# Finding Needles in a Compressive Haystacks
# **CS 754 Course Project**

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## 1 Problem Statement

Our objective for the course project which follows from our literature reference [1] is of compressive classification, which is, that given different categories of compressed data obtained through a measurement system, how do we distinguish between them without reconstructing the original uncompressed signal.

## 2 Literature Review

The method explained in [1], shows that SVM classifiers on compressive measurements obtained through the Delsarte-Goethal frames are near optimal, i.e give close performance as to when done in the non-compressed case. We observed the matrix properties which endorse the usage of the DG frames by proving them to be low-coherence tight frames, which is desirable for good sensing matrices.

We also saw the average case and worst case coherence bounds are optimal which means that the distinction between two points in the original domain is likely to be maintained in the measurement domain as well, similarly low average case coherence, denotes a similar spread in the data in both the uncompressed and compressed domain. Thus we can say that projections obtained using the DG frames is likely to give near optimal performance. Also, proof of preservation of inner products in the compressed domain is provided in Theorem 4.1 of [1].

## 3 Contribution

Our contributions in this project is divided on two different verticals, firstly, the design of optimal sensing matrices, and secondly, observing classification performance on sensed data using SVMs and neural networks.

Before we understand the specifics of the sensing matrices and classification algorithms let us try to understand how the classification pipeline works.

We start with uncompressed data from a dataset, obtain its sparse representation in some basis, and then obtain the compressive measurements for that data using the sensing matrix. We apply the classification algorithm to both the compressed and original data with the same training parameters, and same fraction of the training and test data to infer the classification accuracy, and compare the training loss over different epochs of training. Ideally, we wish to design a sensing matrix which gives the classification accuracy as good as the ones obtained on the original data

Let us start with the first one, design of compressed sensing matrices .

### 3.1 Design of Sensing Matrices

#### 3.1.1 Method 1 : Random Bernoulli

We know that using Bernoulli random matrices with zero mean and variance $(-\frac{1}{\sqrt{m}}, \frac{1}{\sqrt{m}})$, as a sensing matrix, we can get finite error bounds for the reconstruction of the exact signal from the compressed signal. This is derived from the fact that such matrices follow the Restricted Isometry Property(RIP) and are thus distance preserving. Thus, we started off our experimentation with the Bernoulli. The matrix was designed using the Numpy library.

#### 3.1.2 Method 2 : Delsarte-Goethal Frames

The Delsarte-Goethal (DG) frames have been used in the reference literature in [?] as the sensing matrices. The reason for using such matrices as the sensing matrices is due to the Theorem 4.1 in the paper which states that a DG frame is likely to preserve the inner products of two vectors in the compressed domain with some finite probability. Since, SVM classification is driven by inner products thus, using a DG sensing matrix would be a good alternative for the SVM case than using the Bernoulli matrix.

The matrix was designed iteratively by obtaining every element in the matrix using the following formula state in section 4.2 of [?]. Since generating the entire matrix is difficult in one run due to time complexities involved, thus we generate half of the matrix using the DG formula stated in [1], and then we obtain the full matrix by flipping the matrix horizontally and further concatenating with the original matrix.

### 3.2 Classification Algorithms

#### 3.2.1 Support Vector Machines

Support Vectors Machines is the algorithm that has been used in the reference literature [1] to perform classification on both the compressive as well as the original data. The reason why SVMs were used there is due to the advantage of the DG sensing matrix. It has been proved in Theorem 4.1 of [1] that DG matrix is inner product preserving with a probability, given some conditions on the number of compressed samples. Since SVMs perform projection of data onto the kernel space through inner products thus it is safe to say that the projected inner products are also preserved metric wise when sensed with the DG matrix. Thus, the SVM classifier on the sensed data is optimal.

#### 3.2.2 ANNs

An Artificial neural network (ANN) consist of neurons which in turn are responsible for creating layers, these neurons are also called parameters of ANN model. In ANNs, the output from each layer is passed on to the next layer. There are different nonlinear activation functions to each layer, which helps in the learning process and the output of each layer. The weighted association with the neurons and which are responsible for the overall predictions are updated on each epoch. In our ANN model, we have used Adam optimizer and sparse categorical cross entropy as loss function.

