Test germană

4 $T(x) = Rot_{\frac{\pi}{6}} \circ T_{(2,1)}(x)$

$\Rightarrow T^{-1} = \left(Rot_{\frac{\pi}{6}} \circ T_{(2,1)}(x)\right)^{-1} =$

$= T_{(2,1)}^{-1}(x) \circ Rot_{\frac{\pi}{6}}^{-1} =$

$= T_{(-2,-1)}(x) \circ Rot_{-\frac{\pi}{6}}^{4k} =$

$$= \begin{bmatrix} \cos\left(-\frac{\pi}{6}\right) & -\sin\left(-\frac{\pi}{6}\right) \\ \cancel{\cos}\,\sin\left(-\frac{\pi}{6}\right) & \cos\left(-\frac{\pi}{6}\right) \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \end{bmatrix} =$$

$$= \begin{bmatrix} \cos\frac{\pi}{6} & \sin\frac{\pi}{6} \\ -\sin\frac{\pi}{6} & \cos\frac{\pi}{6} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

1. $\mathcal{E}: \frac{x^2}{2} + \frac{y^2}{2} = 1$

$M(0, \sqrt{6})$

$l: y = kx \pm \sqrt{a^2k^2 + b^2}$

$M \in l \Rightarrow \sqrt{6} = \pm\sqrt{2k^2 + 2} \quad (\,)^2$

$6 = 2k^2 + 2$

$2k^2 = 4$

$k^2 = 2 \Rightarrow k = \pm\sqrt{2}$

~~$t_1: y = \sqrt{\ }$~~

$t_1: y = \sqrt{2}x + \sqrt{4+2} \Rightarrow y = \sqrt{2}x + \sqrt{6}$

$t_2: y = -\sqrt{2}x - \sqrt{6} \Rightarrow y = -(\sqrt{2}x + \sqrt{6})$

2 $H: 2x^2 + 3y^2 - 2z^2 - 1 = 0$

$H: \dfrac{x^2}{\frac{1}{2}} + \dfrac{y^2}{\frac{1}{3}} - \dfrac{z^2}{\frac{1}{2}} = 1$

$E: 2x + 3y - 2z - 3 = 0$

$P(x_0, y_0, z_0) \in H \Rightarrow$

$\Rightarrow T_pH: \dfrac{x_0 x}{\frac{1}{2}} + \dfrac{y_0 y}{\frac{1}{3}} - \dfrac{z_0 z}{\frac{1}{2}} = 1$

normal vector of E: $(2, 3, -2)$

normal vector of T_pH: $\left(\dfrac{x_0}{\frac{1}{2}}, \dfrac{y_0}{\frac{1}{3}}, -\dfrac{z_0}{\frac{1}{2}}\right)$

$E \parallel T_pH \Rightarrow$ the normal vectors are proportional:

$$\frac{\cancel{2}}{\frac{x_0}{\frac{1}{\cancel{2}}}} = \frac{\cancel{3}}{\frac{y_0}{\frac{1}{\cancel{3}}}} = \frac{\cancel{-2}}{-\frac{z_0}{\frac{1}{\cancel{2}}}} \Rightarrow$$

$$\Rightarrow \frac{1}{x_0} = \frac{1}{y_0} = \frac{1}{z_0} \Rightarrow$$

$$\Rightarrow \begin{cases} y_0 = x_0 \\ z_0 = x_0 \end{cases}$$

$$P \in H \Rightarrow \cancel{2x_0^2} + 3x_0^2 - \cancel{2x_0^2} = 1$$

$$3x_0^2 = 1 \Rightarrow x_0^2 = \frac{1}{3}$$

$$x_0 = \pm \frac{1}{\sqrt{3}}$$

$$P_1\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \quad P_2\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$$

$$T_{P_1}H : 2 \cdot \frac{1}{\sqrt{3}} x + 3 \cdot \frac{1}{\sqrt{3}} y - 2 \cdot \frac{1}{\sqrt{3}} z = 1$$

$$T_{P_1}H : \frac{2}{\sqrt{3}} x + \frac{3}{\sqrt{3}} y - \frac{2}{\sqrt{3}} z = 1$$

$$T_{P_2}H : -\frac{2}{\sqrt{3}} x - \frac{3}{\sqrt{3}} y + \frac{2}{\sqrt{3}} z = 1$$

