Universitatea Babeş–Bolyai
Facultatea de Matematică şi Informatică

Exam on Dynamical Systems, June 7, 2021

1. (1p) Study the solvability of the boundary value problem

$$x'' + 9x = 2, \quad x(0) = x(3\pi) = 0.$$

Here, as usual, the unknown is the function $t \in \mathbb{R} \mapsto x(t) \in \mathbb{R}$.

2. (0.5p) Study the solvability of the following problem

$$x_{k+2} = e^{x_{k+1}} - 5x_k + 7, \quad k \geq 0, \quad x_0 = 0, \quad x_1 = 0, \quad x_2 = 8.$$

Here, as usual, the unknown is the sequence $k \in \mathbb{N} \mapsto x_k \in \mathbb{R}$.

3. (1p) Find the general solution of the system

$$x' = -7x, \quad y' = x + 7y,$$

using the characteristic equation method for systems.

4. We consider the nonlinear planar system

$$\dot{x} = x + 1, \quad \dot{y} = -3y - xy.$$

a) (0.5p) Find its equilibrium point and the linearized system around it.
b) (2p) Find a first integral.
c) ((0.5p) If the linearized system around the equilibrium point has a global first integral, find it.
d) (0.5p) Represent the phase portrait of the linearized system around the equilibrium point.