

# Challenge: The Catalan Numbers

In this lesson, you will solve your first coding challenge using bottom-up dynamic programming.

## We'll cover the following

- Problem statement
  - Applications of the Catalan numbers
- Input
- Output
- Coding challenge

## Problem statement #

The **Catalan** numbers are a special sequence of numbers given by the following set of formulas:

$$C_0 = 1$$

$$C_n = \sum_{i=0}^{n-1} C_i C_{n-1-i}$$

The first expression gives the base case of the formula. The second expression says that the  $n^{th}$  Catalan number is simply the sum of products of specific Catalan number pairs. These specific pairs are just Catalan numbers with the same distance from either end of the series.

$C_4$  for example would be equal to:

$$C_4 = C_0 C_3 + C_1 C_2 + C_2 C_1 + C_3 C_0$$

You can already see the hefty amount of recursion in this series.

The Catalan numbers form the following series: (1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862...)

## Applications of the Catalan numbers #

The Catalan numbers readily appear in many interesting counting problems.

- The number of ways to put parentheses around  $n$  numbers for multiplication.
- The number of paths to climb up a  $2n \times 2n$  grid without going above the diagonal.
- The number of possible binary trees with  $n$  leaf nodes. This has been shown in the visualization below.

### Catalan Numbers Application

Number of binary trees with  $n$  number of nodes

1 of 14

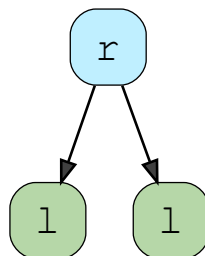
### Catalan Numbers Application

Number of binary trees with  $n$  number of nodes

Leaf nodes = 2

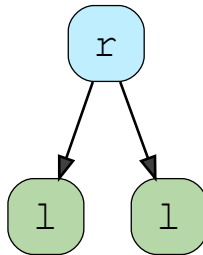
Number of trees with 2 leaf nodes

Leaf nodes = 2



Number of trees with 2 leaf nodes

Leaf nodes = 2



1 way!

Number of trees with 2 leaf nodes

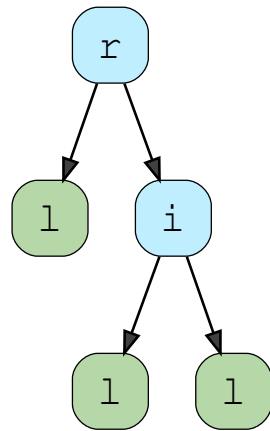
5 of 14

Leaf nodes = 3

Number of trees with 3 leaf nodes

6 of 14

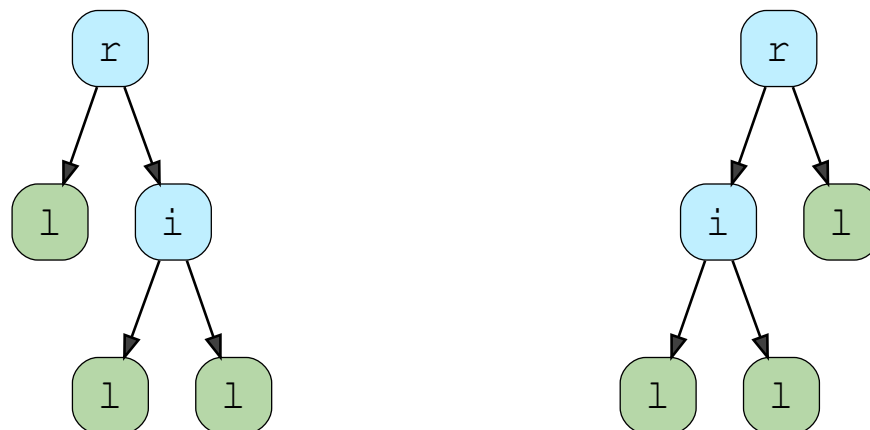
Leaf nodes = 3



Number of trees with 3 leaf nodes

7 of 14

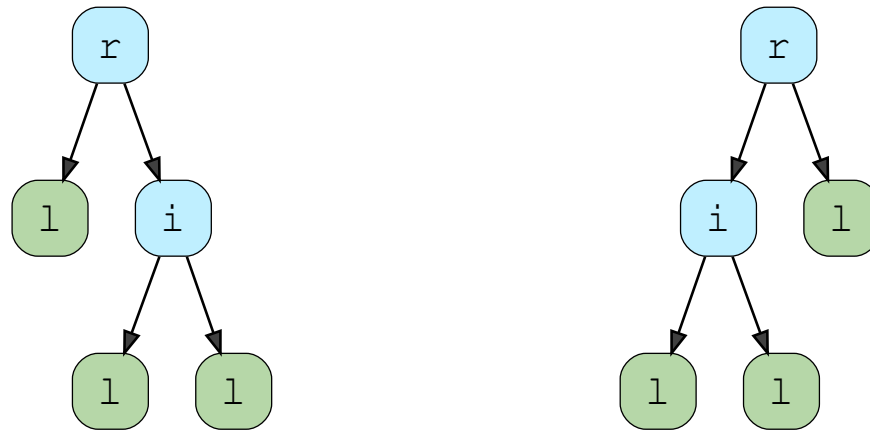
Leaf nodes = 3



Number of trees with 3 leaf nodes

8 of 14

Leaf nodes = 3



2 ways!