## 4 Datasets

We used two datasets to test the classification accuracy of different algorithms on the orignal and compressed data obtained using different sensing matrices.

### 4.1 Broadatz Texture Database

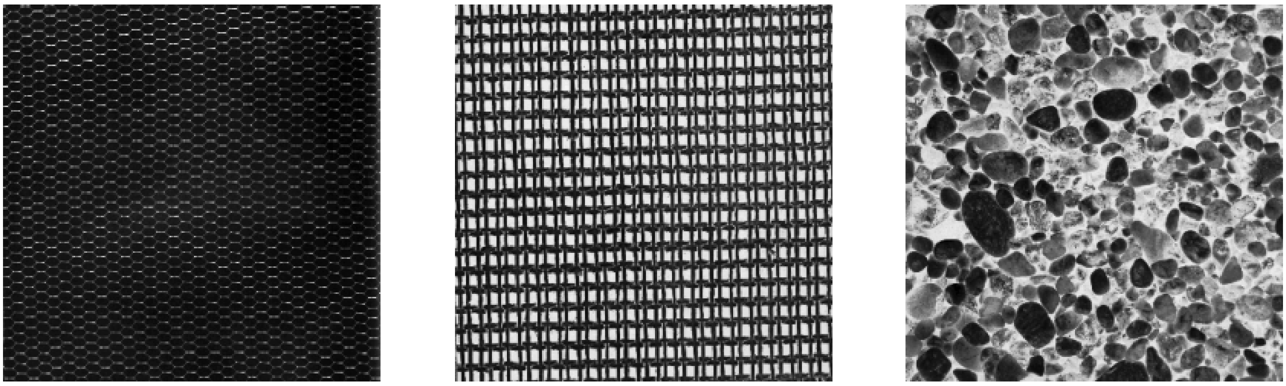

Figure 1: Images from the Broadatz dataset, namely, Horizontal, Vertical and Others

This dataset consists of 112 images of patterns which can be broadly classified into three categories: vertical, horizontal and others. Broadatz is an unlabelled dataset. The labelling method and the how the labelled dataset is created is not mentioned in the reference literature, thus we labelled the dataset manually through visual distinction of different image patterns[3].

The images are grayscale, each of dimension $320 \times 320$ pixels. We downsampled the image by a factor of two in each dimension for computational feasibility. The dataset was split into 56 images for training and 56 for testing in the literature for the SVM classification case, we used a similar split for the ANN based classification as well. The dimensions of the compressed features were $256 \times 1$.

### 4.2 CIFAR-10

The CIFAR-10 dataset consists of 10000 images of animals belonging to 10 different classes. It consists of color images of dimension $32 \times 32 \times 3$ pixels. Compressive classification on CIFAR - 10 is done to get a comparative study of the results on a different dataset than Broadatz [2].

The images were split in 9000 training images and 1000 test images for the SVM and ANN based classification method. The dimension of the compressed features were $32 \times 1$ obtained using different sensing matrices.

## 5 Results and Observations

The following table summarizes the that results were obtained after training on both the SVM classifier as well as the neural network based classifier, for different datasets. We will also observe the metric plots of training loss and accuracy for classification on the actual data and the compressed data, for each of the datasets.

Note that the compression ratio $(\frac{n}{m})$ with the Broadatz dataset was 400 times and that with the CIFAR-10 dataset was 32 times.

Let us observe the results for different datasets over methods we used for classification. Table(1) shows the accuracy on the original uncompressed data and the accuracy on the sensed data on the Broadatz dataset using the SVM and ANN based classifier. Here, we need to observe that for the ANN case the problem is data deficit and thus the model is highly likely to overfit the data with high bias towards a particular class. Also we can see that despite having a high compression ratio, we are getting decent results with the classification using SVMs as well as for the ANNs. Our focus was getting comparable results on the original and compressed data thus we didn't pay much heed to the low classification accuracy. Also due to lack of data, it is likely for the ANN based method to perform poorly.

