ARTIFICIAL INTELLIGENCE

Solving search problems

Uninformed search strategies

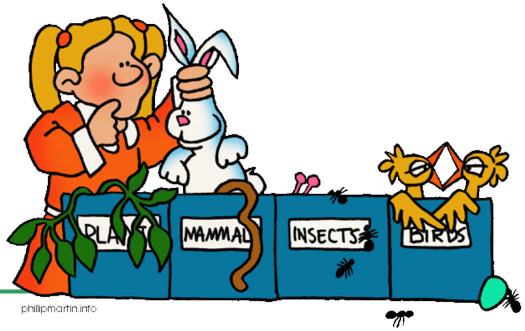
Content

- Problems
- Problem solving
  - Steps of problem solving
- Solving problem by search
  - Steps of solving problem by search
  - Search strategies

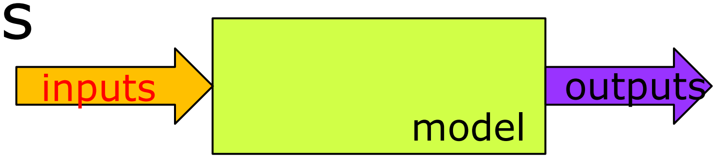
Problems

- Two problem types:
  - Solving in a deterministic manner
    - Computing the sinus of an angle or the square root of a number
  - Solving in a stochastic manner
    - Real-world problems → design of ABS
    - Involve the search of a solution → AI's methods

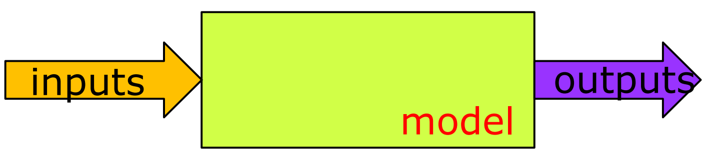
Problems



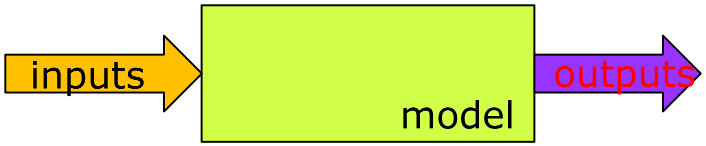
- Tipology
  - Search/optimisation problems



    - Planning, satellite's design
  - Modeling problems



    - Predictions, classifications
  - Simulation problems



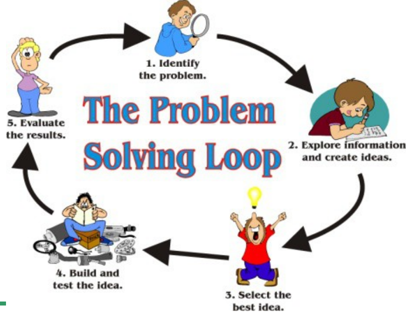
    - Game theory

Problem solving

- Identification of a solution
  - In computer science (AI) → search process
  - In engineering and mathematics → optimisation process

- How?
  - Representation of (partial) solutions → points in the search space
  - Design of a search operators → map a potential solution into another one

Steps in problem solving

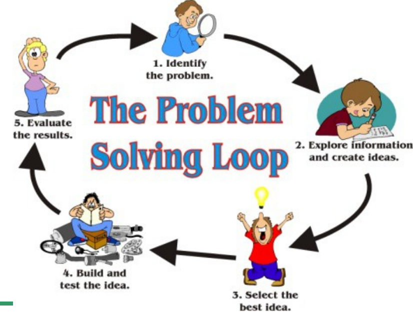


- Problem definition
- Problem analyses
- Selection of a solving technique
  - Search
  - Knowledge representation
  - Abstraction

Solving problems by search

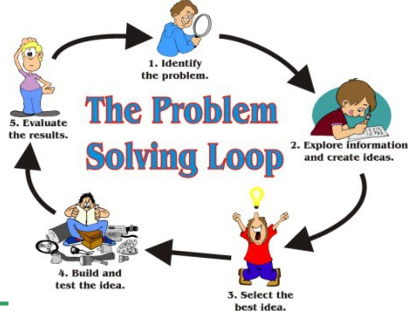
- Based on some objectives
- Composed by actions that accomplish the objectives
  - Each action changes a state of the problem
- More actions that map the initial state of problem into a final state

Steps in solving problems by search
Problem definition

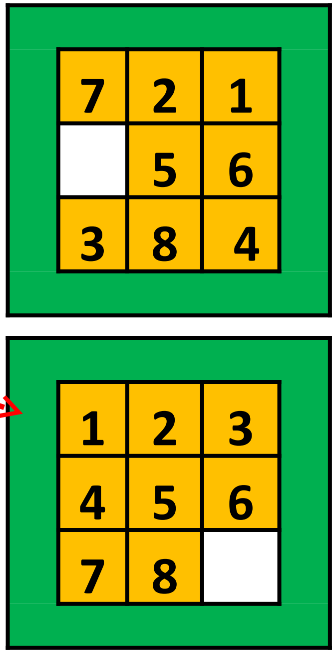


- Problem definition involves:
  - A search space
    - All possible states
    - Representation
      - Explicit – construction of all possible states
      - Default – by using some data structures and some functions (operators)
  - One or more initial state
  - One or more final states
  - One or more paths
    - More successive states
  - A set of rules (actions)
    - Successor functions (operators) – next state after a given one
    - Cost functions that evaluate
      - How a state is mapped into another state
      - An entire path
    - Objective functions that check if a state is final or not

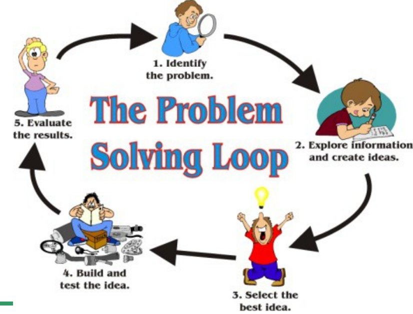
Steps in solving problems by search
Problem definition



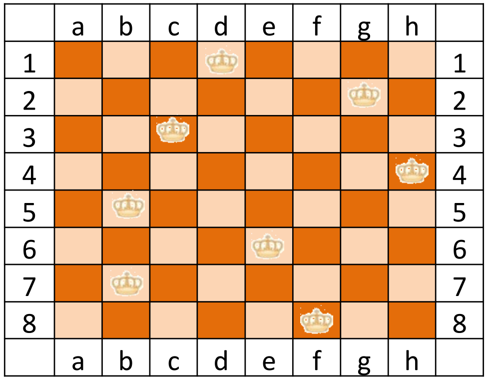
- Examples
  - Puzzle game with 8 pieces
    - State’s space – different board configurations for a game with 8 pieces
    - Initial state – a random configuration
    - Final state – a configuration where all the pieces are sorted in a given manner
    - Rules -> white moves
      - conditions: move inside the table
      - Transformations: the white space is moved up, down, to left or to right
    - Solution - optimal sequence of white moves



