

**Duration: 3 Hours**

**Total Marks: 80**

- N.B.** : 1) Question Number 1 is compulsory  
 2) Solve any three questions from the remaining questions  
 3) Make suitable assumptions if needed  
 4) Assume appropriate data whenever required. State all assumptions clearly.

**Q.1      Solve any four of the following questions.**

- a. What is a tautology? Check whether the following logical expression is tautology? 5  

$$[(p \rightarrow r) \wedge (\neg q \rightarrow p) \wedge \neg r] \rightarrow q$$
- b. State the Pigeonhole principle and show that if any five numbers form 1 to 8 are chosen, then two of them will add to 9. 5
- c. Convert the following into CNF form. 5  

$$(A \rightarrow B) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow C))$$
- d. Given  $S = \{1, 2, \dots, 10\}$  and a relation  $R$  on  $S$ , where  $R = \{(x, y) | x + y = 10\}$ . Is it reflexive, symmetric, transitive, antisymmetric? 5
- e. Define the following terms 5  
 1. Planer graph 2. Cut Vertex 3. Chain 4. Monoid 5. Group

- Q.2** a. Let  $A = \{p, q, r, s\}$  and let  $R = \{(p, p), (p, q), (p, r), (q, p), (q, q), (r, p), (q, r), (r, q), (r, r), (s, s)\}$ . Show that  $R$  is an equivalence relation . Determine the equivalence classes and find the rank of  $R$ . 8

- b. Show that  $A = \{0, 3, 6, 9, 12\}$  is a ring w.r.t. the operation of addition & multiplication modulo 15. 8

- c. Explain two different types of Quantifiers with example? 4  
 Represent the following sentences using First Order logic  
 i) Some students took GenAI.  
 ii) Every student who takes GenAI passes it.

- Q.3** a. What is an Abelian Group? Let  $G = \{1, 2, 3, 4, 5, 6, 7\}$  8

- i) Prepare the composition table w.r.t the operation of multiplication modulo 8.

- ii) Check whether it is an Abelian group? Justify your answer.

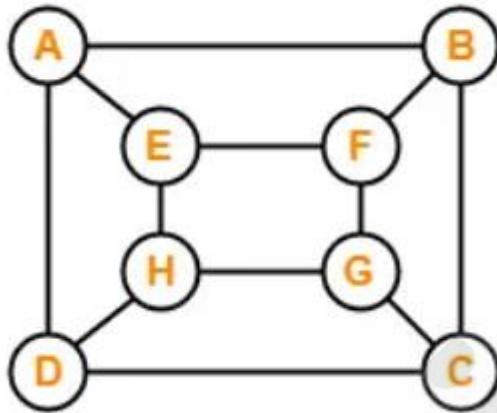
- b. Define minimum hamming distance. Find the code words generated by the parity check matrix  $H$  given below. 8

$$H = \begin{matrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

c

4

Determine whether the following graph is Eulerian or Hamiltonian or both.  
Justify your answer.



- Q.4 a** Define function. What are three different types of functions.. Consider the function  $f$  and  $g$  from  $N \times N$  to  $N$  given by  $f(x,y) = 2x+y$  and  $g(x,y)=xy$ , identify its type. 8

- b** Let  $A= \{a,b,c,d,e\}$  and let  $R$  be a relation on  $A$ . 8

Let  $R=\{(a,a),(a,c),(b,b),(c,d),(c,e),(d,a),(e,b),(e,e)\}$

Compute transitive closure using Warshall's algorithm

- c** Prove using Mathematical Induction that sum of cubes of three consecutive integers is divisible by 9. 4

- Q.5 a** Let  $X=\{1,2,3,4,6,24,36,72\}$  and  $R=\{(x,y) \in R \mid x \text{ divides } y\}$  8

i) Write the pairs in a relation set  $R$ .

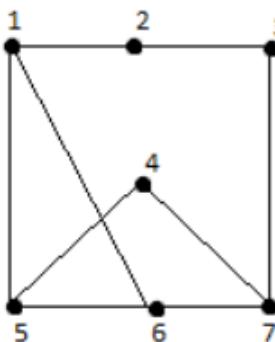
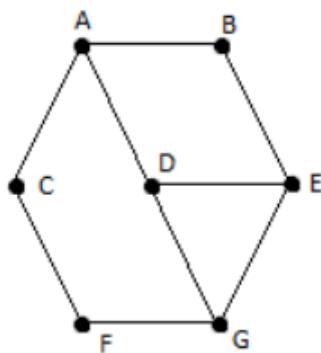
ii) Construct Hasse diagram.

iii) Mention Chains and Anti Chains from above set.

iv) Is it a lattice?

