第五课　二叉树与图

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内容概述

1.5道经典**二叉树与图**的相关题目

预备知识:二叉树基础知识

例1:路径之和2(medium) (二叉树深搜)

例2:最近的公共祖先(medium) (二叉树性质)

例3:二叉树转链表(medium) (二叉树与链表)

预备知识:二叉树层次遍历

例4:侧面观察二叉树(medium) (二叉树宽搜)

预备知识:图的基础知识

例5:课程安排(有向图判断环)(medium)

2.详细讲解题目**解题方法**、**代码实现**

预备知识:二叉树定义

树是n(n>=0)个节点的**有限集**，且这些节点**满足**如下关系:

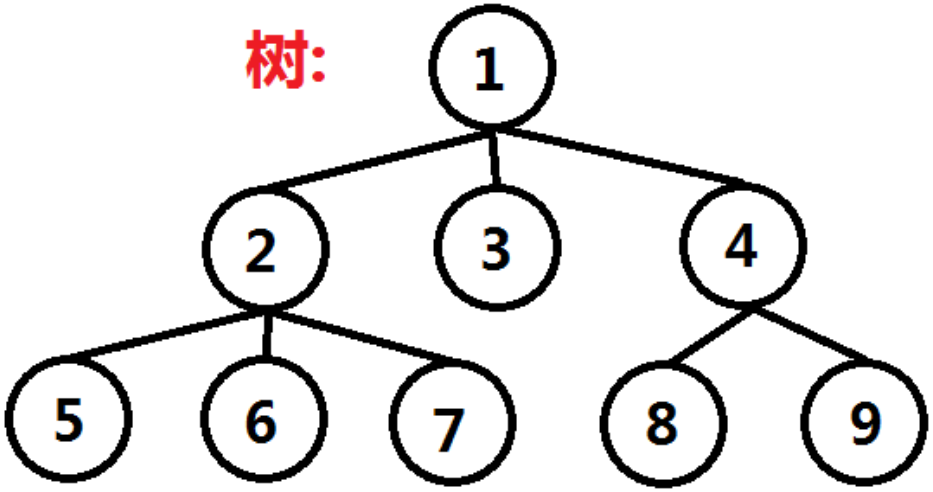
(1)有且仅有一个节点没有父结点，该节点称为树的**根**。

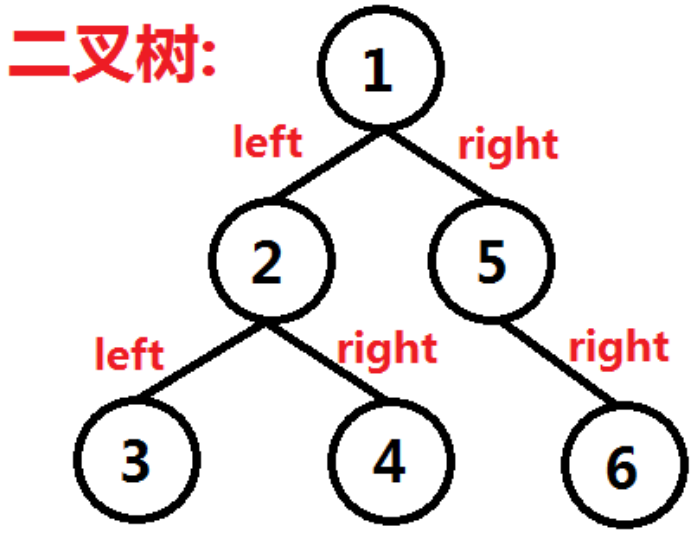
(2)除根外，其余的每个节点都有且仅有一个**父结点**。

(3)树中的每一个节点都构成一个**以它为根**的树。

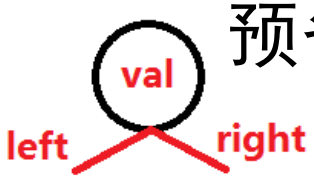
二叉树在满足树的条件时，满足如下条件:

每个节点最多有**两个孩子**(子树)，这两个子树有**左右之分**，次序不可颠倒。

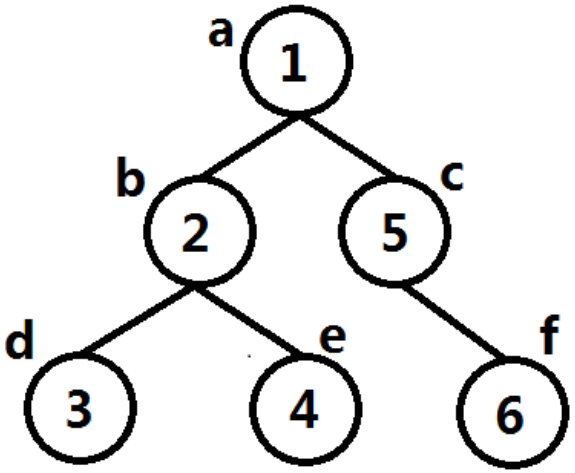




预备知识:二叉树构造



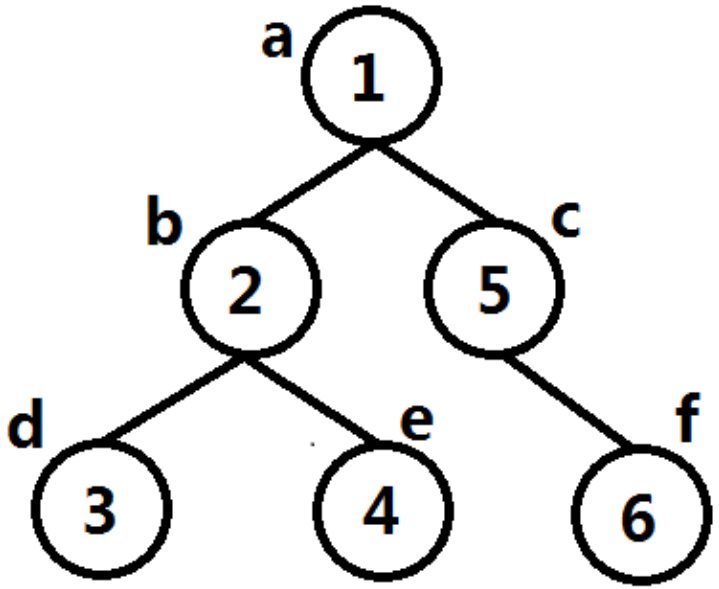

```
#include <stdio.h>

struct TreeNode {                    //二叉树数据结构
    int val;                         //数据域val
    TreeNode *left;
    TreeNode *right;                 //left, right左右子树指针
    TreeNode(int x) : val(x), left(NULL), right(NULL) {}
};

void preorder_print(TreeNode *node,int layer){
    if (!node){                      //正在遍历的节点      //当前节点的层数
        return;
    }
    for (int i = 0; i < layer; i++){
        printf("-----");             //根据层数，打印相应数量的'-'
    }
    printf("[%d]\n", node->val);
    preorder_print(node->left, layer + 1);   //遍历左子树，层数+1
    preorder_print(node->right, layer + 1);  //遍历右子树，层数+1
}

int main(){
    TreeNode a(1);
    TreeNode b(2);
    TreeNode c(5);
    TreeNode d(3);
    TreeNode e(4);
    TreeNode f(6);
    a.left = &b;
    a.right = &c;
    b.left = &d;
    b.right = &e;
    c.right = &f;
    preorder_print(&a, 0);
    return 0;
}
```




```
[1]
-----[2]
----------[3]
----------[4]
-----[5]
----------[6]
```

预备知识:二叉树的深度遍历

```
void traversal(TreeNode *node){
    if (!node){
        return;
    }
    此时访问node称为前序遍历
    traversal(node->left);
    此时访问node称为中序遍历
    traversal(node->right);
    此时访问node称为后序遍历
}
```



前序遍历: a(1), b(2), d(3), e(4), c(5), f(6)

中序遍历: d(3), b(2), e(4), a(1), c(5), f(6)

后序遍历: d(3), e(4), b(2), f(6), c(5), a(1)

预备知识:二叉树的遍历课堂练习

```
void traversal_print1(TreeNode *node,int layer){
    if (!node){
        return;
    }
    traversal_print1(node->left, layer + 1);
    for (int i = 0; i < layer; i++){
        printf("-----");
    }
    printf("[%d]\n", node->val);
    traversal_print1(node->right, layer + 1);
}
```

A x

```
[1]
-----[2]
----------[3]
----------[4]
-----[5]
----------[6]
```

```
void traversal_print2(TreeNode *node,int layer){
    if (!node){
        return;
    }
    traversal_print2(node->left, layer + 1);
    traversal_print2(node->right, layer + 1);
    for (int i = 0; i < layer; i++){
        printf("-----");
    }
    printf("[%d]\n", node->val);
}
```

B y

```
----------[3]
-----[2]
----------[4]
[1]
-----[5]
----------[6]
```

```
void traversal_print3(TreeNode *node,int layer){
    if (!node){
        return;
    }
    for (int i = 0; i < layer; i++){
        printf("-----");
    }
    printf("[%d]\n", node->val);
    traversal_print3(node->left, layer + 1);
    traversal_print3(node->right, layer + 1);
}
```

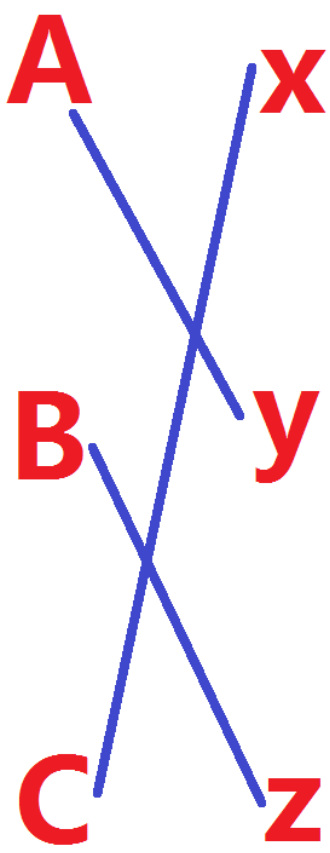
C z

```
----------[3]
----------[4]
-----[2]
----------[6]
-----[5]
[1]
```

A(?) B(?) C(?)

预备知识:二叉树的遍历讲解

```
void traversal_print1(TreeNode *node,int layer){
    if (!node){
        return;
    }
    traversal_print1(node->left, layer + 1);
    for (int i = 0; i < layer; i++){
        printf("-----");
    }
    printf("[%d]\n", node->val);
    traversal_print1(node->right, layer + 1);
}

void traversal_print2(TreeNode *node,int layer){
    if (!node){
        return;
    }
    traversal_print2(node->left, layer + 1);
    traversal_print2(node->right, layer + 1);
    for (int i = 0; i < layer; i++){
        printf("-----");
    }
    printf("[%d]\n", node->val);
}
void traversal_print3(TreeNode *node,int layer){
    if (!node){
        return;
    }
    for (int i = 0; i < layer; i++){
        printf("-----");
    }
    printf("[%d]\n", node->val);
    traversal_print3(node->left, layer + 1);
    traversal_print3(node->right, layer + 1);
}
```




```
[1]
-----[2]
----------[3]
----------[4]
-----[5]
----------[6]
```

```
----------[3]
-----[2]
----------[4]
[1]
-----[5]
----------[6]
```

```
----------[3]
----------[4]
-----[2]
----------[6]
-----[5]
[1]
```

A(y) B(z) C(x)

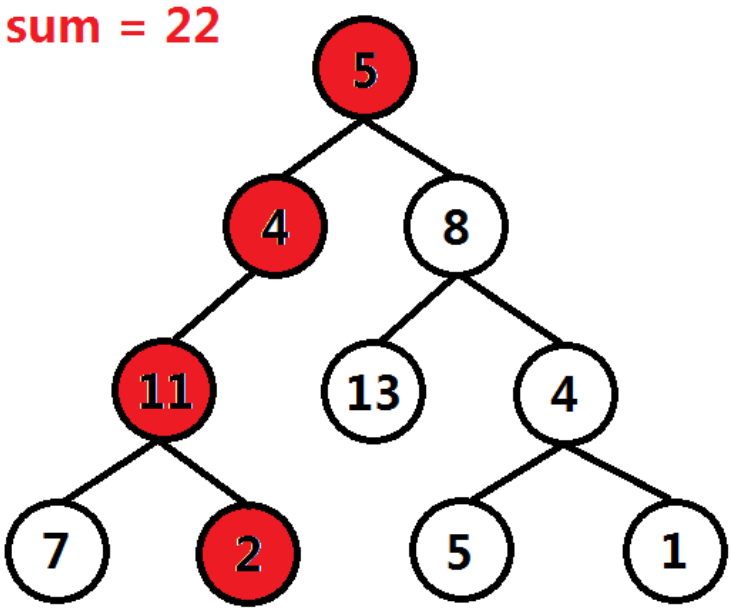
例1:路径之和2

给定一个**二叉树**与整数sum，找出**所有**从根节点到叶结点的**路径**，这些路径上的节点值**累加和为sum**。

```
#include <vector>
struct TreeNode {
    int val;
    TreeNode *left;
    TreeNode *right;
    TreeNode(int x) :
     val(x), left(NULL), right(NULL) {}
};

class Solution {
public:
    std::vector<std::vector<int> >
        pathSum(TreeNode* root, int sum) {

    }
};
```

//树节点数据结构

[[5, 4, 11, 2], [5, 8, 4, 5]]



选自 **LeetCode 113. Path Sum II**

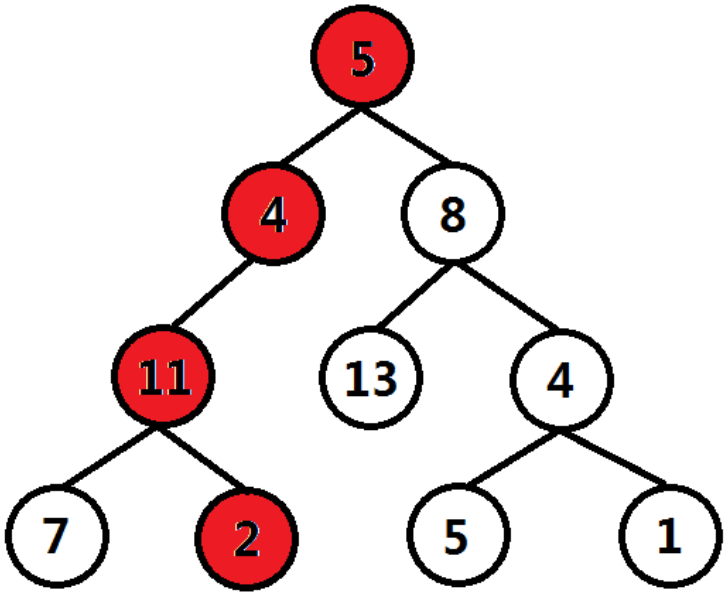
https://leetcode.com/problems/path-sum-ii/description/

难度:**Medium**

例1:思考

深度搜索所有从**根节点**到**叶结点**的**路径**，检查**各路径**上所有节点的**值的和**是否为sum。

sum = 22　[5, 4, 11, 2]　[5, 8, 4, 5]



思考:

1.使用何种数据结构存储**遍历路径**上的节点?

2.在树的**前序遍历**时做什么?**后序遍历**时做什么?

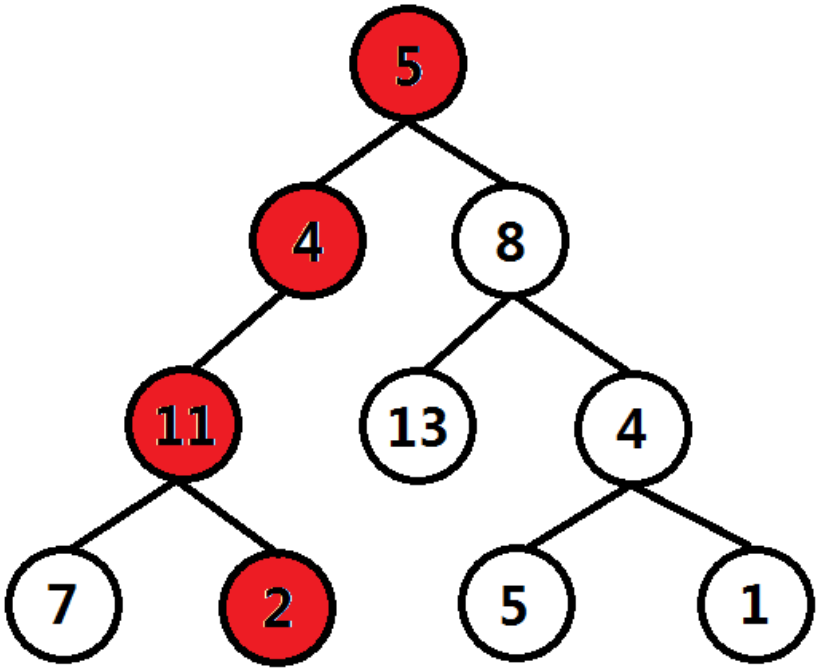
3.如何判断一个节点为**叶结点**?当遍历到叶结点时应该做什么?

例1:算法思路

1.从根节点**深度遍历**二叉树，**先序遍历**时，将该节点值存储至**path栈**中(vector实现)，使用 path_value**累加**节点值。

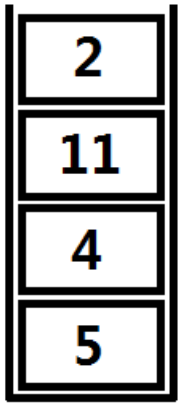
2.当遍历至**叶结点**时，检查**path_value值**是否为sum，若为sum，则将path **push进入**result结果中。

3.在**后续遍历**时，将该节点值从path栈中**弹出**，path_value**减去**节点值。



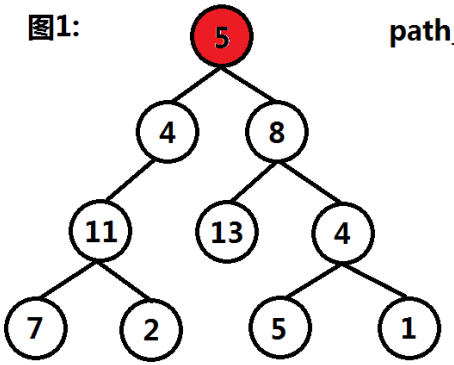
path_value = 22

path_value ?? sum



例1:算法思路

图1:



path_value = 5

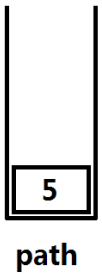
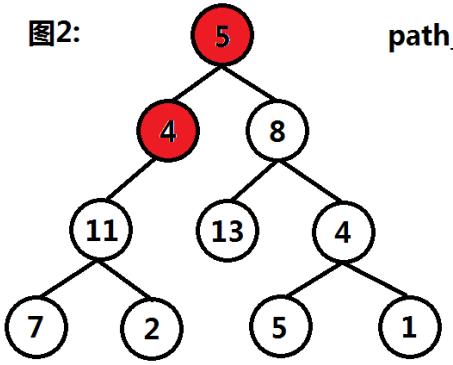


图2:



path_value = 9

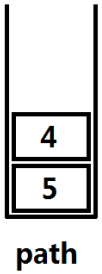
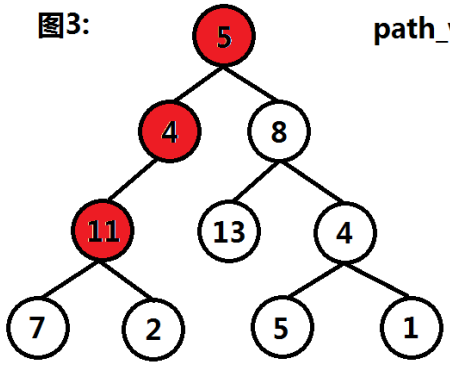


图3:



path_value = 20

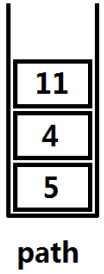
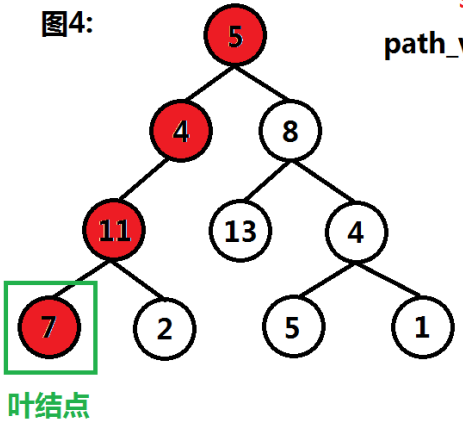


图4:



sum = 22

path_value = 27

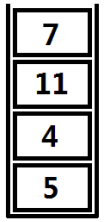
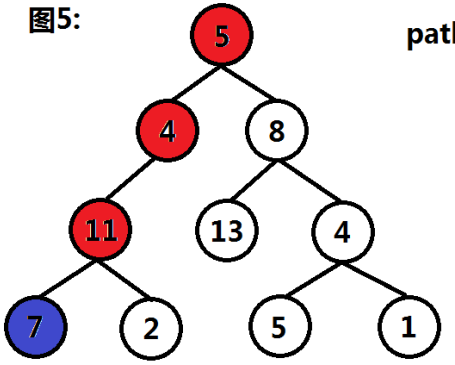