Also, we believe that for compressive measurements to be accurately classified it is a necessity that they be sparse in the signal basis. Since texture images are less likely to be sparse in the DCT domain, thus it is probable that the combined sensing matrix composed of the Bernoulli matrix's product with the DCT signal basis might not follow RIP, and is thus not distance preserving.

| **Sens Mat** | **Classifier** | **Epochs** | **Acc (Org Data)** | **Acc (Sens Data)** |
|---|---|---|---|---|
| Bernoulli | SVM | - | 0.5 | 0.36 |
| Bernoulli | ANN | 15 | 0.37 | 0.34 |
| Delsarte-Goethal | SVM | - | 0.59 | 0.44 |
| Delsarte-Goethal | ANN | 15 | 0.37 | 0.35 |

Table 1: Table showing results of compressive classification over Broadatz

We know that it is important for the signal to be sparse in the measurement domain for compressed sensing to work. In the case of the Broadatz dataset, we were unaware of an orthogonal signal basis where such artificial texture images could be sparsely represented. Still, we know that natural images are usually sparse in the DCT domain, thus to trust the credibility of our proposed solution we performed compressive classification on the CIFAR-10 dataset, where the combined sensing matrix was the combination of the random matrix such as Bernoulli and Delsarte-Goethal frames, along with the DCT basis. Here to maintain generality between the two methods, we used a similar model architecture as we used in the Broadatz dataset for comparision. Note that here the problem is not data sparse but since our model complexity is low comparativel thus we are likely to underfit the data.

As we can clearly observe from the results in Table(2) that the accuracy on the sensed data comes very close to that of the original dataset despite the compression ratio being 32 times.

| **Sens Mat** | **Classifier** | **Epochs** | **Acc (Org Data)** | **Acc(Sens Data)** |
|---|---|---|---|---|
| Bernoulli | SVM | - | 0.39 | 0.34 |
| Bernoulli | ANN | 15 | 0.32 | 0.28 |
| Delsarte-Goethal | SVM | - | 0.39 | 0.29 |
| Delsarte-Goethal | ANN | 15 | 0.32 | 0.26 |

Table 2: Table showing results of compressive classification over CIFAR-10

In the reference literature in [1], the results have been shown as the number of predictions in each category of the Broadatz dataset. Similarly, for comparision we present the results we obtained on our custom labelled Broadatz dataset using the DG frame of required order as the sensing matrix. The results have been shown in Table(3). Similarly, the relative error between the compressed measurement classification and the original measurement classification is $\frac{|8-10|+|28-27|+|20-19|}{56} \approx 7.14\%$.

| **SVM** | **No. of horizontals** | **No. of verticals)** | **No. of others** |
|---|---|---|---|
| Measurement domain | 8 | 28 | 20 |
| Data domain | 10 | 27 | 19 |

Table 3: Table showing results of classification on Broadatz data for DF frames.