- b** Find the number of integers between 1 to 500 that are not divisible by 5,6, or 8? 8

- c** Check whether the following graphs are Isomorphic or not? Justify 4





Paper / Subject Code: 49372 / Discrete Structures & Graph Theory

1T01873 - S.E. Computer Science & Engineering (Artificial Intelligence & Machine Learning) (R-2019)  
SEMESTER - III / 49372 - Discrete Structures & Graph Theory  
QP CODE: 10030226 DATE : 26/05/2023

**(3 Hours)**

Total Marks : 80

N.B. : (1) Question Number 1 is compulsory  
(2) Solve any three questions from the remaining questions  
(3) Make suitable assumptions if needed  
(4) Assume appropriate data whenever required. State all assumptions clearly.

- |    |   |                       |
|----|---|-----------------------|
| 1. | <ol style="list-style-type: none"> <li>Define the following with suitable example</li> <li>a)Ring b) Cyclic Group c) Monoid d)Normal Subgroup e) Planar Graph</li> <li>Check whether <math>[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p</math> is a tautology</li> <li>Determine the number of positive integers n where <math>1 \leq n \leq 100</math> and n is not divisible by 2,3 or 5.</li> <li>Prove by mathematical induction that<br/> <math display="block">2+5+8+\dots+(3n-1) = n(3n+1)/2</math></li> </ol> | 5<br>5<br>5<br>5<br>5 |
| 2  | <ol style="list-style-type: none"> <li>Define Equivalence Relation. Let A be a set of integers, Let R be a Relation on <math>A \times A</math> defined by <math>(a,b) R (c,d)</math> if and only if <math>ad = bc</math>. Prove that R is an Equivalence Relation</li> <li>Let <math>A = \{a, b, c, d, e\}</math></li> </ol>  | 8<br>8                |

$$MR = \begin{vmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{vmatrix}$$

Find the transitive closure of it using Warshall's algorithm.

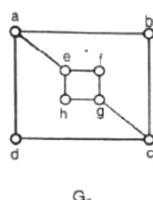
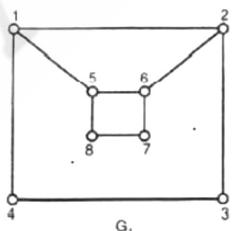
- 3      c Let G be a group. Prove that the identity element ‘e’ is unique. , 4

3      a Prove that set  $G = \{1,2,3,4,5,6\}$  is a finite abelian group of order 6 with respect to multiplication module 7 8

3      b Give the exponential generating function for the sequences 8

3      i)  $\{1,1,1\dots\}$   
 3      ii)  $\{0,1,0,-1,0,1,0,-1\dots\}.$

3      c Determine whether the following graphs are isomorphic. Justify your answer. 4



4. a A Function  $f: \mathbb{R} - \left\{ \frac{7}{3} \right\} \rightarrow \mathbb{R} - \left\{ \frac{4}{3} \right\}$  is defined as  $f(x) = (4x - 5)/(3x - 7)$   
 Prove that f is Bijective and find the rule for  $f^{-1}$

- b Show that  $(2,5)$  encoding function  $e: B^2 \rightarrow B^5$  defined by 8

$$e(00)=00000$$

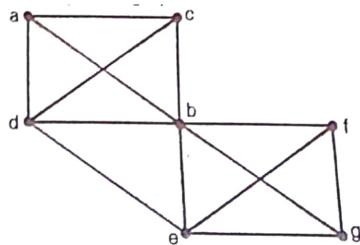
$$e(01)=01110$$

$$e(10)=10101$$

$$e(11)=11011$$

is a group code.

- c Check whether Euler cycle and Euler Path exist in the Graph given below. If yes Mention them 4



5. a Consider the Set  $A=\{1,2,3,4,5,6\}$  under multiplication Modulo 7. 8

1) Prove that it is a Cyclic group.

2) Find the orders and the Subgroups generated by  $\{2,3\}$  and  $\{3,4\}$

- b State and explain the extended Pigeonhole principle. How many friends must you have to guarantee that at least five of them will have birthdays in the same month. 8

- c Functions  $f,g,h$  are defined on a set  $X=\{a,b,c\}$  as 4

$$f=\{(a,b),(b,c),(c,a)\}$$

$$g=\{(a,b),(b,a),(b,b)\}$$

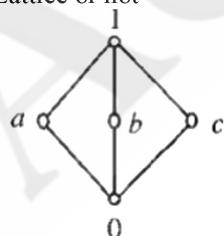
$$h=\{(a,a),(b,b),(c,a)\}$$

i) Find  $fog$ ,  $gof$ . Are they equal?

ii) Find  $fogh$  and  $fogh$ ?

6. a Draw the Hasse Diagram of  $D_{72}$  and  $D_{105}$  and check whether they are Lattice. 8

- b Define Bounded Lattice and Distributive Lattice. Check if the following diagram is a Distributive Lattice or not 8



- c Define the following with suitable example. 4

a) Hamiltonian path    b) Euler Circuit    c) Sub Lattice    d) Group    e) Surjective Function

(3 Hours)

[Total Marks : 80]

N.B.