图5:



path_value = 20

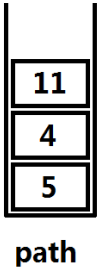
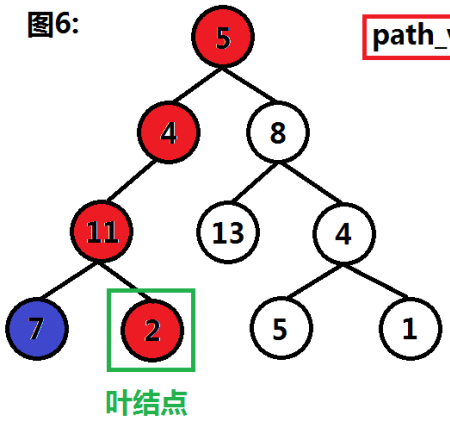
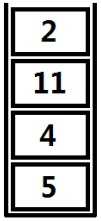


图6:



sum = 22

path_value = 22



例1:算法思路

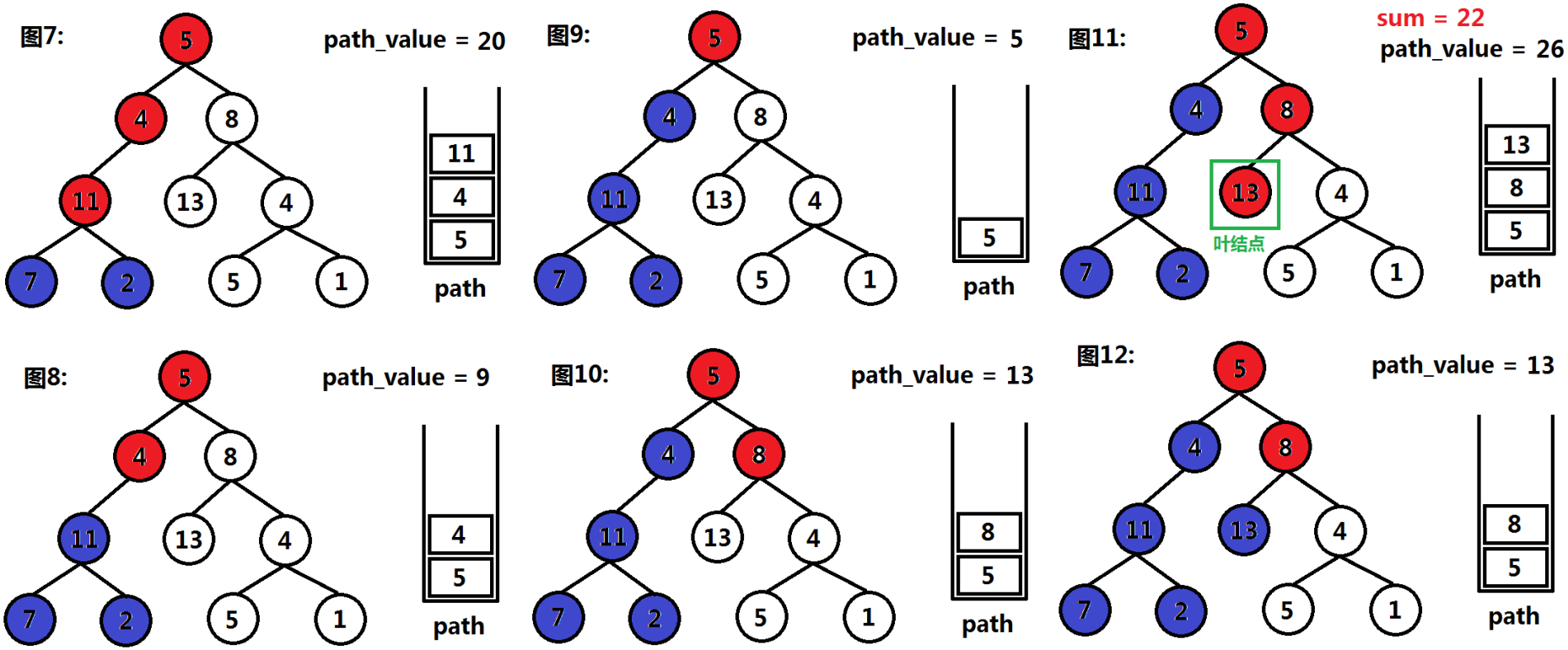
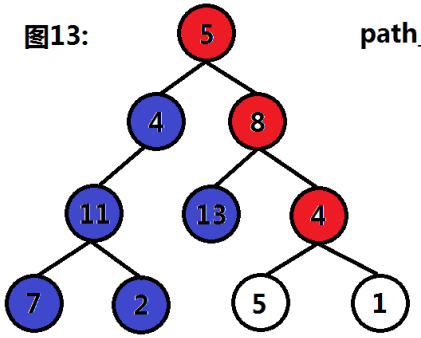


图7:
path_value = 20
path
图9:
path_value = 5
path
图11:
sum = 22
path_value = 26
叶结点
path
图8:
path_value = 9
path
图10:
path_value = 13
path
图12:
path_value = 13
path

例1:算法思路

图13:



path_value = 17

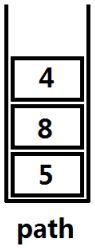
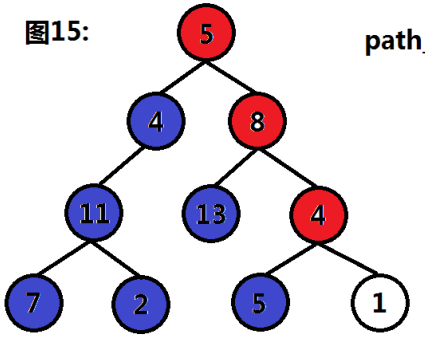


图15:



path_value = 17

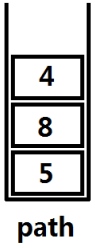
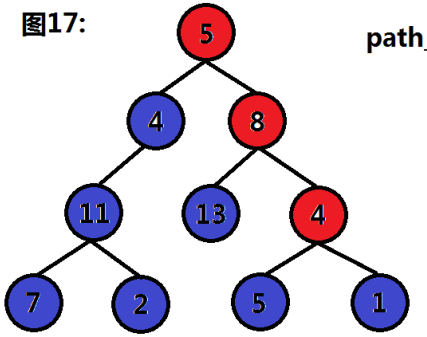


图17:



path_value = 17

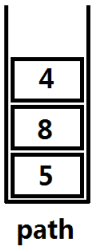
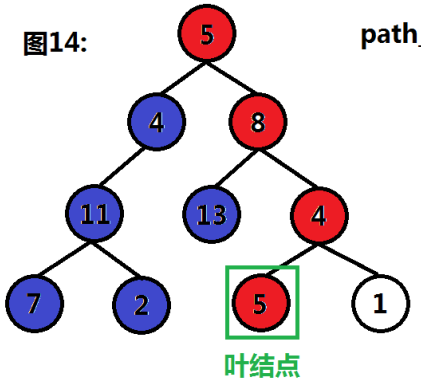


图14:



path_value = 22

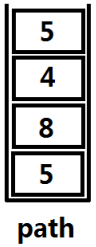
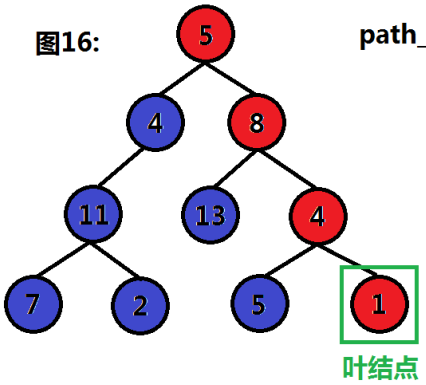


图16:



path_value = 18

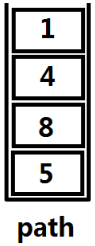
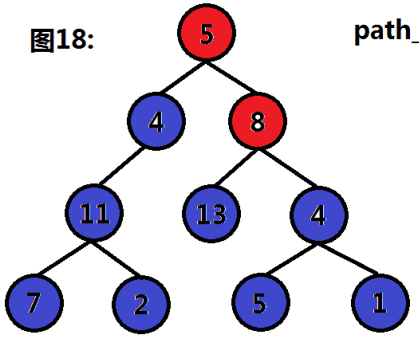
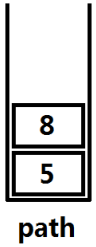


图18:

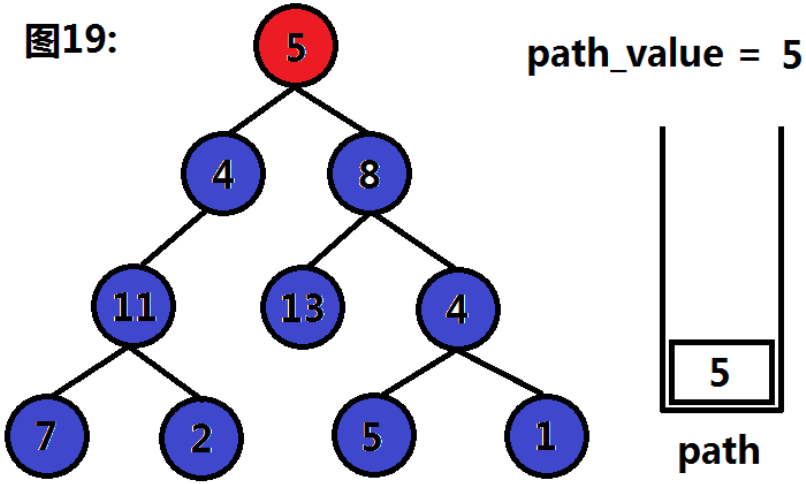


path_value = 13



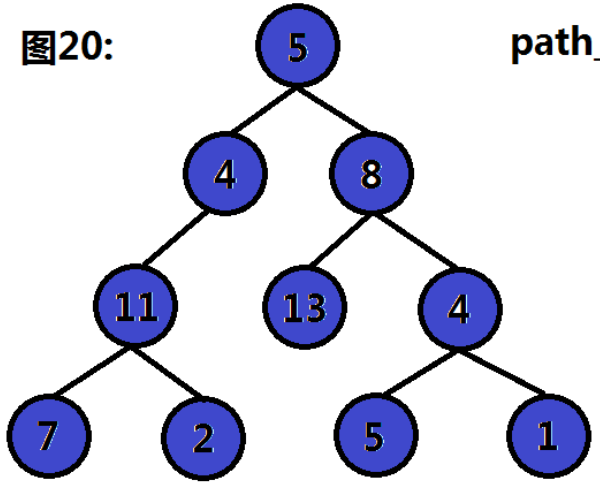
例1:算法思路

图19:

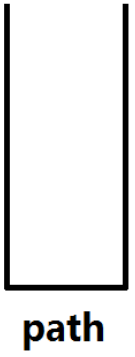


path_value = 5

图20:



path_value = 0



所有路径:

[5, 4, 11, 7] path_value = 27

[5, 4, 11, 2] path_value = 22

[5, 8, 13] path_value = 26

[5, 8, 4, 5] path_value = 22

[5, 4, 11, 1] path_value = 21

例1:课堂练习

```
class Solution {
public:
    std::vector<std::vector<int> > pathSum(TreeNode* root, int sum) {
        std::vector<std::vector<int> > result;
        std::vector<int> path;
        int path_value = 0;
        preorder(root, path_value, sum, path, result);
        return result;
    }
private:
    void preorder(TreeNode *node, int &path_value, int sum,
                  std::vector<int> &path,
                  std::vector<std::vector<int> > &result){
        if (!node){
            return;
        }
        path_value += node->val;
        1
        if (  2  ){
            result.push_back(path);
        }
        preorder(node->left, path_value, sum, path, result);
        preorder(node->right, path_value, sum, path, result);
        3
        path.pop_back();
    }
};
```

//存储满足条件路径的数组

//路径栈与路径值

//遍历一个节点即更新一次路径值

//满足 ?? 条件时，将path添加至结果数组

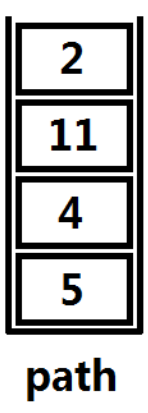
//遍历完成后，将该节点送路径栈中弹出

3分钟填写代码，

有问题随时提出！

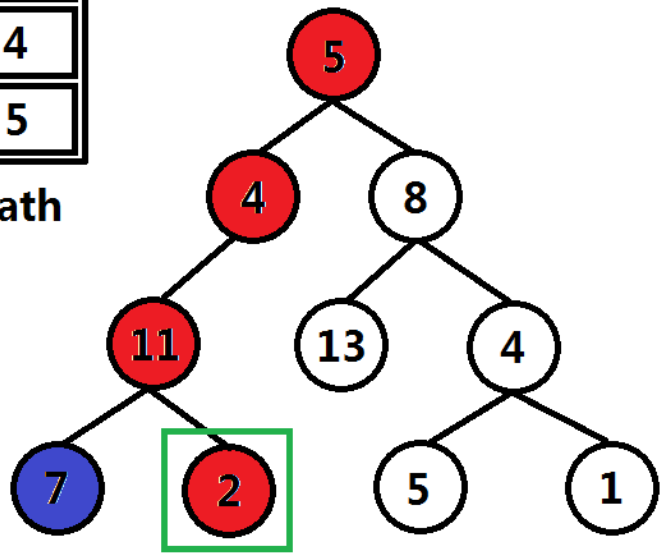
例1:实现

```
void preorder(TreeNode *node, int &path_value, int sum,
              std::vector<int> &path,
              std::vector<std::vector<int> > &result){
    if (!node){
        return;
    }                    //遍历一个节点即更新一次路径值
    path_value += node->val;
    path.push_back(node->val);

    if ( !node->left && !node->right && path_value == sum ){
        result.push_back(path);   //满足 ?? 条件时，将path添加至结果数组
    }
    preorder(node->left, path_value, sum, path, result);
    preorder(node->right, path_value, sum, path, result);
    path_value -= node->val;
    path.pop_back();   //遍历完成后，将该节点送路径栈中弹出
}
```



sum = 22

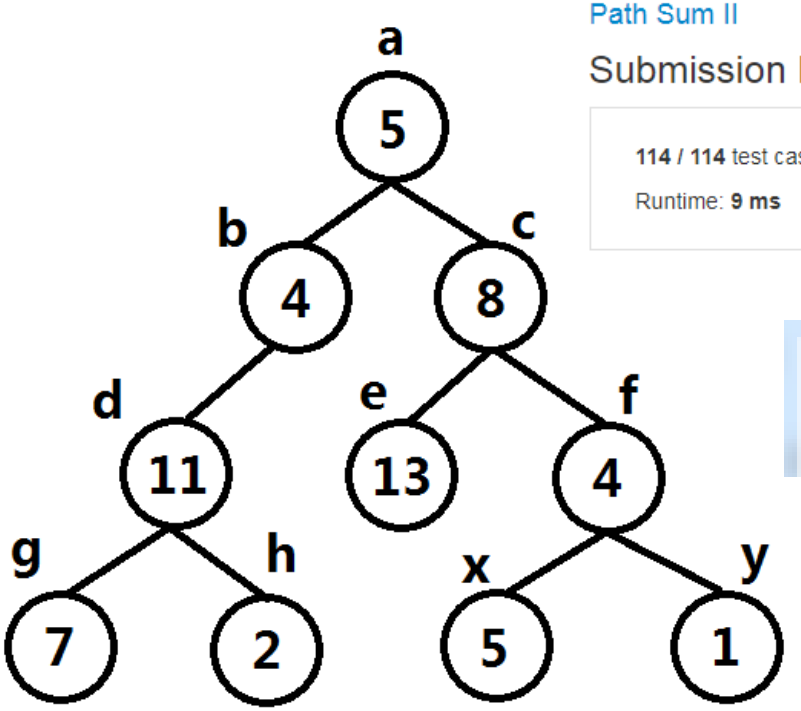
path_value = 22



叶结点

例1:测试与leetcode提交结果

```
int main(){
    TreeNode a(5);
    TreeNode b(4);
    TreeNode c(8);
    TreeNode d(11);
    TreeNode e(13);
    TreeNode f(4);
    TreeNode g(7);
    TreeNode h(2);
    TreeNode x(5);
    TreeNode y(1);
    a.left = &b;
    a.right = &c;
    b.left = &d;
    c.left = &e;
    c.right = &f;
    d.left = &g;
    d.right = &h;
    f.left = &x;
    f.right = &y;
    Solution solve;
    std::vector<std::vector<int> > result = solve.pathSum(&a, 22);
    for (int i = 0; i < result.size(); i++){
        for (int j = 0; j < result[i].size(); j++){
            printf("[%d]", result[i][j]);
        }
        printf("\n");
    }
    return 0;
}
```



Path Sum II

Submission Details

114 / 114 test cases passed. Status: Accepted

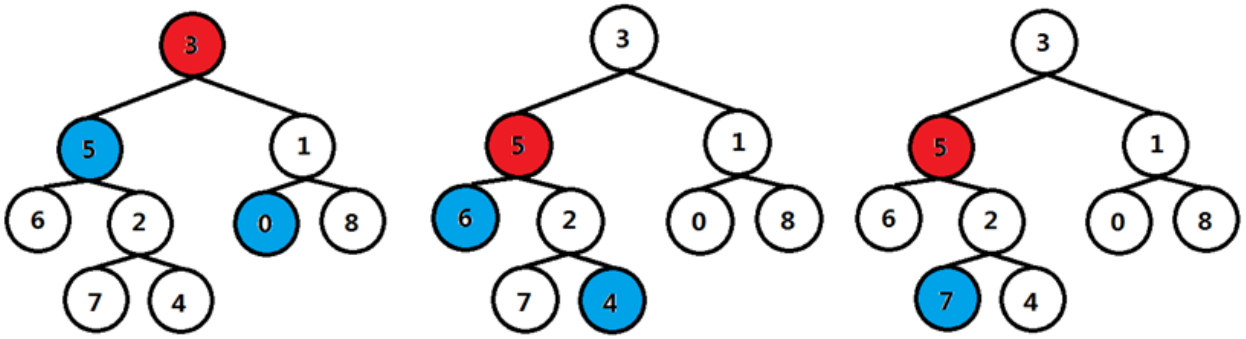
Runtime: 9 ms Submitted: 0 minutes ago

```
[5][4][11][2]
[5][8][4][5]
```

例2:最近的公共祖先

已知二叉树，求二叉树中给定的两个节点的**最近公共祖先**。

最近公共祖先: 两节点v与w的最近公共祖先u，满足在**树上最低(离根最远)**，且v,w两个节点都是u的**子孙**。




```
class Solution {
public:
    TreeNode* lowestCommonAncestor(TreeNode* root, TreeNode* p, TreeNode* q) {
        返回p,q最近公共祖先节点
    }
};
```

选自 **LeetCode 236. Lowest Common Ancestor of a Binary Tree**

https://leetcode.com/problems/lowest-common-ancestor-of-a-binary-tree/description/

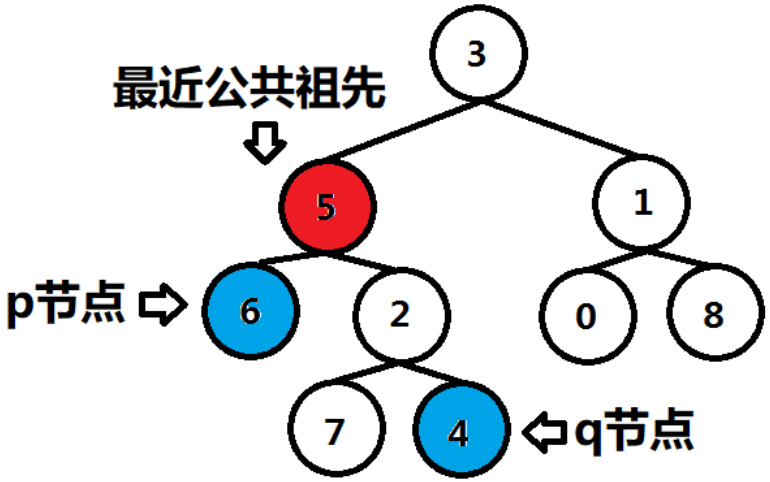
难度:**Medium**

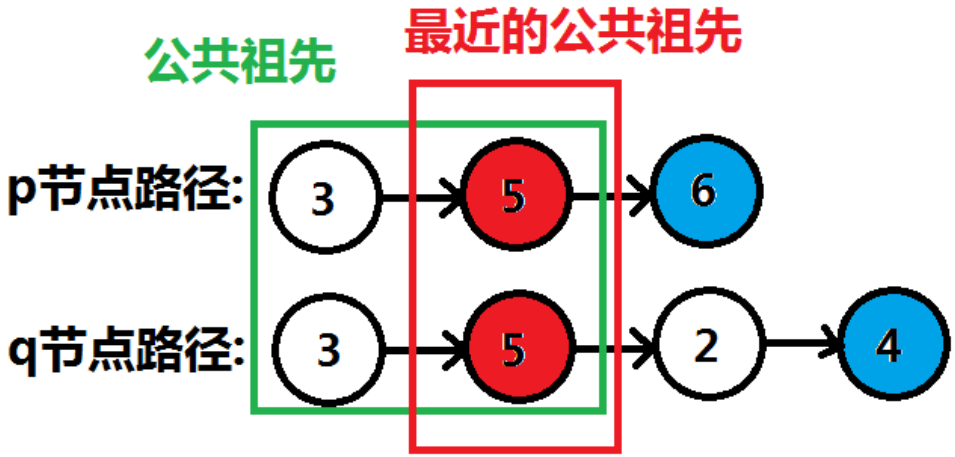
例2:思考与分析

1.两个节点的**公共祖先**一定在从根节点，至这两个节点的路径上。

2.由于求公共祖先中的**最近**公共祖先，那么即**同时出现**在这两条路径上的**离根节点最远**的节点(或离两个最近)。

3.最终算法即:求p节点路径，q节点路径，两路径上**最后一个相同**的节点。





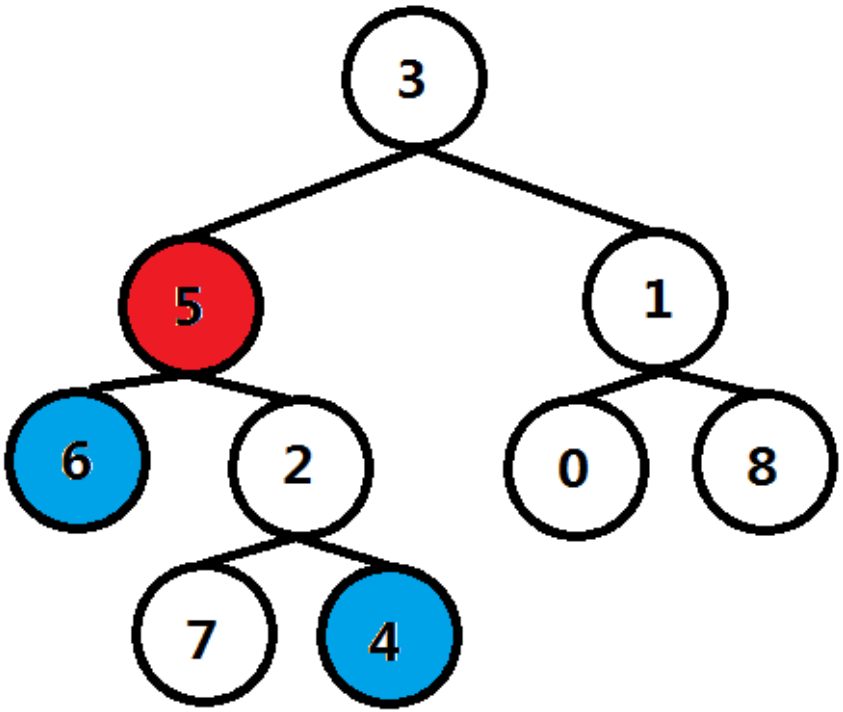
例2:求根节点至某节点路径(深度搜索)