Steps in solving problems by search
Problem definition

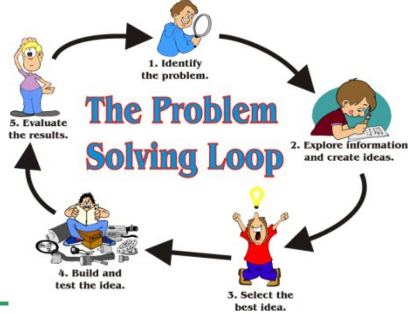


- Examples
  - Queen's problem
    - State's space – different board configurations for a game with n queens



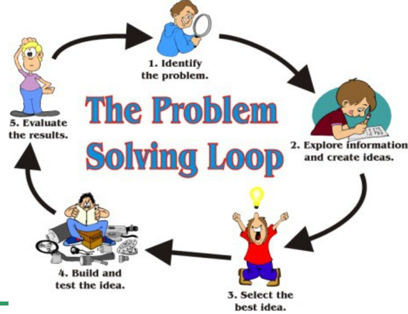
    - Initial state – a configuration without queens
    - Final state – a configuration n queens so that none of them can hit any other in one move
    - Rules -> put a queen on the table
      - conditions: the queen is not hit by any other queen
      - Transformations: put a new queen in a free cell of the table
    - Solution - optimal placement of queens

Steps in solving problems by search
Problem analyse



- The problem can be decomposed?
  - The sub-problems are independent or not?
- The possible state's space is predictable?
- We want a solution or an optimal solution?
- The solution is represented by a single state or by more successive states?
- We require some knowledge for limiting the search or for identifying the solution?
- The problem is conversational or solitary?
  - Human interaction is required for problem solving?

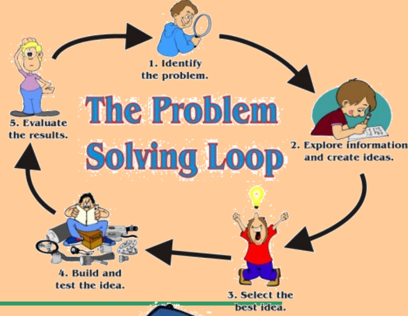
Steps in solving problems by search
Selection of a solving technique



- Solving by moving rules (and control strategy) in the search space until we find a path from the initial state to the final state

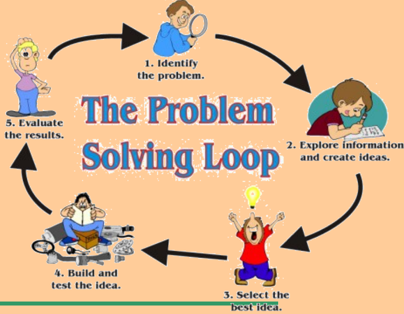
- Solving by search
  - Examination of all possible states in order to identify
    - A path from the initial state to the final state
    - An optimal state
  - The search space = all possible states and the operators that maps the states

Steps in solving problems by search
Selection of a solving technique



- Solving by search
  - More searching strategies → how we select one of them?
    - Computational complexity (temporal and spatial)
    - Completeness → the algorithms always ends and finds a solution (if it exists)
    - Optimal → the algorithms finds the optimal solution (the optimal cost of the path from the initial state to the final state)

Steps in solving problems by search
Selection of a solving technique

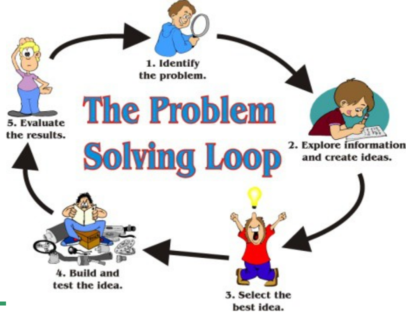


- Solving by search
  - More searching strategies → how we select one of them? → Computational complexity (temporal and spatial)
    - Strategy's performance depends on
      - Time for running
      - Memory for running
      - Size of input data
      - Computer's performance
      - Compiler's quality
    - Can be evaluated by complexity → computational efficiency
      - Spatial → required memory for solution identification
        - *S(n)* – memory used by the best algorithms *A* that solves a decision problem *f* with *n* input data
      - Temporal → required time for solution identification
        - T(n) – running time (number of steps) of the best algorithm A that solves a decision problem *f* with *n* input data

Internal factors

External factors

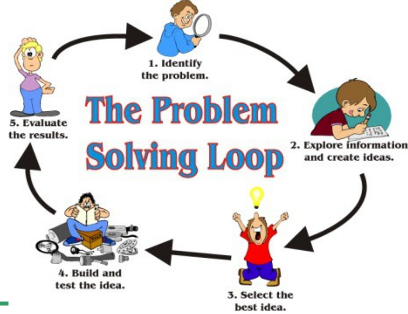
Steps in solving problems by search
Selection of a solving technique



- Problem solving by search can be performed by:
  - Step by step construction of solution

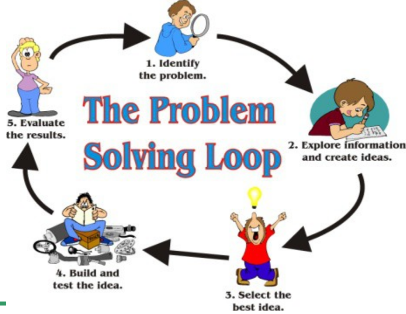
  - Optimal solution identification

Steps in solving problems by search
Selection of a solving technique



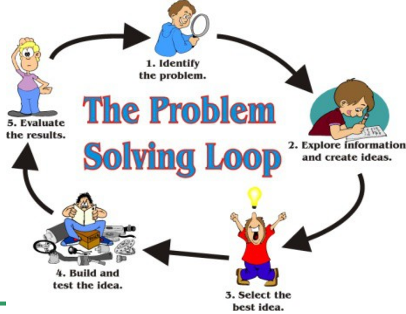
- Problem solving by search can be performed by:
  - Step by step construction of solution
    - Problem's components
      - Initial state
      - Operators (successor functions)
      - Final state
      - Solution = a path (of optimal cost) from the initial state to the final state
    - Search space
      - All the states that can be obtained from the initial state (by using the operators)
      - A state = a component of solution
    - Example
      - Traveling Salesman Problem (TSP)
    - Algorithms
      - Main idea: start with a solution's component and adding new components until a complete solution is obtained
      - Recurrent → until a condition is satisfied
      - The search's history (path from initial state to the final state) is retained in LIFO/FIFO containers
    - Advantages
      - Do not require knowledge (intelligent information)