- 1) **Q.1 is compulsory.**
- 2) Solve any 3 questions out of remaining 5 questions.
- 3) Assumptions made should be clearly stated.
- 4) Draw the figures wherever required.

**Q.1 Solve any four of the following questions.**

- |  |          |
|--|----------|
| a) Prove using Mathematical Induction that $n^3+2n$ is divisible by 3 for all $n \geq 1$   | <b>5</b> |
| b) Explain the following terms with suitable example:<br>i) Partition set<br>ii) Power set.  | <b>5</b> |
| c) State the Pigeonhole principle and show that if any five numbers from 1 to 8 are chosen, then two of them will add to 9.                    | <b>5</b> |
| d) Consider the function $f(x) = 2x-3$ . Find a formula for the composition functions<br>i) $f^2 = f \circ f$<br>ii) $f^3 = f \circ f \circ f$ | <b>5</b> |
| e) Explain the bipartite graph with suitable example.  | <b>5</b> |

**Q.2**

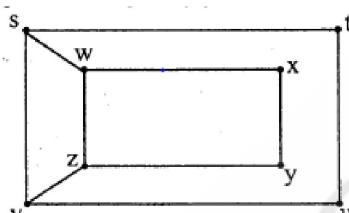
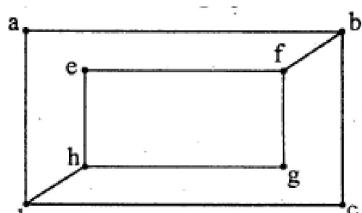
- |  |           |
|--|-----------|
| a) What is a transitive closure? Find the transitive closure of R using Warshall's algorithm where $A=\{1, 2, 3, 4, 5\}$ & $R=\{(x,y) \mid  x-y  = \pm 1\}$                    | <b>10</b> |
| b) What is a ring? Let $A=\{0, 1, 2, 3, 4, 5, 6, 7\}$ . Determine whether a set A with addition modulo 8 & multiplication modulo 8 is a commutative ring? Justify your answer. | <b>10</b> |

**Q.3**

- |   |          |
|---|----------|
| a) A survey in 1986 asked households whether they had a VCR, a CD player or cable TV. 40 had a VCR. 60 had a CD player; and 50 had cable TV. 25 owned VCR and CD player. 30 owned a CD player and had cable TV. 35 owned a VCR and had cable TV. 10 households had all three. How many households had at least one of the three? How many of them had only CD player? | <b>8</b> |
| b) Find the complete solution of a recurrence relation<br>$a_n + 2a_{n-1} = n + 3$ for $n \geq 1$ and with $a_0 = 3$  | <b>6</b> |
| c) Obtain CNF & DNF for the following expression:<br>$p \leftrightarrow (\sim p \vee \sim q)$   | <b>6</b> |

**Q.4**

- a) What is a group? Let  $A=\{3, 6, 9, 12\}$  10  
 i) Prepare the composition table w.r.t. the operation of multiplication modulo 15.  
 ii) Whether it is an abelian group? Justify your answer.  
 iii) Find the inverses of all the elements.  
 iv) Whether it is a cyclic group?
- b) What are the isomorphic graphs? Determine whether following graphs are isomorphic. 10

**Q.5**

- a) Let  $X=\{1, 2, 3, 6, 24, 36\}$  &  $R = \{(x,y) \in R \mid x \text{ divides } y\}$  10  
 i) Write the pairs in a relation set R.  
 ii) Construct the Hasse diagram.  
 iii) What are the Maximal and Minimal elements?  
 iv) Mention Chains and Ant chains from above set.  
 v) Is this poset a lattice?
- b) Define the term bijective function. 5

Let  $f : R \rightarrow (7/5) \rightarrow R - \left(\frac{2}{5}\right)$  be defined by  $f(x) = \frac{2x-3}{5x-7}$ .

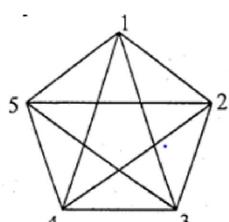
Whether a function is bijective? Justify your answer.

- c) Define minimum hamming distance. Consider  $e : B^3 \rightarrow B^6$ . Find the code words generated by the parity check matrix H given below. 5

$$H = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

**Q.6**

- a) Define with example Euler path, Euler circuit, Hamiltonian path, and Hamiltonian circuit. Determine if the following diagram has Euler circuit and Hamiltonian circuit. Mention the path/circuit. 6



- b) Let  $p$  denote the statement ‘The food is good’,  
 $q$  denote the statement ‘The service is good’ &  
 $r$  denote the statement ‘The rating is 3 star.’

Write the following statements in a symbolic form-

8

- Either food is good or service is good or both.
- The food is good but service is not good.
- If both food & service are good then the rating is 3 star.
- It is not true that a 3 star rating always means good food & good service.

- c) Find out the incidence matrix of following graphs.

6

