Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză, Curs: Dynamical Systems, Semestru: Primăvara 2020

Seminar 1

1. Show that the function $\varphi : \mathbb{R} \to \mathbb{R}$, given by the expression $\varphi(t) = 2e^{3t}$ for all $t \in \mathbb{R}$, is a solution of the Initial Value Problem $\quad x' = 3x, \quad x(0) = 2.$
Represent the corresponding integral curve* and describe its long term behavior**.

**A graphical representation of a solution of some differential equation is called an integral curve or a solution curve of this equation.*

***To describe the long term behavior of some function means to decide whether it is: periodic, oscillatory around some fixed value η^* (i.e. the values of the function changes many many times from values below η^* to values above η^*), bounded, increasing, and to describe how it behaves at $\pm\infty$.*

2. Let $\eta \in \mathbb{R}^*$ be fixed. Show that the function $\varphi : \mathbb{R} \to \mathbb{R}$, $\varphi(t) = \eta \sin t$ for all $t \in \mathbb{R}$, is a solution of the Initial Value Problem $\quad x''+x = 0, \quad x(0) = 0, \quad x'(0) = \eta.$
Represent the corresponding integral curve and describe its long term behavior.
For each $\eta \in \{\frac{\pi}{18}, -\frac{\pi}{18}, \frac{\pi}{2}, \frac{\pi}{3}, 1, 2\}$ describe the movement of a pendulum if $\varphi(t) = \eta \sin t$ is the angle (measured in radians in the trigonometric sense) between the rod and the vertical. We consider a simple gravity pendulum (idealized) that moves along a vertical circle whose radius is equal to the length of the rod. The movement initiates at the moment $t = 0$.

3. Show that the function $\varphi(t) = e^{-2t} \cos t$ for all $t \in \mathbb{R}$, is a solution of the Initial Value Problem $\quad x'' + 4x' + 5x = 0, \quad x(0) = 1, \quad x'(0) = -2.$
Represent this integral curve and describe its long term behavior.
Describe the movement of a pendulum if $\varphi(t) = e^{-2t} \cos t$ is the angle (measured in radians in the trigonometric sense) between the rod and the vertical. We consider a gravity pendulum (idealized) that moves along a vertical circle whose radius is equal to the length of the rod, which is subject to friction. The movement initiates at the moment $t = 0$.

4. Decide whether $\varphi : \mathbb{R} \to \mathbb{R}$, $\varphi(t) = \cos t$ for all $t \in \mathbb{R}$, is a solution of the differential equations $x' + x = 0$ or $x'' - x = 0$ or $x''' + x' = 0$ or $x^{(4)} + x'' = 0$.

5. Find all constant solutions of the differential equations: a) $x' = x - x^3$; b) $x' = \sin x$; c) $x' = \dfrac{x+1}{2x^2+5}$; d) $x' = x^2+x+1$; e) $x' = x+4x^3$; f) $x' = -1+x+4x^3$.

6. i) Let the functions $x_1, x_2, x_3 : \mathbb{R} \to \mathbb{R}$ be defined by $x_1(1) = 1$, $x_2(t) = t$ and $x_3(t) = t^2$ for all $t \in \mathbb{R}$. Prove that they are linearly independent in the linear space $C(\mathbb{R})$ (over the filed $\mathbb{R}$ and with the usual operations).

(ii) Find all $a, b, c \in \mathbb{R}$ such that $x(t) = at^2+bt+c$ is a solution of $x'-5x = 2t^2+3$ or $x'' = 0$ or $x''' = 0$. Write the solutions (of the differential equation) that you found.

7. (i) Let the functions $x_1, x_2, x_3 : \mathbb{R} \to \mathbb{R}$ be such that $x_1(1) = \cos t$, $x_2(t) = \sin t$ and $x_3(t) = e^t$ for all $t \in \mathbb{R}$. Prove that they are linearly independent in the linear space $C(\mathbb{R})$.

(ii) Find all $a, b, c \in \mathbb{R}$ such that $x(t) = a \sin t + b \cos t + c\, e^t$ is a solution of $x' + x = -3 \sin t + 2\, e^t$ or $x'' + 4x = -3 \sin t$ or $x'' + x = -3 \sin t$ or $x'' + x = 0$ or $x''' - x'' + x' - x = 0$. Write the solutions you found.

8. Find $r \in \mathbb{R}$ such that $x(t) = e^{rt}$ is a solution of $x'' - 5x' + 6x = 0$ or $x''' - 5x'' + 6x' = 0$ or $x^{(4)} - 5x''' + 6x'' = 0$ or $x'' + 9x = 0$ or $x'' + x' + x = 0$.

9. Find $r \in \mathbb{R}$ such that $x(t) = t^r$ is a solution on the interval $(0, \infty)$ of $t^2x'' - 4tx' + 6x = 0$ or $t^2x'' + tx' - x = 0$ or $t^2x'' - x = 0$ or $t^2x'' + x = 0$ or $t^2x'' - tx' + x = 0$.

10. Find as many functions $x \in C^1(\mathbb{R})$ as you can such that: a) $x' = x$; b) $x' = 2x$; c) $x' = -x$; d) $x' = ax$, with $a \neq 0$ a real parameter.

Integrate the following differential equations.

a) $x' = 0$; b) $x' = 2t$; c) $x' = \sin t$; d) $x' = 2t + \sin t$; e) $x' = e^{2t} \cos t$;
f) $x' = (t^2 - 5t + 7) \sin t$; g) $x' = e^{t^2}$; h) $x'' = -3$; i) $x''' = 0$.
j) $tx'+x = 0$; k) $tx'+x = 1$; l) $2xx' = -2t$; m) $x'e^t+xe^t = 0$; n) $x'e^{2t}+2xe^{2t} = 0$
We say that $\mu(t) = e^t$ is an *integrating factor* of $x' + x = 0$.
o) $x' + x = 0$; p) $x' + x = 1 + t$; q) $x' + 2x = \sin t$; r) $x' - 2x = 0$;
s) $tx' + 2x = 1$; t) $tx' + 3x = \frac{1}{t^2}$.