1.从根节点**遍历(搜索)**至该节点，**找到**该节点后就**结束搜索**。

2.将遍历过程中遇到的节点**按照顺序**存储起来，这些节点即路径节点。

前序遍历(深度优先遍历)　　正在遍历的节点

```
void preorder(TreeNode *node,
              TreeNode *search){
                                  希望遍历到的节点
    if (!node){
        return;
    } //如果遍历至空指针(叶结点的孩子)，结束

    if (node == search){
      //找到了待搜索节点，做一些操作
    }                                          前序

    preorder(node->left, search);
                                               中序
    preorder(node->right, search);
                                               后续
}
```



例2:求根节点至某节点路径(栈存储路径)

记录节点p路径的过程:

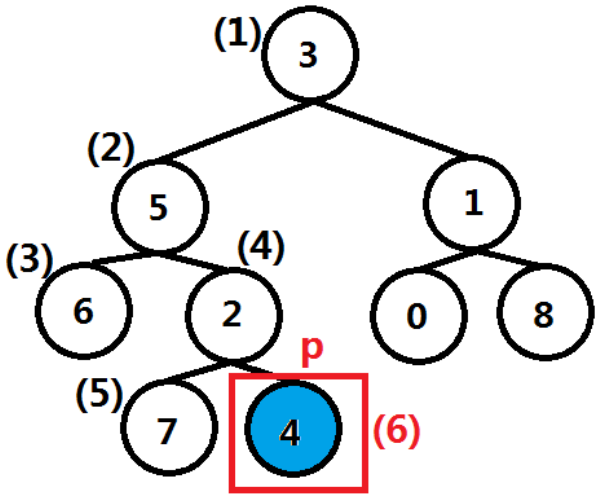
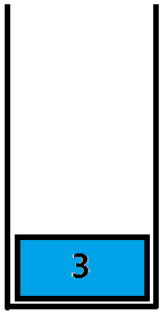
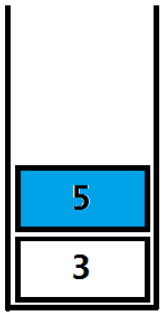


图1



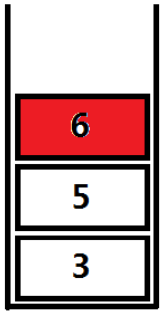
深度遍历节点(1)
前序

图2



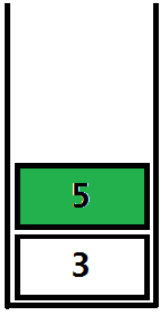
深度遍历节点(2)
前序

图3



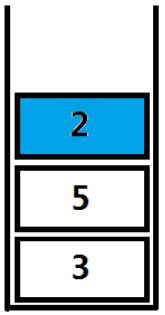
深度遍历节点(3)
前序 (后序)

图4



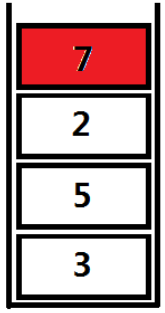
深度遍历节点(2)
中序

图5



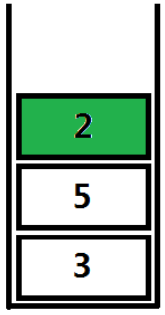
深度遍历节点(4)
前序

图6



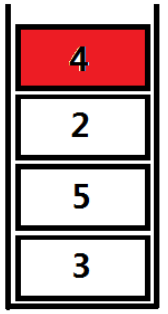
深度遍历节点(5)
前序 (后序)

图7



深度遍历节点 (4)
中序

图8



深度遍历节点(6)
前序 (后序)

例2:求根节点至某节点路径(实现，课堂练习)

```
void preorder(TreeNode* node, //正在遍历的节点
              TreeNode *search, //待搜索节点
              std::vector<TreeNode*> &path, //遍历时的节点路径栈
              std::vector<TreeNode*> &result, //最终搜索到节点search的路径结果
              int &finish){ //记录是否找到节点search的变量，未找到时是0，找到为1
    if (!node || 1 ){ //当node为空或已找到search节点直接返回，结束搜索
        return;
    }
    path.push_back(node); //先序遍历时，将节点压入path栈
    if (node == search){
        finish = 1;       //当找到search节点后，标记finish变量
        2
    }
    preorder(node->left, search, path, result, finish); //深度遍历node左孩子
    preorder(node->right, search, path, result, finish); //深度遍历node右孩子
    3
}
```

3分钟时间填写代码，

有问题随时提出！

例2:求根节点至某节点路径(实现)

```
void preorder(TreeNode* node, //正在遍历的节点
              TreeNode *search, //待搜索节点
              std::vector<TreeNode*> &path, //遍历时的节点路径栈
              std::vector<TreeNode*> &result, //最终搜索到节点search的路径结果
              int &finish){ //记录是否找到节点search的变量，未找到时是0，找到为1
    if (!node || finish){ //当node为空或已找到search节点直接返回，结束搜索
        return;
    }
    path.push_back(node); //先序遍历时，将节点压入path栈
    if (node == search){
        finish = 1;       //当找到search节点后，标记finish变量
        result=path       //将当前的path存储到result中
    }
    preorder(node->left, search, path, result, finish); //深度遍历node左孩子
    preorder(node->right, search, path, result, finish); //深度遍历node右孩子
    path.pop_back();  //结束遍历node时，将node节点弹出path栈
}
```

1

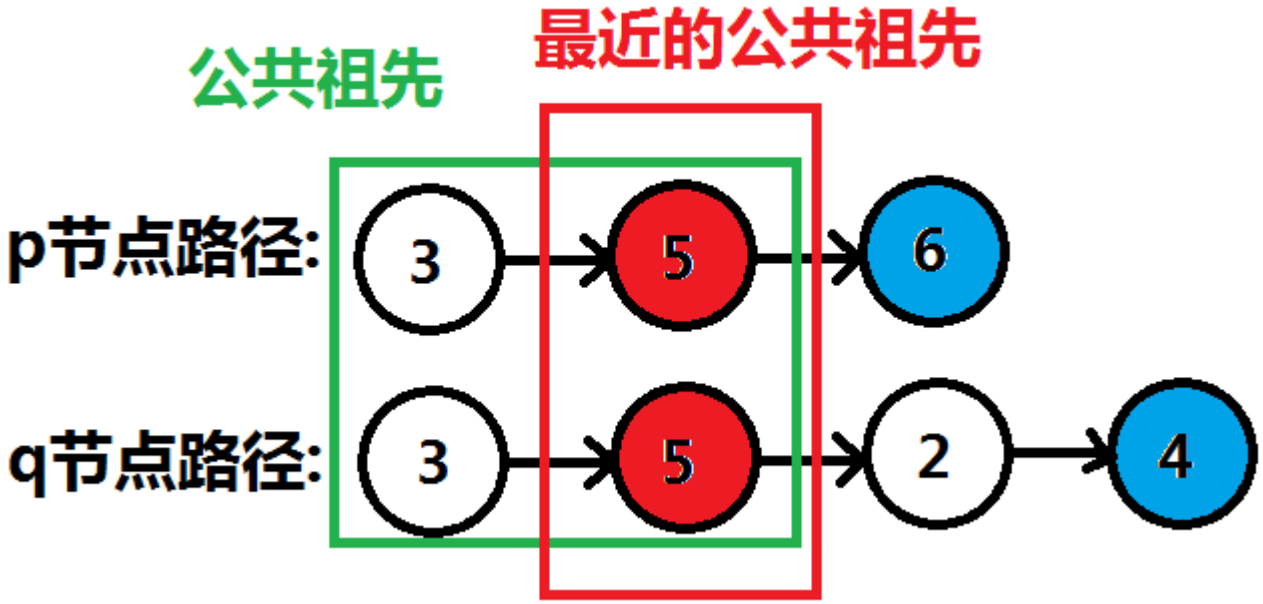
2

3

例2:求两路径上最后一个相同的节点

1.求出**较短**路径的**长度n**。

2.同时遍历p节点的路径与q节点的路径，遍历n个节点，**最后一个**发现的相同节点，即**最近公共祖先**。



例2:整体代码(实现，课堂练习)

```
class Solution {
public:
    TreeNode* lowestCommonAncestor(TreeNode* root, TreeNode* p, TreeNode* q) {
        std::vector<TreeNode*> path;  //声明遍历用的临时栈
        std::vector<TreeNode*> node_p_path; //存储p节点路径
        std::vector<TreeNode*> node_q_path; //存储q节点路径
        int finish = 0; //记录是否完成搜索的变量finish
        1
        path.clear();
        finish = 0;     //清空path、finish，计算q节点路径
        preorder(root, q, path, node_q_path, finish);
        int path_len = 0; //较短路径的长度
        if (2){
            path_len = node_p_path.size();
        }
        else{
            path_len = node_q_path.size();
        }
        TreeNode *result = 0; 同时遍历根到p,q两个节点的的路径上的节点
        for (int i = 0; i < path_len; i++){
            if (3){
                result = node_p_path[i];
            }
        }
        return result;
    }
};
```

3分钟时间填写代码，

有问题随时提出！

例2:整体代码(实现)

```
class Solution {
public:
    TreeNode* lowestCommonAncestor(TreeNode* root, TreeNode* p, TreeNode* q) {
        std::vector<TreeNode*> path;  //声明遍历用的临时栈
        std::vector<TreeNode*> node_p_path;  //存储p节点路径
        std::vector<TreeNode*> node_q_path;  //存储q节点路径
        int finish = 0; //记录是否完成搜索的变量finish
        preorder(root, p, path, node_p_path, finish);
        path.clear();
        finish = 0;     //清空path、finish，计算q节点路径
        preorder(root, q, path, node_q_path, finish);
        int path_len = 0; //较短路径的长度
        if ( node_p_path.size() < node_q_path.size() ){
            path_len = node_p_path.size();
        }
        else{
            path_len = node_q_path.size();
        }
        TreeNode *result = 0; 同时遍历根到p,q两个节点的的路径上的节点
        for (int i = 0; i < path_len; i++){
            if ( node_p_path[i] == node_q_path[i] ){
                result = node_p_path[i];  //找到了最近公共祖先
            }
        }
        return result;
    }
};
```

例2:测试与leetcode提交结果

```
int main(){
    TreeNode a(3);
    TreeNode b(5);
    TreeNode c(1);
    TreeNode d(6);
    TreeNode e(2);
    TreeNode f(0);
    TreeNode x(8);
    TreeNode y(7);
    TreeNode z(4);
    a.left = &b;
    a.right = &c;
    b.left = &d;
    b.right = &e;
    c.left = &f;
    c.right = &x;
    e.left = &y;
    e.right = &z;
    Solution solve;
    TreeNode *result = solve.lowestCommonAncestor(&a, &b, &f);
    printf("lowestCommonAncestor = %d\n", result->val);
    result = solve.lowestCommonAncestor(&a, &d, &z);
    printf("lowestCommonAncestor = %d\n", result->val);
    result = solve.lowestCommonAncestor(&a, &b, &y);
    printf("lowestCommonAncestor = %d\n", result->val);

    return 0;
}
```

构造样例中的树

```
lowestCommonAncestor = 3
lowestCommonAncestor = 5
lowestCommonAncestor = 5
请按任意键继续. . .
```

Lowest Common Ancestor of a Binary Tree

Submission Details

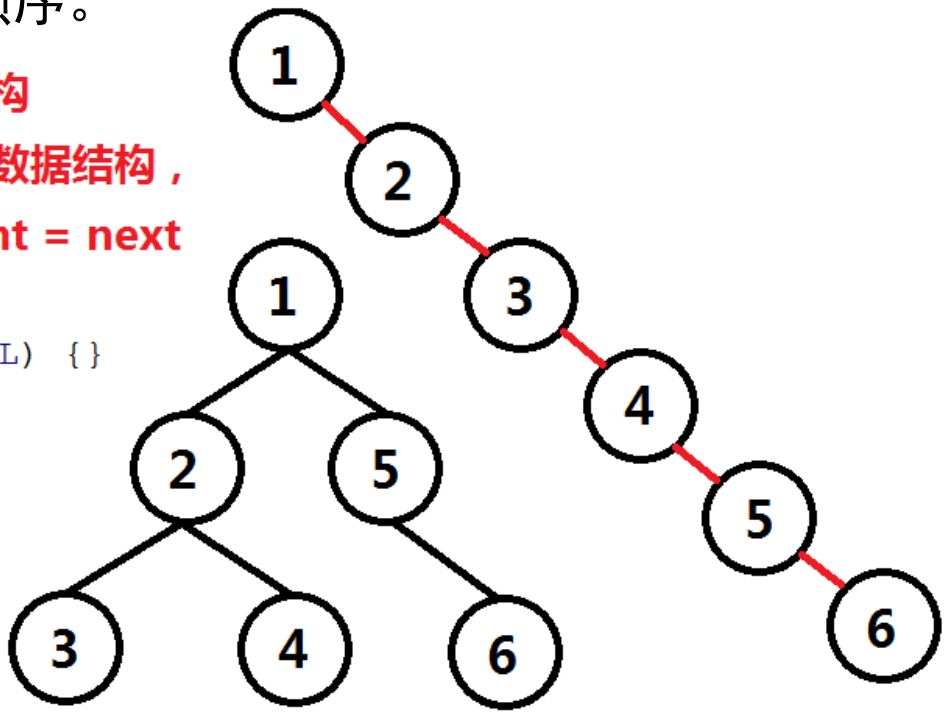
31 / 31 test cases passed. Status: Accepted

Runtime: 19 ms Submitted: 1 minute ago

例3:二叉树转链表

给定一个**二叉树**，将该二叉树**就地(in-place)**转换为**单链表**。单链表中节点顺序为二叉树**前序遍历**顺序。

```
                                    //树节点的数据结构
struct TreeNode {                   //单链表仍使用该数据结构，
    int val;                        即left=NULL,right = next
    TreeNode *left;
    TreeNode *right;
    TreeNode(int x) :
        val(x), left(NULL), right(NULL) {}
};
class Solution {
public:
    void flatten(TreeNode *root)
    }
};
```



选自 **LeetCode 114. Flatten Binary Tree to Linked List**

https://leetcode.com/problems/flatten-binary-tree-to-linked-list/description/

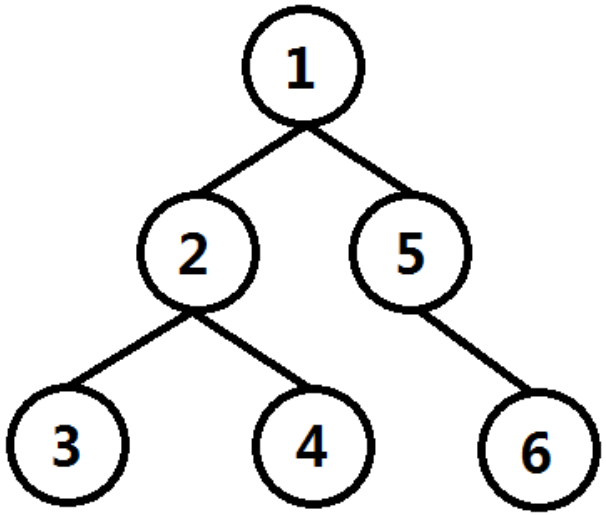
难度:**Medium**

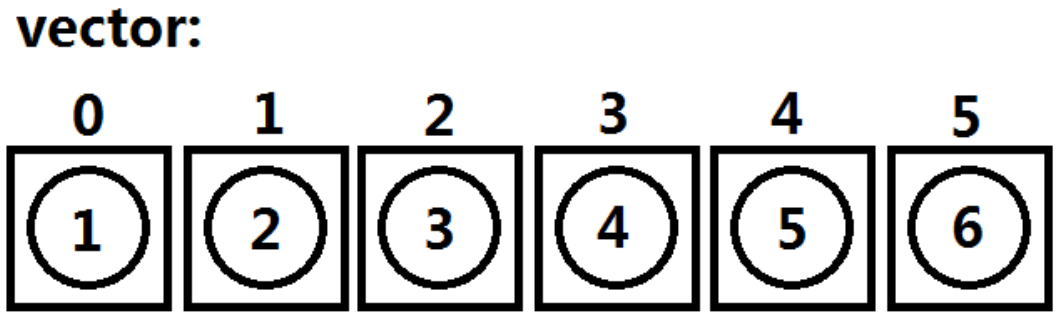
例3:思考

前序遍历二叉树，将节点指针push进入vector，顺序**遍历**vector中的节点，链接相邻两节点，形成链单链表。**(投机取巧)**

该方法虽然可通过题目，但不满足**就地(in-place)转换**的条件。

若**就地(in-place)转换**应该如何做？





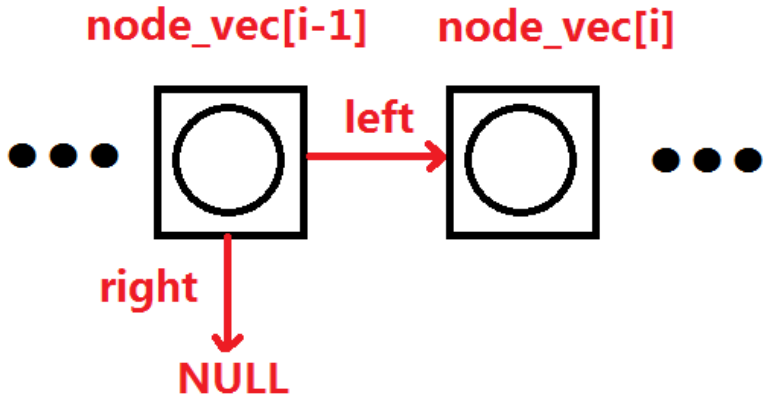
按顺序遍历vector，将前面的节点左指针置空右指针与后面的节点相连。

例3:方法1课堂练习

```
#include <vector>
class Solution {
public:
    void flatten(TreeNode *root) {
        std::vector<TreeNode *> node_vec;
        preorder(root, node_vec);
        for (int i = 1; i < node_vec.size(); i++){
            1
            2
        }
    }
private:
    void preorder(TreeNode *node, std::vector<TreeNode *> &node_vec){
        if (!node){
            return;
        }
            3
        preorder(node->left, node_vec);
        preorder(node->right, node_vec);
    }
};
```

3分钟填写代码，
有问题随时提出！

例3:方法1实现

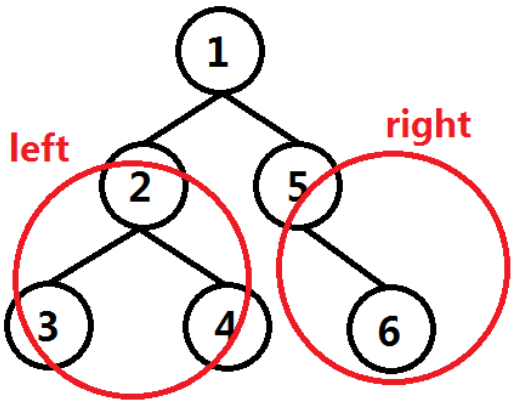


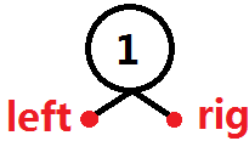

```
#include <vector>
class Solution {
public:
    void flatten(TreeNode *root) {
        std::vector<TreeNode *> node_vec;
        preorder(root, node_vec);
        for (int i = 1; i < node_vec.size(); i++){
            node_vec[i-1]->left = NULL;
            node_vec[i-1]->right = node_vec[i];
        }
    }
private:
    void preorder(TreeNode *node, std::vector<TreeNode *> &node_vec){
        if (!node){
            return;
        }
        node_vec.push_back(node);
        preorder(node->left, node_vec);
        preorder(node->right, node_vec);
    }
};
```

