y2522li_a1q4

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```
# Standard imports
import numpy as np
np.seterr(all='ignore');  # allows floating-point exceptions
import matplotlib.pyplot as plt
```

1.1 Q4: Upper Bound on Error

$$\frac{\left|(a \otimes b) \ominus c - (ab - c)\right|}{|ab - c|} \tag{1}$$

$$= \frac{\left|ab(1+\delta_1) \ominus c - (ab - c)\right|}{|ab - c|} \tag{2}$$

$$= \frac{\left|[ab(1+\delta_1) - c](1+\delta_2) - (ab - c)\right|}{|ab - c|} \tag{3}$$

$$= \frac{\left|ab(1+\delta_1) - c + [ab(1+\delta_1) - c]\delta_2 - (ab - c)\right|}{|ab - c|} \tag{4}$$

$$= \frac{\left|ab(1+\delta_1) + ab(1+\delta_1)\delta_2 - c\delta_2 - ab\right|}{|ab - c|} \tag{5}$$

$$= \frac{\left|ab + ab\delta_1 + ab\delta_2 + ab\delta_1\delta_2 - c\delta_2 - ab\right|}{|ab - c|} \tag{6}$$

$$= \frac{\left|ab\delta_1(1+\delta_2) + (ab - c)\delta_2\right|}{|ab - c|} \tag{7}$$

$$\text{By Triangle Inequality, } \leq \frac{|ab|}{|ab - c|}\,|\delta_1(1+\delta_2)| + |\delta_2| \tag{8}$$

$$\text{Since } \delta_i \leq E, \leq \frac{|ab|}{|ab - c|}E(1+E) + E \tag{9}$$

$$\tag{10}$$

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