Motion of a Charged Particle in Dipole Field
Computer Project

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1 Introduction

Imagine that you have a "point dipole" at the origin $\vec{p} = p_0\hat{z}$. Assume that this dipole is not allowed to move. A positive point charge q is released from rest at a point in the xy plane. What happens to it? That is what you are going to investigate here.

2 Part I: Just Think....

To imagine what would happen, it is important to visualize the electric field of the dipole at origin.

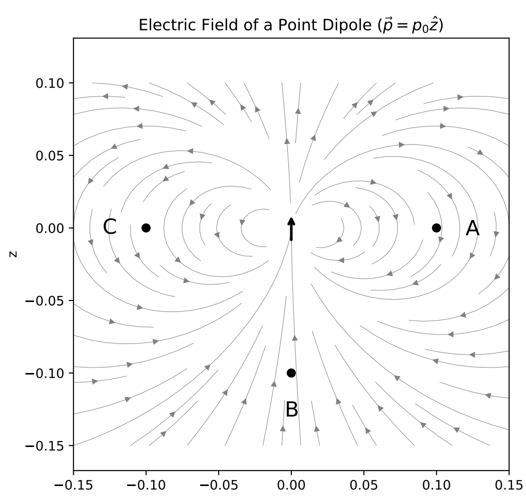

Figure 1: Electric field lines of a point dipole oriented along the z-axis.

If a positive test charge was released at A at rest, it would follow the electric field lines to reach B. At B it has inertia and would keep moving to C. In the absence of resistive forces it would reach C perfectly, where it would repeat the path, oscillating for eternity. Thus my intuition suggests some kind of oscillation.

3 Part II: Numerical Calculation of the Motion

The analytical solution involves calculating electric field due to the dipole, the force on the charge and consecutively the motion.

We know electric field due to a dipole is:

$$\vec{E} = \frac{1}{4\pi\epsilon_0 r^3} \cdot (3(\vec{p}\cdot\hat{r})\hat{r} \; - \; \vec{p}) \tag{1}$$

Here $\hat{r}$ is $\frac{y\hat{y}+z\hat{z}}{\sqrt{y^2+z^2}}$. 3D motion is not considered in this project, thus all motion will be restricted to the yz plane. Denoting $\sqrt{y^2+z^2}$ as r leads us to these equations for the electric field:

$$\vec{E_y} = \frac{3p_0 y}{4\pi\epsilon_0 r^3}\hat{y} \tag{2}$$

$$\vec{E_z} = \frac{p_0(3z^2 - r^2)}{4\pi\epsilon_0 r^5}\hat{z} \tag{3}$$

The acceleration of the charge was calculated using $\vec{a} = \frac{q\vec{E}}{m}$. Numerically for each time t in a series of time frames, the acceleration $\vec{a}$ was calculated, the velocity vector $\vec{v}$ was calculated by adding $\vec{a}dt$. The distance vector was also updated by $\vec{v}dt$. Thus beginning from initial conditions, it is quite easy to use a looping Python code to get a trajectory.

With mass $m = 10^{-6}kg$ and dipole moment $p_0 = 10^{-9}Cm$, this is the trajectory for the charge $q = 10^{-9}C$ at $y = 0.1m$ and $z = 0$ with no initial velocity.

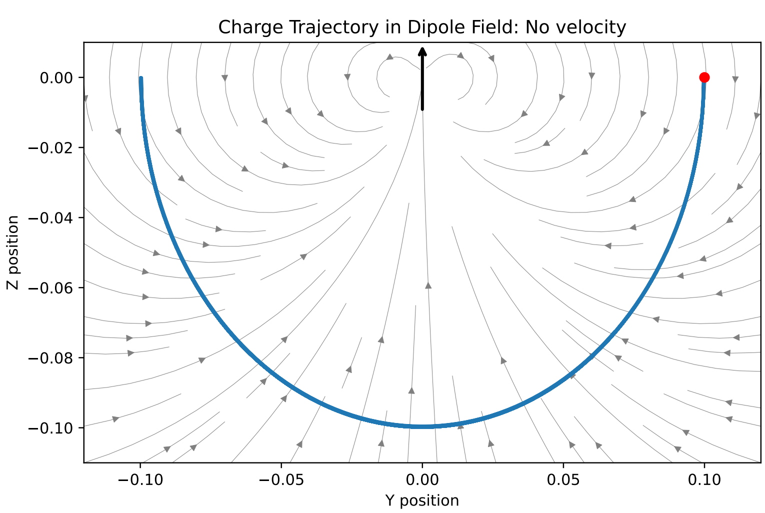

Figure 2: Charge trajectory for $p_0 = 10^{-9}Cm$.