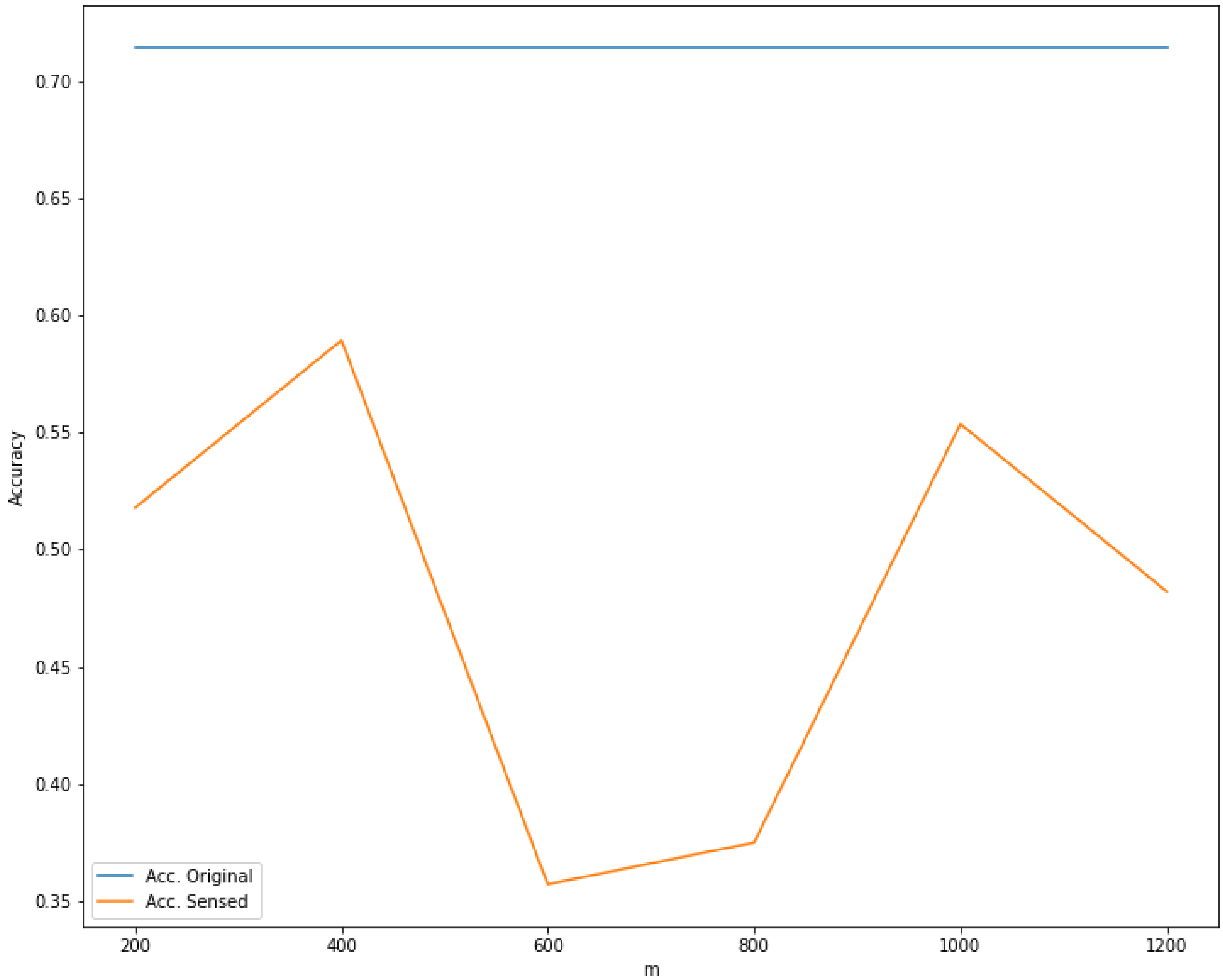


Figure 2: Figure showing the variation of classification accuracy of SVM on the Broadatz dataset in the compressed domain for different number of compressed measurements (m). Measurements obtained using the Bernoulli sensing matrix.

The figure (2) shows the plot of the accuracy of the original data along with the variation of classification accuracy with the change in the number of compressed measurements (m) for the Bernoulli sensing matrix over the Broadatz dataset. Ideally, this should be monotonically increasing if the conditions for compressed sensing and those of compressive classification are met, but as we know that texture images are aritifical images and are thus less likely to be sparse in the DCT basis. Thus our overall sensing matrix which a product of the Bernoulli sensing matrix and the DcT basis is not likely to follow RIP. If the sensing matrix doesn't follow RIP, then it is not distance preserving in the compressed domain, and thus for increasing number of measurements there is no certainity about the behaviour of the classification algorithms on the compressed measurements.

In figure(3), due to the inability of the sensing matrix obtained as a product of Bernoulli random matrix along with the DCT coefficients to follow RIP, there is no consistent trend that the classification accuracy for the compressed data follows with respect to the original data. Furthermore, due to the task being data-deficit for the Broadatz case, the classification accuracy on the original data is also very unstable since the batch size is 1 (stochastic gradient descent).

