

1 Pure Mathematics 1

1.2 Functions

Candidates should be able to:

- understand the terms function, domain, range, one-one function, inverse function and composition of functions
- identify the range of a given function in simple cases, and find the composition of two given functions
- determine whether or not a given function is one-one, and find the inverse of a one-one function in simple cases
- illustrate in graphical terms the relation between a one-one function and its inverse
- understand and use the transformations of the graph of $y = f(x)$ given by
 $y = f(x) + a$, $y = f(x + a)$,
 $y = af(x)$, $y = f(ax)$ and simple combinations of these.

Notes and examples

e.g. range of $f : x \mapsto \frac{1}{x}$ for $x \geq 1$ and

range of $g : x \mapsto x^2 + 1$ for $x \in \mathbb{R}$. Including the condition that a composite function gf can only be formed when the range of f is within the domain of g .

e.g. finding the inverse of

$$h : x \mapsto (2x + 3)^2 - 4 \text{ for } x < -\frac{3}{2}.$$

Sketches should include an indication of the mirror line $y = x$.

Including use of the terms 'translation', 'reflection' and 'stretch' in describing transformations. Questions may involve algebraic or trigonometric functions, or other graphs with given features.

1.3 Coordinate geometry

Candidates should be able to:

- find the equation of a straight line given sufficient information
- interpret and use any of the forms $y = mx + c$, $y - y_1 = m(x - x_1)$, $ax + by + c = 0$ in solving problems
- understand that the equation $(x - a)^2 + (y - b)^2 = r^2$ represents the circle with centre (a, b) and radius r
- use algebraic methods to solve problems involving lines and circles
- understand the relationship between a graph and its associated algebraic equation, and use the relationship between points of intersection of graphs and solutions of equations.

Notes and examples

e.g. given two points, or one point and the gradient.

Including calculations of distances, gradients, midpoints, points of intersection and use of the relationship between the gradients of parallel and perpendicular lines.

Including use of the expanded form $x^2 + y^2 + 2gx + 2fy + c = 0$.

Including use of elementary geometrical properties of circles, e.g. tangent perpendicular to radius, angle in a semicircle, symmetry.

Implicit differentiation is not included.

e.g. to determine the set of values of k for which the line $y = x + k$ intersects, touches or does not meet a quadratic curve.

1 Pure Mathematics 1

1.4 Circular measure

Candidates should be able to:

- understand the definition of a radian, and use the relationship between radians and degrees
- use the formulae $s = r\theta$ and $A = \frac{1}{2}r^2\theta$ in solving problems concerning the arc length and sector area of a circle.

Notes and examples

Including calculation of lengths and angles in triangles and areas of triangles.

1.5 Trigonometry

Candidates should be able to:

- sketch and use graphs of the sine, cosine and tangent functions (for angles of any size, and using either degrees or radians)
- use the exact values of the sine, cosine and tangent of 30° , 45° , 60° , and related angles
- use the notations $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$ to denote the principal values of the inverse trigonometric relations
- use the identities $\frac{\sin \theta}{\cos \theta} \equiv \tan \theta$ and $\sin^2 \theta + \cos^2 \theta \equiv 1$
- find all the solutions of simple trigonometrical equations lying in a specified interval (general forms of solution are not included).

Notes and examples

Including e.g. $y = 3 \sin x$, $y = 1 - \cos 2x$,
 $y = \tan\left(x + \frac{1}{4}\pi\right)$.

e.g. $\cos 150^\circ = -\frac{1}{2}\sqrt{3}$, $\sin \frac{3}{4}\pi = \frac{1}{\sqrt{2}}$.

No specialised knowledge of these functions is required, but understanding of them as examples of inverse functions is expected.

e.g. in proving identities, simplifying expressions and solving equations.

e.g. solve $3 \sin 2x + 1 = 0$ for $-\pi < x < \pi$,
 $3 \sin^2 \theta - 5 \cos \theta - 1 = 0$ for $0^\circ \leq \theta \leq 360^\circ$.

1.6 Series

Candidates should be able to:

- use the expansion of $(a + b)^n$, where n is a positive integer
- recognise arithmetic and geometric progressions
- use the formulae for the n th term and for the sum of the first n terms to solve problems involving arithmetic or geometric progressions
- use the condition for the convergence of a geometric progression, and the formula for the sum to infinity of a convergent geometric progression.

Notes and examples

Including the notations $\binom{n}{r}$ and $n!$

Knowledge of the greatest term and properties of the coefficients are not required.

Including knowledge that numbers a, b, c are 'in arithmetic progression' if $2b = a + c$ (or equivalent) and are 'in geometric progression' if $b^2 = ac$ (or equivalent).