Steps in solving problems by search
Selection of a solving technique

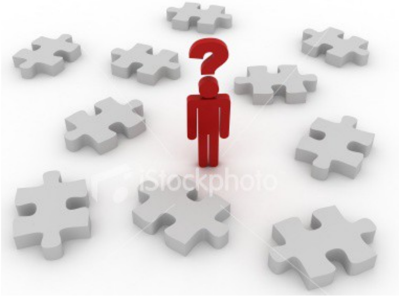


- Problem solving by search can be performed by:
  - Optimal solution identification
    - Problem's components
      - Conditions (constraints) that must be satisfied by the solution
      - Evaluation function for a potential solution → optimum identification
    - Search space
      - All possible and complete solutions
      - State = a complete solution
    - Example
      - Queen's problem
    - Algorithms
      - Main idea: start with a state that doesn't respect some conditions and change it for eliminating these violations
      - Iterative → a single state is retained and the algorithm tries to improve it
      - The searches history is not retained
    - Advantages
      - Simple
      - Requires a small memory
      - Can find good solutions in (continuous) search spaces very large (where other algorithms can not be utilised)

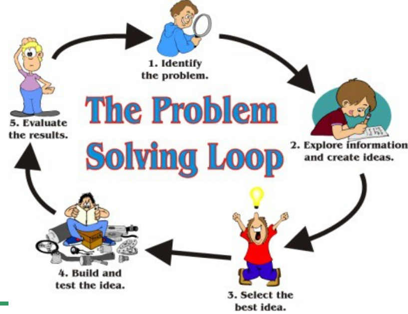
Steps in solving problems by search
Selection of a solving technique



- Solving problem by search involves:
  - Very complex algorithms (NP-complete problems)
  - Search in an exponential space

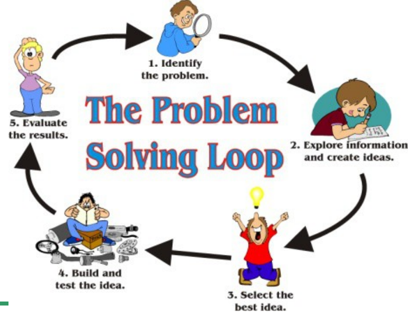


Steps in solving problems by search
Selection of a solving technique



- Topology of search strategies:
  - Solution **generation**
    - **Constructive** search
      - Solution is identified step by step
      - Ex. TSP
    - **Perturbative** search
      - A possible solution is modified in order to obtain another possible solution
      - Ex. SAT - Propositional Satisfaction Problem
  - Search space **navigation**
    - **Systematic** search
      - The entire search space is visited
        - Solution identification (if it exists) → complete algorithms
    - **Local** search
      - Moving from a point of the search space into a neighbor point → incomplete algorithms
      - A state can be visited more times
  - **Certain** items of the search
    - **Deterministic** search
      - Algorithms that exactly identify the solution
    - **Stochastic** search
      - Algorithms that approximate the solution
  - Search space **exploration**
    - **Sequential** search
    - **Parallel** search

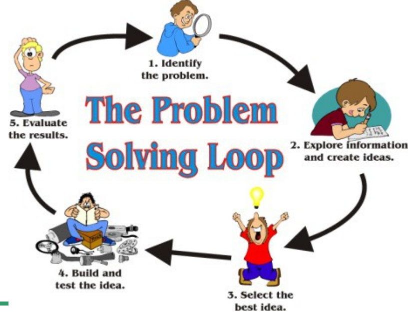
Steps in solving problems by search
Selection of a solving technique



Topology of search strategies:

- **Number of objectives**
  - **Single-objective** search
    - The solution must respect a single condition/constraint
  - **Multi-objective** search
    - The solution must respect more conditions/constraints
- **Number of solutions**
  - **single-modal search**
    - There is a single optimal solution
  - **multi-modal search**
    - There are more optimal solutions
- **Algorithm**
  - Search over a **finite number of steps**
  - **Iterative search**
    - The algorithms converge through the optimal solutions
  - **Heuristic search**
    - The algorithms provide an approximation of the solution
- Search **mechanism**
  - **traditional search**
  - **modern search**
- where the search takes **place**
  - **local search**
  - **global search**

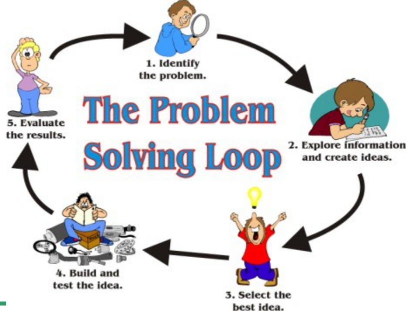
Steps in solving problems by search
Selection of a solving technique



Topology of search strategies:

- Type **(linearity) of constraints**
  - **Linear search**
  - **non-linear search**
    - Clasical (deterministic)
      - Direct – based on evaluation of the objective function
      - Indirect – based on derivative (I and/or II) of the objective function
    - Enumeration-based
      - How solution is identified
        - Uninformed – the solution is the final state
        - Informed – deals with an evaluation function for a possible solution
      - Search space type
        - Complete – the space is finite (if solution exists, then it can be found)
        - Incomplete – the space is infinite
    - Stochastic search
      - Based on random numbers
- **Agents** involves in search
  - Search by **a single agent** → without obstacle for achieving the objectives
  - **Adversarial search** → the opponent comes with some uncertainty

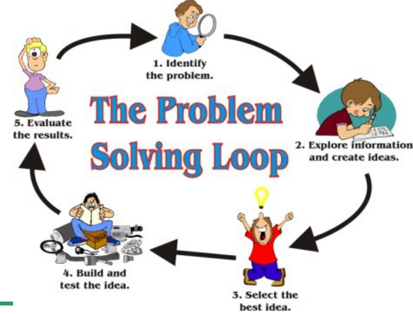
Steps in solving problems by search
Selection of a solving technique



Example

- Topology of search strategies
  - Solution generation
    - **Constructive search**
    - Perturbative search
  - Search space navigation
    - **Systematic search**
    - Local search
  - Certain items of the search
    - **Deterministic search**
    - Stochastic search
  - Search space exploration
    - **Sequential search**
    - Parallel search

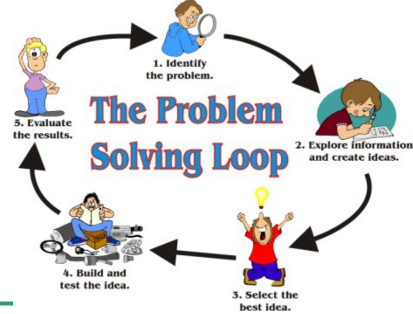
Steps in solving problems by search
Selection of a solving technique