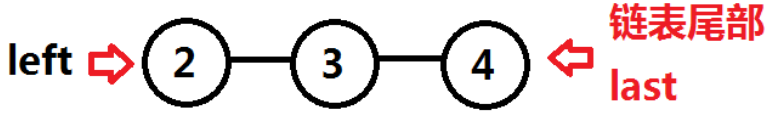
例3:算法思路(方法2整体)

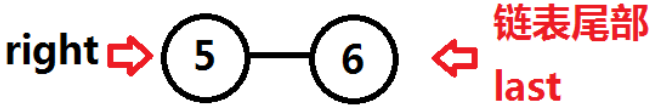
```
//将以node为根的树转为链表(拉直)，并传出必要信息
void preorder(TreeNode *node,  ●●●          ){
    if (!node){
        return;
    }
    if (  页节点                          ){
        do something
    }
    前序，访问node时
    do something
    if (有左子树){          必要信息
        preorder(left,  ●●●   );
        中序，完成左孩子的访问
    }
    if (有右子树){          必要信息
        preorder(right,  ●●●   );
        后序，完成右孩子的访问
    }
}
```

必要信息



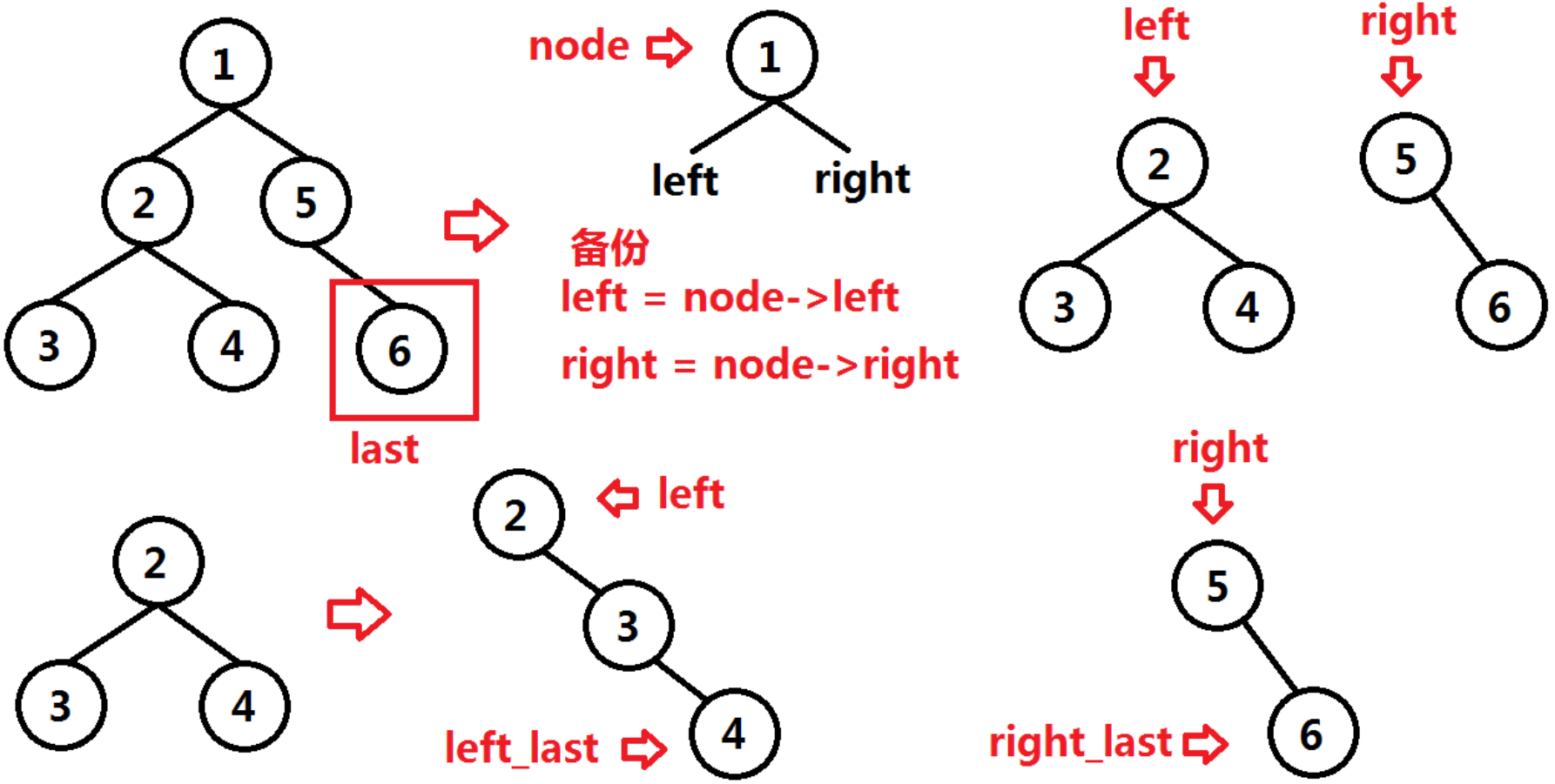




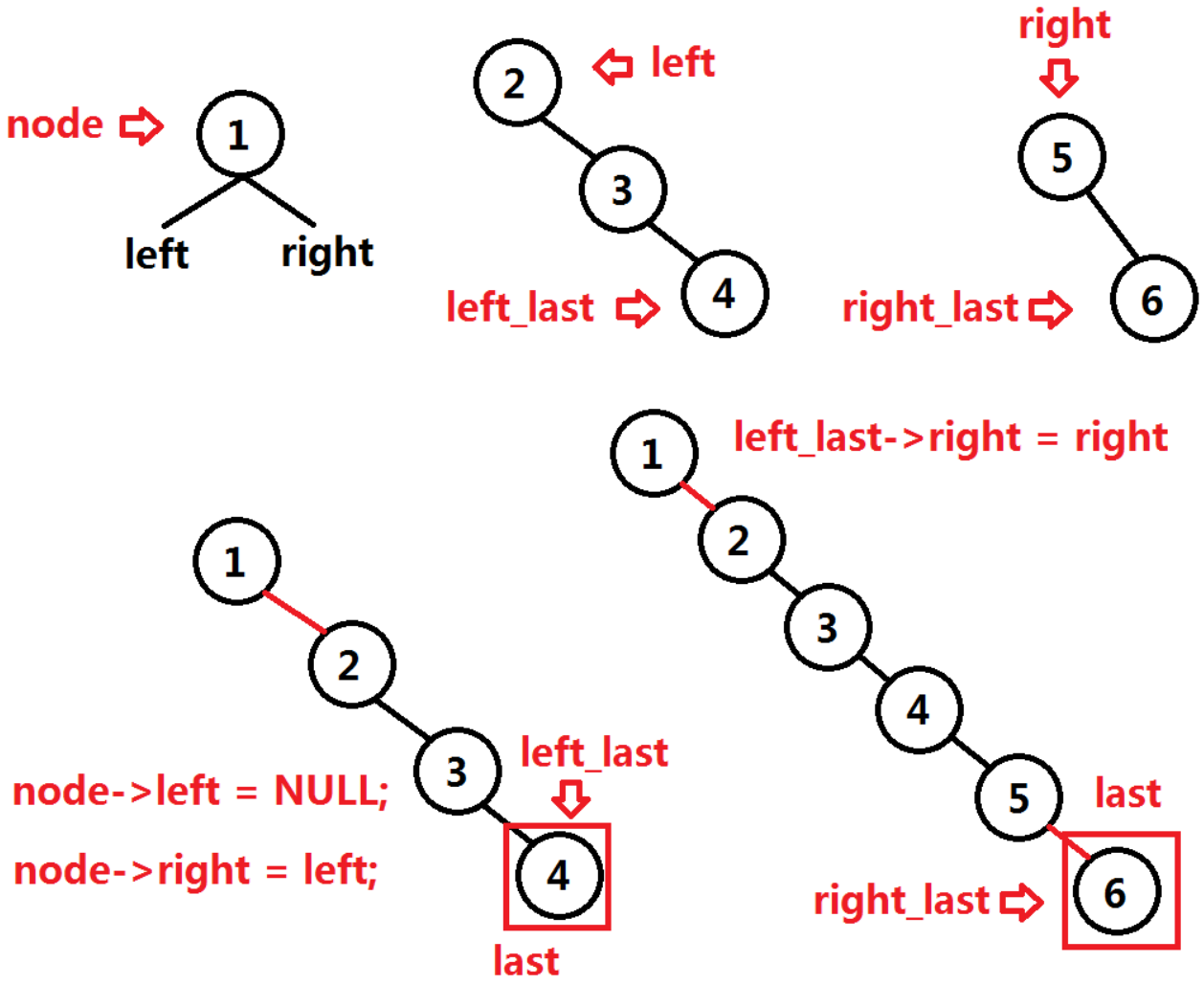


必要信息 ●●● 即为链表尾部last指针

例3:算法思路(拆解并解决子问题)



例3:算法思路(解决当前问题)



例3:方法2课堂练习

```
class Solution {
public:
    void flatten(TreeNode *root) {
        TreeNode *last = NULL;
        preorder(root, last);
    }
private:
    void preorder(TreeNode *node, TreeNode *&last){
        if (!node){
            return;
        }
        if (          1          ){
                      2
            return;
        }
        TreeNode *left = node->left;
        TreeNode *right = node->right;
        TreeNode *left_last = NULL;
        TreeNode *right_last = NULL;
        if (left){
            preorder(left, left_last);
            node->left = NULL;
                      3
            last = left_last;
        }
        if (right){
            preorder(right, right_last);
            if (left_last){
                      4
            }
                      5
        }
    }
};
```

//当前子树的先序遍历的最后一个节点，传引用会传出

// 当前的节点

//备份左右指针

//左右子树最后一个节点

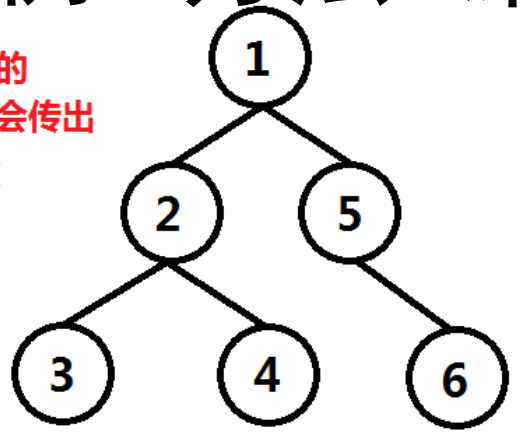
//若有左子树，递归将左子树转换单链表

//左指针赋空

//将该节点的last保存为左子树的last

//若有右子树，递归将右子树转换单链表

//若node找到左子树最后一个节点(有左子树)



5分钟填写代码，

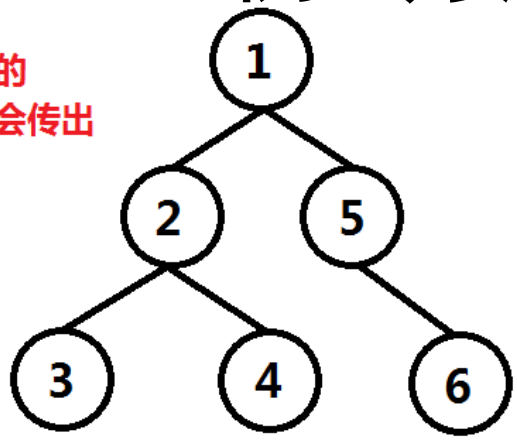
有问题随时提出！

例3:方法2实现

```
class Solution {
public:
    void flatten(TreeNode *root) {
        TreeNode *last = NULL;
        preorder(root, last);
    }
private:
    void preorder(TreeNode *node, TreeNode *&last){
        if (!node){
            return;
        }
        if ( !node->left && !node->right ){
            last = node;
            return;
        }
        TreeNode *left = node->left;
        TreeNode *right = node->right;
        TreeNode *left_last = NULL;
        TreeNode *right_last = NULL;
        if (left){
            preorder(left, left_last);
            node->left = NULL;
            node->right = left;
            last = left_last;
        }
        if (right){
            preorder(right, right_last);
            if (left_last){
                left_last->right = right;
            }
            last = right_last;
        }
    }
};
```

//当前子树的先序遍历的最后一个节点，传引用会传出

// 当前的节点



//备份左右指针

//左右子树最后一个节点

//若有左子树，递归将左子树转换单链表

//左指针赋空

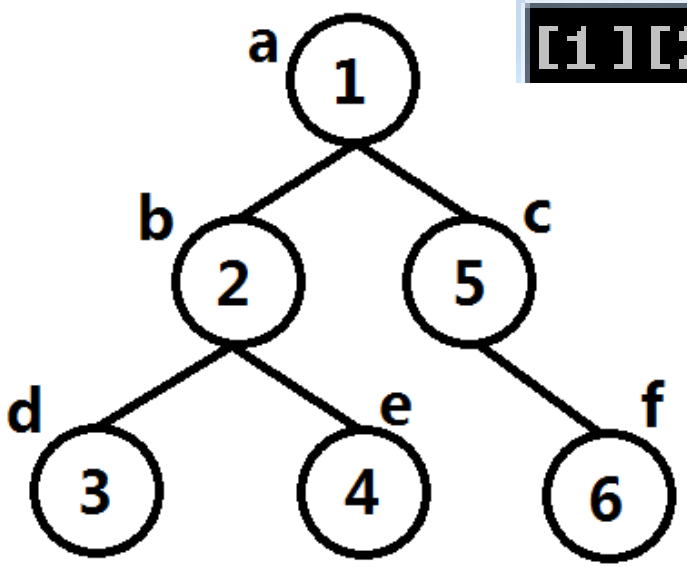
//将该节点的last保存为左子树的last

//若有右子树，递归将右子树转换单链表

//若node找到左子树最后一个节点(有左子树)

例3:测试与leetcode提交结果

```
int main(){
    TreeNode a(1);
    TreeNode b(2);
    TreeNode c(5);
    TreeNode d(3);
    TreeNode e(4);
    TreeNode f(6);
    a.left = &b;
    a.right = &c;
    b.left = &d;
    b.right = &e;
    c.right = &f;
    Solution solve;
    solve.flatten(&a);
    TreeNode *head = &a;
    while(head){
        if (head->left){
            printf("ERROR\n");
        }
        printf("[%d]", head->val);
        head = head->right;
    }
    printf("\n");
    return 0;
}
```




```
[1][2][3][4][5][6]
```

Flatten Binary Tree to Linked List

Submission Details

225 / 225 test cases passed. Status: Accepted

Runtime: **6 ms** Submitted: **3 hours, 4 minutes ago**

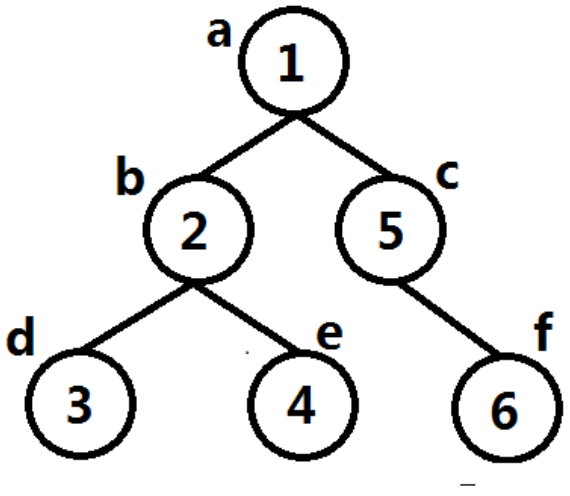
课间休息10分钟

有问题提出！

预备知识:二叉树层次遍历

二叉树**层次遍历**，又称为**宽度优先搜索**，按树的层次依次访问树的结点。层次遍历使用**队列**对遍历节点进行**存储**，先进入队列的结点，**优先遍历**拓展其左孩子与右孩子。

```
设置队列 Q
将根节点 push 进Q
while(Q不空){
    取出队列头部节点node
    对node访问
    将node的左、右孩子push进队列
}
```

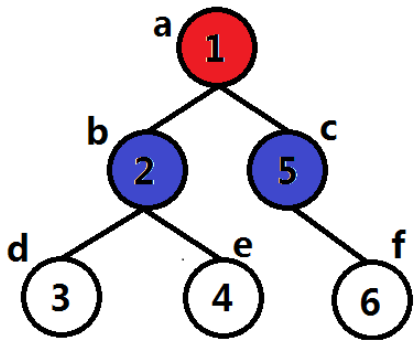


层次遍历:

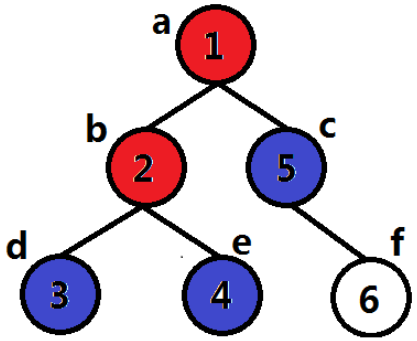
a(1), b(2), c(5), d(3), e(4), f(6)

预备知识:二叉树层次遍历

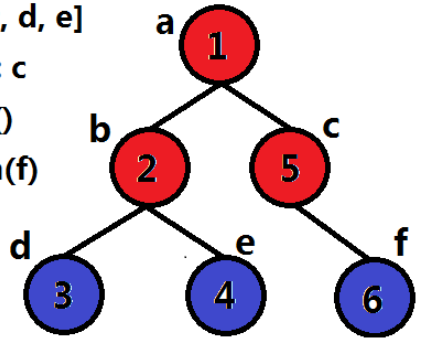
1. Q.push(a)
   Q = [a]
   search: a
   Q.pop()
   Q.push(b)
   Q.push(c)



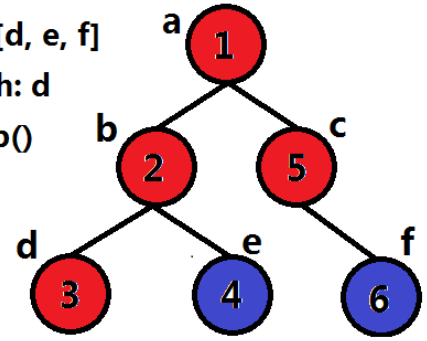
2. Q = [b, c]
   search: b
   Q.pop()
   Q.push(d)
   Q.push(e)



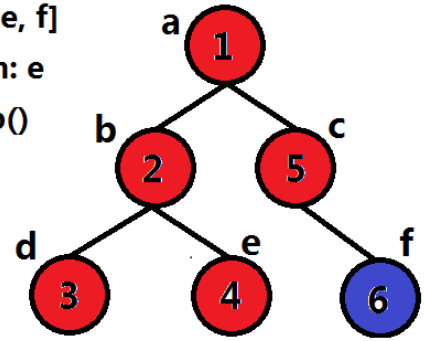
3. Q = [c, d, e]
   search: c
   Q.pop()
   Q.push(f)



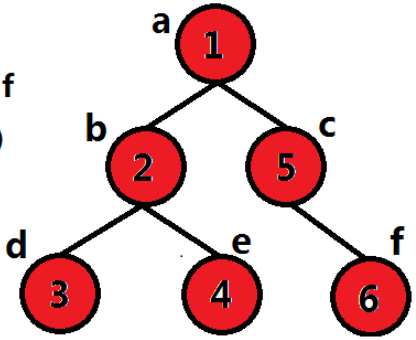
4. Q = [d, e, f]
   search: d
   Q.pop()



5. Q = [e, f]
   search: e
   Q.pop()

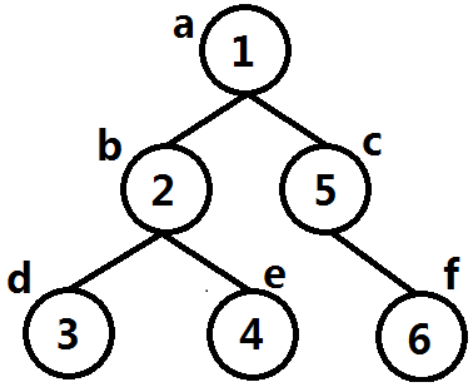


6. Q = [f]
   search: f
   Q.pop()
7. Q = []



预备知识:二叉树层次遍历，课堂练习

```
#include <stdio.h>
#include <vector>
#include <queue>

struct TreeNode {    //二叉树数据结构
    int val;
    TreeNode *left;
    TreeNode *right;
    TreeNode(int x) : val(x), left(NULL), right(NULL) {}
};
void BFS_print(TreeNode* root){ //宽度优先搜索二叉树
    std::queue<TreeNode *> Q;
    Q.push(root);
    while(!Q.empty()){
        TreeNode *node = Q.front();
        Q.pop();
        printf("[%d]\n", node->val);
        if (node->left){
            Q.push(node->left);
        }
        if (node->right){
            Q.push(node->right);
        }
    }
}
```



层次遍历:

a(1), b(2), c(5), d(3), e(4), f(6)