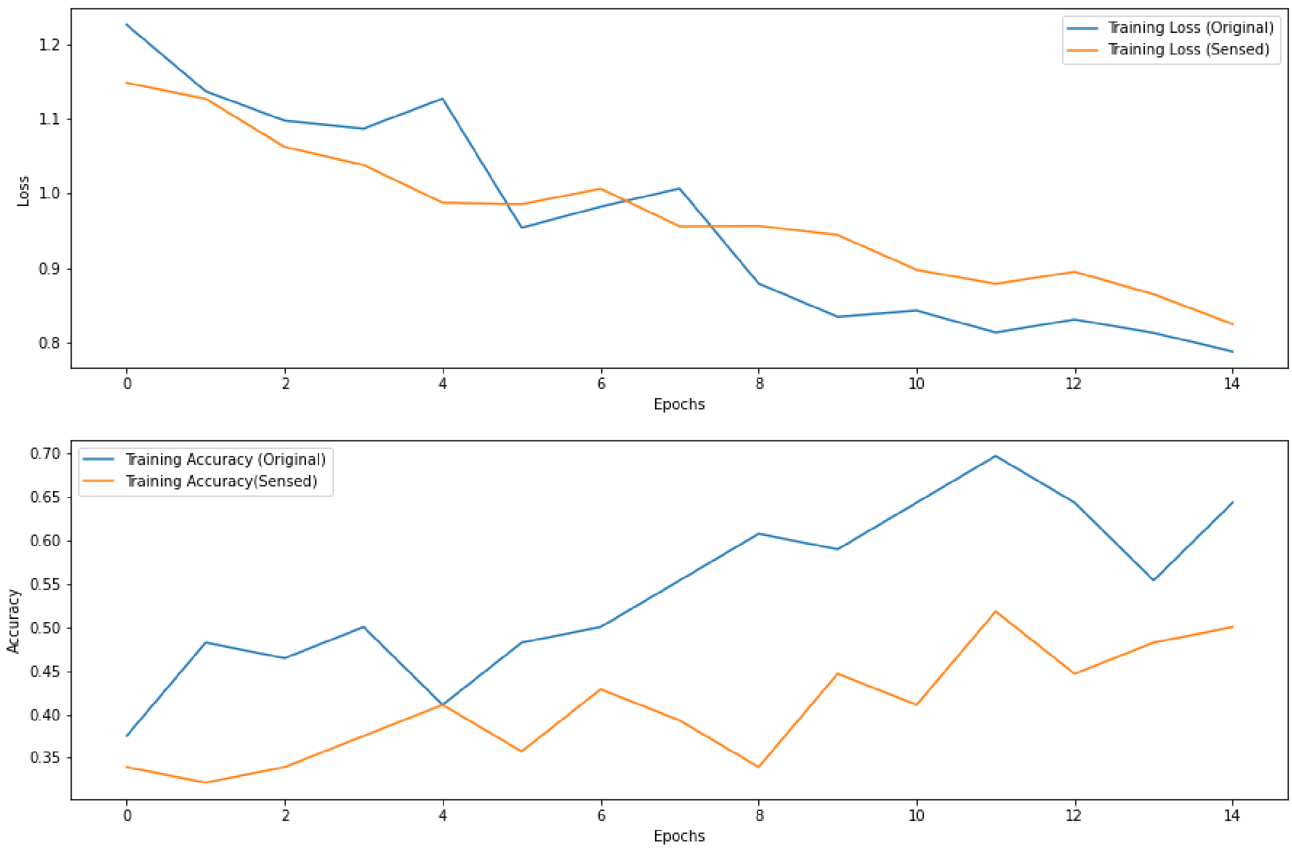


Figure 3: Figure showing the training accuracy of the ANN classification method on compressive measurements of the Broadatz dataset for 15 training epochs. Measurements obtained using the Bernoulli sensing matrix.

In the case of the classification accuracies as seen in figure(4), when the sensing matrix is product of the Delsarte-Goethal frame of order 11 with the DCT basis, we see the training accuracy for the sensed case is often less than that obtained on the original data. Thus comparatively for the DCT basis, the DG frame of order 11 is more distance preserving than Bernoulli, as can be inferred from the graph.