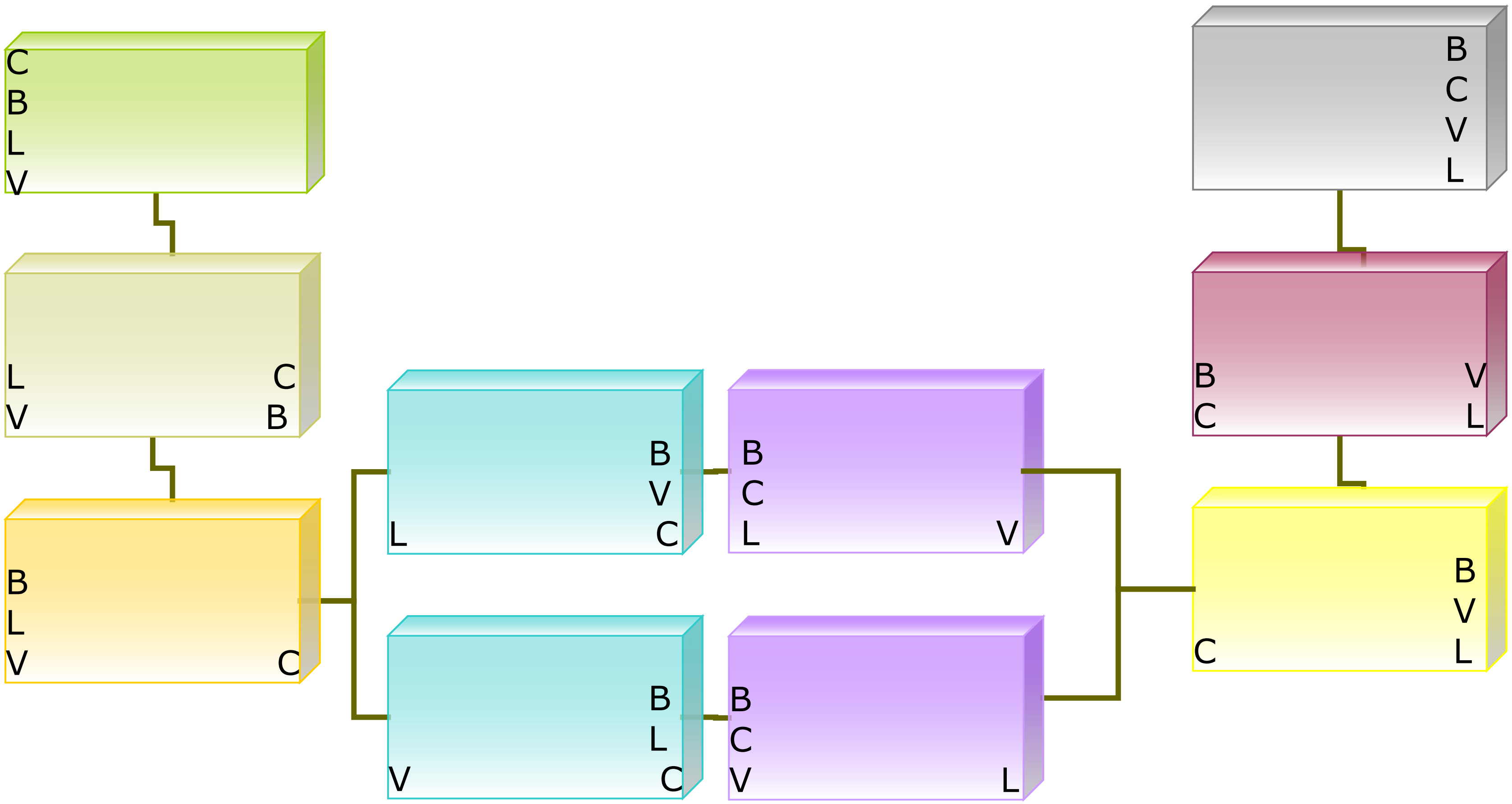


Example
- Constructive, global, determinist, sequential search

- Problem"capra, varza şi lupul"
  - Input :
    - A goat, a cabbage and a wolf on a river-side
    - A boat with a boater
  - Output:
    - Move all the passengers on the other side of the river
    - Taking into account:
      - The boat has only 2 places
      - It is not possible to rest on the same side:
        - The goat and the cabbage
        - The wolf and the goat

Steps in solving problems by search
Selection of a solving technique





Search strategies – Basic elements

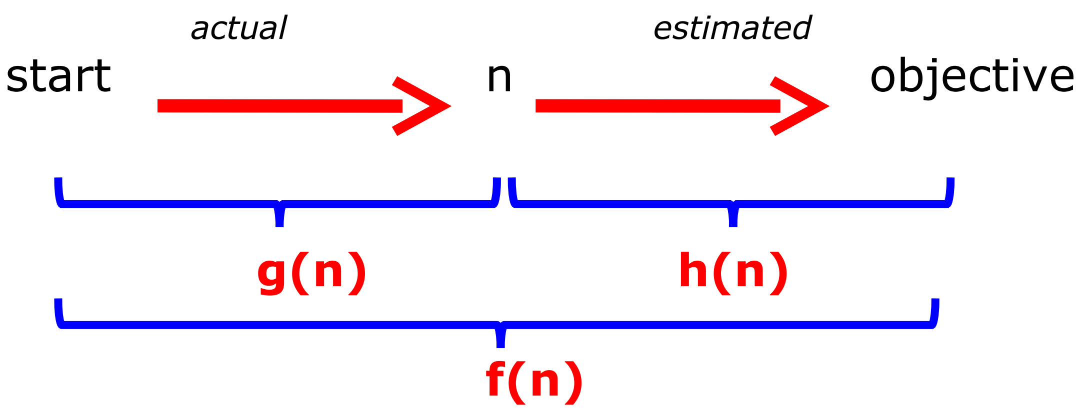
- Abstract data types (ADTs)
  - ADT list → linear structure
  - ADT tree → hierarchic structure
  - ADT graph → graph-based structure
- ADT
  - Domain and operations
  - Representation

Uninformed search strategies (USS)

- Characteristics
  - Are NOT based on problem specific information
  - Are general
  - Blind strategies
  - Brute force methods
- Topology
  - Order of node exploration:
    - USS in linear structures
      - Linear search
      - Binary search
    - USS in non-linear structures
      - Breadth-first search
        - Uniform cost search (branch and bound)
      - Depth first search
        - Limited depth first search
        - iterative deepening depth-first search
      - Bidirectional search

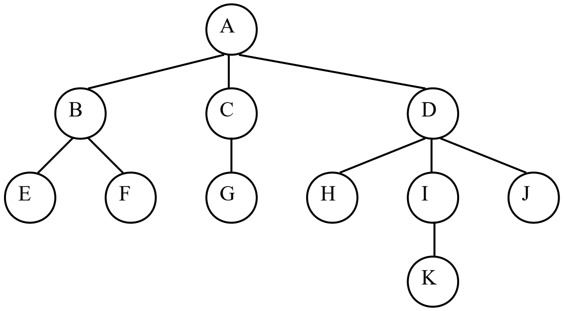
SS in tree-based structures

- Basic elements
  - *f(n)* – evaluation function for estimating the cost of a solution through node (state) *n*
  - *h(n)* – evaluation function for estimating the cost of a solution path from node (state) *n* to the final node (state)
  - *g(n)* – evaluation function for estimating the cost of a solution path from the initial node (state) to node (state) *n*
  - *f(n) = g(n) + h(n)*



USS in tree-based structures
Breadth-first search – BFS

- **Basic elements**
  - All the nodes of depth *d* are visited before all the nodes of depth *d+1*
  - All children of current node are added into a FIFO list (queue)
- **Examplu**



