Exercises 2.2

2. Prove that the square of an odd integer is an odd integer:

   $Odd\ numbers\ can\ be\ expressed\ as\ 2n\ +\ 1$
   $Theorem\ is\ true\ if\ (2n\ +\ 1)^2\ =\ 2k\ +\ 1$
   $(2n\ +\ 1)^2\ =\ 2k\ +\ 1$
   $4n^2\ +\ 4n\ +\ 1\ =\ 2k\ +\ 1$
   $2(2n^2\ +\ 2n)\ +\ 1\ =\ 2k\ +\ 1$

   L.H.S. resembles R.H.S and $2n^2\ +\ 2n$ must evaluate to some integer k since n is also an integer so the statement is true.

3. Create your own example of a large number of true cases but that are not universally true:
   "Every number can be spelt in English without using the letter 'C'"

4. Extend the formula $n^2\ -\ n\ +\ 41$ to negative numbers:

n = -1	43
n = -2	47
n = -3	53
n = -4	61
n = -5	71
…	…
n = -39	1601

   The formula behaves nearly identical to when applied to positive numbers, with the negative numbers "running ahead" or approaching infinity faster because the negative sign gets flipped on the second term. Which means the composite numbers appear sooner with negatives than with positives.

5. Is it true that every positive integer is either a prime or a composite number?

   A prime number is only divisible by itself and one. So it has only two factors. A composite number is a number with more than two factors. So this statement

claims that every positive integer either has two factors or more than two factors. Number 1 is a positive integer that has only one factor. In other words, it is neither prime nor composite. Since there exists a positive integer that is neither prime nor composite, the statement is false.

6. Do you think that -7 should be called a prime? Why?

   If we say its factors are -1 and -7, then it has only two factors. But the definition of prime numbers, as far as I know, insist they are also divisible by 1. -7 is divisible by 1. But if we count 1 as a factor, then it has more than two factors. It is also divisible by 7, bringing its factors up to four.

   When -7 is divided by 1 and 7, the result is negative (-7 and -1, respectively). Should these be counted as factors if the results are negative? Or perhaps -1 and -7 should not be counted as factors in the first place since the result of division is positive in those cases.

   In anycase, we agree 7 is a prime number. Yet, it seems to me to be perfectly divisible by -1 and -7 too. The answers in that case are negative as well. If 7 is a prime number regardless, then -7 could be considered a prime number regardless as well. But I don't know if we should call it prime, as I imagine the current definition has some utility to it.