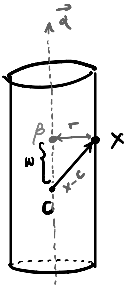
# Ray – Cylinder Intersection




- $\vec{a}$ is normalized
- We found $w$ using the projection formula $\left\| \left((x-c)\cdot a\right)\cdot a \right\| = (x-c)\cdot a \, \|a\|$
  $= (x-c)\cdot a$
  because $\|a\| = 1$.
- Pythagora's rule for the triangle:

$$r^2 + w^2 = |x-c|^2$$

$$r^2 + \left((x-c)\cdot a\right)^2 - (x-c)^T(x-c) = 0$$

replace $x$ with the ray's equation:

$$r^2 + \left((o+td-c)\cdot a\right)^2 - (o+td-c)^T(o+td-c) = 0$$

Set $q = o - c$ :

$$r^2 + \left((td+q)\cdot a\right)^2 - (td+q)^T(td+q) = 0$$

$$r^2 + \left((td+q)^T a\right)^2 - \left(t^2 d^T d + t d^T q + t q^T d + q^T q\right) = 0$$

$$r^2 + \left(t d^T a + q^T a\right)^2 - \left(t^2 d^T d + t d^T q + t q^T d + q^T q\right) = 0$$

$$r^2 + \left(t^2 d^T a d^T a + 2t d^T a q^T a + q^T a q^T a\right) - t^2 d^T d - t d^T q - t q^T d - q^T q = 0$$

$$t^2 \underbrace{\left(d^T a d^T a - d^T d\right)}_{a} + t \underbrace{\left(2 d^T a q^T a - d^T q - q^T d\right)}_{b} + \underbrace{\left(r^2 + q^T a q^T a - q^T q\right)}_{c} = 0$$

And then we solved the quadratic equation.

## Normal Derivation.

After solving for $t$ in the previous section, we could express the location of intersection point using $t$.

In addition, we could solve for $\omega$ and make use of the result to express the location of point $\beta$. (can be found in the graph in last page).

With both location of $\alpha$ and $\beta$, we could easily express the normal using formula $(\alpha - \beta)$.

## Point Alternation.

If one intersection point actually lands outside of the reachable distance from the center of the cylinder, we need to change to another intersection point and test if it's within the height of cylinder.

1. We would compute $\omega$ to see if it's within half of the height. If not, this means the current intersection point doesn't land on the cylinder.

2. We will proceed to change to another unchecked intersection point to make sure we don't reuse invalid intersection point.