  - Visiting order: A, B, C, D, E, F, G, H, I, J, K
- **Algorithm**

```
bool BFS(elem, list){
        found = false;
        visited = Φ;
        toVisit = {start};          //FIFO list
        while((toVisit != Φ) && (!found)){
                node = pop(toVisit);
                visited = visited U {node};
                if (node == elem)
                    found = true;
                else{
                    aux = Φ;
                    for all (unvisited) children of node do{
                        aux = aux U {child};
                    }
                }
                toVisit = toVisit U aux;
        }  //while
        return found;
}
```

Vizitate deja	De vizitat
Φ	A
A	B, C, D
A, B	C, D, E, F
A, B, C	D, E, F, G
A, B, C, D	E, F, G, H, ,I, J
A, B, C, D, E	F, G, H, I, J
A, B, C, D, E, F	G, H, I, J
A, B, C, D, E, F, G	H, I, J
A, B, C, D, E, F, G, H	I, J
A, B, C, D, E, F, G, H, I	J, K
A, B, C, D, E, F, G, H, I, J	K
A, B, C, D, E, F, G, H, I, J, K	Φ

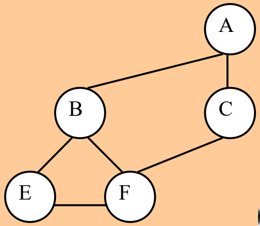
USS in tree-based structures
Breadth-first search – BFS

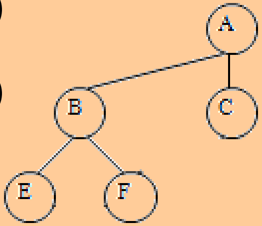
- **Search analyse:**
  - Time complexity:
    - *b* – ramification factor (number of children of a node)
    - *d* – length (depth) of solution
    - $T(n) = 1 + b + b^2 + \ldots + b^d \Rightarrow O(b^d)$
  - Space complexity
    - $S(n) = T(n)$
  - Completness
    - If solution exists, then BFS finds it
  - Optimality
    - No

- **Advantages**
  - Finds the shortest path to the objective node (the shallowest solution)

- **Disadvantages**
  - Generate and retain a tree whose size exponentially increases (with depth of objective node)
  - Exponential time and space complexity
  - Russel&Norving experiment
  - Works only for small search spaces

- **Applications**
  - Identification of connex components in a graph
  - Identification of the shortest path in a graph
  - Optimisation in transport networks → algorithm Ford-Fulkerson
  - Serialisation/deseralisation of a binary tree (vs. serialization in a sorted manner) allows efficiently reconstructing of the tree
  - Collection copy (garbage collection) → algorithm Cheney



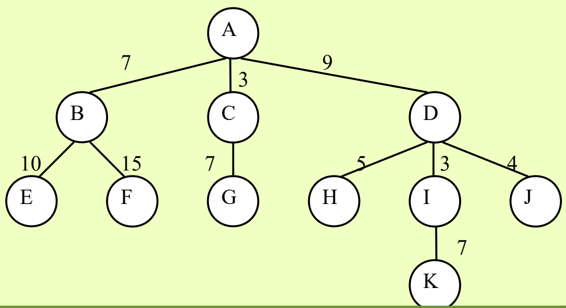


Vizitate deja	De vizitat
Φ	B
B	A, E, F
B, A	E, F, C
B, A, E	F, C
B, A, E, F	C
B, A, E, F, C	Φ

USS in tree-based structures
Uniform cost search – UCS

- **Basic elements**
  - BFS +special expand procedure (based on the cost of links between nodes)
  - All the nodes of depth *d* are visited before all the nodes of depth *d+1*
  - All children of current node are added into a FIFO ordered list
    - The nodes of minimum cost are firstly expanded
    - When a path to the final state is obtained, it became a candidat to the optimal solution
  - *Branch and bound* algorithm

- **Example**
  - Visiting oreder: A, C, B, D, G, E, F, I, H, J, K



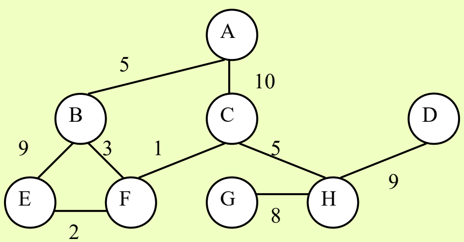
- **Algorithm**

```
bool UCS(elem, list){
        found = false;
        visited = Φ;
        toVisit = {start};       //FIFO sorted list
        while((toVisit != Φ) && (!found)){
                node = pop(toVisit);
                visited = visited U {node};
                if (node== elem)
                    found = true;
                else
                    aux = Φ;
                for all (unvisited) children of node do{
                    aux = aux U {child};
                }  // for
                toVisit = toVisit U aux;
                TotalCostSort(toVisit);
        }  //while
        return found;
}
```

visited	toVisit
Φ	A
A	C(3), B(7), D(9)
A, C	B(7), D(9), G(3+7)
A, C, B	D(9), G(10), E(7+10), F(7+15)
A, C, B, D	G(10), I(9+3), J(9+4) ,H(9+5), E(17), F(22)
A, C, B, D, G	I(12), J(13) ,H(14), E(17), F(22)
A, C, B, D, G, I	J(13) ,H(14), E(17), F(22), **K(9+3+7)**
A, C, B, D, G, I, J	H(14), E(17), F(22), **K(19)**
A, C, B, D, G, I, J, H	E(17), F(22), **K(19)**
A, C, B, D, G, I, J, H, E	F(22), **K(19)**
A, C, B, D, G, I, J, H, E, F	K(19)
A, C, B, D, G, I, J, H, E, F, K	Φ

USS in tree-based structures
Uniform cost search – UCS

- **Complexity analyse**
  - Time complexity
    - *b* – ramification factor
    - *d* - length (depth) of solution
    - $T(n) = 1 + b + b^2 + \ldots + b^d \Rightarrow O(b^d)$
  - Space complexity
    - $S(n) = T(n)$
  - Completness
    - yes – if solutions exists, then UCS finds it
  - Optimality
    - Yes
- **Advantages**
  - Finding the minimum cost path to the objective node
- **Disadvantages**
  - Exponential time and space complexity
- **Applications**
  - Shortest path → Dijkstra algorithm

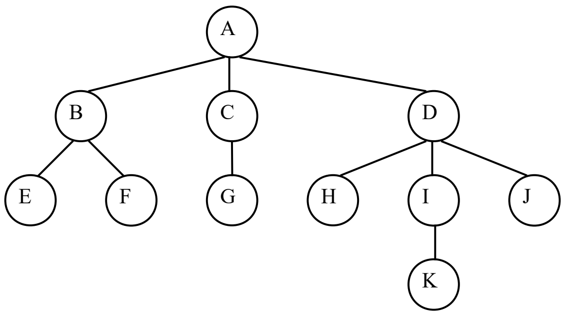


Vizitate deja	De vizitat
Φ	A(0)
A(0)	B(5), C(10)
A(0), B(5)	F(8), C(10), E(14)
A(0), B(5), F(8)	C(9), E(10)
A(0), B(5), F(8), C(9)	E(10), H(14)
A(0), B(5), F(8), C(9), E(10)	H(14)

