Universitatea Babeş-Bolyai, Facultatea de Matematică şi Informatică

Secţia: Informatică engleză, Curs: Dynamical Systems, Semestru: Primăvara 2021

Dynamical Systems 2020/21
Exam

1. (1.25p) Find a fundamental matrix solution of the linear homogeneous differential system
$$x' = 2x + y, \quad y' = -2x + 4y.$$
2. (1p) Using the integrating factor method find the general solution of the differential equation $x' - 3\lambda x = e^{3t}$, discussing with respect to the parameter $\lambda \in \mathbb{R}$.
3. Let $f : \mathbb{R} \to \mathbb{R}$ be a C^1 function with $f(-7) > 0$, $f(0) = -1$ and $f(7) > 0$.

   (a) (0.5p) Justify that the dynamical system $\dot{x} = f(x)$ does not have a global attractor equilibrium point.

   (b) (0.25p) Give a simple example of such function f.

   (c) (1p) Consider f from (b). Represent the phase portrait in $\mathbb{R}^2 \setminus \{(0,0)\}$ of the system given in polar coordinates
$$\dot{\rho} = f(\rho), \quad \dot{\theta} = -2.$$

   (d) (1p) Transform in cartesian coordinates the system from (b).

4. (2p) Find the values of $h > 0$ such that the attractor equilibrium point of $\dot{x} = x^2 - 5x + 4$ is also an attractor fixed point of the discrete dynamical system associated to the Euler's numerical formula with stepsize $h > 0$ for the given differential equation.