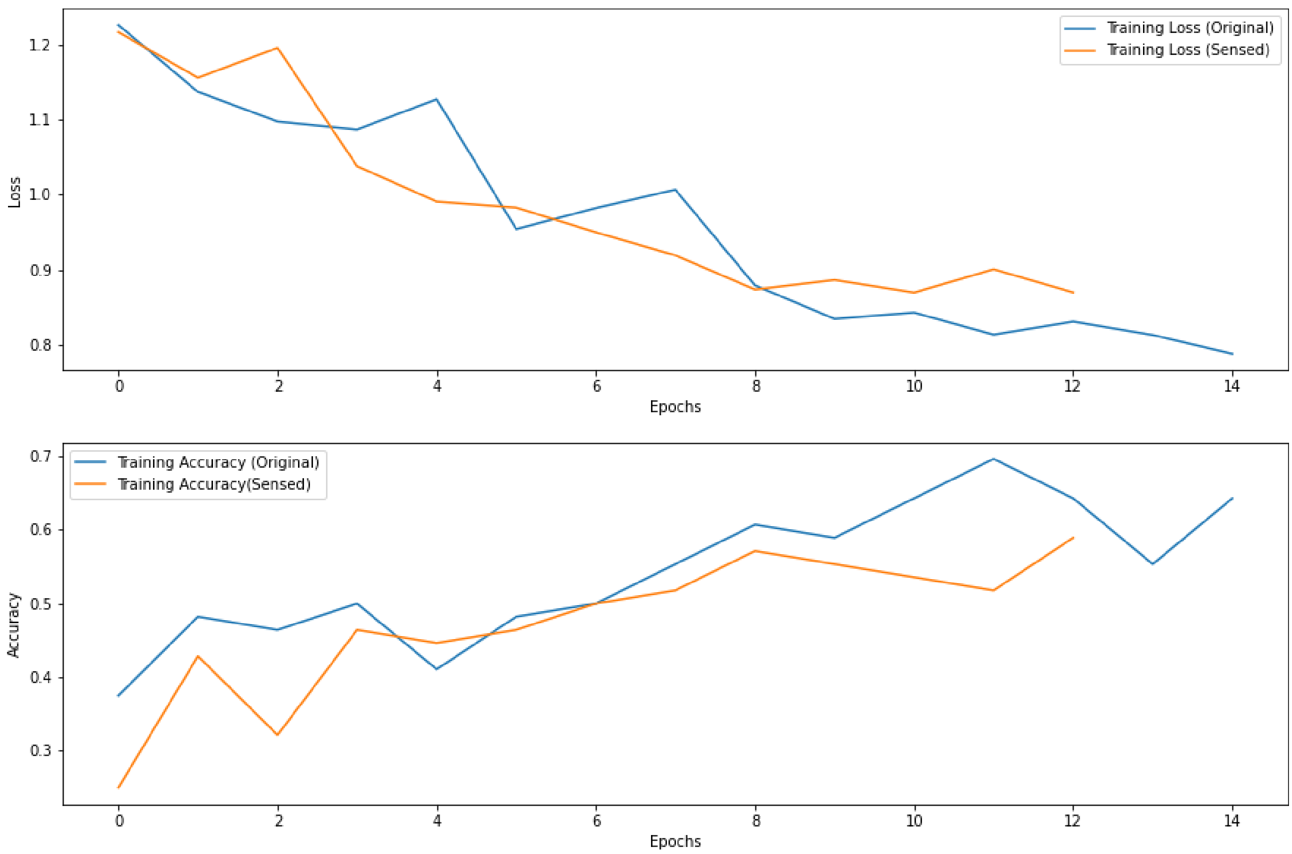


Figure 4: Figure showing the training accuracy of the ANN classification method on compressive measurements of the Broadatz dataset for 15 training epochs. Measurements obtained using the Delsarte-Goethal frame of order 11 as sensing matrix.

The figure(5), shows the plot of the training accuracy over the CIFAR-10 dataset with the sensing matrix as a product of the Bernoulli random matrix and the DCT basis. Since natural images are sparse in the DCT basis, thus it is likely for the sensing matrix in this case to follow RIP, and be distance preserving. Thus, we see a consistent trend of the classification accuracy in the sensed domain with the classification accuracy in the original domain.

