Prime Factors

In this lesson, we'll discuss about prime factorization of prime numbers.

We'll cover the following

- Representation
- Prime Factors

Representation

Any integer can be represented as the product of a power of primes. For example:

- $6 = 2 \times 3$
- $24 = 2^3 \times 3$
- $3087 = 3^2 \times 7^3$

Breaking an integer into its prime factors with their corresponding powers is called prime factorization.

Prime factorization of an integer is a very common problem and will reoccur in a wide range of topics, hence it is important to know an efficient way to do it.

Prime Factors

Property: *An integer N will have at most one prime factor $\geq \sqrt{N}$.*

If an integers n has m prime factors $(p_1 < p_2 < ... < p_m)$. Then either,

- All prime factors are less than or equal to $\sqrt{N}$ or,
- All prime factors except p_m are less than or equal to $\sqrt{N}$.

Proof: Using contradiction for N, if the above statement is not true, then there must be two prime factors, p_1 and p_2, such that:

$$p_1 \geq \sqrt{N}, p_2 \geq \sqrt{N}$$

But since $p_1 \neq p_2$, both can't be equal to $\sqrt{N}$.

In that case $p_1 \times p_2 > N$, and hence they can't be factors of N.

Property: *If N has a prime factor $p > \sqrt{N}$. Then the power of p in prime factorization of N is* 1

Proof: Again, by contradiction, if the power is greater than 1, then $p \times p > N$, which is not possible.

In the next lesson, we'll see how to find the prime factorization of a number.