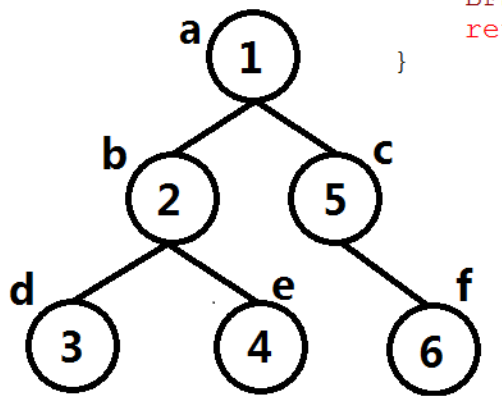
```
int main(){
    TreeNode a(1);
    TreeNode b(2);
    TreeNode c(5);
    TreeNode d(3);
    TreeNode e(4);
    TreeNode f(6);
    a.left = &b;
    a.right = &c;
    b.left = &d;
    b.right = &e;
    c.right = &f;
    BFS_print(&a);
    return 0;
}
```

```
[1]
[2]
[5]
[3]
[4]
[6]
```

预备知识:二叉树层次遍历，实现

```
#include <stdio.h>
#include <vector>
#include <queue>

struct TreeNode {    //二叉树数据结构
    int val;
    TreeNode *left;
    TreeNode *right;
    TreeNode(int x) : val(x), left(NULL), right(NULL) {}
};
void BFS_print(TreeNode* root){ //宽度优先搜索二叉树
    std::queue<TreeNode *> Q;
    Q.push(root);
    while(!Q.empty()){
        TreeNode *node = Q.front();
        Q.pop();
        printf("[%d]\n", node->val);
        if (node->left){
            Q.push(node->left);
        }
        if (node->right){
            Q.push(node->right);
        }
    }
}
```

```
int main(){
    TreeNode a(1);
    TreeNode b(2);
    TreeNode c(5);
    TreeNode d(3);
    TreeNode e(4);
    TreeNode f(6);
    a.left = &b;
    a.right = &c;
    b.left = &d;
    b.right = &e;
    c.right = &f;
    BFS_print(&a);
    return 0;
}
```



层次遍历:

a(1), b(2), c(5), d(3), e(4), f(6)

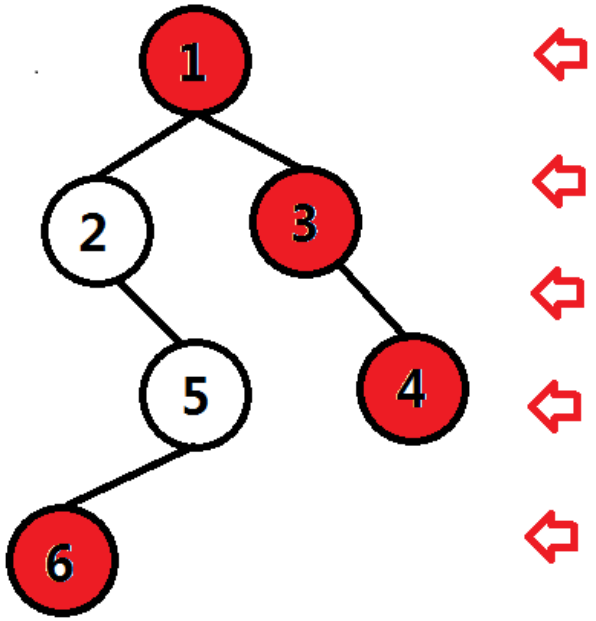
```
[1]
[2]
[5]
[3]
[4]
[6]
```

例4:侧面观察二叉树

给定一个**二叉树**，假设从该二叉树的**右侧**观察它，将观察到的节点按照**从上到下**的顺序输出。

```
struct TreeNode {
    int val;
    TreeNode *left;
    TreeNode *right;
    TreeNode(int x) :
        val(x), left(NULL), right(NULL) {}
};
class Solution {
public:
    std::vector<int> rightSideView(TreeNode* root) {
    }
};
```

[1, 3, 4, 6]



选自 **LeetCode 199. Binary Tree Right Side View**

https://leetcode.com/problems/binary-tree-right-side-view/description/

难度:**Medium**

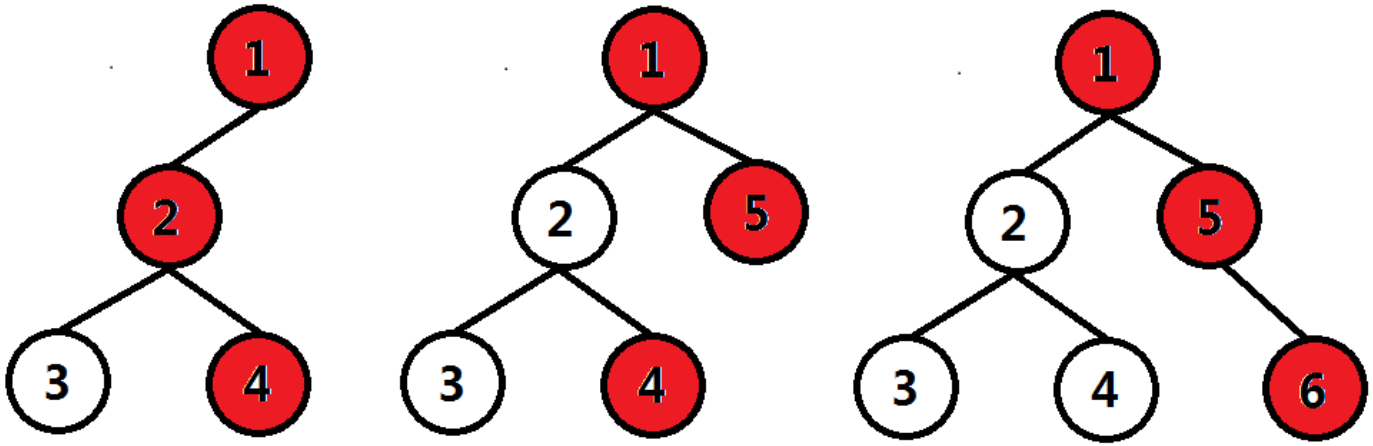
例4:思考与分析

从二叉树的**右侧**观察它，将观察到的节点按照**从上到下**的顺序输出，就是求**层次遍历二叉树**，每个层中的**最后一个节点**。

第0层

第1层

第2层



[1,2,4] [1,5,4] [1,5,6]

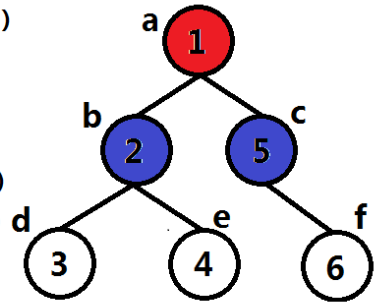
思考:层次遍历时，如何记录每一层中出现的**最后一个节点**?

例4:算法思路

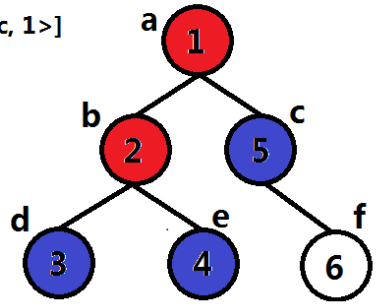
层次遍历时，将**节点与层数**绑定为pair，压入队列时，将节点与层数**同时**压入队列，并记录每一层中出现的**最后一个节点**。

在**层次遍历**中，每一层中的**最后一个节点最后遍历到**，**随时更新**对每层的最后一个节点即可。

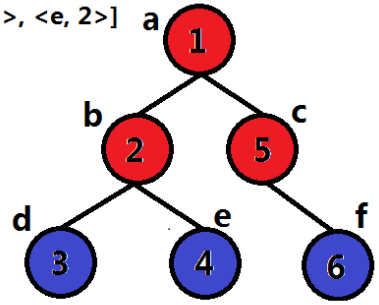
1. Q.push(<a, 0>)
   Q = [<a, 0>]
   search: a
   Q.pop()
   Q.push(<b, 1>)
   Q.push(<c, 1>)
   view[0] = 1(a)



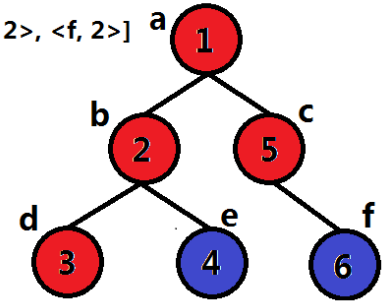
2. Q = [<b, 1>, <c, 1>]
   search: b
   Q.pop()
   Q.push(<d, 2>)
   Q.push(<e, 2>)
   view[1] = 2(b)



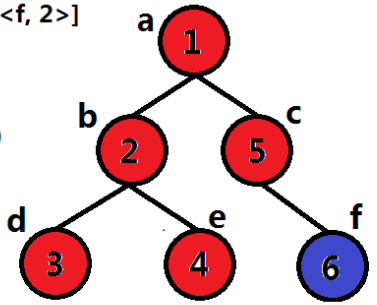
3. Q = [<c, 1>, <d, 2>, <e, 2>]
   search: c
   Q.pop()
   Q.push(<f, 2>)
   view[1] = 5(c)



4. Q = [<d, 2>, <e, 2>, <f, 2>]
   search: d
   Q.pop()
   view[2] = 3(d)

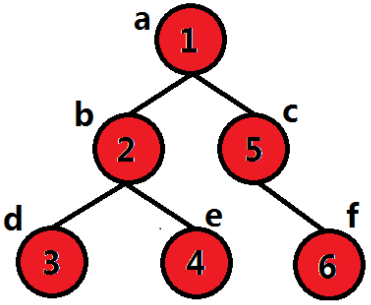


5. Q = [<e, 2>, <f, 2>]
   search: e
   Q.pop()
   view[2] = 4(e)



6. Q = [<f, 2>]
   search: f
   Q.pop()
   view[2] = 6(f)

7. Q = []



例4:课堂练习

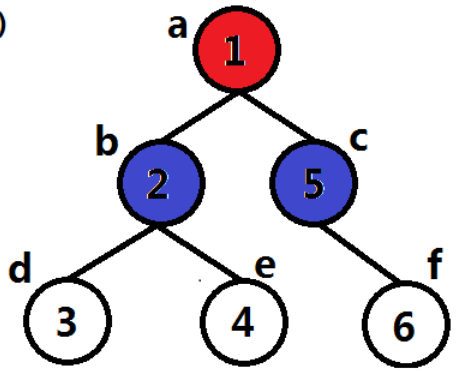
```
class Solution {
public:
    std::vector<int> rightSideView(TreeNode* root) {
        std::vector<int> view; //按层遍历的最后一个节点
        std::queue<std::pair<TreeNode *, int> > Q;
                        //宽度优先搜索队列<节点，层数>
        if (root){
            Q.push(std::make_pair(root, 0));
        }       //根节点非空时，将<root, 0> push进入队列
        while(!Q.empty()){
            TreeNode *node = Q.front().first; //搜索节点
            int depth = Q.front().second;  //待搜索节点的层数
            Q.pop();
            if ( 1 ){
                view.push_back(node->val);
            }
            else{
                2
            }
            if ( 3 ){
                Q.push(std::make_pair(node->left, 4 ));
            }
            if (node->right){
                Q.push(std::make_pair(node->right, 5 ));
            }
        }
        return view;
    }
};
```

5分钟填写代码，
有问题随时提出！

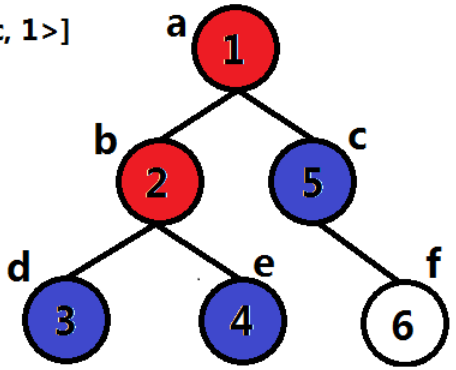
例4:实现

```
class Solution {
public:
    std::vector<int> rightSideView(TreeNode* root) {
        std::vector<int> view; //按层遍历的最后一个节点
        std::queue<std::pair<TreeNode *, int> > Q;
                              //宽度优先搜索队列<节点，层数>
        if (root){
            Q.push(std::make_pair(root, 0));
        }       //根节点非空时，将<root, 0> push进入队列

        while(!Q.empty()){
            TreeNode *node = Q.front().first; //搜索节点
            int depth = Q.front().second;  //待搜索节点的层数
            Q.pop();
            if ( view.size() == depth ){
                view.push_back(node->val);
            }
            else{
                view[depth] = node->val;
            }
            if ( node->left ){
                Q.push(std::make_pair(node->left, depth + 1));
            }
            if (node->right){
                Q.push(std::make_pair(node->right, depth + 1));
            }
        }
        return view;
    }
};
```

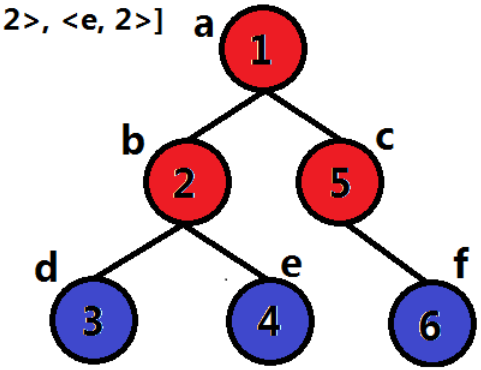
1. Q.push(<a, 0>)
   Q = [<a, 0>]
   search: a
   Q.pop()
   Q.push(<b, 1>)
   Q.push(<c, 1>)
   view[0] = 1(a)



2. Q = [<b, 1>, <c, 1>]
   search: b
   Q.pop()
   Q.push(<d, 2>)
   Q.push(<e, 2>)
   view[1] = 2(b)

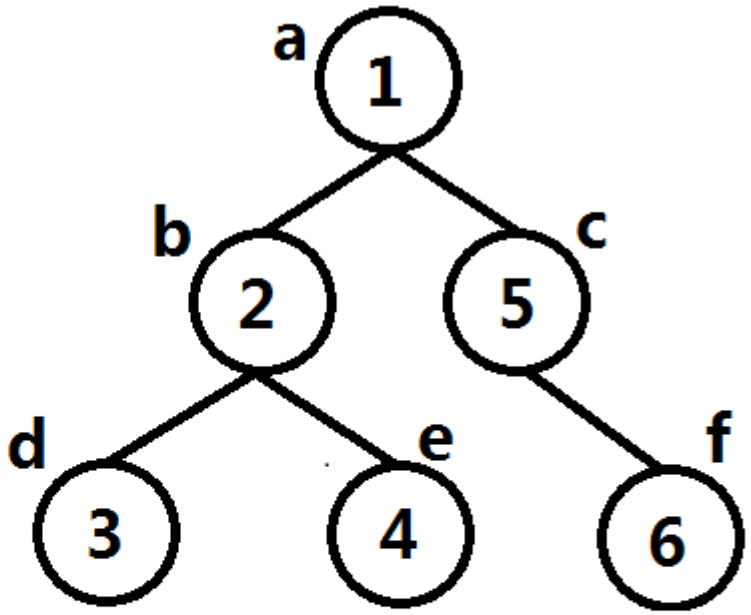


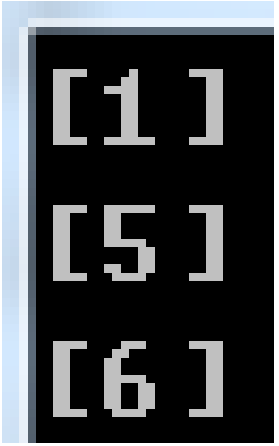
3. Q = [<c, 1>, <d, 2>, <e, 2>]
   search: c
   Q.pop()
   Q.push(<f, 2>)
   view[1] = 5(c)



例4:测试与leetcode提交结果

```
int main(){
    TreeNode a(1);
    TreeNode b(2);
    TreeNode c(5);
    TreeNode d(3);
    TreeNode e(4);
    TreeNode f(6);
    a.left = &b;
    a.right = &c;
    b.left = &d;
    b.right = &e;
    c.right = &f;
    Solution solve;
    std::vector<int> result = solve.rightSideView(&a);
    for (int i = 0; i < result.size(); i++){
        printf("[%d]\n", result[i]);
    }
    return 0;
}
```





Binary Tree Right Side View

Submission Details

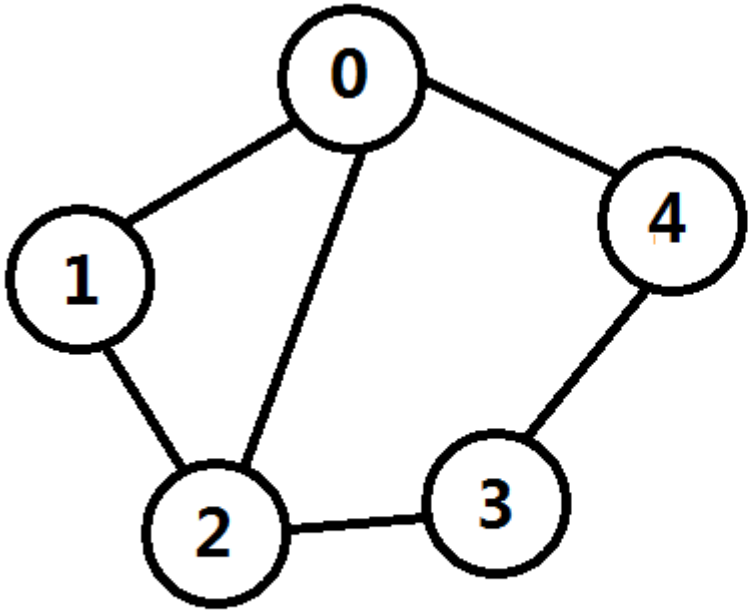
210 / 210 test cases passed. Status: Accepted

Runtime: **3 ms** Submitted: **0 minutes ago**

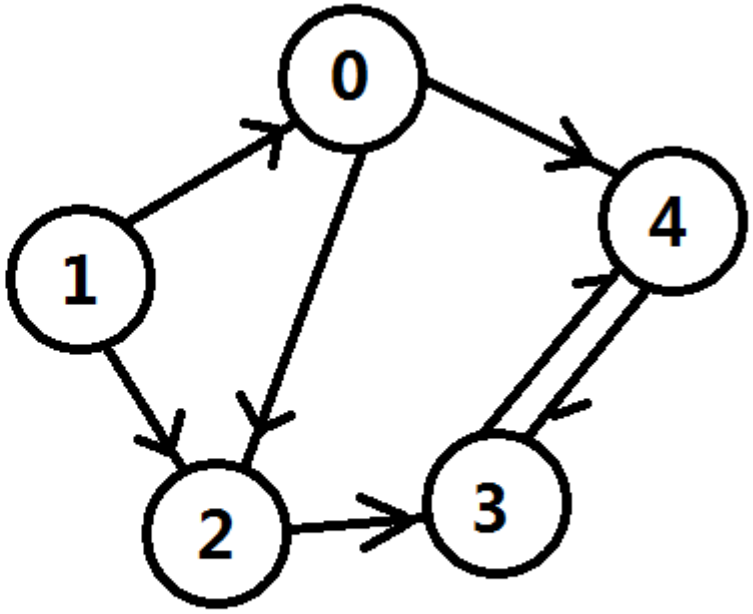
预备知识:图的定义

图（Graph）是由**顶点**的有穷非空集合和**顶点之间边**的集合组成，通常表示为：G(V, E)，其中，G表示一个图，V是图G中**顶点**的集合，E是图G中**边**的集合。

图分**无向图**与**有向图**，根据图的边长，又分**带权图**与**不带权图**。



无向图

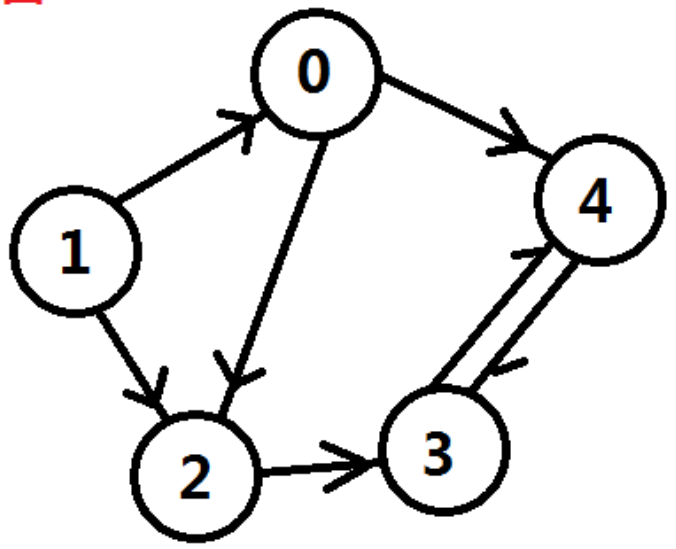


有向图

预备知识:图的构造与表示(临接矩阵)