If the dipole moment increases 10 times to $p_0 = 10 \cdot 10^{-9} Cm$. The result is unexpected. The charge still oscillates but has a radial velocity away from the dipole. This is very initially strange and interesting. The inference is that the field lines are not perfectly spherical, which now is obvious. It has a tiny net radial component as well.

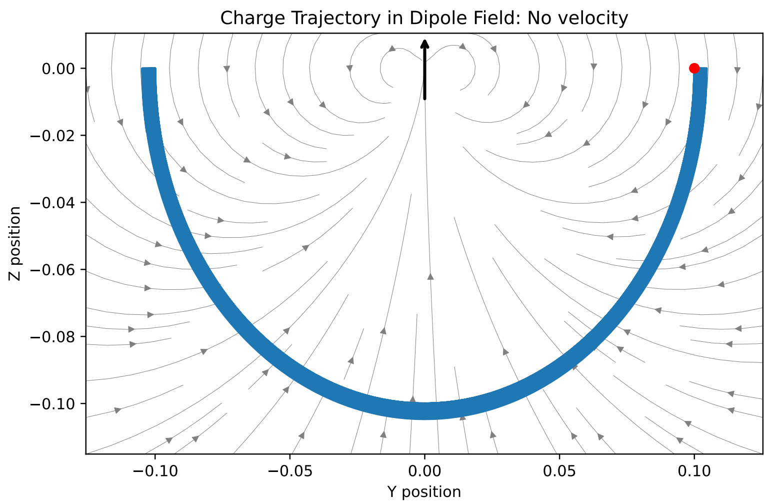

Figure 3: Charge trajectory for $p_0 = 10 \cdot 10^{-9} Cm$.

There are some enlightening trajectories as well, especially when the initial velocities are perturbed.

When there is an inward y-velocity, it initially travels inward, increasing the angular velocity. But the centripetal acceleration is no longer enough to keep the charge at the same distance, and thus it springs outward.

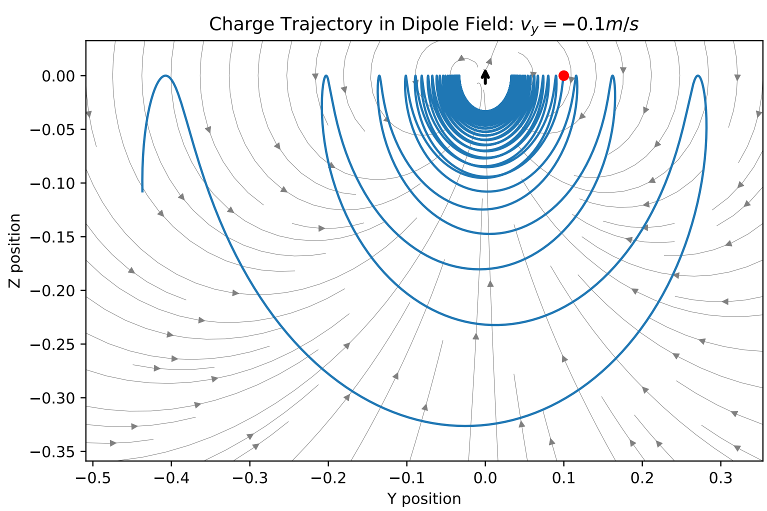

Figure 4: Charge trajectory for $v_y = -0.1 m/s$.

The result is as expected in case of outward y-velocity.

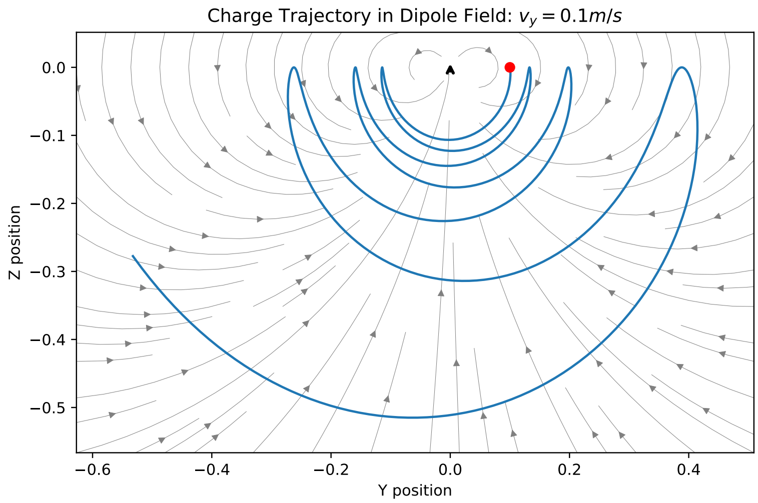

Figure 5: Charge trajectory for $v_y = 0.1m/s$.

If the velocity is too high, it escapes without too many oscillations.

