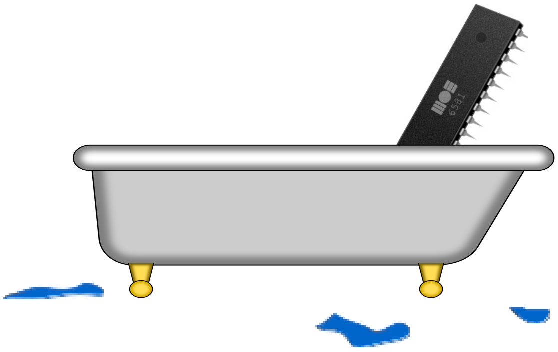
Om Mistry

ENGR 123 Lab 4 2023:

A Chip in a Tub



Purpose of the Lab

In this lab we will practice working with probability density functions (pdfs) and cumulative density functions (cdfs) in an engineering context. We also hope to learn a few new concepts and develop our skills with spreadsheet applications.

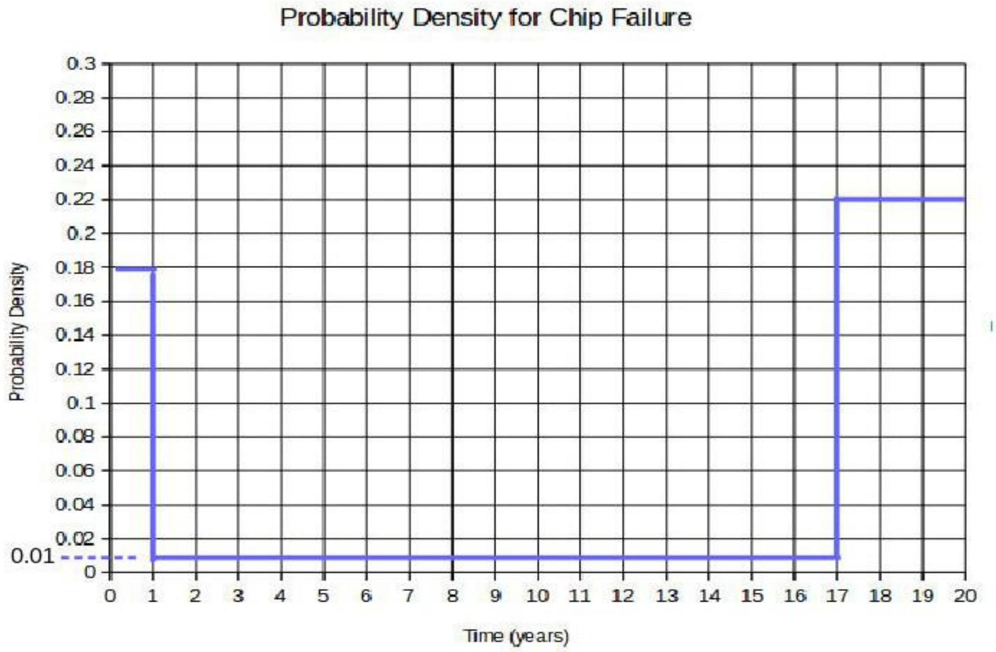
The Bathtub Curve

Devices including living organisms have finite lifetimes. It turns out that the so-called “bathtub curve” (see link below) describing the probability density for device failure versus time is nearly universal: the details of the shape vary from one device to another but overall look somewhat similar.

Link to a good sketch of a “bathtub curve” with some recommended reading:
https://users.ece.cmu.edu/~koopman/des_s99/electronic_electrical/

We can understand this conceptually as follows. There is a high rate of failure initially because some of the devices will have flaws. This is sometimes called “infant mortality” or “burn-in failure.” Specimens that survive this early period then have quite a low failure rate until they start to approach the end of their usual lifetimes, and then the failure rate increases as they wear out. Again, it is remarkable that this behaviour, or some version of it at least, is seen in almost all devices.

The length of time a microchip operates before it fails is a continuous random variable. The graph below gives the failure probability density versus time for an imaginary chip. Note that this is somewhat simplified: the probability density would not suddenly drop to zero at 20 years for example. We will explore these simplifications in Challenge 1.