Questions may involve more than one progression.

1 Pure Mathematics 1

1.7 Differentiation

Candidates should be able to:

- understand the gradient of a curve at a point as the limit of the gradients of a suitable sequence of chords, and use the notations $f'(x)$, $f''(x)$, $\frac{dy}{dx}$, and $\frac{d^2y}{dx^2}$ for first and second derivatives
- use the derivative of x^n (for any rational n), together with constant multiples, sums and differences of functions, and of composite functions using the chain rule
- apply differentiation to gradients, tangents and normals, increasing and decreasing functions and rates of change
- locate stationary points and determine their nature, and use information about stationary points in sketching graphs.

Notes and examples

Only an informal understanding of the idea of a limit is expected.

e.g. includes consideration of the gradient of the chord joining the points with x coordinates 2 and $(2 + h)$ on the curve $y = x^3$. Formal use of the general method of differentiation from first principles is not required.

e.g. find $\frac{dy}{dx}$, given $y = \sqrt{2x^3 + 5}$.

Including connected rates of change, e.g. given the rate of increase of the radius of a circle, find the rate of increase of the area for a specific value of one of the variables.

Including use of the second derivative for identifying maxima and minima; alternatives may be used in questions where no method is specified.

Knowledge of points of inflexion is not included.

1.8 Integration

Candidates should be able to:

- understand integration as the reverse process of differentiation, and integrate $(ax + b)^n$ (for any rational n except -1), together with constant multiples, sums and differences
- solve problems involving the evaluation of a constant of integration
- evaluate definite integrals
- use definite integration to find
 - the area of a region bounded by a curve and lines parallel to the axes, or between a curve and a line or between two curves
 - a volume of revolution about one of the axes.

Notes and examples

e.g. $\int (2x^3 - 5x + 1)dx$, $\int \frac{1}{(2x + 3)^2} dx$.

e.g. to find the equation of the curve through $(1, -2)$ for which $\frac{dy}{dx} = \sqrt{2x + 1}$.

Including simple cases of 'improper' integrals, such as $\int_0^1 x^{-\frac{1}{2}} dx$ and $\int_1^\infty x^{-2} dx$.

A volume of revolution may involve a region not bounded by the axis of rotation, e.g. the region between $y = 9 - x^2$ and $y = 5$ rotated about the x -axis.

2 Pure Mathematics 2 (for Paper 2)

Knowledge of the content for Paper 1: Pure Mathematics 1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions.

2.1 Algebra

Candidates should be able to:

- understand the meaning of $|x|$, sketch the graph of $y = |ax + b|$ and use relations such as $|a| = |b| \Leftrightarrow a^2 = b^2$ and $|x - a| < b \Leftrightarrow a - b < x < a + b$ when solving equations and inequalities
- divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero)
- use the factor theorem and the remainder theorem.

Notes and examples

Graphs of $y = |f(x)|$ and $y = f(|x|)$ for non-linear functions f are not included.

e.g. $|3x - 2| = |2x + 7|$, $2x + 5 < |x + 1|$

e.g. to find factors and remainders, solve polynomial equations or evaluate unknown coefficients.

Including factors of the form $(ax + b)$ in which the coefficient of x is not unity, and including calculation of remainders.

2.2 Logarithmic and exponential functions

Candidates should be able to:

- understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base)
- understand the definition and properties of e^x and $\ln x$, including their relationship as inverse functions and their graphs
- use logarithms to solve equations and inequalities in which the unknown appears in indices
- use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept.

Notes and examples

Including knowledge of the graph of $y = e^{kx}$ for both positive and negative values of k .

e.g. $2^x < 5$, $3 \times 2^{3x-1} < 5$, $3^{x+1} = 4^{2x-1}$.

e.g.

$y = kx^n$ gives $\ln y = \ln k + n \ln x$ which is linear in $\ln x$ and $\ln y$

$y = k(a^x)$ gives $\ln y = \ln k + x \ln a$ which is linear in x and $\ln y$.

2 Pure Mathematics 2

2.3 Trigonometry

Candidates should be able to:

- understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude
- use trigonometrical identities for the simplification and exact evaluation of expressions, and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of
 - $\sec^2 \theta \equiv 1 + \tan^2 \theta$ and $\operatorname{cosec}^2 \theta \equiv 1 + \cot^2 \theta$
 - the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$
 - the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$
 - the expression of $a \sin \theta + b \cos \theta$ in the forms $R \sin(\theta \pm \alpha)$ and $R \cos(\theta \pm \alpha)$.

