1) Messages arrive at an electronic message center at random times, as rare events, 1 message every 6 minutes.

    a) (1 point) Find the probability of receiving at least 5 messages during one hour.

    b) (1 point) During one hour, knowing that at least 6 messages were received, what is the probability that 8 messages arrived?

    c) (1 point) How many messages can be expected to arrive in 90 minutes?

2) (1.5 points) Let $X \in N(0,1)$. Find the pdf of $Y = 2X - 7$.

3) Let $X_1, X_2, ..., X_n$ be a random sample drawn from a distribution with pdf $f(x; \mu) = \dfrac{1}{3\mu} e^{-\frac{x}{3\mu}}$, for $x > 0$, $E(X) = 3\mu$, $V(X) = 9\mu^2$, with $\mu > 0$, unknown.

    a) (0.5 points) Find the method of moments estimator, $\hat{\mu}$, for μ.

    b) (1.5 points) Find the maximum likelihood estimator, $\bar{\mu}$, for μ.

    c) (1 point) Is $\bar{\mu}$ an absolutely correct estimator? Explain.

    d) (1.5 points) At the significance level $\alpha \in (0,1)$, find a most powerful test for testing $H_0 : \mu = 1$ against $H_1 : \mu = 2$.