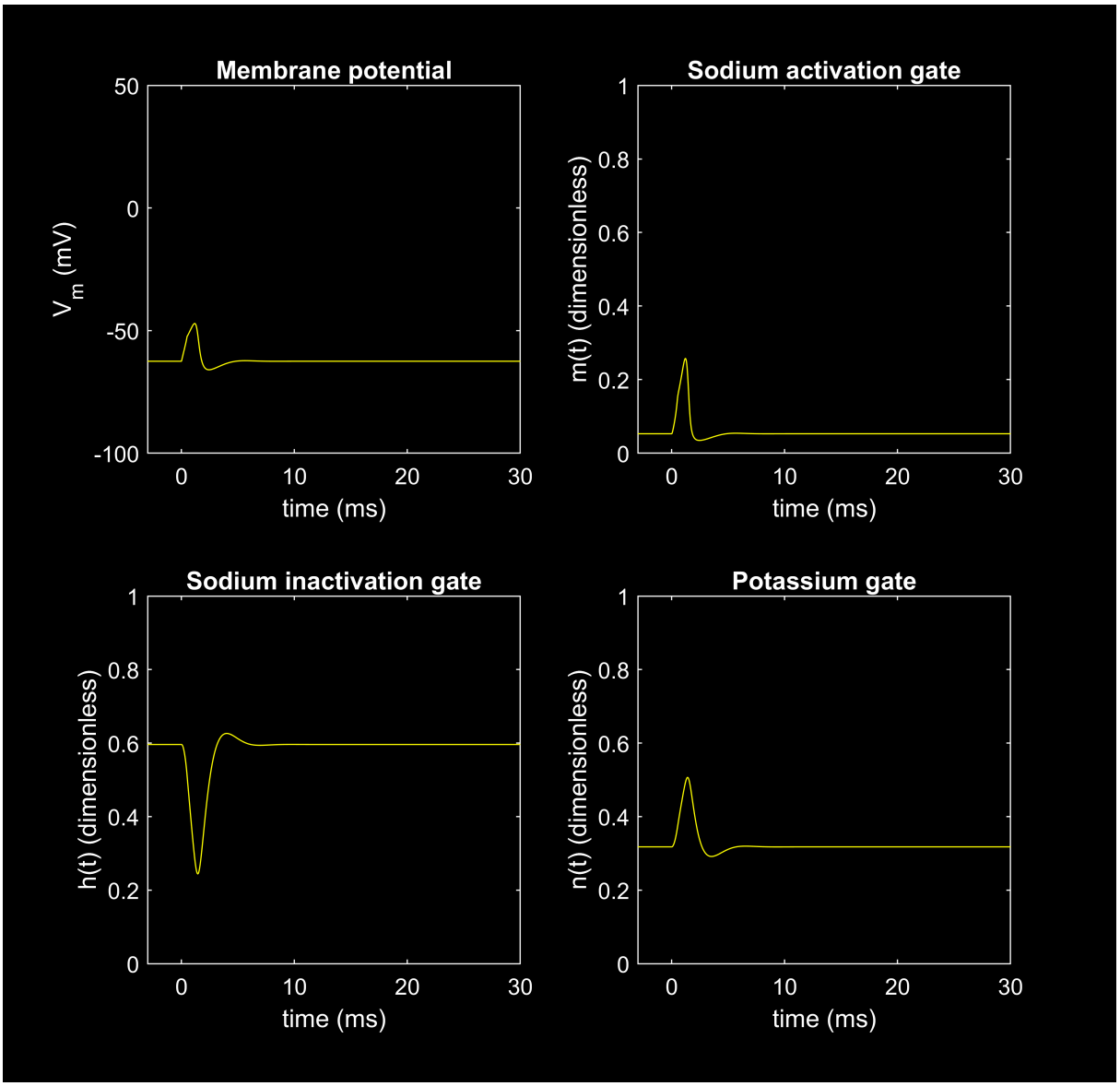

```
vclamp = 0;
amp1 = 20;
width1 = 0.5;
tempc = 25;
hhmplot(0,30,0);
```


Membrane potential
50
0
-50
-100
V m (mV)
0
10
20
30
time (ms)
Sodium activation gate
1
0.8
0.6
0.4
0.2
0
m(t) (dimensionless)
0
10
20
30
time (ms)
Sodium inactivation gate
1
0.8
0.6
0.4
0.2
0
h(t) (dimensionless)
0
10
20
30
time (ms)
Potassium gate
1
0.8
0.6
0.4
0.2
0
n(t) (dimensionless)
0
10
20
30
time (ms)