CORE 1 (15 marks)

Based on your tutor's presentation, indicate the "burn-in", normal operating life, and wear-out periods on the graph. What are the probabilities that the chip fails during each of these periods?

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Burn-in:
Infant mortality, it will fail quite early.
On the graph it's the initial area that has a probability density of 0.18. It stops after the first year.

Normal operating life:
This goes from year 1 all the way until year 17, the probability density throughout this lifetime is 0.01 and the probability of failure is 0.16.

Wear-out:
The wear-out period on the graph begins at the 17th year and goes until after. The probability density is increased to 0.22.The probability of failure here is 0.66.

You can see that the chip has a risk of death at the very first year and it's very likely to die after the 17th year.
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CORE 2 (15 marks)

What is the probability that the chip will fail eventually? How should this be reflected in the pdf graph? Check that it is correct (show your calculations).

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The chip has a risk of failing of 0.18 in the first year, then during the next 16 years the risk of failing is 0.16 and then finally the risk of failing from the 17th to 20th year is 0.66.

0.18 * 1 = 0.18 Burn-in
0.01 * 16 = 0.16 Normal operating life
0.22 * 3 = 0.66 Wear-out

0.18 + 0.16 + 0.66 = 1

The probability that the chip will fail in the end is 1 because it should add up to 1.

After checking the graph I can see that my calculations are correct and the graph also correctly portrays what
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CORE 3 (15 marks)
What is the probability that the chip will fail at any particular instant? For example, what is the probability that the chip fails at exactly 7 years, 232 days, 1 hour, 3 minutes, and 2.7 seconds? Explain. What are the units on the vertical axis of your pdf graph?

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At any particular instance the probability of the chip failing will be 0. This is because if we give the probability at any second the total probability of the chip failing will be great than 1. This is incorrect as probability can't exceed 1. However if instead of any particular instance we have a specific range of time. We can figure out the probability that the chip will fail in that specific time period.

So the probability that the ship will fail in 7 years, 232 days, 1 hour, 3 minutes and 2.7 seconds is 0.0.

The units on the vertical axis of my PDF graph are Probability over years, this is because.
Unit of (y) axis * Time(years) = Probability
So then
Unit of (y) axis = Probability / Time(years)
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COMPLETION 1 (20 marks)

Use your pdf graph to find the cdf and graph it. Briefly explain your methods. Hint: you might calculate the cdf for one year intervals.