Notes and examples

e.g. simplifying $\cos(x - 30^\circ) - 3 \sin(x - 60^\circ)$.

e.g. solving $\tan \theta + \cot \theta = 4$, $2 \sec^2 \theta - \tan \theta = 5$, $3 \cos \theta + 2 \sin \theta = 1$.

2.4 Differentiation

Candidates should be able to:

- use the derivatives of e^x , $\ln x$, $\sin x$, $\cos x$, $\tan x$, together with constant multiples, sums, differences and composites
- differentiate products and quotients
- find and use the first derivative of a function which is defined parametrically or implicitly.

Notes and examples

e.g. $\frac{2x-4}{3x+2}$, $x^2 \ln x$, $x e^{1-x^2}$.

e.g. $x = t - e^{2t}$, $y = t + e^{2t}$.

e.g. $x^2 + y^2 = xy + 7$.

Including use in problems involving tangents and normals.

2.5 Integration

Candidates should be able to:

- extend the idea of 'reverse differentiation' to include the integration of e^{ax+b} , $\frac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$ and $\sec^2(ax+b)$
- use trigonometrical relationships in carrying out integration
- understand and use the trapezium rule to estimate the value of a definite integral.

Notes and examples

Knowledge of the general method of integration by substitution is not required.

e.g. use of double-angle formulae to integrate $\sin^2 x$ or $\cos^2(2x)$.

Including use of sketch graphs in simple cases to determine whether the trapezium rule gives an over-estimate or an under-estimate.

2 Pure Mathematics 2

2.6 Numerical solution of equations

Candidates should be able to:

- locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change
- understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation
- understand how a given simple iterative formula of the form $x_{n+1} = F(x_n)$ relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy.

Notes and examples

e.g. finding a pair of consecutive integers between which a root lies.

Knowledge of the condition for convergence is not included, but an understanding that an iteration may fail to converge is expected.

3 Pure Mathematics 3 (for Paper 3)

Knowledge of the content of Paper 1: Pure Mathematics 1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions.

3.1 Algebra

Candidates should be able to:

- understand the meaning of $|x|$, sketch the graph of $y = |ax + b|$ and use relations such as $|a| = |b| \Leftrightarrow a^2 = b^2$ and $|x - a| < b \Leftrightarrow a - b < x < a + b$ when solving equations and inequalities
- divide a polynomial, of degree not exceeding 4, by a linear or quadratic polynomial, and identify the quotient and remainder (which may be zero)
- use the factor theorem and the remainder theorem
- recall an appropriate form for expressing rational functions in partial fractions, and carry out the decomposition, in cases where the denominator is no more complicated than
 - $(ax + b)(cx + d)(ex + f)$
 - $(ax + b)(cx + d)^2$
 - $(ax + b)(cx^2 + d)$
- use the expansion of $(1 + x)^n$, where n is a rational number and $|x| < 1$.

Notes and examples

Graphs of $y = |f(x)|$ and $y = f(|x|)$ for non-linear functions f are not included.

e.g. $|3x - 2| = |2x + 7|$, $2x + 5 < |x + 1|$.

e.g. to find factors and remainders, solve polynomial equations or evaluate unknown coefficients.

Including factors of the form $(ax + b)$ in which the coefficient of x is not unity, and including calculation of remainders.

Excluding cases where the degree of the numerator exceeds that of the denominator

Finding the general term in an expansion is not included.

Adapting the standard series to expand e.g. $(2 - \frac{1}{2}x)^{-1}$ is included, and determining the set of values of x for which the expansion is valid in such cases is also included.

3 Pure Mathematics 3

3.2 Logarithmic and exponential functions

Candidates should be able to:

- understand the relationship between logarithms and indices, and use the laws of logarithms (excluding change of base)
- understand the definition and properties of e^x and $\ln x$, including their relationship as inverse functions and their graphs
- use logarithms to solve equations and inequalities in which the unknown appears in indices
- use logarithms to transform a given relationship to linear form, and hence determine unknown constants by considering the gradient and/or intercept.

Notes and examples

Including knowledge of the graph of $y = e^{kx}$ for both positive and negative values of k .

e.g. $2^x < 5$, $3 \times 2^{3x-1} < 5$, $3^{x+1} = 4^{2x-1}$.

e.g.

$y = kx^n$ gives $\ln y = \ln k + n \ln x$ which is linear in $\ln x$ and $\ln y$.

$y = k(a^x)$ gives $\ln y = \ln k + x \ln a$ which is linear in x and $\ln y$.