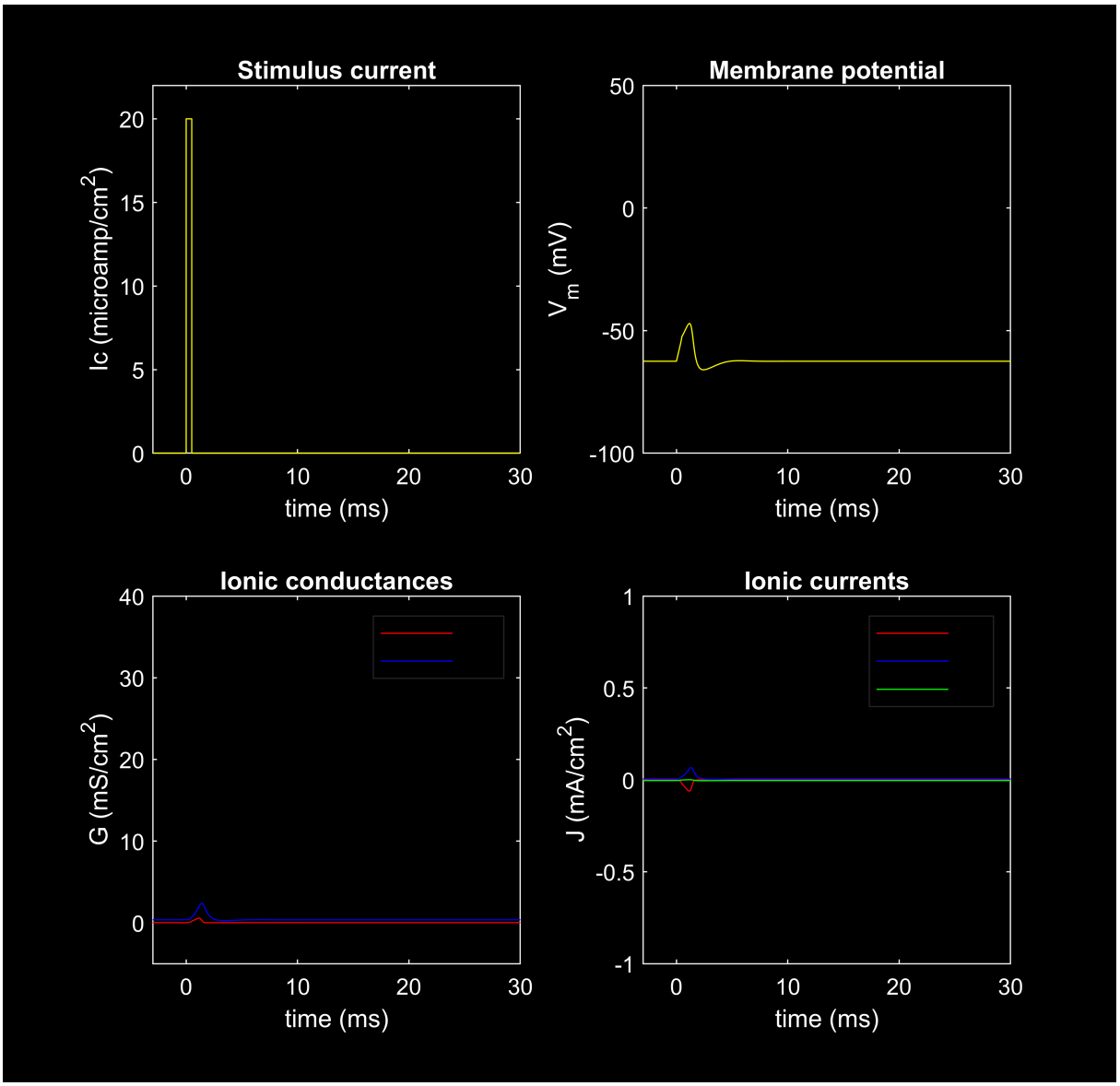
```
hhsplot(0,30);
```



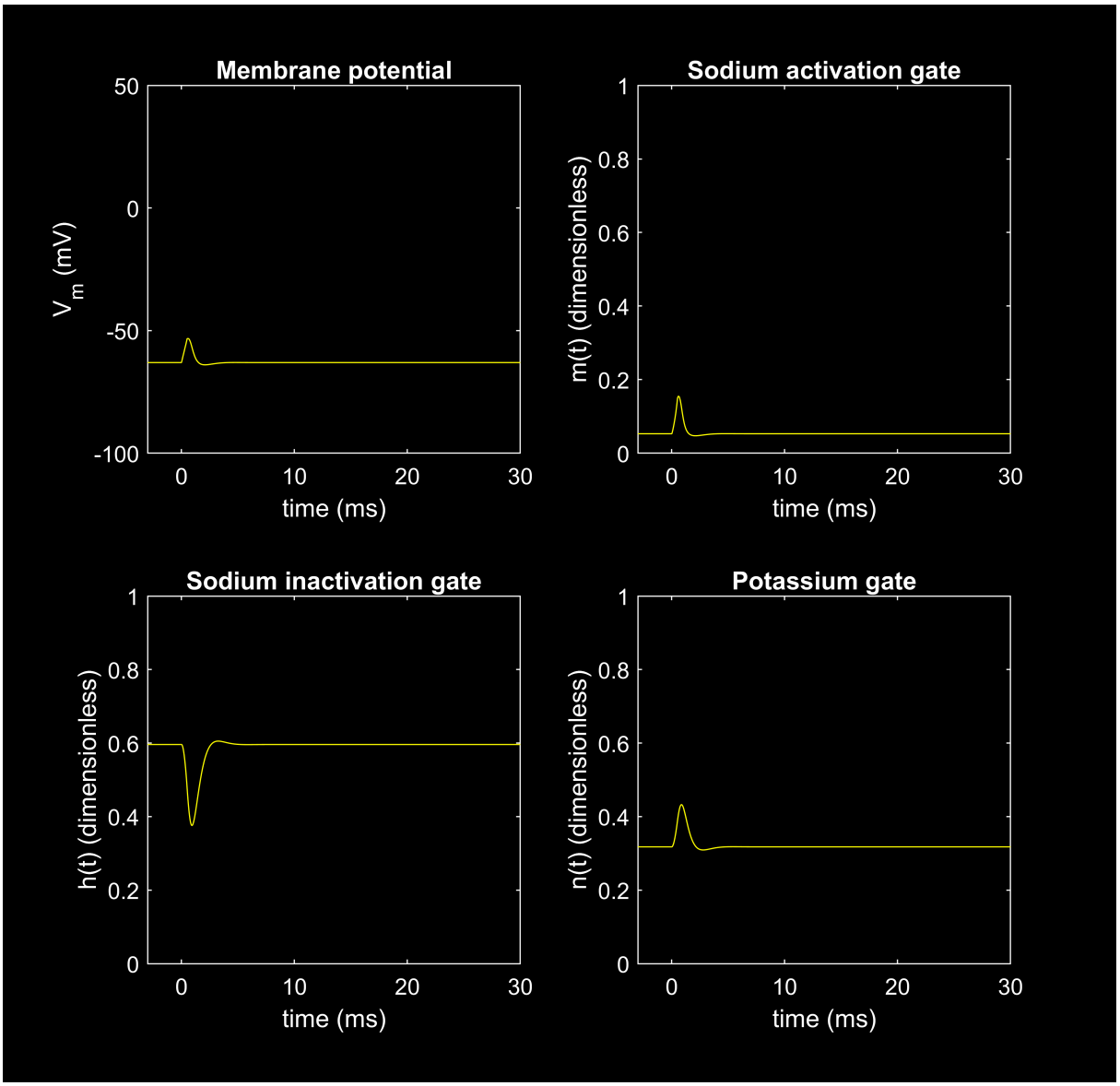

```
vclamp = 0;
amp1 = 20;
width1 = 0.5;
tempc = 26;
hhmplot(0,30,0);
```


Membrane potential
50
0
-50
-100
V m (mV)
0
10
20
30
time (ms)
Sodium activation gate
1
0.8
0.6
0.4
0.2
0
m(t) (dimensionless)
0
10
20
30
time (ms)
Sodium inactivation gate
1
0.8
0.6
0.4
0.2
0
h(t) (dimensionless)
0
10
20
30
time (ms)
Potassium gate
1
0.8
0.6
0.4
0.2
0
n(t) (dimensionless)
0
10
20
30
time (ms)

