Kelvin-Helmholtz setup

We consider a shear layer and assume that there is some characteristic total velocity difference $2V_0$, with characteristic shear-layer depth $2d_0$, and total density difference $2\rho_0$. Choosing the length-scale d_0, density-scale ρ_0 and time-scale d_0/V_0 the governing equations are

$$\frac{D\boldsymbol{u}}{Dt} = -\nabla p - Ri\rho\hat{\boldsymbol{z}} + \frac{1}{Re}\Delta\boldsymbol{u},$$
$$\frac{D\rho}{Dt} = \frac{1}{RePr}\Delta\rho,$$
$$\nabla\cdot\boldsymbol{u} = 0,$$

where $Re = d_0V_0/\nu, Pr = \nu/\kappa$ and the bulk Richardson number $Ri = \frac{g\delta\rho d_0}{\rho_0 V_0^2}$. Consider a horizontally periodic box of length $[0, 4\pi]$ in the spanwise/streamwise directions and height $[-\pi, \pi]$ in the vertical. Following (Winters et al., 1995) we set $Pr = 1, Re = 10^4$ and $Ri = 0.1$ while boundary conditions in the vertical are stress-free for velocity and Dirichlet for the density field. As initial conditions we choose:

$$u(z) = \tanh(z/d), \qquad \rho(z) = tanh(Rz/d),$$

to which we add the velocity perturbation

$$\psi = exp(-z^2/d^2)[\cos(8\pi x) + \cos(20\pi x)]$$

such that

$$\boldsymbol{u} = u(z)\hat{\boldsymbol{x}} + \epsilon\,(\partial_z\psi, -\partial_x\psi)$$

In computations we choose $d = 1/28$ and $\epsilon = 0.05$. The system is solved in two dimensions using a Fourier-Chebyshev psuedo-spectral method on a $N_x, N_z = 1024, 512$ grid with 3/2 dealiasing. A stiffly stable RK443 time-stepper is used with $\Delta t_{max} = 5e-03$