3.3 Trigonometry

Candidates should be able to:

- understand the relationship of the secant, cosecant and cotangent functions to cosine, sine and tangent, and use properties and graphs of all six trigonometric functions for angles of any magnitude
- use trigonometrical identities for the simplification and exact evaluation of expressions, and in the course of solving equations, and select an identity or identities appropriate to the context, showing familiarity in particular with the use of
 - $\sec^2 \theta \equiv 1 + \tan^2 \theta$ and $\operatorname{cosec}^2 \theta \equiv 1 + \cot^2 \theta$
 - the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$
 - the formulae for $\sin 2A$, $\cos 2A$ and $\tan 2A$
 - the expression of $a \sin \theta + b \cos \theta$ in the forms $R \sin(\theta \pm \alpha)$ and $R \cos(\theta \pm \alpha)$.

Notes and examples

e.g. simplifying $\cos(x - 30^\circ) - 3 \sin(x - 60^\circ)$.

e.g. solving $\tan \theta + \cot \theta = 4$, $2 \sec^2 \theta - \tan \theta = 5$, $3 \cos \theta + 2 \sin \theta = 1$.

3 Pure Mathematics 3

3.4 Differentiation

Candidates should be able to:

- use the derivatives of e^x , $\ln x$, $\sin x$, $\cos x$, $\tan x$, $\tan^{-1} x$, together with constant multiples, sums, differences and composites
- differentiate products and quotients
- find and use the first derivative of a function which is defined parametrically or implicitly.

Notes and examples

Derivatives of $\sin^{-1} x$ and $\cos^{-1} x$ are not required.

e.g. $\frac{2x-4}{3x+2}$, $x^2 \ln x$, xe^{1-x^2} .

e.g. $x = t - e^{2t}$, $y = t + e^{2t}$.

e.g. $x^2 + y^2 = xy + 7$.

Including use in problems involving tangents and normals.

3.5 Integration

Candidates should be able to:

- extend the idea of 'reverse differentiation' to include the integration of e^{ax+b} , $\frac{1}{ax+b}$, $\sin(ax+b)$, $\cos(ax+b)$, $\sec^2(ax+b)$ and $\frac{1}{x^2+a^2}$
- use trigonometrical relationships in carrying out integration
- integrate rational functions by means of decomposition into partial fractions
- recognise an integrand of the form $\frac{kf'(x)}{f(x)}$, and integrate such functions
- recognise when an integrand can usefully be regarded as a product, and use integration by parts
- use a given substitution to simplify and evaluate either a definite or an indefinite integral.

Notes and examples

Including examples such as $\frac{1}{2+3x^2}$.

e.g. use of double-angle formulae to integrate $\sin^2 x$ or $\cos^2(2x)$.

Restricted to types of partial fractions as specified in topic 3.1 above.

e.g. integration of $\frac{x}{x^2+1}$, $\tan x$.

e.g. integration of $x \sin 2x$, $x^2 e^{-x}$, $\ln x$, $x \tan^{-1} x$.

e.g. to integrate $\sin^2 2x \cos x$ using the substitution $u = \sin x$.

3 Pure Mathematics 3

3.6 Numerical solution of equations

Candidates should be able to:

- locate approximately a root of an equation, by means of graphical considerations and/or searching for a sign change
- understand the idea of, and use the notation for, a sequence of approximations which converges to a root of an equation
- understand how a given simple iterative formula of the form $x_{n+1} = F(x_n)$ relates to the equation being solved, and use a given iteration, or an iteration based on a given rearrangement of an equation, to determine a root to a prescribed degree of accuracy.

Notes and examples

e.g. finding a pair of consecutive integers between which a root lies.

Knowledge of the condition for convergence is not included, but an understanding that an iteration may fail to converge is expected.

3.7 Vectors

Candidates should be able to:

- use standard notations for vectors, i.e.

$$\begin{pmatrix} x \\ y \end{pmatrix}, x\mathbf{i} + y\mathbf{j}, \begin{pmatrix} x \\ y \\ z \end{pmatrix}, x\mathbf{i} + y\mathbf{j} + z\mathbf{k}, \overrightarrow{AB}, \mathbf{a}$$

- carry out addition and subtraction of vectors and multiplication of a vector by a scalar, and interpret these operations in geometrical terms
- calculate the magnitude of a vector, and use unit vectors, displacement vectors and position vectors
- understand the significance of all the symbols used when the equation of a straight line is expressed in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, and find the equation of a line, given sufficient information
- determine whether two lines are parallel, intersect or are skew, and find the point of intersection of two lines when it exists
- use formulae to calculate the scalar product of two vectors, and use scalar products in problems involving lines and points.