USS in tree-based structures depth-first search – DFS

- **Basic elements**
  - Expand a child and depth search until
    - The final node is reached or
    - The node is a leaf
  - Coming back in the most recent node that must be explored
  - All the children of the current node are added in a LIFO list (stack)

- **Examplue**
  - Visiting order: A, B, E, F, C, G, D, H, I, K, J



Vizitate deja	De vizitat
Φ	A
A	B, C, D
A, B	E, F, C, D
A, B, E	F, C, D
A, B, E, F	C, D
A, B, E, F, C	G, D
A, B, E, F, C, G	D
A, B, E, F, C, G, D	H, I, J
A, B, E, F, C, G, D, H	I, J
A, B, E, F, C, G, D, H, I	K, J
A, B, E, F, C, G, D, H, I, K	J
A, B, E, F, C, G, D, H, I, K, J	Φ

- **Algorithm**

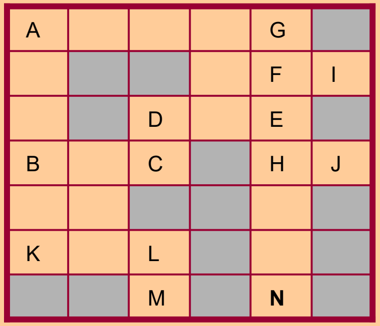
```
bool DFS(elem, list){
      found = false;
      visited = Φ;
      toVisit = {start};      //LIFO list
      while((toVisit != Φ) && (!found)){
            node = pop(toVisit);
            visited = visited U {node};
            if (node== elem)
               found = true;
            else{
                  aux = Φ;
                  for all (unvisited) children of node do{
                     aux = aux U {child};
                  }
                  toVisit = aux U toVisit;
            }
      }  //while
      return found;
}
```

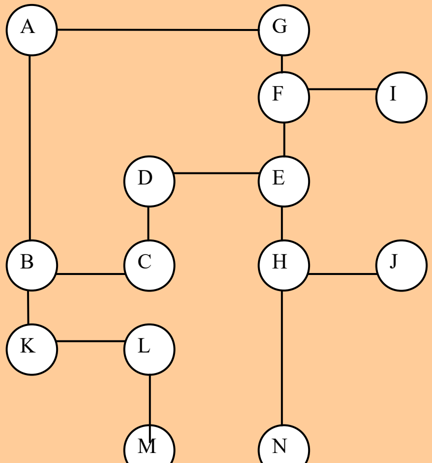
USS in tree-based structures
depth-first search – DFS

- **Complexity analyse**
  - Time complexity
    - b - ramification factor
    - d^{max} – maximal length (depth) of explored tree
    - $T(n) = 1 + b + b^2 + \ldots + b^{dmax} \Rightarrow O(b^{dmax})$
  - Space complexity
    - $S(n) = b * d_{max}$
  - Completness
    - No → the algorithm does not end for infinite paths (there is no sufficient memory for all the nodes that are visited already)
  - Optimality
    - No → depth search can find a longer path than the optimal one

- **Advantages**
  - Finding the shortest path with minimal resources (recursive version)

- **Disadvantages**
  - Dead paths
    - Infinite cycles
    - Longer solution than the optimal one
- **Applications**
  - Maze problem
  - Identification of connex components
  - Topological sorting
  - Testing the graph planarity





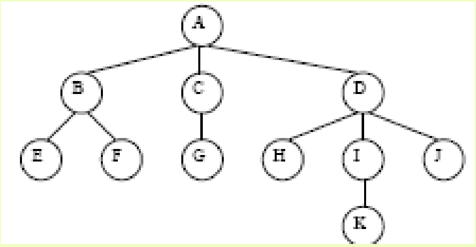
USS in tree-based structures
depth-first search – DFS

```
bool DFS_edges(elem, list){
   discovered = Φ;
   back = Φ;
   toDiscover = Φ;     //LIFO
   for (all neighbours of start) do
         toDiscover = toDiscover U {(start, neighbour)}
   found = false;
   visited = {start};
   while((toDiscover != Φ) && (!found)){
         edge = pop(toDiscover);
         if (edge.out !є visited){
            discovered = discovered U {edge};
            visited = visited U {edge.out}
            if (edge.out == end)
                  found = true;
            else{
                  aux = Φ;
                  for all neighbours of edge.out  do{
                  aux = aux U {(edge.out, neighbour)};
            }
            toDiscover = aux U toDiscover;
         }
         else
            back = back U {edge}
   }  //while
   return found;
}
```

Muchia	Muchii vizitate deja	Muchii de vizitat	înapoi	Noduri vizitate
	Φ	AB, AF	Φ	A
AB	AB	BC, BK, AF	Φ	A, B
BC	AB, BC	CD, BK, AF	Φ	A, B, C
CD	AB. BC, CD	DE, BK, AF	Φ	A, B, C, D
DE	AB, BC, CD, DE	EF, EH, BK, AF	Φ	A, B, C, D, E
EF	AB, BC, CD, DE, EF	FI, FG, EH, BK, AF	Φ	A, B, C, D, E, F
FI	AB, BC, CD, DE, EF, FI	FG, EH, BK, AF	Φ	A, B, C, D, E, F, I
FG	AB, BC, CD, DE, EF, FI, FG	GA, EH, BK, AF	Φ	A, B, C, D, E, F, I, G
GA	AB, BC, CD, DE, EF, FI, FG	EH, BK, AF	GA	A, B, C, D, E, F, I, G
EH	AB, BC, CD, DE, EF, FI, FG	HJ, HN, BK, AF	GA	A, B, C, D, E, F, I, G, H
HJ	AB, BC, CD, DE, EF, FI, FG, HJ	HN, BK, AF	GA	A, B, C, D, E, F, I, G, H, J
HN	AB, BC, CD, DE, EF, FI, FG, HI, HN	BK, AF	GA	A, B, C, D, E, F, I, G, H, N

USS in tree-based structures depth-limited search – DLS

- **Basic elements**
  - DFS + maximal depth that limits the search (d_{lim})
  - Solved the completeness problems of DFS
- **Example**
  - $d_{lim} = 2$
  - Visiting order: A, B, E, F, C, G, D, H, I, J



Vizitate deja	De vizitat
Φ	A
A	B, C, D
A, B	E, F, C, D
A, B, E	F, C, D
A, B, E, F	C, D
A, B, E, F, C	G, D
A, B, E, F, C, G	D
A, B, E, F, C, G, D	H, I, J
A, B, E, F, C, G, D, H	I, J
A, B, E, F, C, G, D, H, I	J
A, B, E, F, C, G, D, H, I, K, J	Φ

- **Algorithm**

```
bool DLS(elem, list, dlim){
        found = false;
        visited = Φ;
        toVisit = {start}; //LIFO list
        while((toVisit != Φ) && (!found)){
                node = pop(toVisit);
                visited = visited U {node};
                if (node.depth <= dlim){
                        if (node == elem)
                            found = true;
                        else{
                            aux = Φ;
                            for all (unvisited) children of node do{
                                        aux = aux U {child};
                            }
                            toVisit = aux U toVisit;
                        }//if found
                }//if dlim
        } //while
        return found;
}
```

