1. Determine the intersection of the ellipsoid

$$\mathcal{E}_{4,2\sqrt{3},2} : \frac{x^2}{16} + \frac{y^2}{12} + \frac{z^2}{4} - 1 = 0 \quad \text{with the line} \quad \ell = \begin{bmatrix} 4 \\ -6 \\ -2 \end{bmatrix} + \left\langle \begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix} \right\rangle.$$

Write down the equations of the tangent planes in the intersection points.

2. Determine the intersection of the ellipsoid

$$\mathcal{E}_{2,3,4} : \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} = 1$$

with planes parallel to the coordinate planes. Treat the various cases separately.

3. Determine the intersection of the ellipsoid

$$\mathcal{E}_{2,\sqrt{3},3} : \frac{x^2}{4} + \frac{y^2}{3} + \frac{z^2}{9} = 1 \quad \text{with the line} \quad \ell : x = y = z.$$

Write down the equations of the tangent planes in the intersection points.

4. Determine the tangent planes to the ellipsoid

$$\mathcal{E}_{2,3,2\sqrt{2}} : \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{8} = 1$$

which are parallel to the plane $\pi : 3x - 2y + 5z + 1 = 0$.

5. Use the classification of quadrics to determine what surfaces are described by the following equations

1. $xz + xy + yz = 1$
2. $x^2 - 2xz - y^2 - z^2 = 1$
3. $xz + xy + yz = -1$
4. $5x^2 + 3y^2 + xz = 1$

6. Determine the points P of the ellipsoid

$$\mathcal{E} : \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

for which the tangent space $T_P\mathcal{E}$ intersects the coordinate axis in congruent segments.

7. Show that the line

$$\begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix} + \left\langle \begin{bmatrix} 0 \\ -1 \\ 2 \end{bmatrix} \right\rangle \quad \text{is tangent to the quadric} \quad \frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{16} - 1 = 0$$

and determine the tangency point.

8. Prove that the intersection of a quadric in $\mathbb{E}^3$ with a plane is either the empty set or a point or a line or two lines or an ellipse or a hyperbola or a parabola.

9. Prove that the intersection of an ellipsoid with a plane is either the empty set or a point or an ellipse.

10. Show that the ellipsoid $\mathcal{E}_{a,b,b}$ is the locus of points for which the sum of the distances to two given points is constant. Such a surface is called *ellipsoid of revolution*.

11. Use a parametrization of an ellipse and a rotation matrix to deduce a parametrization of an ellipsoid of revolution.

12. For the surface $\mathcal{S}$ with parametrization

$$\mathcal{S}: \begin{cases} x = 4\cos(s)\cos(t) \\ y = 4\sin(s)\cos(t) \\ z = 2\sin(t) \end{cases} \qquad s \in [0, 2\pi[\quad t \in [-\frac{\pi}{2}, \frac{\pi}{2}[$$

- Give an equation of $\mathcal{S}$.
- Find the parameters of the point $P(3, \sqrt{3}, 1)$.
- Calculate a parametrization of the tangent plane $T_P\mathcal{S}$ using partial derivatives.
- Give an equation of $T_P\mathcal{S}$.

13. Prove that the intersection of an elliptic cone with a plane is either a point or a line or an ellipse or a hyperbola or a parabola.