Number of trees with 3 leaf nodes

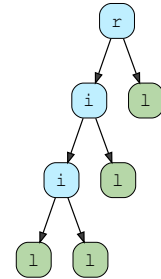
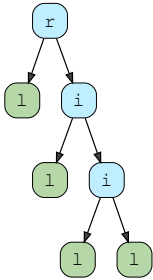
9 of 14

Leaf nodes = 4

Number of trees with 4 leaf nodes

10 of 14

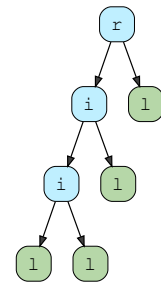
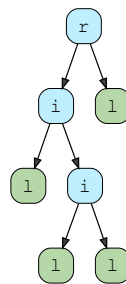
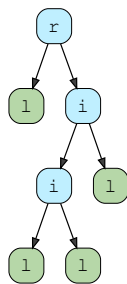
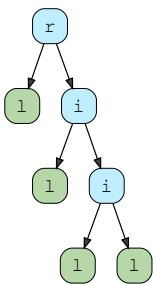
Leaf nodes = 4



Number of trees with 4 leaf nodes

11 of 14

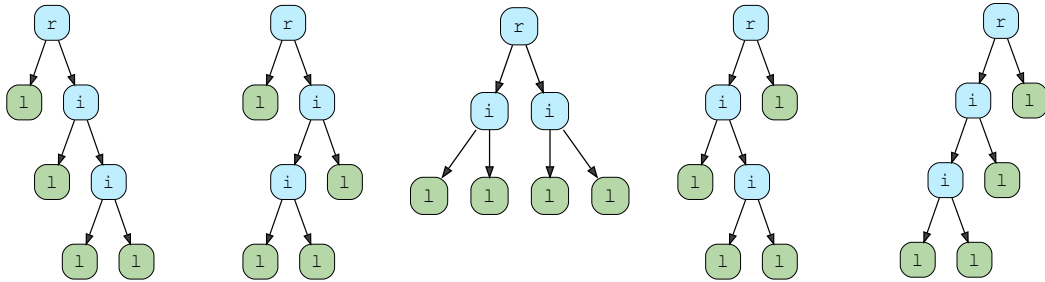
Leaf nodes = 4



Number of trees with 4 leaf nodes

12 of 14

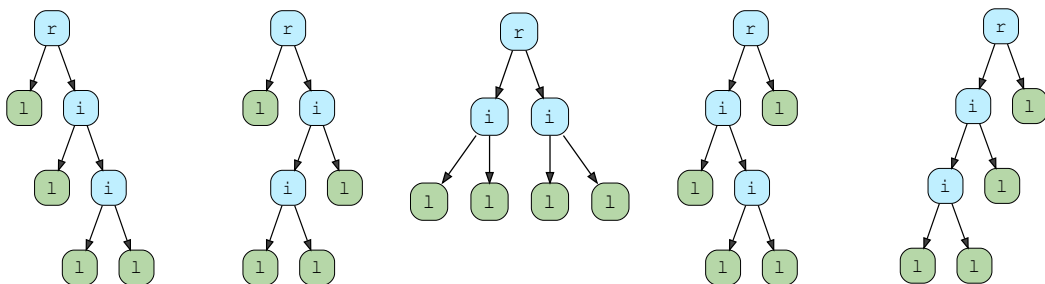
Leaf nodes = 4



Number of trees with 4 leaf nodes

13 of 14

Leaf nodes = 4



5 ways!

Number of trees with 4 leaf nodes

14 of 14



## Input #

Your program will take as input a number, `n`. `n` is a non-negative integer i.e. `n`  $\geq 0$ .

```
n = 4
```

## Output #

Your program will evaluate `n`<sup>th</sup> Catalan number and return it.

```
catalan(4) = 14
catalan(6) = 132
```

## Coding challenge #

You should write a bottom-up algorithm for this challenge and you should use tabulation. We will test your solution with bigger numbers and if tabulation is not done properly, tests might be timed out.

If you plan to write a recursive solution first, you may check its correctness by setting `stressTesting` to `False`. If you plan to write a top-down algorithm with memoization first, you may check its correctness by setting `testforBottomUp` to `False`. If you only write a bottom-up algorithm, you can test without changing these variables.

Think about the problem, devise your strategy, and then write your code. Best of luck!

```
def catalan(n):
    # write your code here
    return 0

stressTesting = True # to only check if your recursive solution is correct, set it to false
testforBottomUp = True # to test a top down implementation set it to false
```





Look at equation, to evaluate n-th Catalan number you will have  
to loop over all previous ones



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In the next lesson, we will review the solution to this challenge.