Databases

Lecture 6

Functional Dependencies. Normal Forms (III)

Sabina S. CS

* See recap lecture example with schema decomposition.

Example 10. The following relation stores the exam session schedule:

EX_SCHEDULE[Date, Hour, Faculty_member, Room, Group]

- the following restrictions are expressed via <u>key definitions</u> and <u>functional dependencies</u>:

  1. a group of students has at most one exam per day

  => *{Date, Group}* is a key

  2. on a certain date and time, a faculty member has at most one exam

  => *{Faculty_member, Date, Hour}* is a key

  3. on a certain date and time, there is at most one exam in a room

  => *{Room, Date, Hour}* is a key

  4. a faculty member doesn't change the room in a day

  => the following dependency holds: *{Faculty_member, Date}* → *{Room}*

- all attributes appear in at least one key, i.e., there are no non-prime attributes
- given the normal forms' definitions specified thus far, the relation is in 3NF
- objective: eliminate the *{Faculty_member, Date}* $\rightarrow$ *{Room}* functional dependency

Definition. A relation is in the Boyce-Codd normal form (BCNF) if and only if every determinant (for a functional dependency) is a key (informal definition - simplifying assumption: determinants are not too big; only non-trivial functional dependencies are considered).

- to eliminate the functional dependency, the original relation must be decomposed into:

EX_SCHEDULE'[Date, Hour, Faculty_member, Group],

ROOM_ALLOCATION[Faculty_member, Date, Room]

- these relations don't contain other functional dependencies, i.e., they are in BCNF
- however, the key associated with the 3rd constraint, *{Room, Date, Hour}*, does not exist anymore
- if this constraint is to be kept, it needs to be checked in a different manner (e.g., through the program)

- R[A] - a relation
- F - a set of functional dependencies
- α – a subset of attributes

- problems

I. compute the closure of F: F^+

II. compute the closure of a set of attributes under a set of functional dependencies, e.g., the closure of α under F: α^+

III. compute the minimal cover for a set of dependencies

- R[A] - a relation
- F - a set of functional dependencies
- problems

I. compute the closure of F: F^+

- the set F^+ contains all the functional dependencies implied by F
- F implies a functional dependency f if f holds on every relation that satisfies F
- the following 3 rules can be repeatedly applied to compute F^+ (Armstrong's Axioms):
  - α, β, γ - subsets of attributes of A

  1. reflexivity: if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
  2. augmentation: if $\alpha \rightarrow \beta$, then $\alpha\gamma \rightarrow \beta\gamma$
  3. transitivity: if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$

- these rules are complete (they generate all dependencies in the closure) and sound (no erroneous functional dependencies can be derived)

- R[A] - a relation
- F - a set of functional dependencies
- problems

I. compute the closure of F: F^+

- the following rules can be derived from Armstrong's Axioms:

4. union: if $\alpha \to \beta$ and $\alpha \to \gamma$, then $\alpha \to \beta\gamma$

$$\left.\begin{array}{l} \alpha \to \beta \underset{\text{augmentation}}{=>} \alpha\alpha \to \alpha\beta \\ \\ \alpha \to \gamma \underset{\text{augmentation}}{=>} \alpha\beta \to \beta\gamma \end{array}\right\} \underset{\text{transitivity}}{=>} \quad \alpha \to \beta\gamma$$

- R[A] - a relation
- F - a set of functional dependencies
- problems

I. compute the closure of F: F^+

- the following rules can be derived from Armstrong's Axioms:

5. decomposition: if $\alpha \rightarrow \beta\gamma$, then $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$

$$\left.\begin{array}{l}\alpha \rightarrow \beta\gamma \\ \beta\gamma \rightarrow \beta \text{ (reflexivity)}\end{array}\right\} \underset{\text{transitivity}}{=>} \quad \alpha \rightarrow \beta \text{ (}\alpha \rightarrow \gamma \text{ can similarly be shown to hold)}$$

- R[A] - a relation
- F - a set of functional dependencies
- problems

I. compute the closure of F: F^+

- the following rules can be derived from Armstrong's Axioms:

6. pseudotransitivity: if $\alpha \to \beta$ and $\beta\gamma \to \delta$, then $\alpha\gamma \to \delta$

$$\left.\begin{array}{l} \alpha \to \beta \Rightarrow \alpha\gamma \to \beta\gamma \\ \beta\gamma \to \delta \end{array}\right\} \underset{\text{transitivity}}{\Rightarrow} \quad \alpha\gamma \to \delta$$