Notes and examples

e.g. 'OABC is a parallelogram' is equivalent to $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{OC}$.

The general form of the ratio theorem is not included, but understanding that the midpoint of AB has position vector $\frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})$ is expected.

In 2 or 3 dimensions.

e.g. finding the equation of a line given the position vector of a point on the line and a direction vector, or the position vectors of two points on the line.

Calculation of the shortest distance between two skew lines is not required. Finding the equation of the common perpendicular to two skew lines is also not required.

e.g. finding the angle between two lines, and finding the foot of the perpendicular from a point to a line; questions may involve 3D objects such as cuboids, tetrahedra (pyramids), etc.

Knowledge of the vector product is not required.

3 Pure Mathematics 3

3.8 Differential equations

Candidates should be able to:

- formulate a simple statement involving a rate of change as a differential equation
- find by integration a general form of solution for a first order differential equation in which the variables are separable
- use an initial condition to find a particular solution
- interpret the solution of a differential equation in the context of a problem being modelled by the equation.

Notes and examples

The introduction and evaluation of a constant of proportionality, where necessary, is included.

Including any of the integration techniques from topic 3.5 above.

Where a differential equation is used to model a 'real-life' situation, no specialised knowledge of the context will be required.

3.9 Complex numbers

Candidates should be able to:

- understand the idea of a complex number, recall the meaning of the terms real part, imaginary part, modulus, argument, conjugate, and use the fact that two complex numbers are equal if and only if both real and imaginary parts are equal
- carry out operations of addition, subtraction, multiplication and division of two complex numbers expressed in Cartesian form $x + iy$
- use the result that, for a polynomial equation with real coefficients, any non-real roots occur in conjugate pairs
- represent complex numbers geometrically by means of an Argand diagram
- carry out operations of multiplication and division of two complex numbers expressed in polar form $r(\cos \theta + i \sin \theta) \equiv re^{i\theta}$
- find the two square roots of a complex number
- understand in simple terms the geometrical effects of conjugating a complex number and of adding, subtracting, multiplying and dividing two complex numbers
- illustrate simple equations and inequalities involving complex numbers by means of loci in an Argand diagram

Notes and examples

Notations $\operatorname{Re} z$, $\operatorname{Im} z$, $|z|$, $\arg z$, z^* should be known. The argument of a complex number will usually refer to an angle θ such that $-\pi < \theta \leq \pi$, but in some cases the interval $0 \leq \theta < 2\pi$ may be more convenient. Answers may use either interval unless the question specifies otherwise.

For calculations involving multiplication or division, full details of the working should be shown.

e.g. in solving a cubic or quartic equation where one complex root is given.

Including the results $|z_1 z_2| = |z_1| |z_2|$ and $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$, and corresponding results for division.

e.g. the square roots of $5 + 12i$ in exact Cartesian form. Full details of the working should be shown.

e.g. $|z - a| < k$, $|z - a| = |z - b|$, $\arg(z - a) = \alpha$.

4 Mechanics (for Paper 4)

Questions set will be mainly numerical, and will aim to test mechanical principles without involving difficult algebra or trigonometry. However, candidates should be familiar in particular with the following trigonometrical results:

$$\sin(90^\circ - \theta) \equiv \cos \theta, \cos(90^\circ - \theta) \equiv \sin \theta, \tan \theta \equiv \frac{\sin \theta}{\cos \theta}, \sin^2 \theta + \cos^2 \theta \equiv 1.$$

Knowledge of algebraic methods from the content for Paper 1: Pure Mathematics 1 is assumed.

This content list refers to the equilibrium or motion of a 'particle'. Examination questions may involve extended bodies in a 'realistic' context, but these extended bodies should be treated as particles, so any force acting on them is modelled as acting at a single point.

Vector notation will not be used in the question papers.

4.1 Forces and equilibrium

Candidates should be able to:

- identify the forces acting in a given situation
- understand the vector nature of force, and find and use components and resultants
- use the principle that, when a particle is in equilibrium, the vector sum of the forces acting is zero, or equivalently, that the sum of the components in any direction is zero
- understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component
- use the model of a 'smooth' contact, and understand the limitations of this model
- understand the concepts of limiting friction and limiting equilibrium, recall the definition of coefficient of friction, and use the relationship $F = \mu R$ or $F \leq \mu R$, as appropriate
- use Newton's third law.

Notes and examples

e.g. by drawing a force diagram.

Calculations are always required, not approximate solutions by scale drawing.

Solutions by resolving are usually expected, but equivalent methods (e.g. triangle of forces, Lami's Theorem, where suitable) are also acceptable; these other methods are not required knowledge, and will not be referred to in questions.