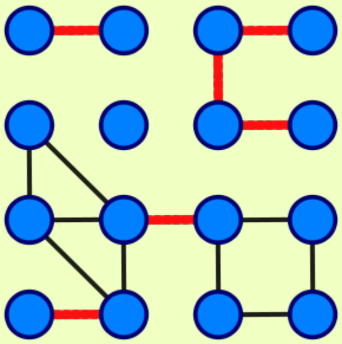
USS in tree-based structures depth-limited search – DLS

- **Complexity analyse**
  - Time complexity:
    - *b* – ramification factor
    - d^{lim} – limit of length (depth) allowed for the explored tree
    - $T(n) = 1 + b + b^2 + \ldots + b^{dlim} \Rightarrow O(b^{dlim})$
  - Space complexity
    - $S(n) = b * d_{lim}$
  - Completeness
    - Yes, but ⇔ $d_{lim} > d$, where *d* = length (path) of optimal solution
  - Optimality
    - No → DLS can find a longer path than the optimal one

- **Advantages**
  - Solves the completeness problems of DFS

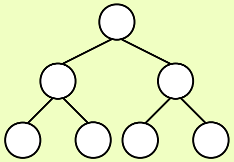
- **Disadvantages**
  - How to choose a good limit d_{lim}?

- **Applications**
  - Identification of bridges in a graph

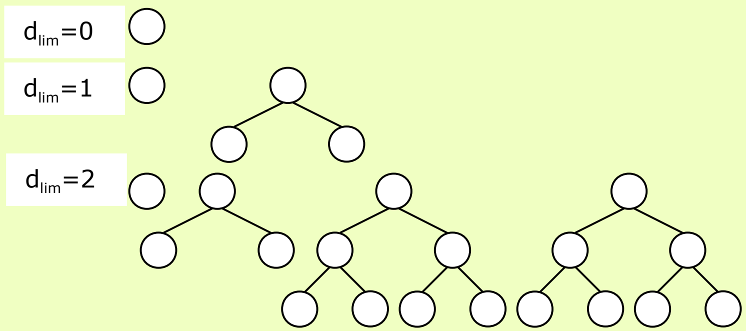


USS in tree-based structures
iterative deepening depth search – IDDS

- **Basic elements**
  - U DLS(d_{lim}), where d_{lim} = 1, 2, 3, ..., d_{max}
  - Solves the identification of the optimal limit d_{lim} from DLS
  - Usually, it works when:
    - The search space is large
    - The length (depth) of solution is known



- **Example**



- **Algorithm**

```
bool IDS(elem, list){
      found = false;
      dlim = 0;
      while ((!found) && (dlim < dmax)){
            found = DLS(elem, list, dlim);
            dlim++;
      }
      return found;
}
```

USS in tree-based structures
iterative deepening depth search – IDDS

- **Complexity analyse**
  - Time complexity:
    - b^{dmax} nodes at depth d_{max} are expanded once => 1 * b^{dmax}
    - b^{dmax-1} nodes at depth d_{max}*-1* are expanded twice => *2 * ($b^{dmax - 1}$)*
    - ...
    - *b* nodes at depth 1are expanded d_{max} times => d_{max} * b^1
    - 1 node (the root) at depth 0 is expanded d_{max}+1 times => *(d_{max}+1)*b^0*

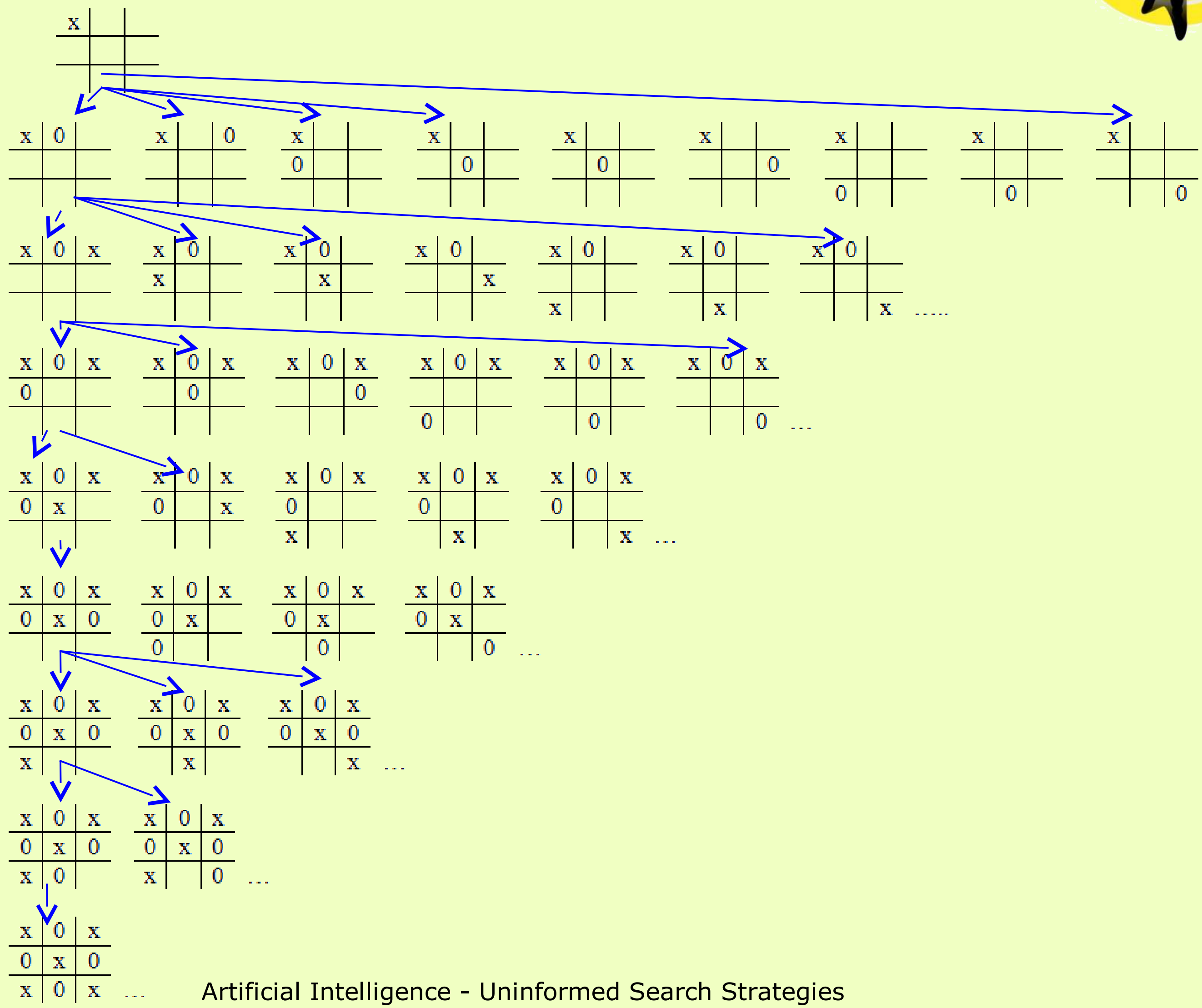
    $$T(n) = \sum_{i=0}^{d_{\max}} (i+1) b^{d_{\max}-1} \Rightarrow O(b^{d_{\max}})$$