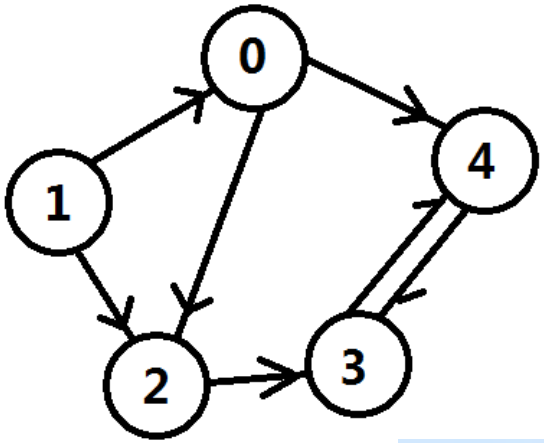
```
#include <stdio.h>

int main(){
    const int MAX_N = 5; //一共5个顶点
    int Graph[MAX_N][MAX_N] = {0}; //使用邻接矩阵表示
    Graph[0][2] = 1;
    Graph[0][4] = 1;  //将图连通，且不带权
    Graph[1][0] = 1;
    Graph[1][2] = 1;  //一般用临接矩阵表示稠密图
    Graph[2][3] = 1;
    Graph[3][4] = 1;
    Graph[4][3] = 1;
    printf("Graph:\n");
    for (int i = 0; i < MAX_N; i++){
        for (int j = 0; j < MAX_N; j++){
            printf("%d ", Graph[i][j]);
        }
        printf("\n");
    }
    return 0;
}
```

```
Graph:
0 0 1 0 1
1 0 1 0 0
0 0 0 1 0
0 0 0 0 1
0 0 0 1 0
```



预备知识:图的构造与表示(临接表)

```
#include <stdio.h>
#include <vector>

//图的邻接表数据结构
struct GraphNode{
    int label;                          //图的顶点的值
    std::vector<GraphNode *> neighbors; //相邻节点指针数组
    GraphNode(int x) : label(x) {};
};

int main(){
    const int MAX_N = 5;
    GraphNode *Graph[MAX_N]; //5个顶点

    for (int i = 0; i < MAX_N; i++){
        Graph[i] = new GraphNode(i);
    }
    //添加边
    Graph[0]->neighbors.push_back(Graph[2]);
    Graph[0]->neighbors.push_back(Graph[4]);
    Graph[1]->neighbors.push_back(Graph[0]);
    Graph[1]->neighbors.push_back(Graph[2]);
    Graph[2]->neighbors.push_back(Graph[3]);
    Graph[3]->neighbors.push_back(Graph[4]);
    Graph[4]->neighbors.push_back(Graph[3]);

    printf("Graph:\n");
    for (int i = 0; i < MAX_N; i++){
        printf("Label(%d) : ", i);
        for (int j = 0; j < Graph[i]->neighbors.size(); j++){
            printf("%d ", Graph[i]->neighbors[j]->label);
        }
        printf("\n");
    }
    for (int i = 0; i < MAX_N; i++){
        delete Graph[i];
    }

    return 0;
}
```

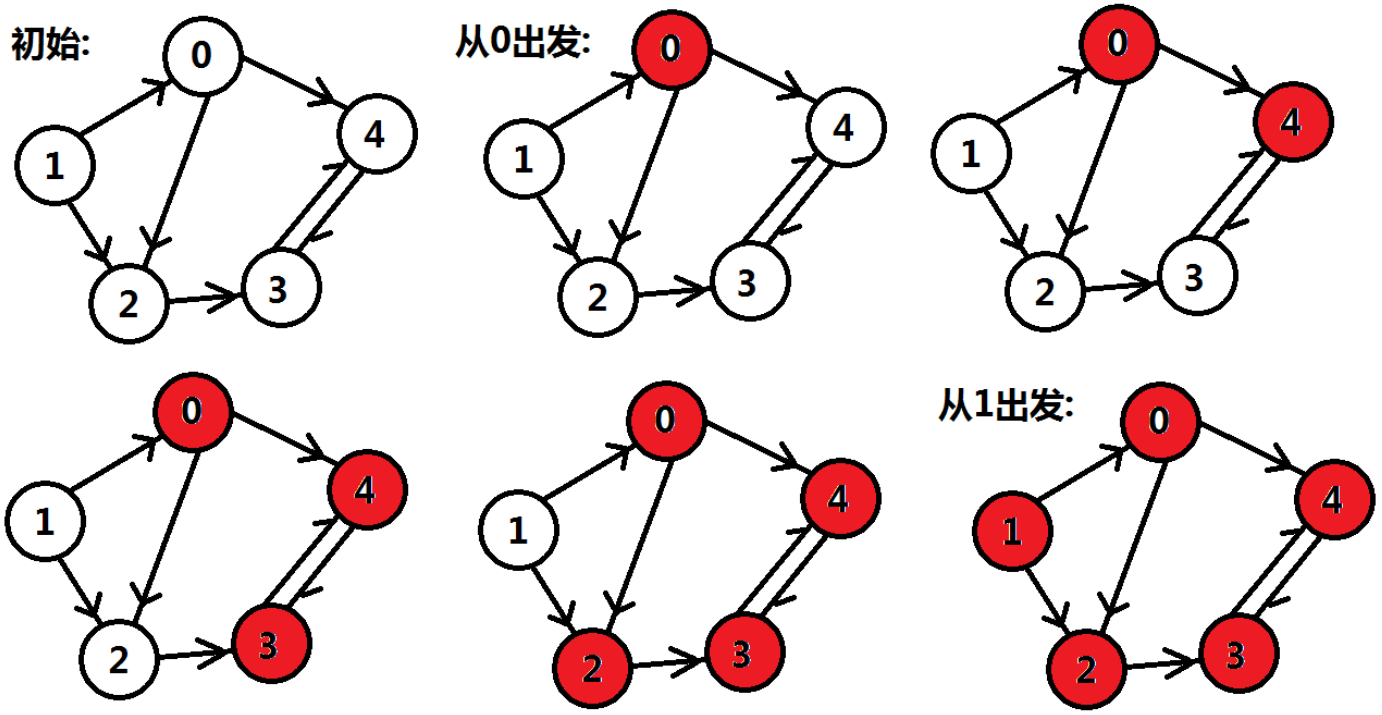


有向图

```
Graph:
Label(0) : 2 4
Label(1) : 0 2
Label(2) : 3
Label(3) : 4
Label(4) : 3
```

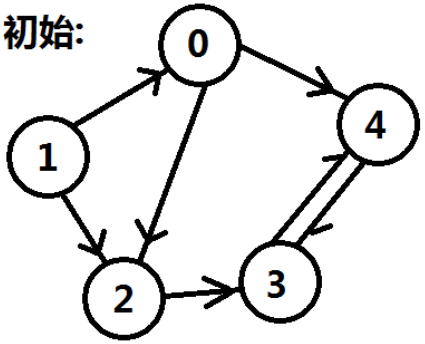
预备知识:图的深度优先遍历

从图中**某个顶点v**出发，首先访问该顶点，然后**依次**从它的各个未被访问的**邻接点**出发**深度优先搜索**遍历图，直至图中所有和v有**路径相通且未被访问**的顶点都被访问到。 若此时尚有其他顶点**未被访问**到，则另选一个未被访问的顶点作**起始点**，**重复**上述过程，直至图中**所有顶点**都被访问到为止。

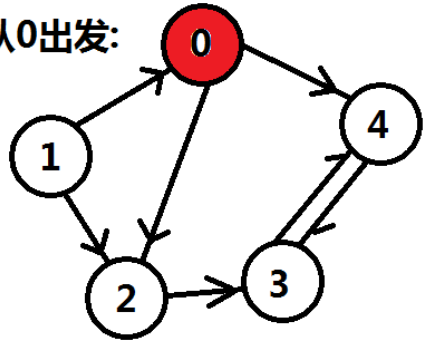


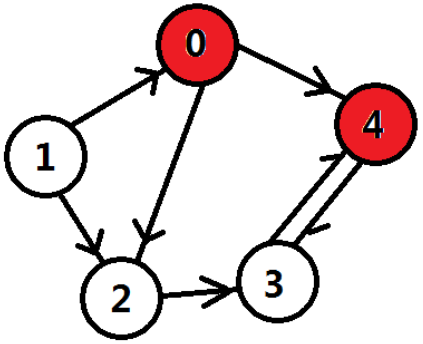
```
#include <stdio.h>
#include <vector>   //图的邻接表数据结构

struct GraphNode{   //图的顶点的值        //相邻节点指针数组
    int label;
    std::vector<GraphNode *> neighbors;
    GraphNode(int x) : label(x) {};
};
void DFS_graph(GraphNode *node, int visit[]){
    1
    printf("%d ", node->label);
    for (int i = 0; i < node->neighbors.size(); i++){
        if (  2  ){
            DFS_graph(node->neighbors[i], visit);
        }
    }
}
int main(){
    const int MAX_N = 5;      //创建图的顶点
    GraphNode *Graph[MAX_N];
    for (int i = 0; i < MAX_N; i++){
        Graph[i] = new GraphNode(i);
    }
                //添加图的边，注意添加边的顺序
    Graph[0]->neighbors.push_back(Graph[4]);
    Graph[0]->neighbors.push_back(Graph[2]);
    Graph[1]->neighbors.push_back(Graph[0]);
    Graph[1]->neighbors.push_back(Graph[2]);
    Graph[2]->neighbors.push_back(Graph[3]);
    Graph[3]->neighbors.push_back(Graph[4]);
    Graph[4]->neighbors.push_back(Graph[3]);
    int visit[MAX_N] = {0};  //标记已访问的顶点
    for (int i = 0; i < MAX_N; i++){
        if (  3  ){
            printf("From label(%d) : ", Graph[i]->label);
            DFS_graph(Graph[i], visit);
            printf("\n");
        }
    }
    for (int i = 0; i < MAX_N; i++){
        delete Graph[i];
    }

    return 0;
}
```

初始:



从0出发:






```
From label(0) : 0 4 3 2
From label(1) : 1
```

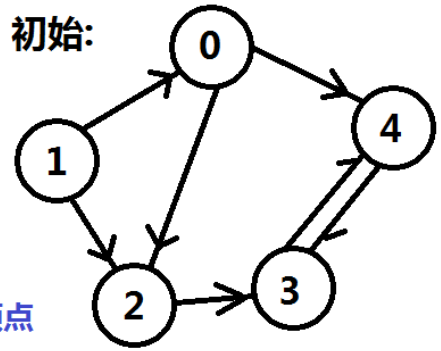
预备知识：图的深度优先遍历，课堂练习

3分钟填写代码，**有问题随时提出！**

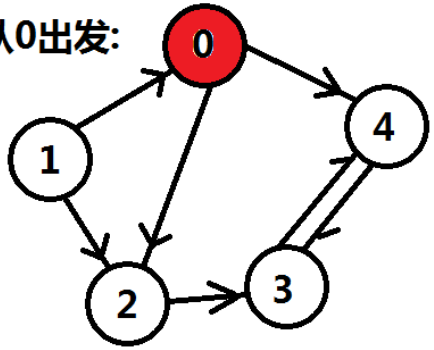
预备知识：图的深度优先遍历，实现

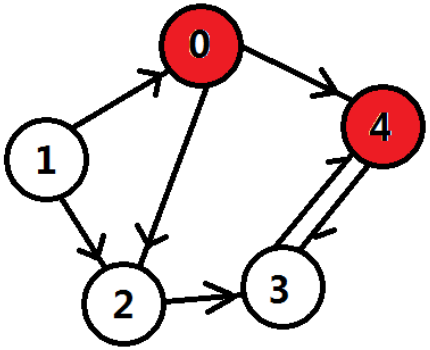
```
#include <stdio.h>
#include <vector>   //图的邻接表数据结构

struct GraphNode{   //图的顶点的值        //相邻节点指针数组
    int label;
    std::vector<GraphNode *> neighbors;
    GraphNode(int x) : label(x) {};
};
void DFS_graph(GraphNode *node, int visit[]){
    visit[node->label] = 1;          //标记已访问的顶点
    printf("%d ", node->label);      //访问相邻的且没有被访问的顶点
    for (int i = 0; i < node->neighbors.size(); i++){
        if (visit[node->neighbors[i]->label] == 0){
            DFS_graph(node->neighbors[i], visit);
        }
    }
}
int main(){
    const int MAX_N = 5;     //创建图的顶点
    GraphNode *Graph[MAX_N];
    for (int i = 0; i < MAX_N; i++){
        Graph[i] = new GraphNode(i);
    }
                //添加图的边，注意添加边的顺序
    Graph[0]->neighbors.push_back(Graph[4]);
    Graph[0]->neighbors.push_back(Graph[2]);
    Graph[1]->neighbors.push_back(Graph[0]);
    Graph[1]->neighbors.push_back(Graph[2]);
    Graph[2]->neighbors.push_back(Graph[3]);
    Graph[3]->neighbors.push_back(Graph[4]);
    Graph[4]->neighbors.push_back(Graph[3]);
    int visit[MAX_N] = {0};  //标记已访问的顶点
    for (int i = 0; i < MAX_N; i++){
        if (visit[i] == 0){ //顶点没有被标记才会访问
            printf("From label(%d) : ", Graph[i]->label);
            DFS_graph(Graph[i], visit);
            printf("\n");
        }
    }
    for (int i = 0; i < MAX_N; i++){
        delete Graph[i];
    }

    return 0;
}
```

初始：



从0出发：

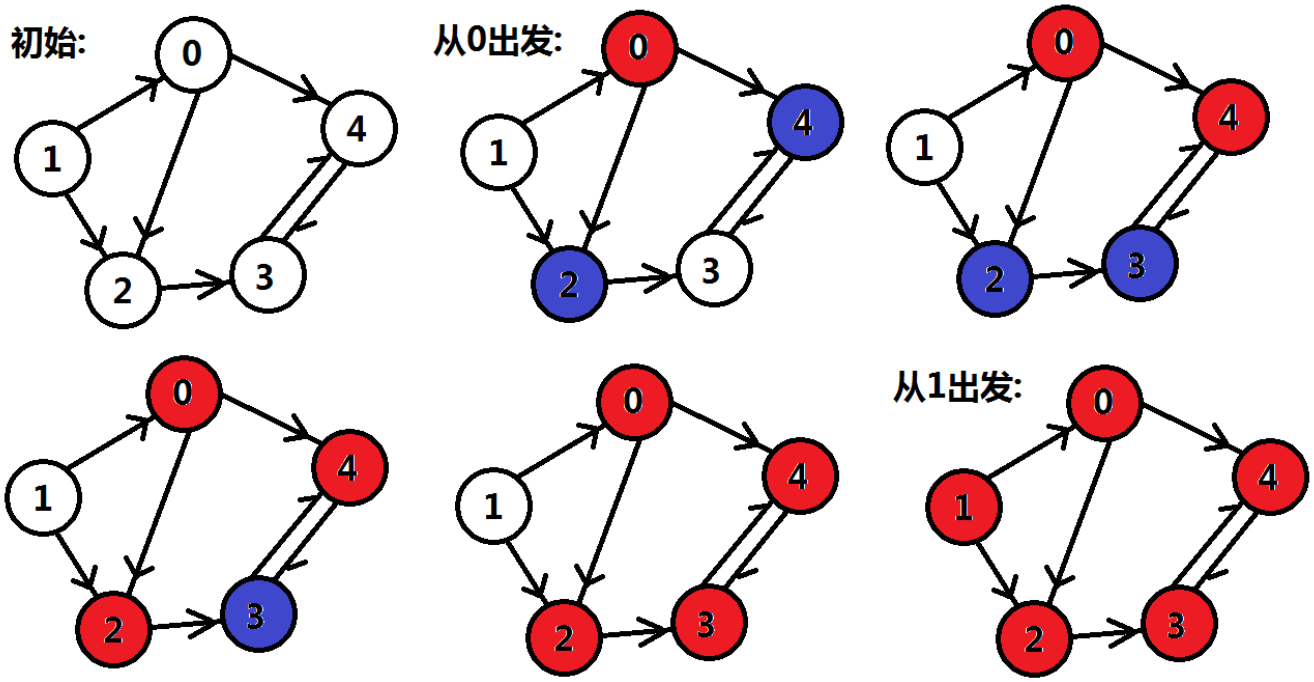





```
From label(0) : 0 4 3 2
From label(1) : 1
```

预备知识:图的宽度优先遍历

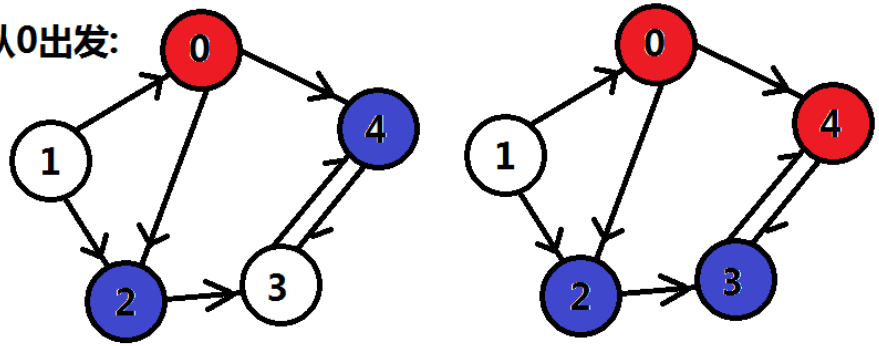
从图中**某个顶点v**出发，在访问了v之后**依次访问**v的各个**未曾访问过的**邻接点，然后**分别**从这些邻接点出发**依次访问**它们的邻接点，并使得“**先被访问**的顶点的邻接点**先于后被访问**的顶点的邻接点被访问”，直至图中所有**已被访问的顶点的邻接点**都被访问到。如果此时图中尚有顶点**未被访问**，则需要**另选**一个未曾被访问过的顶点作为新的起始点，**重复**上述过程，直至图中**所有顶点**都被访问到为止。



预备知识:图的宽度优先遍历，课堂练习

```
void BFS_graph(GraphNode *node, int visit[]){
    std::queue<GraphNode *> Q;
    Q.push(node);
    visit[node->label] = 1;
    while(  1  ){
        GraphNode *node = Q.front();
        Q.pop();
        printf("%d ", node->label);
        for (int i = 0; i < node->neighbors.size(); i++){
            if (  2  ){
                  3
                visit[node->neighbors[i]->label] = 1;
            }
        }
    }
}
```

从0出发:




```
int main(){
    const int MAX_N = 5;          //创建图的顶点
    GraphNode *Graph[MAX_N];
    for (int i = 0; i < MAX_N; i++){
        Graph[i] = new GraphNode(i);
    }
                                  //添加图的边，注意添加边的顺序
    Graph[0]->neighbors.push_back(Graph[4]);
    Graph[0]->neighbors.push_back(Graph[2]);
    Graph[1]->neighbors.push_back(Graph[0]);
    Graph[1]->neighbors.push_back(Graph[2]);
    Graph[2]->neighbors.push_back(Graph[3]);
    Graph[3]->neighbors.push_back(Graph[4]);
    Graph[4]->neighbors.push_back(Graph[3]);
    int visit[MAX_N] = {0};       //标记已访问的顶点
    for (int i = 0; i < MAX_N; i++){
        if (visit[i] == 0){
            printf("From label(%d) : ", Graph[i]->label);
            BFS_graph(Graph[i], visit);
            printf("\n");
        }
    }

    for (int i = 0; i < MAX_N; i++){
        delete Graph[i];
    }

    return 0;
}
```

```
From label(0) : 0 4 2 3
From label(1) : 1
```

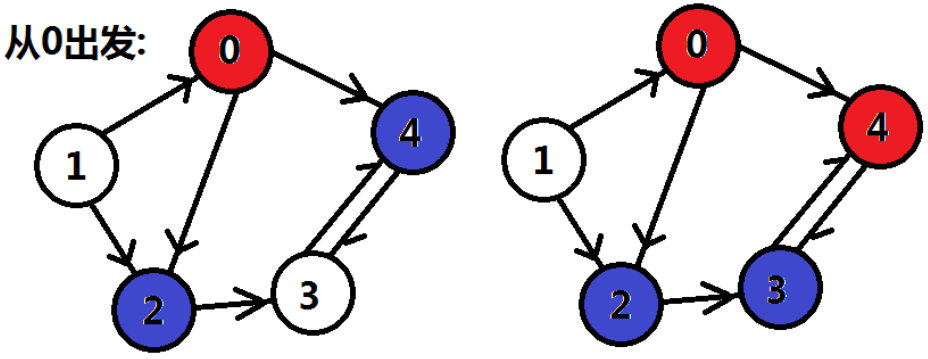
预备知识:图的宽度优先遍历，实现

```
void BFS_graph(GraphNode *node, int visit[]){
    std::queue<GraphNode *> Q;
    Q.push(node);
    visit[node->label] = 1;
    while( !Q.empty() ){
        GraphNode *node = Q.front();
        Q.pop();
        printf("%d ", node->label);
        for (int i = 0; i < node->neighbors.size(); i++){
            if ( visit[node->neighbors[i]->label] == 0 ){
                Q.push(node->neighbors[i]);
                visit[node->neighbors[i]->label] = 1;
            }
        }
    }
}
```

//宽度优先搜索使用队列，

队列不空即一直循环

从0出发:




```
int main(){
    const int MAX_N = 5;          //创建图的顶点
    GraphNode *Graph[MAX_N];
    for (int i = 0; i < MAX_N; i++){
        Graph[i] = new GraphNode(i);
    }
                                  //添加图的边，注意添加边的顺序
    Graph[0]->neighbors.push_back(Graph[4]);
    Graph[0]->neighbors.push_back(Graph[2]);
    Graph[1]->neighbors.push_back(Graph[0]);
    Graph[1]->neighbors.push_back(Graph[2]);
    Graph[2]->neighbors.push_back(Graph[3]);
    Graph[3]->neighbors.push_back(Graph[4]);
    Graph[4]->neighbors.push_back(Graph[3]);
    int visit[MAX_N] = {0};       //标记已访问的顶点
    for (int i = 0; i < MAX_N; i++){
        if (visit[i] == 0){
            printf("From label(%d) : ", Graph[i]->label);
            BFS_graph(Graph[i], visit);
            printf("\n");
        }
    }

    for (int i = 0; i < MAX_N; i++){
        delete Graph[i];
    }

    return 0;
}
```

```
From label(0) : 0 4 2 3
From label(1) : 1
```

例5:课程安排

已知有n个课程，标记从0至n-1，**课程之间**是**有依赖**关系的，例如希望完成A课程，可能需要先完成B课程。已知n个课程的依赖关系，求**是否可以**将n个课程**全部完成**。

```
class Solution {
public:
    bool canFinish(int numCourses,
        std::vector<std::pair<int, int> >& prerequisites) {
    }
};
```

//课程数量

//课程依赖关系<课程1, 课程2>，代表课程1依赖课程2

如: 2, [[1, 0]]，函数返回true.

