1. For each of the following, find the matrix $M \in SO(2)$ which diagonalizes the given symmetric matrix:

1. $\begin{bmatrix} 6 & 2 \\ 2 & 9 \end{bmatrix}$
2. $\begin{bmatrix} 5 & -13 \\ -15 & 5 \end{bmatrix}$
3. $\begin{bmatrix} 7 & -2 \\ -2 & 5/3 \end{bmatrix}$

2. For each of the symmetric matrices A in the previous exercise write down a quadratic equation with associated matrix A.

3. For each of the following equations write down the associated matrix.

1. $-x^2 + xy - y^2 = 0$,
2. $6xy + x - y = 0$.

4. Bring the equations from the previous exercise in canonical form.

5. Decide if the following equations describe an ellipse, a hyperbola or a parabola.

1. $x^2 - 4xy + 2y^2 = 1$,
2. $x^2 + 4xy + 2y^2 = 2$,
3. $x^2 + 4xy + 4y^2 = 3$.

6. Using the classification of quadrics, decide what surfaces are described by the following equations.

1. $x^2 + 2y^2 + z^2 + xy + yz + zx = 1$,
2. $xy + yz + zx = 1$,
3. $x^2 + xy + yz + zx = 1$,
4. $xy + yz + zx = 0$.

7. Consider the rotation R_{90° of $\mathbb{E}^2$ around the origin and the translation $T_{\mathbf{v}}$ of $\mathbb{E}^2$ with vector $\mathbf{v}(1,0)$.

1. Give the algebraic form of the isometries R_{90°, $T_{\mathbf{v}}$ and $T_{\mathbf{v}} \circ R_{90^\circ}$.
2. Determine the equations of the hyperbola $\mathcal{H} : \frac{x^2}{4} - \frac{y^2}{9} - 1 = 0$ and the parabola $\mathcal{P} : y^2 - 8x = 0$ after transforming them with R_{90° and with $T_{\mathbf{v}} \circ R_{90^\circ}$ respectively.

8. Let $\boldsymbol{e}$ and $\boldsymbol{f}$ be two orthonormal bases of a $\mathbb{V}^n$. Show that $\mathrm{M}_{\boldsymbol{e},\boldsymbol{f}}$ is orthogonal, i.e. that $\mathrm{M}_{\boldsymbol{e},\boldsymbol{f}} \in O(n)$.

9. Let $\boldsymbol{e} = (\mathbf{e}_1, \ldots, \mathbf{e}_n)$ be an orthonormal basis of $\mathbb{V}^n$. If π is a permutation of $\{1, \ldots, n\}$ let $\pi(\boldsymbol{e}) = (\mathbf{e}_{\pi(1)}, \ldots, \mathbf{e}_{\pi(n)})$. Show that $\mathrm{M}_{\boldsymbol{e},\pi(\boldsymbol{e})} \in O(n)$.