  - Space complexity
    - *S(n) = b * d_{max}*
  - Completness
    - yes
  - Optimality
    - yes

- **Advantages**
  - Requires linear memory
  - The goal state is obtained by a minimal path
  - Faster than BFS and DFS
- **Disadvantages**
  - Requires to know the solution depth

- **Applications**
  - Tic tac toe game

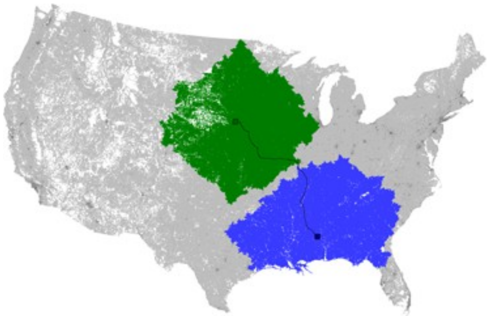
USS in tree-based structures
iterative deepening depth search – IDDS



USS in tree-based structures
bi-directional search – BDS

- **Basic elements**
  - 2 parallel search strategies
    - *forward*: from root to leaves
    - *backward*: from leaves to root

    that end when they meet
  - any SS can be used in a direction
  - Requires establishing:
    - the parents and the children of each node
    - the meeting point
- **Example**



- **Algorithm**
  - Depend on the SS used

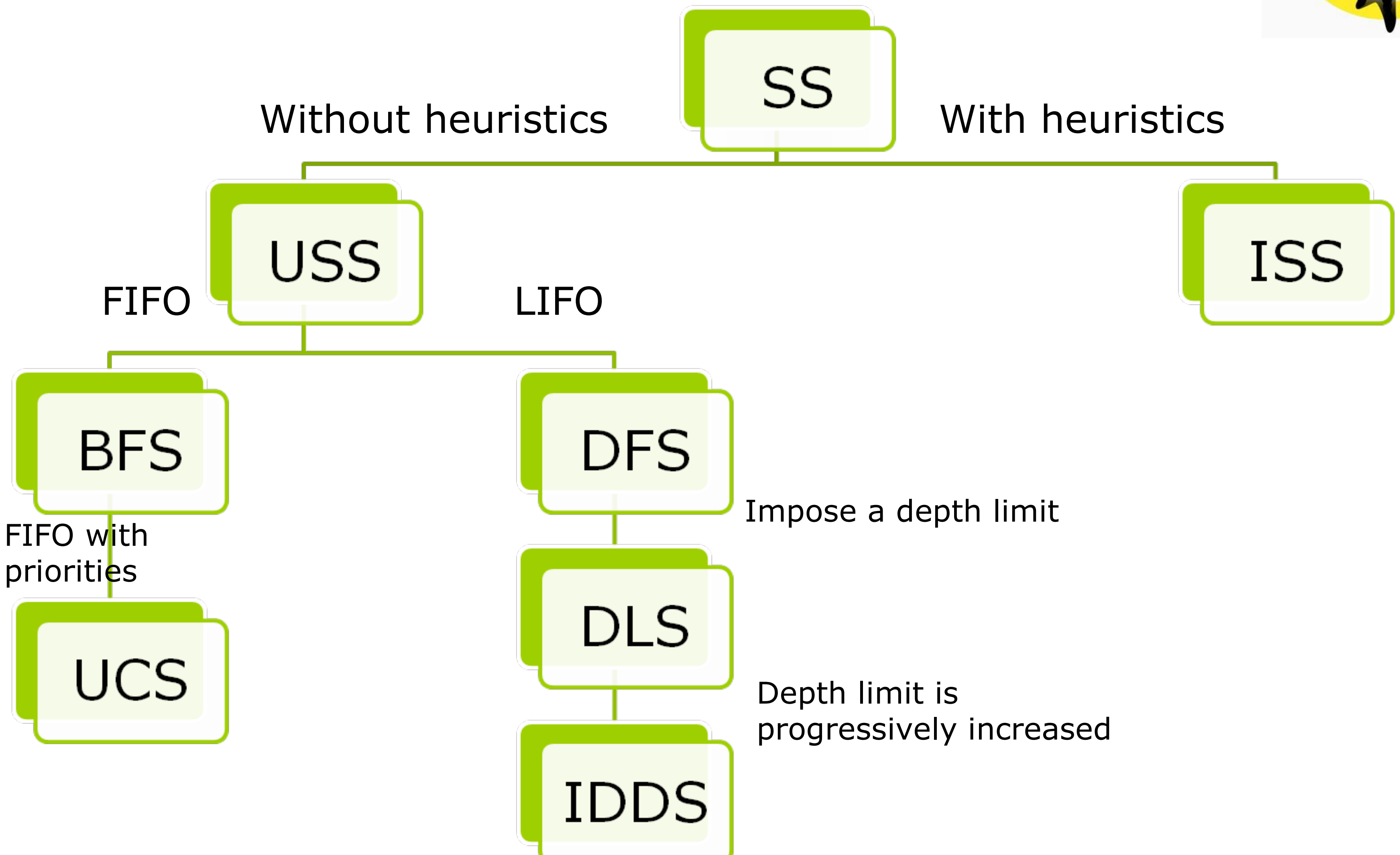
USS in tree-based structures
bi-directional search – BDS

- **Complexity analyse**
  - Time complexity
    - b – ramification factor
    - d – solution length (depth)
    - $O(b^{d/2}) + O(b^{d/2}) \Rightarrow O(b^{d/2})$
  - Space complexity
    - S(n) = T(n)
  - Completeness
    - yes
  - Optimality
    - yes

- **Advantages**
  - Good time and space complexity
- **Disadvantages**
  - Each state must be reversed
    - From had to tail
    - From tail to head
  - Difficult to implement
  - Identification of parents and children for all the nodes
  - The final state must be known

- **Applications**
  - Partitioning problem
  - Shortest path

USS in tree-based structures



USS in tree-based structures

Comparison of performances

SS	Time complexity	Space complexity	Completeness	Optimality
BFS	$O(b^d)$	$O(b^d)$	Yes	Yes
UCS	$O(b^d)$	$O(b^d)$	Yes	Yes
DFS	$O(b^{dmax})$	$O(b*d_{max})$	No	No
DLS	$O(b^{dlim})$	$O(b*d_{lim})$	Yes, if $d_{lim} > d$	No
IDS	$O(b^d)$	$O(b*d)$	Da	Yes
BDS	$O(b^{d/2})$	$O(b^{d/2})$	Yes	Yes