Theory Exercise

We know.

$$\begin{cases} C_b = (1-\alpha_0)C_0 + \alpha_0 C^1 \\ C^i = (1-\alpha_i)C_i + \alpha_i C^{i+1} \end{cases}$$

$$\begin{aligned}
C_b &= (1-\alpha_0)C_0 + \alpha_0 C^1 \\
&= (1-\alpha_0)C_0 + \alpha_0\left[(1-\alpha_1)C_1 + \alpha_1 C^2\right] \\
&= (1-\alpha_0)C_0 + (1-\alpha_1)\alpha_0 C_1 + \alpha_0\alpha_1 C^2 \\
&= (1-\alpha_0)C_0 + (1-\alpha_1)\alpha_0 C_1 + \cdots \alpha_0\cdots\alpha_{i-1}C^i \\
&= (1-\alpha_0)C_0 + (1-\alpha_1)\alpha_0 C_1 + \cdots \alpha_0\cdots\alpha_{i-1}\left[(1-\alpha_i)C_i + \alpha_i C^{i+1}\right] \\
&= (1-\alpha_0)C_0 + (1-\alpha_1)\alpha_0 C_1 + \cdots (1-\alpha_i)(\alpha_0\cdots\alpha_{i-1})C_i + \alpha_0\cdots\alpha_{i-1}\alpha_i C^{i+1} \\
&= \sum_{i=0}^{+\infty}(1-\alpha_i)\left(\prod_{k=0}^{i-1}\alpha_k\right)C_i
\end{aligned}$$

Simplification:

$$\sum_{i=1}^{N}(1-\alpha_i)\left(\prod_{k=0}^{i-1}\alpha_k\right)C_i$$

$$= (1-\alpha_0)C_0 + (1-\alpha_1)\alpha_0 C_1 + (1-\alpha_2)\alpha_0\alpha_1 C_2 \cdots (1-\alpha_N)\alpha_0\alpha_1\cdots\alpha_{N-1}C_N$$

for simplified function:

because for at most N reflections, no intersections occur for N+1, N+2,...

hence $C_{N+1} = C_{N+2} = \dots = 0$.

- Render Light function:

We use the obtained expression in our render_light function.
More specifically, we set N= NUM_REFLECTIONS and using a for-loop we iterate for N.

For each pixel we get its color by:

1. Adding the ambient component
2. For every light, we add the diffuse and Specular component

$\left.\begin{array}{l}\text{1. Adding the ambient component}\\ \text{2. For every light, we add the diffuse and specular component}\end{array}\right\} = C_0$

3. We get α_i as m.mirror (reflection coeffs) for material m at reflection i
4. We achieve the iterative approach using:

$$\text{pix\_color} \mathrel{+}= (1. - \underbrace{\text{m.mirror}}_{\alpha_i}) * \underbrace{\text{reflection\_weight}}_{\alpha_0 \alpha_1 \dots \alpha_{i-1}} * \underbrace{\text{temp\_pix\_color}}_{\substack{C_i \\ \text{(ambient + diffuse +} \\ \text{specular)}}}$$