Terminology such as 'about to slip' may be used to mean 'in limiting equilibrium' in questions.

e.g. the force exerted by a particle on the ground is equal and opposite to the force exerted by the ground on the particle.

4 Mechanics

4.2 Kinematics of motion in a straight line

Candidates should be able to:

- understand the concepts of distance and speed as scalar quantities, and of displacement, velocity and acceleration as vector quantities
- sketch and interpret displacement–time graphs and velocity–time graphs, and in particular appreciate that
 - the area under a velocity–time graph represents displacement,
 - the gradient of a displacement–time graph represents velocity,
 - the gradient of a velocity–time graph represents acceleration
- use differentiation and integration with respect to time to solve simple problems concerning displacement, velocity and acceleration
- use appropriate formulae for motion with constant acceleration in a straight line.

Notes and examples

Restricted to motion in one dimension only.

The term 'deceleration' may sometimes be used in the context of decreasing speed.

Calculus required is restricted to techniques from the content for Paper 1: Pure Mathematics 1.

Questions may involve setting up more than one equation, using information about the motion of different particles.

4.3 Momentum

Candidates should be able to:

- use the definition of linear momentum and show understanding of its vector nature
- use conservation of linear momentum to solve problems that may be modelled as the direct impact of two bodies.

Notes and examples

For motion in one dimension only.

Including direct impact of two bodies where the bodies coalesce on impact.

Knowledge of impulse and the coefficient of restitution is not required.

4 Mechanics

4.4 Newton's laws of motion

Candidates should be able to:

- apply Newton's laws of motion to the linear motion of a particle of constant mass moving under the action of constant forces, which may include friction, tension in an inextensible string and thrust in a connecting rod
- use the relationship between mass and weight
- solve simple problems which may be modelled as the motion of a particle moving vertically or on an inclined plane with constant acceleration
- solve simple problems which may be modelled as the motion of connected particles.

Notes and examples

If any other forces resisting motion are to be considered (e.g. air resistance) this will be indicated in the question.

$W = mg$. In this component, questions are mainly numerical, and use of the approximate numerical value $10 \text{ (ms}^{-2}\text{)}$ for g is expected.

Including, for example, motion of a particle on a rough plane where the acceleration while moving up the plane is different from the acceleration while moving down the plane.

e.g. particles connected by a light inextensible string passing over a smooth pulley, or a car towing a trailer by means of either a light rope or a light rigid tow-bar.

4.5 Energy, work and power

Candidates should be able to:

- understand the concept of the work done by a force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force
- understand the concepts of gravitational potential energy and kinetic energy, and use appropriate formulae
- understand and use the relationship between the change in energy of a system and the work done by the external forces, and use in appropriate cases the principle of conservation of energy
- use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion
- solve problems involving, for example, the instantaneous acceleration of a car moving on a hill against a resistance.

Notes and examples

$$W = Fd \cos \theta;$$

Use of the scalar product is not required.

Including cases where the motion may not be linear (e.g. a child on a smooth curved 'slide'), where only overall energy changes need to be considered.

Including calculation of (average) power as

$$\frac{\text{Work done}}{\text{Time taken}} \\ P = Fv.$$

5 Probability & Statistics 1 (for Paper 5)

Questions set will be mainly numerical, and will test principles in probability and statistics without involving knowledge of algebraic methods beyond the content for Paper 1: Pure Mathematics 1.

Knowledge of the following probability notation is also assumed: $P(A)$, $P(A \cup B)$, $P(A \cap B)$, $P(A|B)$ and the use of A' to denote the complement of A .

5.1 Representation of data

Candidates should be able to:

Notes and examples

- select a suitable way of presenting raw statistical data, and discuss advantages and/or disadvantages that particular representations may have
- draw and interpret stem-and-leaf diagrams, box-and-whisker plots, histograms and cumulative frequency graphs
- understand and use different measures of central tendency (mean, median, mode) and variation (range, interquartile range, standard deviation)
- use a cumulative frequency graph
- calculate and use the mean and standard deviation of a set of data (including grouped data) either from the data itself or from given totals $\sum x$ and $\sum x^2$, or coded totals $\sum(x - a)$ and $\sum(x - a)^2$, and use such totals in solving problems which may involve up to two data sets.

Including back-to-back stem-and-leaf diagrams.

e.g. in comparing and contrasting sets of data.

e.g. to estimate medians, quartiles, percentiles, the proportion of a distribution above (or below) a given value, or between two values.