2, [[1, 0], [0, 1]]，函数返回false.

选自 **LeetCode 207. Course Schedule**
https://leetcode.com/problems/course-schedule/description/
难度:**Medium**

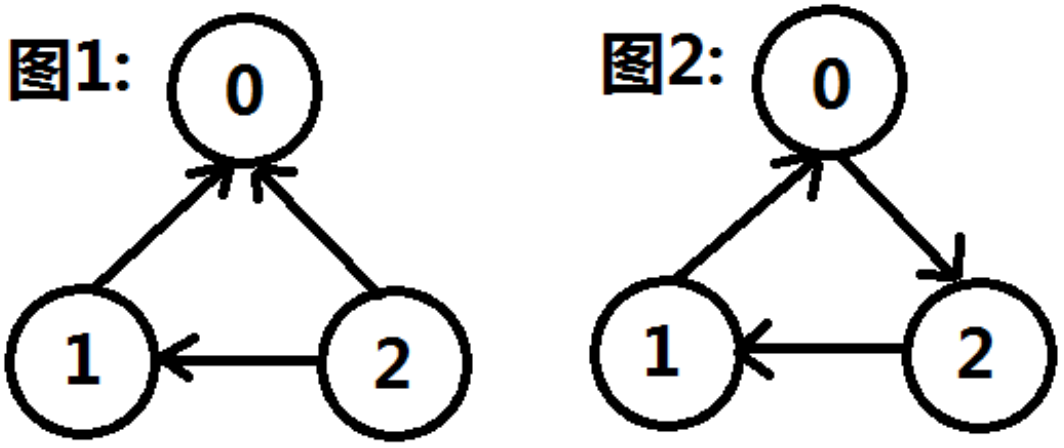
例5:分析

n个课程，它们之间有m个依赖关系，可以看成**顶点**个数为n，**边**个数为m的**有向图**。

图1:n = 3, m = [[0, 1], [0, 2], [1, 2]]；**可以**完成。

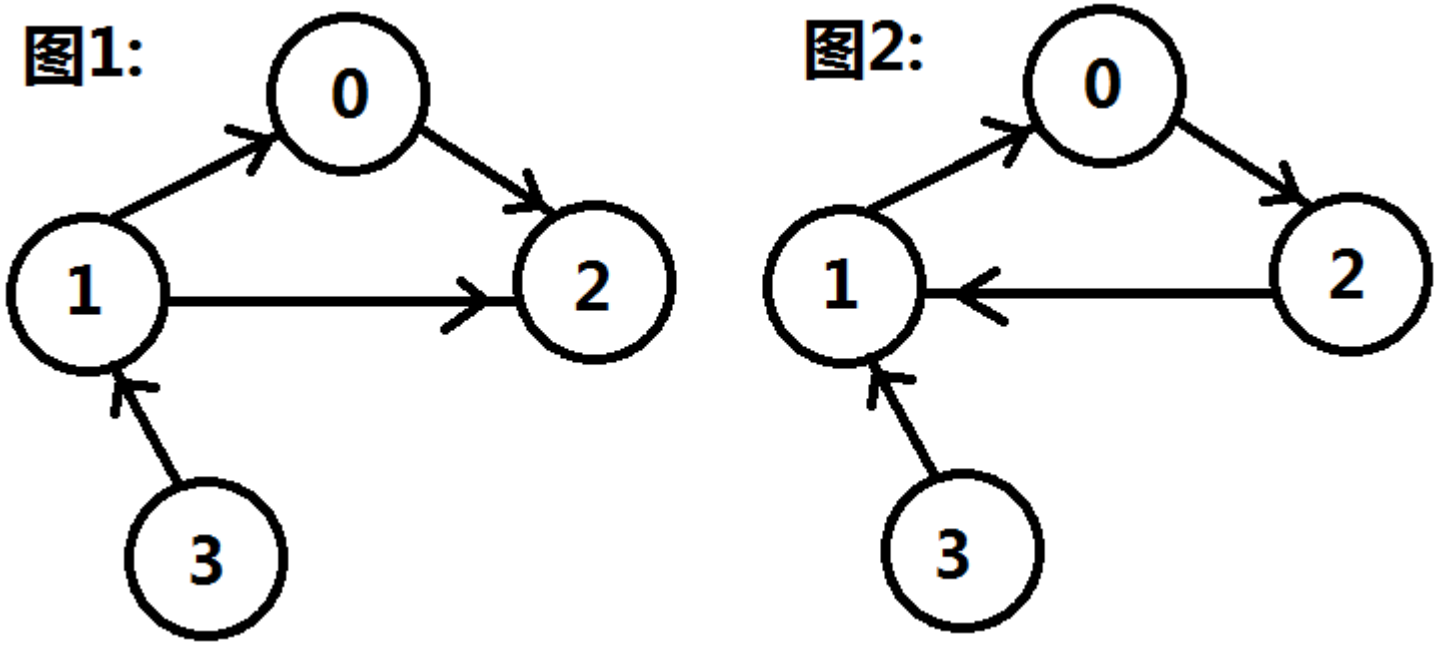
图2:n = 3, m = [[0, 1], [1, 2], [2, 0]]；**不可以**完成。

故，若**有向图无环**，则可以完成全部课程，否则不能。问题转换成，构建图，并**判断图是否有环**。



例5:方法1，深度优先搜索

在**深度优先搜索**时，如果正在搜索某一顶点(还未退出该顶点的递归深度搜索)，又**回到了**该顶点，即证明图有环。
如下图:



例5:算法思路(无环)

初始:

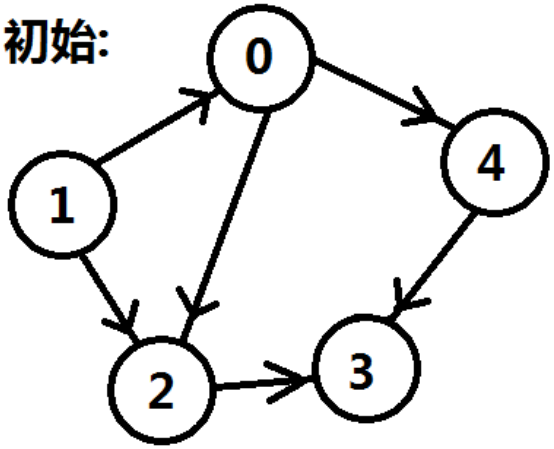


图1:

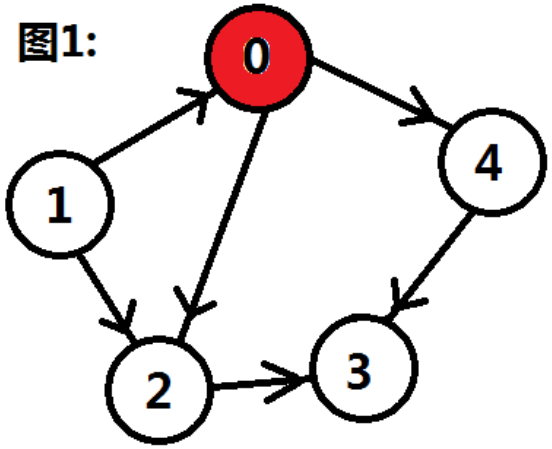


图2:

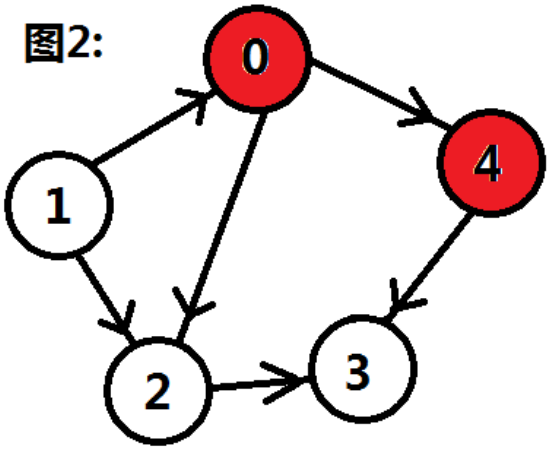


图3:

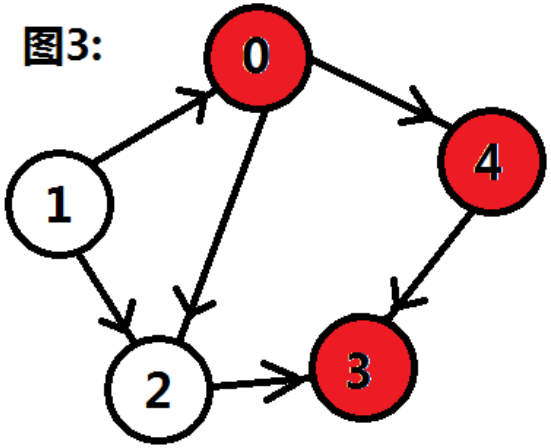


图4:

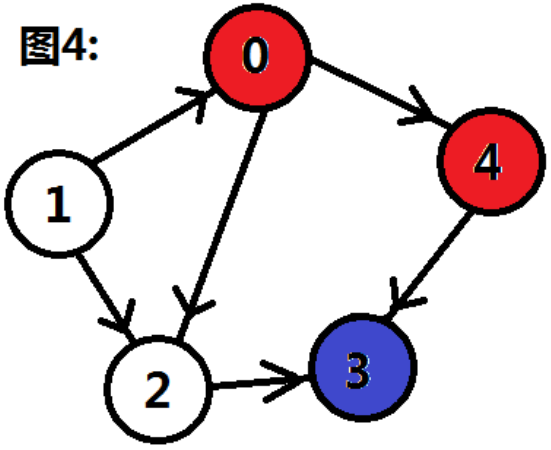
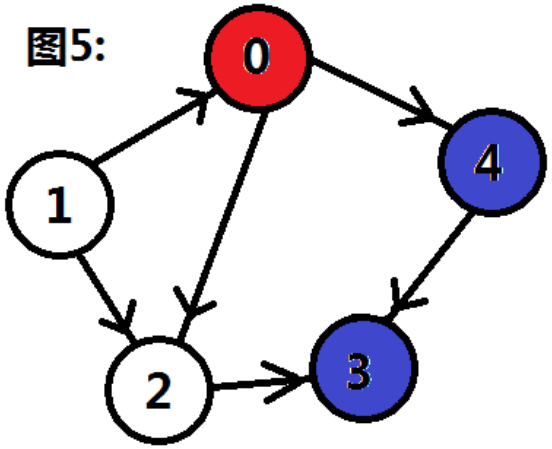
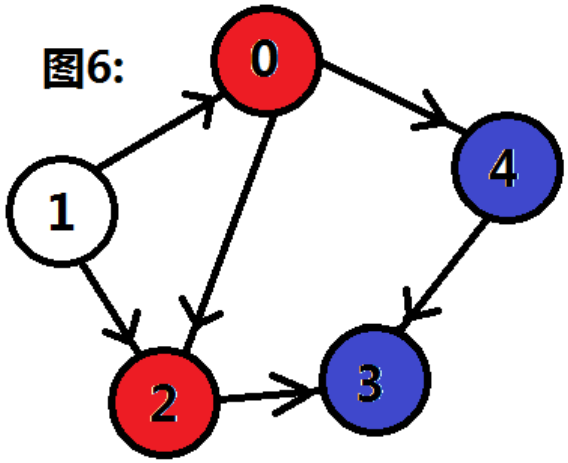
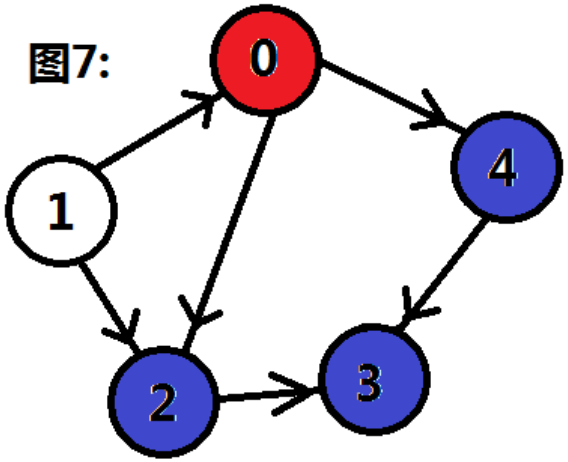


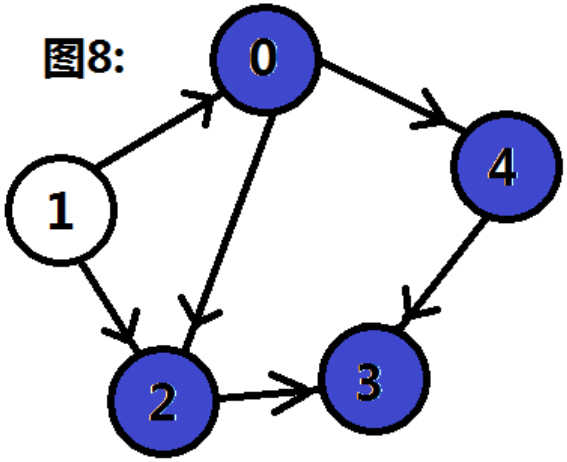
图5:

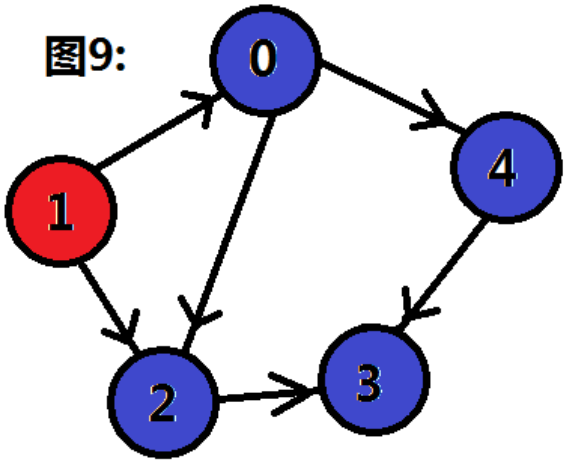


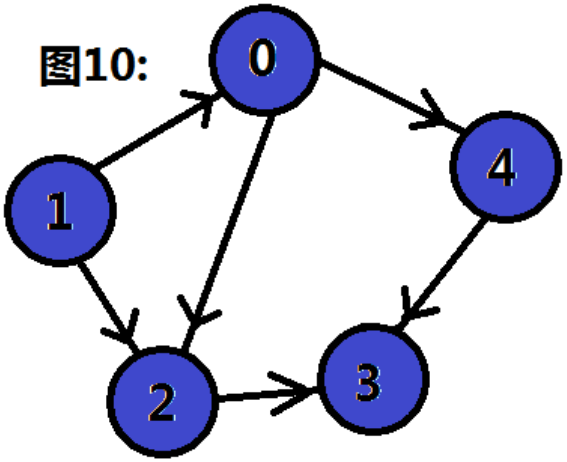
例5:算法思路(无环)



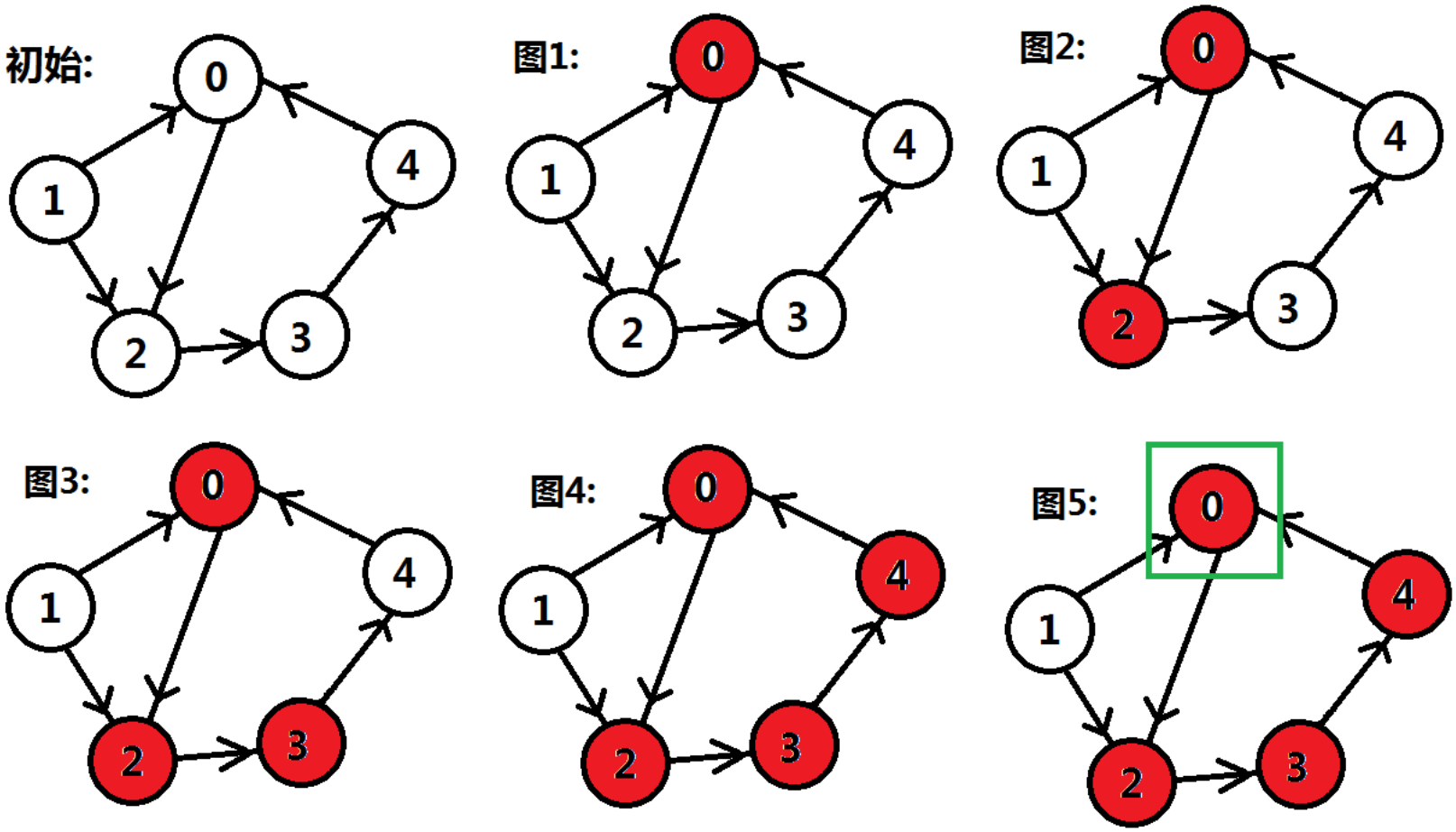








例5:算法思路(有环)