- $\alpha, \beta, \gamma, \delta$ - subsets of attributes of A

- R[A] - a relation
- F - a set of functional dependencies
- α – a subset of attributes
- problems

II. compute the closure of a set of attributes under a set of functional dependencies

- determine the closure of α under F, denoted as α^+
- α^+ - the set of attributes that are functionally dependent on α under F

- R[A] - a relation
- F - a set of functional dependencies
- α – a subset of attributes
- problems

II. compute the closure of a set of attributes under a set of functional dependencies

- algorithm

closure := α**;**

repeat until there is no change:

  for every functional dependency $\beta \rightarrow \gamma$ **in F**

    if $\beta \subseteq$ **closure**

      then closure := closure $\cup\ \gamma$**;**

- R[A] - a relation
- F - a set of functional dependencies
- problems

III. compute the minimal cover for a set of dependencies

Definition: F, G - two sets of functional dependencies; F and G are equivalent (notation $F \equiv G$) if $F^+ = G^+$.

- R[A] - a relation
- F - a set of functional dependencies
- problems

III. compute the minimal cover for a set of dependencies

Definition: F - set of functional dependencies; a minimal cover for F is a set of functional dependencies F_M that satisfies the following conditions:

1. $F_M \equiv F$
2. the right side of every dependency in F_M has a single attribute;
3. the left side of every dependency in F_M is irreducible (i.e., no attribute can be removed from the determinant of a dependency in F_M without changing F_M's closure);
4. no dependency f in F_M is redundant (no dependency can be discarded without changing F_M's closure).

* <u>closure of a set of functional dependencies</u>

P1. Let $R[ABCDEF]$ be a relational schema and S a set of functional dependencies over R, $S = \{A \rightarrow B, A \rightarrow C, CD \rightarrow E, CD \rightarrow F, D \rightarrow E\}$.

Show the following FDs are in S^+: $A \rightarrow BC, CD \rightarrow EF, AD \rightarrow E, AD \rightarrow F$.

$$\left.\begin{array}{l} A \rightarrow B \\ A \rightarrow C \end{array}\right\}_{\text{union}} \Rightarrow A \rightarrow BC$$

$$\left.\begin{array}{l} CD \rightarrow E \\ CD \rightarrow F \end{array}\right\}_{\text{union}} \Rightarrow CD \rightarrow EF$$

$$A \rightarrow C \underset{\text{augmentation}}{\Rightarrow} \left.\begin{array}{l} AD \rightarrow CD \\ CD \rightarrow E \end{array}\right\}_{\text{transitivity}} \Rightarrow AD \rightarrow E$$

* <u>closure of a set of functional dependencies</u>

P1. Let $R[ABCDEF]$ be a relational schema and S a set of functional dependencies over R, $S = \{A \rightarrow B, A \rightarrow C, CD \rightarrow E, CD \rightarrow F, D \rightarrow E\}$.

Show the following FDs are in S^+: $A \rightarrow BC, CD \rightarrow EF, AD \rightarrow E, AD \rightarrow F$.

$$\left.\begin{array}{l} A \rightarrow C \\ CD \rightarrow F \end{array}\right\} \Rightarrow AD \rightarrow F$$

pseudotransitivity

* <u>closure of a set of attributes under a set of functional dependencies</u>

P2. Let $R[ABCDEF]$ be a relational schema, S a set of functional dependencies over R and α a subset of attributes of the set of attributes of R, $S = \{A \to B, A \to C, CD \to E, CD \to F, D \to E\}$, $\alpha = \{A, D\}$. Compute α^+.

$\alpha^+ = \{A, D\}$

$A \to B$ => $\alpha^+ = \{A, B, D\}$

$A \to C$ => $\alpha^+ = \{A, B, C, D\}$

$CD \to E$ => $\alpha^+ = \{A, B, C, D, E\}$

$CD \to F$=> $\alpha^+ = \{A, B, C, D, E, F\}$

$D \to E, E$ already in α^+

- iterate over all dependencies one more time, α^+ remains unchanged
- $\alpha^+ = \{A, B, C, D, E, F\}$

* <u>minimal cover for a set of functional dependencies</u>

P3. Let $R[ABCD]$ be a relational schema and S a set of functional dependencies over R, S={$A \rightarrow BC$, $B \rightarrow C$, $A \rightarrow B$, $AB \rightarrow C$, $AC \rightarrow D$}. Compute a minimal cover of S.