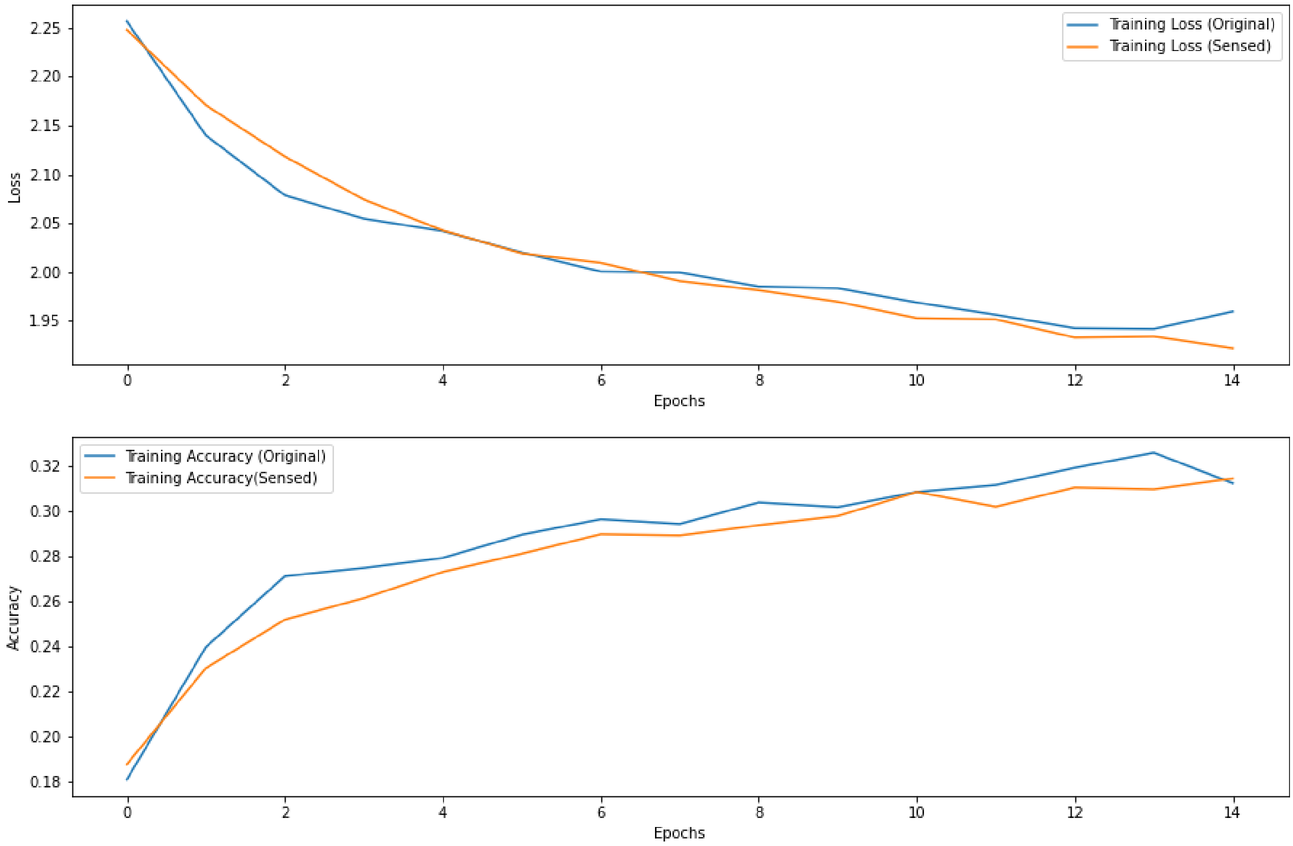


Figure 5: Figure showing the training accuracy of the ANN classification method on compressive measurements of the CIFAR-10 dataset for 15 training epochs. Measurements obtained using the Bernoulli sensing matrix.

We observe a similar trend of the classification accuracy in the measurement domain and the original domain for the case when the sensing matrix is a DG frame of order 5 over the CIFAR -10 dataset.