5.2 Permutations and combinations

Candidates should be able to:

Notes and examples

- understand the terms permutation and combination, and solve simple problems involving selections
- solve problems about arrangements of objects in a line, including those involving
 - repetition (e.g. the number of ways of arranging the letters of the word 'NEEDLESS')
 - restriction (e.g. the number of ways several people can stand in a line if two particular people must, or must not, stand next to each other).

Questions may include cases such as people sitting in two (or more) rows.

Questions about objects arranged in a circle will not be included.

5 Probability & Statistics 1

5.3 Probability

Candidates should be able to:

- evaluate probabilities in simple cases by means of enumeration of equiprobable elementary events, or by calculation using permutations or combinations
- use addition and multiplication of probabilities, as appropriate, in simple cases
- understand the meaning of exclusive and independent events, including determination of whether events A and B are independent by comparing the values of $P(A \cap B)$ and $P(A) \times P(B)$
- calculate and use conditional probabilities in simple cases.

Notes and examples

e.g. the total score when two fair dice are thrown.
e.g. drawing balls at random from a bag containing balls of different colours.

Explicit use of the general formula

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ is not required.

e.g. situations that can be represented by a sample space of equiprobable elementary events, or a tree diagram. The use of $P(A|B) = \frac{P(A \cap B)}{P(B)}$ may be required in simple cases.

5.4 Discrete random variables

Candidates should be able to:

- draw up a probability distribution table relating to a given situation involving a discrete random variable X , and calculate $E(X)$ and $\text{Var}(X)$
- use formulae for probabilities for the binomial and geometric distributions, and recognise practical situations where these distributions are suitable models
- use formulae for the expectation and variance of the binomial distribution and for the expectation of the geometric distribution.

Notes and examples

Including the notations $B(n, p)$ and $\text{Geo}(p)$. $\text{Geo}(p)$ denotes the distribution in which $p_r = p(1 - p)^{r-1}$ for $r = 1, 2, 3, \dots$.

Proofs of formulae are not required.

5 Probability & Statistics 1

5.5 The normal distribution

Candidates should be able to:

- understand the use of a normal distribution to model a continuous random variable, and use normal distribution tables
- solve problems concerning a variable X , where $X \sim N(\mu, \sigma^2)$, including
 - finding the value of $P(X > x_1)$, or a related probability, given the values of x_1 , μ , σ .
 - finding a relationship between x_1 , μ and σ given the value of $P(X > x_1)$ or a related probability
- recall conditions under which the normal distribution can be used as an approximation to the binomial distribution, and use this approximation, with a continuity correction, in solving problems.

Notes and examples

Sketches of normal curves to illustrate distributions or probabilities may be required.

For calculations involving standardisation, full details of the working should be shown.

$$\text{e.g. } Z = \frac{(X - \mu)}{\sigma}$$

n sufficiently large to ensure that both $np > 5$ and $nq > 5$.

6 Probability & Statistics 2 (for Paper 6)

Knowledge of the content of Paper 5: Probability & Statistics 1 is assumed, and candidates may be required to demonstrate such knowledge in answering questions. Knowledge of calculus within the content for Paper 3: Pure Mathematics 3 will also be assumed.

6.1 The Poisson distribution

Candidates should be able to:

- use formulae to calculate probabilities for the distribution $Po(\lambda)$
- use the fact that if $X \sim Po(\lambda)$ then the mean and variance of X are each equal to λ
- understand the relevance of the Poisson distribution to the distribution of random events, and use the Poisson distribution as a model
- use the Poisson distribution as an approximation to the binomial distribution where appropriate
- use the normal distribution, with continuity correction, as an approximation to the Poisson distribution where appropriate.

Notes and examples

Proofs are not required.

The conditions that n is large and p is small should be known; $n > 50$ and $np < 5$, approximately.

The condition that λ is large should be known; $\lambda > 15$, approximately.

6.2 Linear combinations of random variables

Candidates should be able to:

- use, when solving problems, the results that
 - $E(aX + b) = aE(X) + b$ and $\text{Var}(aX + b) = a^2 \text{Var}(X)$
 - $E(aX + bY) = aE(X) + bE(Y)$
 - $\text{Var}(aX + bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y)$ for independent X and Y
 - if X has a normal distribution then so does $aX + b$
 - if X and Y have independent normal distributions then $aX + bY$ has a normal distribution
 - if X and Y have independent Poisson distributions then $X + Y$ has a Poisson distribution.

Notes and examples

Proofs of these results are not required.

6 Probability & Statistics 2

6.3 Continuous random variables

Candidates should be able to:

- understand the concept of a continuous random variable, and recall and use properties of a probability density function
- use a probability density function to solve problems involving probabilities, and to calculate the mean and variance of a distribution.