3. $P : 9x^2 - 9y^2 - z = 0$

$F : x + 2z - 1 = 0$

$$P : 9x^2 - 9y^2 - z = 0$$

$$9x^2 - 9y^2 = z$$

$$(3x + 3y)(3x - 3y) = z$$

$$l_{\alpha_1}: \begin{cases} 3x+3y = \alpha z \\ \alpha(3x+3y) = 1 \end{cases}$$

$$\omega = \begin{vmatrix} i & j & k \\ 3 & 3 & -\alpha \\ \alpha 3 & -3\alpha & 0 \end{vmatrix} = 0 - k(9\alpha) + j \cdot 3\alpha^2 -$$

$$- k \cdot 9\alpha - 0 - i \cdot 3\alpha^2 =$$

$$= i(-3\alpha^2) + j(3\alpha^2) +$$
$$+ k(-18\alpha)$$

~~$\omega \parallel E \Rightarrow$~~

$l_{\alpha_1} \parallel E \Rightarrow \omega \parallel \vec{n} \Rightarrow$

$\Rightarrow -3\alpha^2 - 36\alpha - 1 = 0 \quad | \cdot (-1)$

$3\alpha^2 + 36\alpha + 1 = 0$

$\Delta = 1260 - 12 = 1248$

$$\alpha_{1,2} = \frac{-36 \pm 4\sqrt{78}}{6} =$$

$$= \frac{-18 \pm 2\sqrt{78}}{3}$$

1248	2
624	2
312	2
156	2
78	2
39	3
13	

$$l_{\alpha_2}: \begin{cases} \alpha(3x+3y) = 1 \\ 3x - 3y = \alpha z \end{cases}$$

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$$\omega = \begin{vmatrix} i & j & k \\ 3\alpha & 3\alpha & 0 \\ 3 & -3 & \alpha \end{vmatrix} = i\left(-3\alpha^2\right) + k\cdot(-3\ \ldots) + 0 + \cancel{i} - k(9\alpha) + 0 + j\left(3\alpha^2\right)$$

$$= i\left(-3\alpha^2\right) + j\left(3\alpha^2\right) + k\left(-18\alpha\right)$$

$l_{\alpha_2} \parallel E \Rightarrow \omega \parallel \pi \Rightarrow$

$\Rightarrow -3\alpha^2 - 36\alpha - 1 = 0$

Same as for the first family of generators $\Rightarrow$

$$l_{\alpha_{1,1}}: \begin{cases} 3x + 3y = \dfrac{-18 + 2\sqrt{78}}{3} \cdot z \\ -\dfrac{18 + 2\sqrt{78}}{3}(3x - 3y) = 1 \end{cases}$$

$$l_{\alpha_{1,2}}: \begin{cases} 3x + 3y = -\dfrac{18 - 2\sqrt{78}}{3} \cdot z \\ -\dfrac{18 - 2\sqrt{78}}{3}(3x - 3y) = 1 \end{cases}$$

$$l_{\alpha_{2,1}}: \begin{cases} -\dfrac{18 + 2\sqrt{78}}{3}(3x + 3y) = 1 \\ 3x - 3y = -\dfrac{18 + 2\sqrt{78}}{3} \cdot z \end{cases}$$

$$l_{\alpha_{2,2}}: \begin{cases} -\dfrac{18 - 2\sqrt{78}}{3}(3x + 3y) = 1 \\ 3x - 3y = -\dfrac{18 - 2\sqrt{78}}{3} \cdot z \end{cases}$$