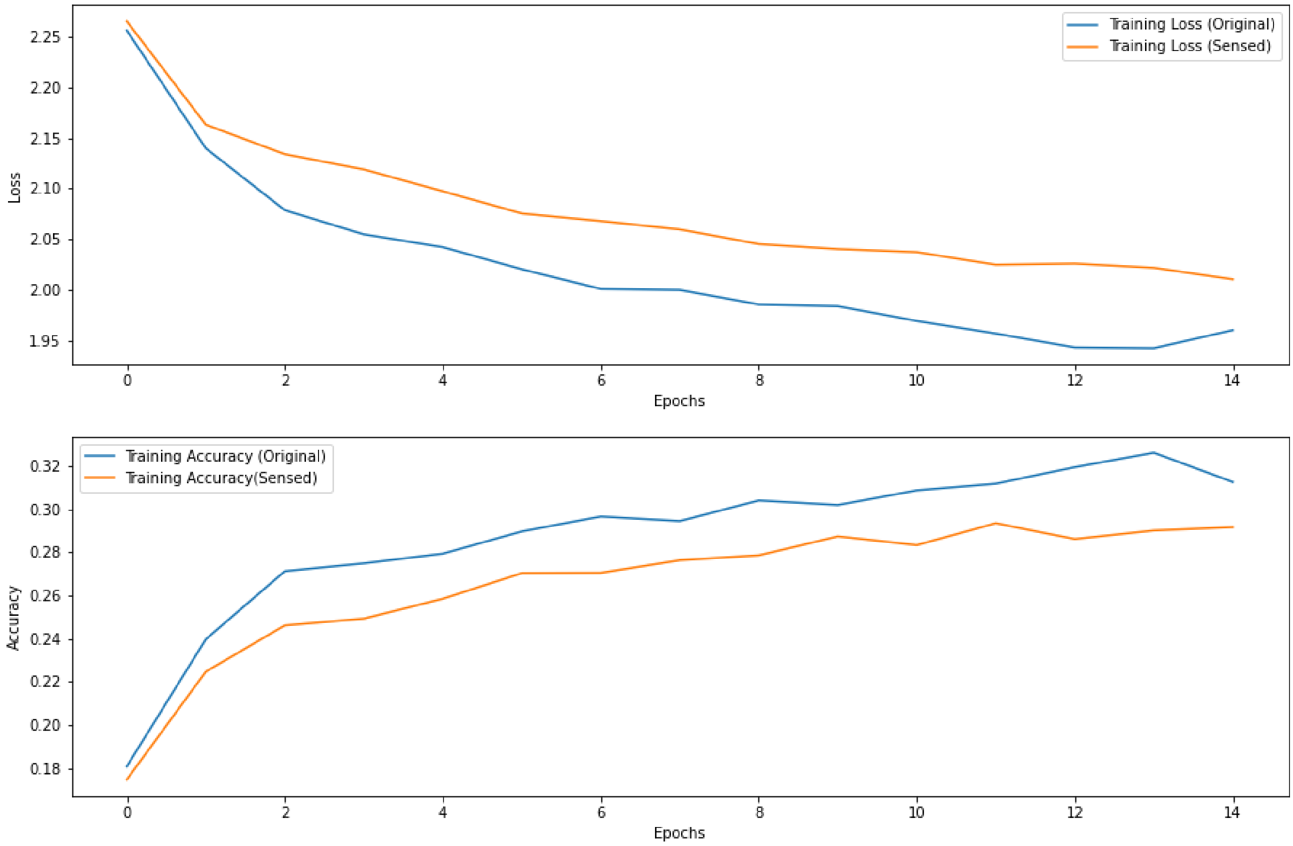


Figure 6: Figure showing the training accuracy of the ANN classification method on compressive measurements of the CIFAR-10 dataset for 15 training epochs. Measurements obtained using the Delsarte-Goethal frame of order 5 as sensing matrix.

# References

[1] Robert Calderbank and Sina Jafarpour. Finding needles in compressed haystacks. In *2012 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, pages 3441–3444. IEEE, 2012.

[2] University of toronto. CIFAR10 dataset. https://www.cs.toronto.edu/ kriz/cifar.html. [Online; accessed 30-April-2022].

[3] SIPI. Broadatz dataset. https://sipi.usc.edu/database/database.php?volume=textures. [Online; accessed 20-April-2022].