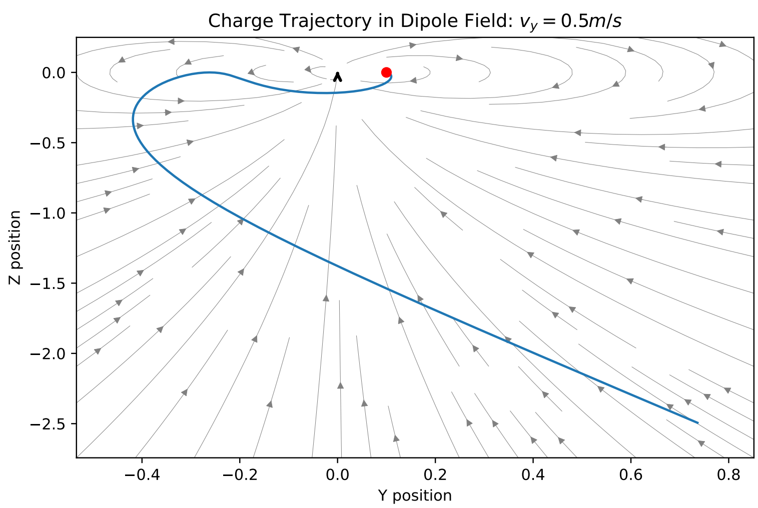

Figure 6: Charge trajectory for $v_y = 0.5m/s$.

The results are as expected with outward z-velocity.

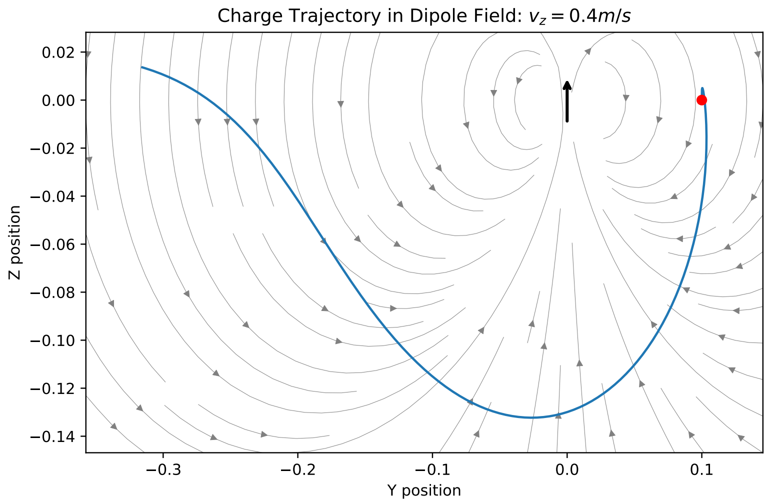

Figure 7: Charge trajectory for $v_z = 0.4m/s$.

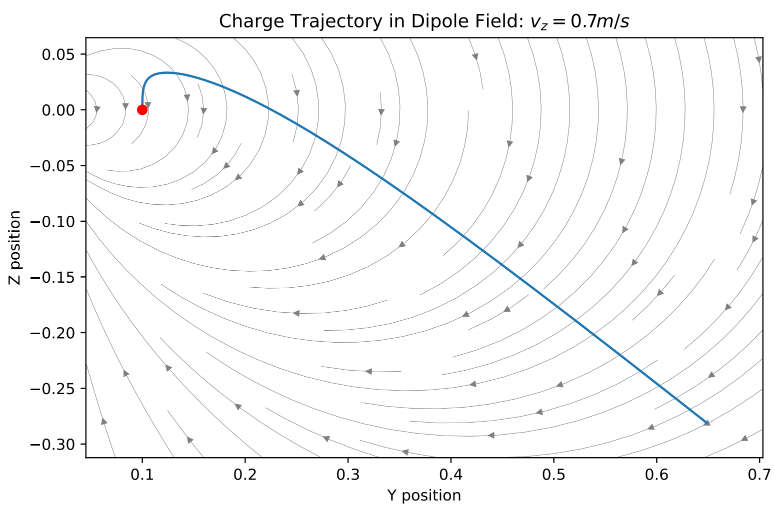

Figure 8: Charge trajectory for $v_z = 0.7m/s$.

4 Conclusion

This brings us to the end of a very insightful project. Newton's laws are thus at least computationally true for classical physics. The solution methodology is quite primitive, with Python iterating the unknowns over and over again. But in each step, I used a little bit of Physics, which is beautiful. Often analytically we solve these equations, but hardly every visualize them. With this project, I have a newfound tool in my tool box along with theory.

I want to thank Prof. Fulvio Melia for this intriguing project, and Sir Timothy John Berners-Lee for creating the WWW and internet of course.