- decomposition: $A \rightarrow BC$ => $A \rightarrow B$, $A \rightarrow C$

=> $A \rightarrow B$

$A \rightarrow C$

$B \rightarrow C$

$\cancel{A \rightarrow B}$ - can be eliminated

$AB \rightarrow C$

$AC \rightarrow D$

* minimal cover for a set of functional dependencies

P3. Let $R[ABCD]$ be a relational schema and S a set of functional dependencies over R, S={$A \to BC$, $B \to C$, $A \to B$, $AB \to C$, $AC \to D$}. Compute a minimal cover of S.

- augmentation: $A \to C$ => $A \to AC$
- transitivity: $A \to AC$, $AC \to D$ => $A \to D$

  => C in $AC \to D$ is redundant

=> $A \to B$

$A \to C$

$B \to C$

$AB \to C$

$A \to D$

* <u>minimal cover for a set of functional dependencies</u>

P3. Let $R[ABCD]$ be a relational schema and S a set of functional dependencies over R, S={$A \to BC$, $B \to C$, $A \to B$, $AB \to C$, $AC \to D$}. Compute a minimal cover of S.

- augmentation: $A \to C$ => $AB \to CB$
- decomposition: $AB \to CB$ => $AB \to C$

  => can eliminate $AB \to C$

=> $A \to B$

$A \to C$

$B \to C$

$A \to D$

* <u>minimal cover for a set of functional dependencies</u>

P3. Let $R[ABCD]$ be a relational schema and S a set of functional dependencies over R, S={$A \to BC$, $B \to C$, $A \to B$, $AB \to C$, $AC \to D$}. Compute a minimal cover of S.

- transitivity: $A \to B$, $B \to C$ => $A \to C$

  => can eliminate $A \to C$

=> $A \to B$

$B \to C$

$A \to D$

Example 11. Consider relation DFM[Department, FacultyMembers, MeetingDates], with repeating attributes *FacultyMembers* and *MeetingDates*.

- a possible instance is given below:

Department	FacultyMembers	MeetingDates
Computer Science	FCS1 FCS2 … FCSm	DCS1 DCS2 … DCSn
Mathematics	FM1 FM2 … FMp	DM1 DM2 … DMq

- eliminate repeating attributes (such that the relation is at least in 1NF) - replace DFM by a relation DFM' in which *FacultyMember* and *MeetingDate* are scalar attributes:

Department	FacultyMember	MeetingDate
Computer Science	FCS1	DCS1
Computer Science	FCS1	DCS2
…	…	…
Computer Science	FCS1	DCSn
Computer Science	FCS2	DCS1
Computer Science	FCS2	DCS2
…	…	…
Mathematics	FM1	DM1
…	…	…
Mathematics	FMp	DMq

Department	FacultyMember	MeetingDate
Computer Science	FCS1	DCS1
Computer Science	FCS1	DCS2
…	…	…
Computer Science	FCS1	DCSn
Computer Science	FCS2	DCS1
Computer Science	FCS2	DCS2
…	…	…
Mathematics	FM1	DM1
…	…	…
Mathematics	FMp	DMq

- in this table, each faculty member has the same meeting dates
- therefore, when adding / changing / removing rows, additional checks must be carried out

References

- [Ta13] ȚÂMBULEA, L., Curs Baze de date, Facultatea de Matematică și Informatică, UBB, 2013-2014
- [Ra02] RAMAKRISHNAN, R., GEHRKE, J., Database Management Systems (3rd Edition), McGraw-Hill, 2002
- [Da03] DATE, C.J., An Introduction to Database Systems (8th Edition), Addison-Wesley, 2003
- [Ga09] GARCIA-MOLINA, H., ULLMAN, J., WIDOM, J., Database Systems: The Complete Book (2nd Edition), Pearson Education, 2009
- [Ha96] HANSEN, G., HANSEN, J., Database Management And Design (2nd Edition), Prentice Hall, 1996
- [Ra02S] RAMAKRISHNAN, R., GEHRKE, J., Database Management Systems, Slides for the 3rd Edition, http://pages.cs.wisc.edu/~dbbook/openAccess/thirdEdition/slides/slides3ed.html
- [Si11] SILBERSCHATZ, A., KORTH, H., SUDARSHAN, S., Database System Concepts (6th Edition), McGraw-Hill, 2011
- [Si19S] SILBERSCHATZ, A., KORTH, H., SUDARSHAN, S., Database System Concepts, Slides for the 7th Edition, http://codex.cs.yale.edu/avi/db-book/
- [Ul11] ULLMAN, J., WIDOM, J., A First Course in Database Systems, http://infolab.stanford.edu/~ullman/fcdb.html