1. Let $ABCD$ be a quadrilateral. Let M and N be the midpoints of two opposite sides respectively. Let P be the midpoint of $[MN]$. Prove that $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} + \overrightarrow{PD} = 0$.

2. Let ABC and $A'B'C'$ be two triangles in $\mathbb{E}^3$ with centroids G and G' respectively. Show that

$$\overrightarrow{AA'} + \overrightarrow{BB'} + \overrightarrow{CC'} = 3\overrightarrow{GG'}.$$

3. Let ABC be a triangle. Consider the points $C' \in [AB]$ and $B' \in [AC]$ such that $\overrightarrow{AC'} = \lambda\overrightarrow{BC'}$ and $\overrightarrow{AB'} = \mu\overrightarrow{CB'}$. Let M be the intersection point of BB' and CC'. Show that

$$\overrightarrow{OM} = \frac{\overrightarrow{OA} - \lambda\overrightarrow{OB} - \mu\overrightarrow{OC}}{1-\lambda-\mu}.$$

for any point O outside the plane of the triangle.

4. Let ABC be a triangle with centroid G, orthocenter H, incenter I and circumcenter Q. Show that

1. $\overrightarrow{OG} = \frac{\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}}{3}$
2. $\overrightarrow{OI} = \frac{a\overrightarrow{OA}+b\overrightarrow{OB}+c\overrightarrow{OC}}{a+b+c}$
3. $\overrightarrow{OH} = \frac{\tan(\hat{A})\overrightarrow{OA}+\tan(\hat{B})\overrightarrow{OB}+\tan(\hat{C})\overrightarrow{OC}}{\tan(\hat{A})+\tan(\hat{B})+\tan(\hat{C})}$
4. $\overrightarrow{OQ} = \frac{\sin(2\hat{A})\overrightarrow{OA}+\sin(2\hat{B})\overrightarrow{OB}+\sin(2\hat{C})\overrightarrow{OC}}{\sin(2\hat{A})+\sin(2\hat{B})+\sin(2\hat{C})}$

for any point O outside the plane of the triangle.

5. Let BOB' be an angle. Consider $A \in [O,B]$, $A' \in [O,B']$ and $m.n \in \mathbb{R}$ such that $\overrightarrow{OB} = m\overrightarrow{OA}$ and $\overrightarrow{OB'} = n\overrightarrow{OA'}$. Let $M = AB' \cap A'B$ and $N = AA' \cap BB'$. Show that

$$\overrightarrow{OM} = m\frac{1-n}{1-mn}\overrightarrow{OA} + n\frac{1-m}{1-mn}\overrightarrow{OA'}$$

and

$$\overrightarrow{ON} = m\frac{n-1}{n-m}\overrightarrow{OA} + n\frac{m-1}{m-n}\overrightarrow{OA'}.$$

6. Let $OAEBDC$ be a complete quadrilateral. Let M, N, P be the midpoints of the diagonals $[OB]$, $[AC]$ and $[ED]$ respectively. Show that M, N, P are collinear.

7. Let ABC be a triangle with centroid G, orthocenter H and circumcenter Q. Let A' be such that $[AA']$ is a diameter of the circumcenter. Show that

1. $\overrightarrow{QA} + \overrightarrow{QB} + \overrightarrow{QC} = \overrightarrow{QH}$,
2. $\overrightarrow{HA} + \overrightarrow{HC} = \overrightarrow{HA'}$,
3. $\overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC} = 2\overrightarrow{HQ}$,
4. $\overrightarrow{HA} + \overrightarrow{HB} + \overrightarrow{HC} = 3\overrightarrow{HG}$,
5. the points H, G, Q are collinear and $2|GQ| = |HG|$.