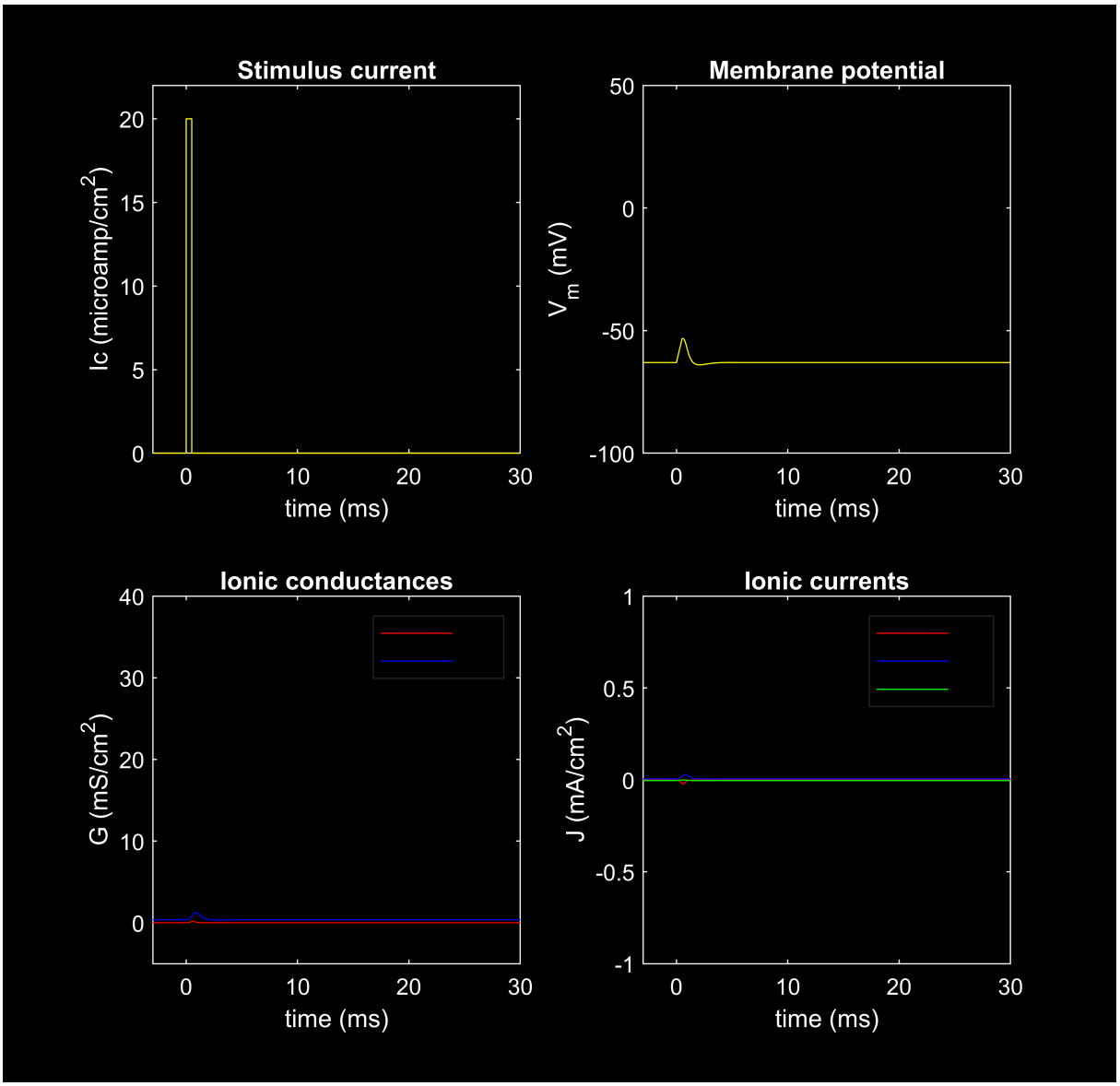
```
hhsplot(0,30);
```




```
vclamp = 0;
amp1 = 20;
width1 = 0.5;
tempc = 30;
hhmplot(0,30,0);
```


Membrane potential
50
0
-50
-100
V m (mV)
0
10
20
30
time (ms)
Sodium activation gate
1
0.8
0.6
0.4
0.2
0
m(t) (dimensionless)
0
10
20
30
time (ms)
Sodium inactivation gate
1
0.8
0.6
0.4
0.2
0
h(t) (dimensionless)
0
10
20
30
time (ms)
Potassium gate
1
0.8
0.6
0.4
0.2
0
n(t) (dimensionless)
0
10
20
30
time (ms)

```
hhsplot(0,30);
```



Action potential amplitude as well as the threshhold is affected by the temperature.