Notes and examples

For density functions defined over a single interval only; the domain may be infinite,

e.g. $\frac{3}{x^4}$ for $x \geq 1$.

Including location of the median or other percentiles of a distribution by direct consideration of an area using the density function.

Explicit knowledge of the cumulative distribution function is not included.

6.4 Sampling and estimation

Candidates should be able to:

- understand the distinction between a sample and a population, and appreciate the necessity for randomness in choosing samples
- explain in simple terms why a given sampling method may be unsatisfactory
- recognise that a sample mean can be regarded as a random variable, and use the facts that $E(\bar{X}) = \mu$ and that $\text{Var}(\bar{X}) = \frac{\sigma^2}{n}$
- use the fact that (\bar{X}) has a normal distribution if X has a normal distribution
- use the Central Limit Theorem where appropriate
- calculate unbiased estimates of the population mean and variance from a sample, using either raw or summarised data
- determine and interpret a confidence interval for a population mean in cases where the population is normally distributed with known variance or where a large sample is used
- determine, from a large sample, an approximate confidence interval for a population proportion.

Notes and examples

Including an elementary understanding of the use of random numbers in producing random samples.

Knowledge of particular sampling methods, such as quota or stratified sampling, is not required.

Only an informal understanding of the Central Limit Theorem (CLT) is required; for large sample sizes, the distribution of a sample mean is approximately normal.

Only a simple understanding of the term 'unbiased' is required, e.g. that although individual estimates will vary the process gives an accurate result 'on average'.

6 Probability & Statistics 2

6.5 Hypothesis tests

Candidates should be able to:

- understand the nature of a hypothesis test, the difference between one-tailed and two-tailed tests, and the terms null hypothesis, alternative hypothesis, significance level, rejection region (or critical region), acceptance region and test statistic
- formulate hypotheses and carry out a hypothesis test in the context of a single observation from a population which has a binomial or Poisson distribution, using
 - direct evaluation of probabilities
 - a normal approximation to the binomial or the Poisson distribution, where appropriate
- formulate hypotheses and carry out a hypothesis test concerning the population mean in cases where the population is normally distributed with known variance or where a large sample is used
- understand the terms Type I error and Type II error in relation to hypothesis tests
- calculate the probabilities of making Type I and Type II errors in specific situations involving tests based on a normal distribution or direct evaluation of binomial or Poisson probabilities.

Notes and examples

Outcomes of hypothesis tests are expected to be interpreted in terms of the contexts in which questions are set.

4 Details of the assessment

Relationship between components

Candidates build their knowledge of the mathematics content as they progress through the course.

Paper 1: Pure Mathematics 1 is the foundation for all other components.

Paper 2: Pure Mathematics 2 and Paper 3: Pure Mathematics 3 build on the subject content for Paper 1: Pure Mathematics 1.

Paper 4: Mechanics and Paper 5: Probability & Statistics 1 components assume prior knowledge of the Paper 1: Pure Mathematics 1 content.

Paper 5: Probability & Statistics 1 is the foundation for studying Paper 6: Probability & Statistics 2.

Candidates may not take both Paper 2 and Paper 3 in the same examination series. Paper 2 and Paper 3 are taken in alternative routes through the qualification – Paper 2 is for AS Level only, and Paper 3 is for A Level. Paper 2 subject content is largely a subset of the Paper 3 subject content.

Examination information

All components are assessed by written examinations which are externally marked. Sample assessment materials are available on our website at www.cambridgeinternational.org showing the question style and level of the examination papers.

Application of mathematical techniques

As well as demonstrating the appropriate techniques, candidates need to apply their knowledge in solving problems. Individual examination questions may involve ideas and methods from more than one section of the subject content for that component.

The main focus of examination questions will be the AS & A Level Mathematics subject content. However, in examination questions, candidates may need to make use of prior knowledge and mathematical techniques from previous study, as listed in section 3 of this syllabus.

Structure of the question paper

All questions in the examination papers are compulsory. An approximate number of questions for each paper is given in the Assessment overview in section 2 of this syllabus. Questions are of varied lengths and often contain several parts, labelled (a), (b), (c), which may have sub-parts (i), (ii), (iii), as needed. Some questions might require candidates to sketch graphs or diagrams, or draw accurate graphs.

Answer space

Candidates answer on the question paper. All working should be shown neatly and clearly in the spaces provided for each question. New questions often start on a fresh page, so more answer space may be provided than is needed. If additional space is required, candidates should use the lined page at the end of the question paper, where the question number or numbers must be clearly shown.