1. Determine the intersection of the hyperboloid

$$\mathcal{H}^1_{4,3,1} : \frac{x^2}{16} + \frac{y^2}{9} - \frac{z^2}{1} = 1 \quad \text{with the line} \quad \ell = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix} + \langle \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix} \rangle.$$

Write down the equations of the tangent planes in the intersection points.

2. Determine the tangent plane of the hyperboloid

$$\mathcal{H}^1_{2,3,1} : \frac{x^2}{4} + \frac{y^2}{9} - \frac{z^2}{1} = 1$$

in the point $M(2,3,1)$. Show that the tangent plane intersects the surface in two lines.

3. Determine the generators of the hyperboloid

$$\frac{x^2}{36} + \frac{y^2}{9} - \frac{x^2}{4} = 1$$

which are parallel to the plane $x + y + z = 0$.

4. Determine the intersection of the hyperboloid

$$\mathcal{H}^2_{2,1,3} : \frac{x^2}{4} + \frac{y^2}{1} - \frac{z^2}{9} = -1 \quad \text{with the line} \quad \ell = \begin{bmatrix} 3 \\ 1 \\ 6 \end{bmatrix} + \langle \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \rangle.$$

Write down the equations of the tangent planes in the intersection points.

5. Determine the intersection of the paraboloid

$$\mathcal{P}^h_{2,\frac{1}{2}} : x^2 - 4y^2 = 4z \quad \text{with the line} \quad \ell = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} + \langle \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix} \rangle.$$

Write down the equations of the tangent planes in the intersection points.

6. Determine the tangent plane of

1. the elliptic paraboloid $\frac{x^2}{5} + \frac{y^2}{3} = z$ and of
2. the hyperbolic paraboloid $x^2 - \frac{y^2}{4} = z$

which are parallel to the plane $x - 3y + 2z - 1 = 0$.

7. Determine the plane which contains the line

$$\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + \langle \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \rangle \quad \text{and is tangent to the quadric} \quad x^2 + 2y^2 - z^2 + 1 = 0.$$

8. Show that the parabolid $\mathcal{P}^e_{p,p}$ is the locus of points for which the distance from a point equals the distance to a plane. Such a surface is called *elliptic paraboloid of revolution*.

9. Use a parametrization of a parabola and a rotation matrix to deduce a parametrization of an elliptic paraboloid of revolution.

10. For the surface $\mathcal{S}$ with parametrization

$$\mathcal{S} : \begin{cases} x = \sqrt{1+t^2}\cos(s) \\ y = \sqrt{1+t^2}\sin(s) \\ z = 2t \end{cases}$$

- Give the equation of $\mathcal{S}$.
- Find the parameters of the point $P(1,1,2)$.
- Calculate a parametrization of the tangent plane $T_P\mathcal{S}$ using partial derivatives.
- Give the equation of $T_P\mathcal{S}$.

11. For the surface $\mathcal{S}$ with parametrization

$$\mathcal{S} : \begin{cases} x = s \\ y = t \\ z = s^2 - t^2 \end{cases}$$

- Give the equation of $\mathcal{S}$.
- Find the parameters of the point $P(1,1,0)$.
- Calculate a parametrization of the tangent plane $T_P\mathcal{S}$ using partial derivatives.
- Give the equation of $T_P\mathcal{S}$.

12. Determine the generators of the paraboloid

$$4x^2 - 9y^2 = 36z$$

containing the point $P(3\sqrt{2}, 2, 1)$.

13. Determine the generators of the paraboloid

$$\frac{x^2}{16} - \frac{y^2}{4} = z$$

which are parallel to the plane $3x + 2y - 4z = 0$.

14. Which of the following is a hyperboliod?

1. $\mathcal{S} : 2xz + 2xy + 2yz = 1$
2. $\mathcal{S} : 5x^2 + 3y^2 + xz = 1$
3. $\mathcal{S} : 2xy + 2yz + y + z = 2$