RMSProp Algorithm

The RMSProp (Root Mean Square Propagation) algorithm is an adaptive learning rate optimization method designed to improve the convergence of gradient-based optimization. Below is the algorithm along with explanations for each parameter and variable.

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input : α (alpha), γ (lr), θ_0 (params), $f(\theta)$ (objective)
λ (weight decay), μ (momentum), *centered*, ϵ (epsilon)
initialize : $v_0 \leftarrow 0$ (square average), $\mathbf{b}_0 \leftarrow 0$ (buffer), $g_0^{ave} \leftarrow 0$

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for $t = 1$ **to** $\ldots$ **do**
$g_t \leftarrow \nabla_\theta f_t(\theta_{t-1})$
$if\ \lambda \neq 0$
$g_t \leftarrow g_t + \lambda\theta_{t-1}$
$v_t \leftarrow \alpha v_{t-1} + (1-\alpha)g_t^2$
$\tilde{v_t} \leftarrow v_t$
$if\ centered$
$g_t^{ave} \leftarrow g_{t-1}^{ave}\alpha + (1-\alpha)g_t$
$\tilde{v_t} \leftarrow \tilde{v_t} - \left(g_t^{ave}\right)^2$
$if\ \mu > 0$
$\mathbf{b}_t \leftarrow \mu\mathbf{b}_{t-1} + g_t/\left(\sqrt{\tilde{v_t}} + \epsilon\right)$
$\theta_t \leftarrow \theta_{t-1} - \gamma\mathbf{b}_t$
$else$
$\theta_t \leftarrow \theta_{t-1} - \gamma g_t/\left(\sqrt{\tilde{v_t}} + \epsilon\right)$

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return θ_t

Explanation of Parameters and Variables

- α (**alpha**): The decay rate for the moving average of squared gradients. It controls how quickly the past gradients are forgotten. Typical values are between 0.9 and 0.99.
- γ (**lr**): The learning rate. It determines the step size of the parameter updates. A smaller learning rate leads to slower but more stable convergence.
- θ_0 (**params**): The initial parameters of the model. These are the values that the optimization algorithm will adjust to minimize the objective function.
- $f(\theta)$ (**objective**): The objective function (or loss function) that the algorithm aims to minimize. It is a function of the parameters θ.
- λ (**weight decay**): The weight decay coefficient. It adds a penalty proportional to the squared magnitude of the parameters to the objective function, encouraging smaller parameter values.
- μ (**momentum**): The momentum coefficient. It accelerates the optimization process by adding a fraction of the previous update to the current update. If $\mu = 0$, momentum is not used.
- **centered**: A boolean flag indicating whether to use the centered version of RMSProp. If enabled, the algorithm subtracts the mean of the gradients from the squared gradients.
- ϵ (**epsilon**): A small constant added to the denominator to improve numerical stability. It prevents division by zero.
- v_0 (**square average**): The initial value for the moving average of squared gradients. It is typically initialized to zero.

- $\mathbf{b}_0$ (**buffer**): The initial value for the momentum buffer. It is typically initialized to zero.
- g_0^{ave}: The initial value for the moving average of gradients (used in the centered version). It is typically initialized to zero.
- g_t: The gradient of the objective function with respect to the parameters at time step t.
- v_t: The moving average of squared gradients at time step t.
- $\tilde{v}_t$: The adjusted moving average of squared gradients (used in the centered version).
- g_t^{ave}: The moving average of gradients (used in the centered version).
- $\mathbf{b}_t$: The momentum buffer at time step t.
- θ_t: The updated parameters at time step t.