例5:方法1，调用代码

```
class Solution {
public:
    bool canFinish(int numCourses,
        std::vector<std::pair<int, int> >& prerequisites) {
        std::vector<GraphNode*> graph;
        std::vector<int> visit;
        for (int i = 0; i < numCourses; i++){
            graph.push_back(new GraphNode(i));
            visit.push_back(-1);
        }
        for (int i = 0; i < prerequisites.size(); i++){
            GraphNode *begin = graph[prerequisites[i].second];
            GraphNode *end = graph[prerequisites[i].first];
            begin->neighbors.push_back(end);
        }
        for (int i = 0; i < graph.size(); i++){
            if (visit[i] == -1 && !DFS_graph(graph[i], visit)){
                return false;
            }
        }
        for (int i = 0; i < numCourses; i++){
            delete graph[i];
        }
        return true;
    }
};
```

pair<课程1，课程2> 课程1依赖课程2

//邻接表

//节点访问状态,-1没有访问过,0代表正在访问,1代表已完成访问

//创建图的节点，并赋访问状态为空

//创建图，连接图的顶点

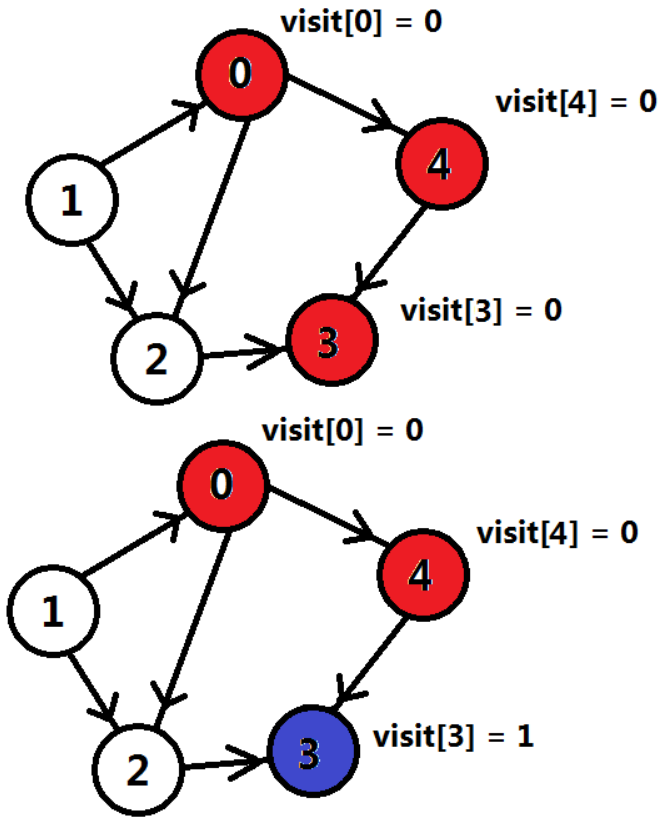
//课程2指向课程1

//如果节点没访问过，进行DFS，如果DFS遇到环，返回无法完成

//返回可以完成

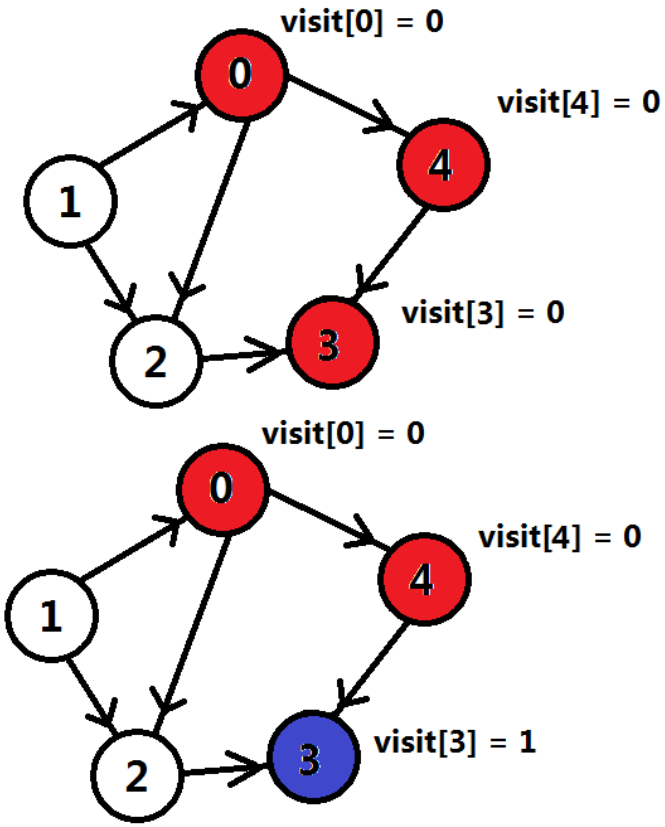
例5:方法1，课堂练习

```
#include <vector>
struct GraphNode{
    int label;
    std::vector<GraphNode *> neighbors;
    GraphNode(int x) : label(x) {};
};
    //节点访问状态,-1没有访问过,0代表正在访问,1代表已完成访问
bool DFS_graph(GraphNode *node, std::vector<int> &visit){
    1
    for (int i = 0; i < node->neighbors.size(); i++){
        if (  2  ){
            if (DFS_graph(node->neighbors[i], visit) == 0){
                3
            }
        }
        else if (visit[node->neighbors[i]->label] == 0){
            4
        }
    }
    5
    return true;
}
```



例5:方法1，实现

```
#include <vector>
struct GraphNode{
    int label;
    std::vector<GraphNode *> neighbors;
    GraphNode(int x) : label(x) {};
};
      //节点访问状态,-1没有访问过,0代表正在访问,1代表已完成访问
bool DFS_graph(GraphNode *node, std::vector<int> &visit){
      visit[node->label] = 0;
    for (int i = 0; i < node->neighbors.size(); i++){
        if ( visit[node->neighbors[i]->label] == -1 ){
            if (DFS_graph(node->neighbors[i], visit) == 0){
                return false;
            }
        }
        else if (visit[node->neighbors[i]->label] == 0){
                return false;
        }
    }
      visit[node->label] = 1;
    return true;
}
```



例5:方法2，拓扑排序(宽度优先搜索)

在**宽度优先搜索**时，只将**入度**为0的点添加至队列。当完成一个顶点的搜索(从队列取出)，它指向的所有顶点**入度都减1**，若此时某顶点入度为0则**添加**至队列，若完成宽度搜索后，所有的点入度都为0，则**图无环**，否则**有环**。

图1:

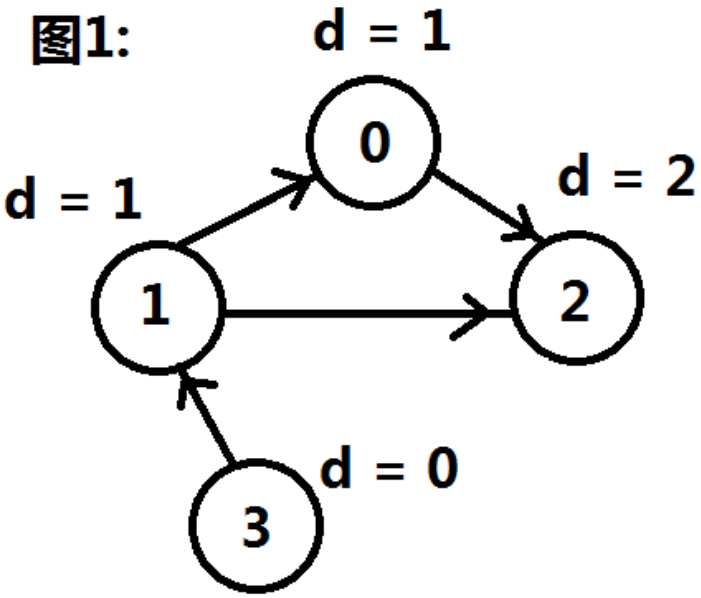
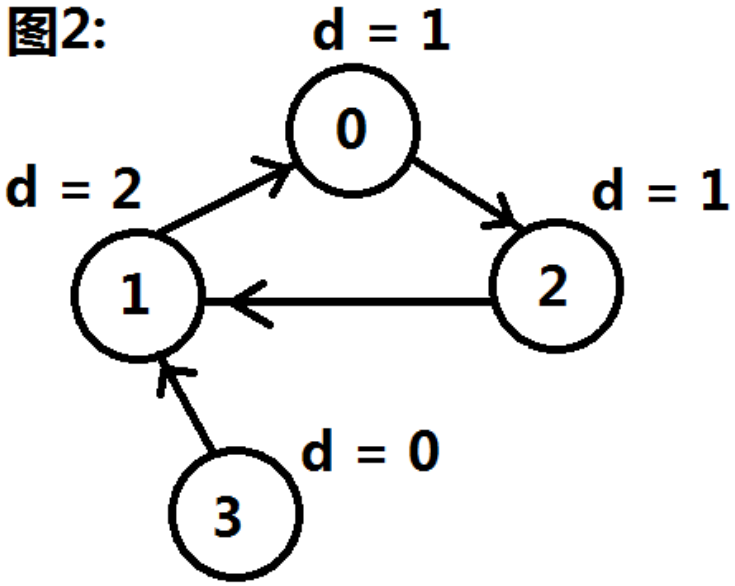
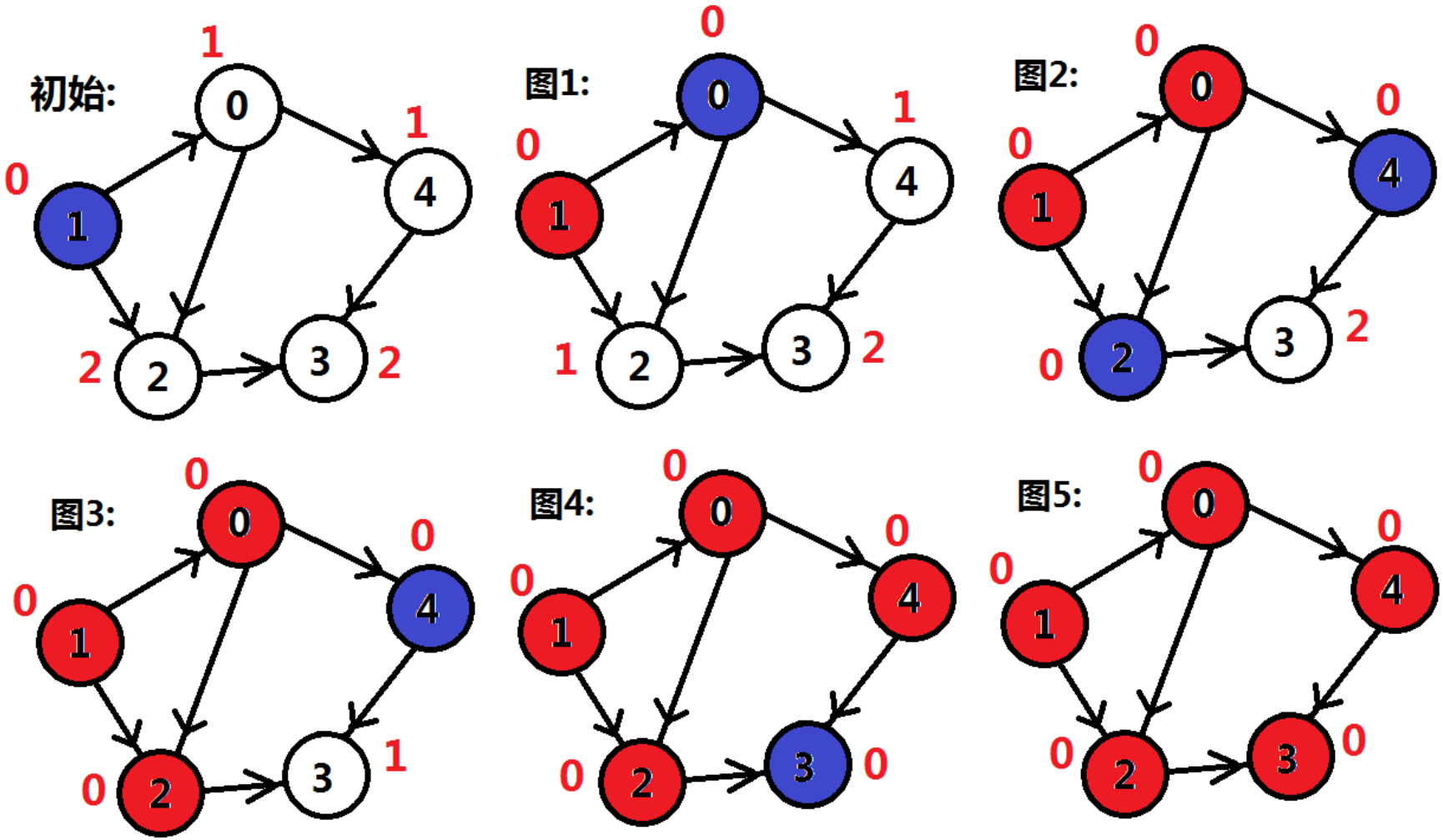


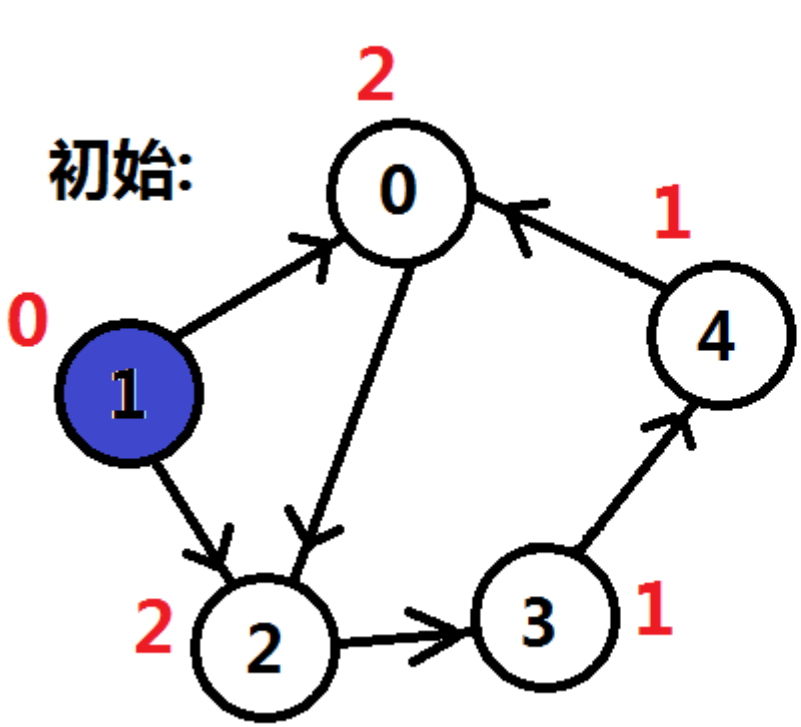
图2:

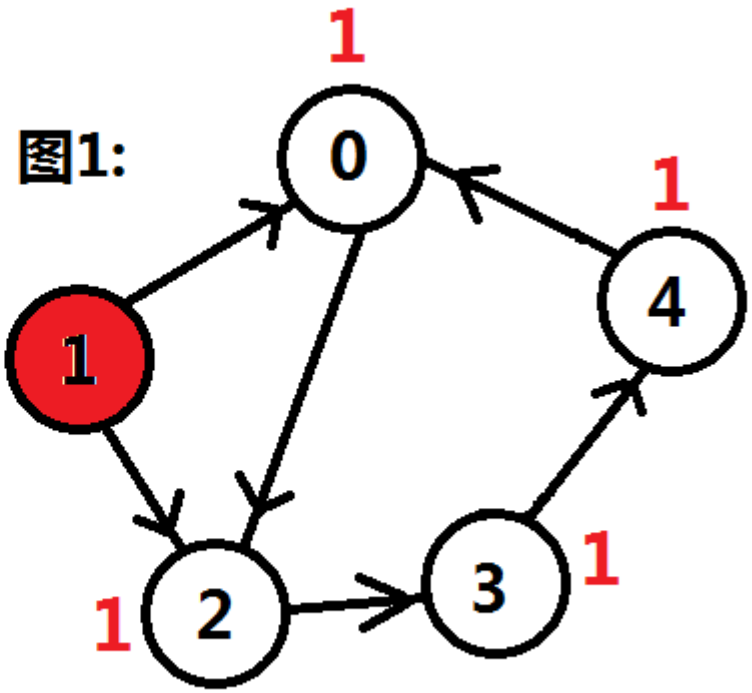


例5:算法思路(无环)


初始:
0
1
0
1
2
2
3
4
2
图1:
0
0
1
1
2
3
4
1
2
图2:
0
0
1
0
2
3
4
0
2
图3:
0
0
1
0
2
3
4
0
1
图4:
0
0
1
0
2
3
4
0
0
图5:
0
0
1
0
2
3
4
0
0

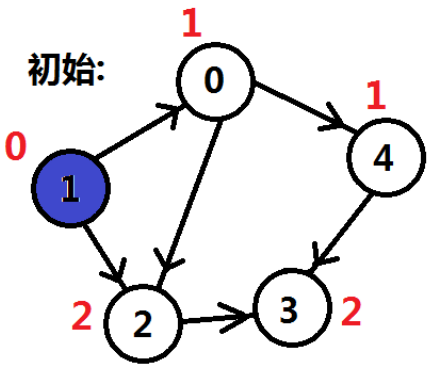
例5:算法思路(有环)

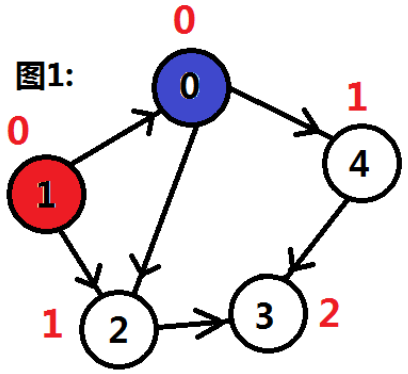


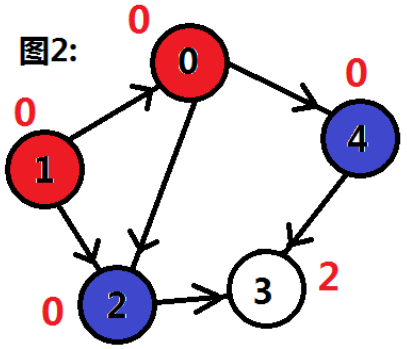


例5:方法2，课堂练习

```
class Solution {
public:
    bool canFinish(int numCourses,
        std::vector<std::pair<int, int> >& prerequisites) {
        std::vector<GraphNode*> graph;
        std::vector<int> degree; //入度数组
        for (int i = 0; i < numCourses; i++){
            degree.push_back(0);
            graph.push_back(new GraphNode(i));
        }
        for (int i = 0; i < prerequisites.size(); i++){
            GraphNode *begin = graph[prerequisites[i].second];
            GraphNode *end = graph[prerequisites[i].first];
            begin->neighbors.push_back(end);
            1                                  //入度++，即pair<课程1，课程2>
                                               课程1的入度++
        }
        std::queue<GraphNode *> Q;
        for (int i = 0; i < numCourses; i++){
            if (  2  ){
                Q.push(graph[i]);
            }
        }
        while(!Q.empty()){
            GraphNode *node = Q.front();
            Q.pop();
            for (int i = 0; i < node->neighbors.size(); i++){
                  3
                if (  4  ){
                    Q.push(node->neighbors[i]);
                }
            }
        }
        for (int i = 0; i < graph.size(); i++){
            delete graph[i];
        }
        for (int i = 0; i < degree.size(); i++){
            if (degree[i]){
                  5
            }
        }
        return true;
    }
};
```



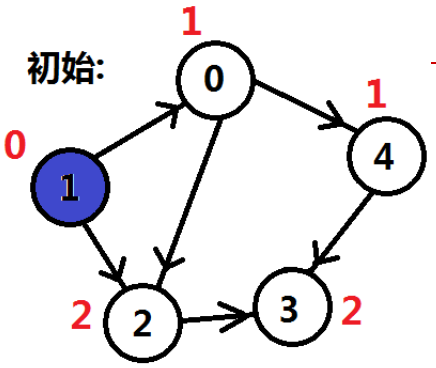


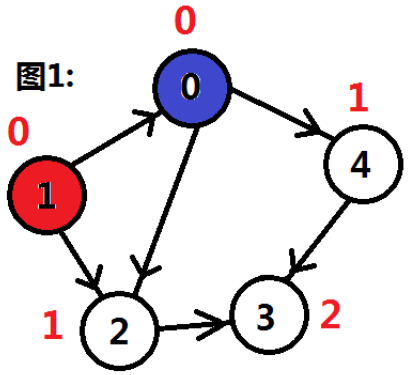


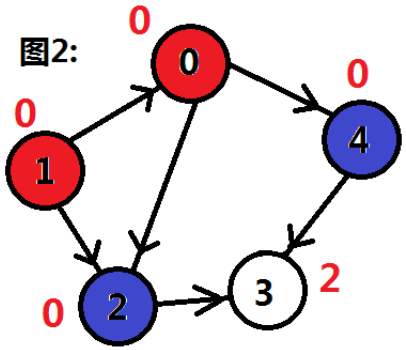
例5:方法2，实现

```
class Solution {
public:
    bool canFinish(int numCourses,
        std::vector<std::pair<int, int> >& prerequisites) {
        std::vector<GraphNode*> graph;
        std::vector<int> degree; //入度数组
        for (int i = 0; i < numCourses; i++){
            degree.push_back(0);
            graph.push_back(new GraphNode(i));
        }
        for (int i = 0; i < prerequisites.size(); i++){
            GraphNode *begin = graph[prerequisites[i].second];
            GraphNode *end = graph[prerequisites[i].first];
            begin->neighbors.push_back(end);
            degree[prerequisites[i].first]++;
        }
        std::queue<GraphNode *> Q;
        for (int i = 0; i < numCourses; i++){
            if ( degree[i] == 0 ){
                Q.push(graph[i]);
            }
        }
        while(!Q.empty()){
            GraphNode *node = Q.front();
            Q.pop();
            for (int i = 0; i < node->neighbors.size(); i++){
                degree[node->neighbors[i]->label]--;
                if ( degree[node->neighbors[i]->label] == 0 ){
                    Q.push(node->neighbors[i]);
                }
            }
        }
        for (int i = 0; i < graph.size(); i++){
            delete graph[i];
        }
        for (int i = 0; i < degree.size(); i++){
            if (degree[i]){
                return false;
            }
        }
        return true;
    }
};
```

//入度++，即pair<课程1，课程2>
课程1的入度++

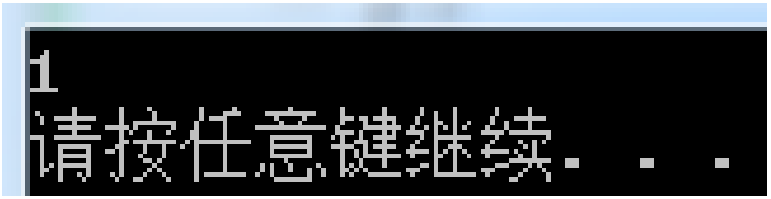






例5:测试与leetcode提交结果

```
int main(){
    std::vector<std::pair<int, int> > prerequisites;
    prerequisites.push_back(std::make_pair(1, 0));
    prerequisites.push_back(std::make_pair(2, 0));
    prerequisites.push_back(std::make_pair(3, 1));
    prerequisites.push_back(std::make_pair(3, 2));
    Solution solve;
    printf("%d\n", solve.canFinish(4, prerequisites));
    return 0;
}
```



Course Schedule

Submission Details

37 / 37 test cases passed. Status: Accepted

Runtime: **13 ms** Submitted: **39 minutes ago**

结束

非常感谢大家！

林沐