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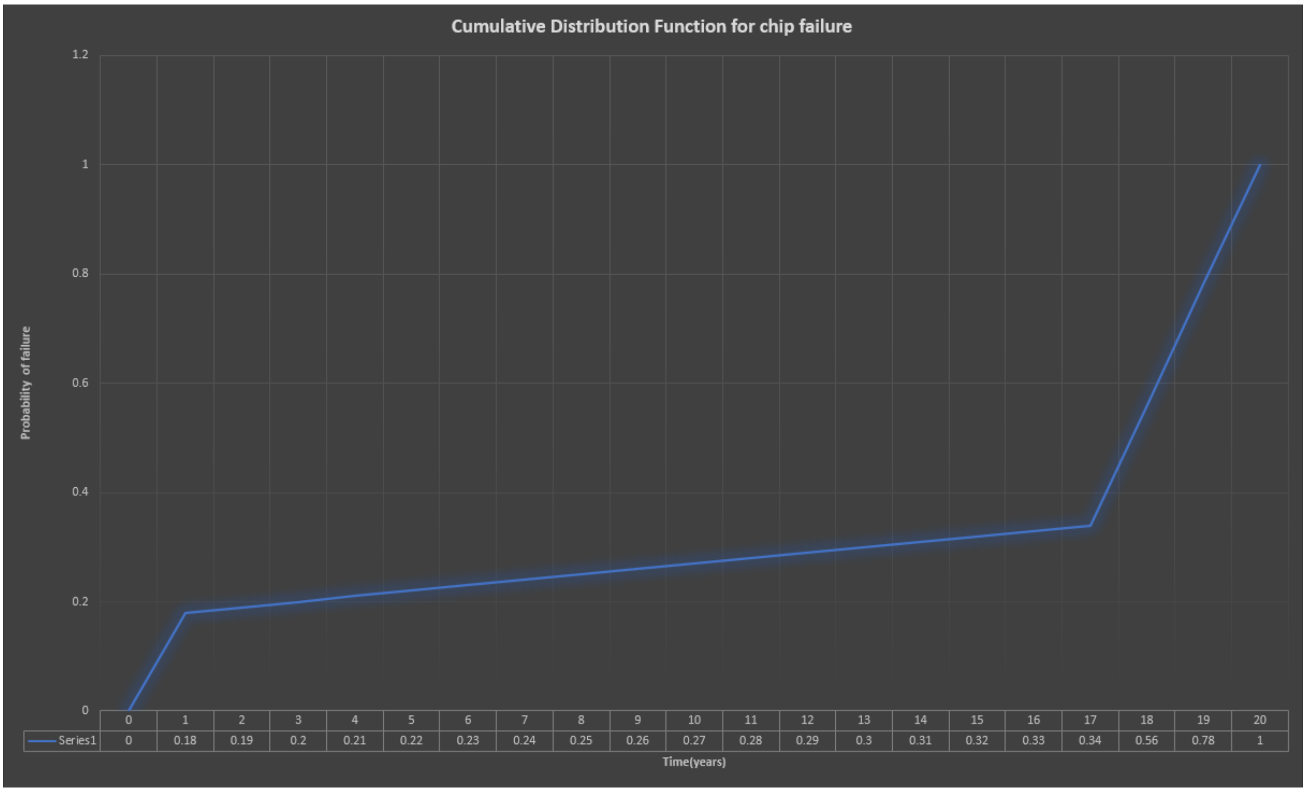

For converting pdf to cdf. The probability of failing in the second year is the probability of the first year added on to the probability of the second year. Every year it increases by adding the previous year probability together with the current year's probability.
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COMPLETION 2 (10 marks)

A “used” microchip that has been “burned in” can actually be more valuable than a new one. Such chips are used in military and space applications for example. Explain this in terms of your pdf and/or cdf.

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In terms of my pfd and cdf. The reason why a “burned-in” chip would be more valuable than a normal one is because a “burned-in” chip has gone past the initial stage of Infant Mortality. Thus increasing the lifespan of a successfully burnt-in chip. This would mean that the chip would have a incredibly higher change of surviving an extremely long time. Which would be very useful for important work such as military or space applications. You can see that after the “burn-in” period on the cdf graph the line doesn’t increase as much per year, it’s a slight upward increase in angle. And in the pdf graph you can see how it’s lowered itself very close to the 0.0 mark. This shows how unlikely it is that the chip will fail.
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CHALLENGE 1 (5 marks)

The pdf data you have been given is somewhat realistic but there are aspects of it that are artificial. The probability density during the chip’s operating life would be much lower than 0.01 per year, the probability density would not change discontinuously, and there would be some small probability of the chip lasting a very long time. How might you modify your bathtub curve to take these into account? Make a sketch by hand of your modified bathtub curve.

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CHALLENGE 2 (10 marks)

Did you miss the news? Proxima Centauri is our nearest neighbor star apart from the sun, at 4.2 light years from Earth. And the planet discovered orbiting it is in the possibly habitable zone, orbiting close enough but not too close to the star, allowing for the possibility of liquid water. Wow. It’s only a bit more than four light years away!

Have a look at your pdf graph (the original data, not the data as modified in challenge 1). Design a “burn-in” programme for a spacecraft set to visit the newly discovered planet circling Proxima Centauri. [One-way, robot, 15 years travel time from the robot’s perspective. Note this is WAY beyond any technology we are likely to have any time soon.] How long should the burn-in period be?

Comment on whether you think this is an adequately reliable chip for such a mission. Remember a spacecraft has many chips.

Now the hard part: model a pdf for the burned-in chip.

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CHALLENGE 3 (10 Marks)

The text files on the course page for this lab give simulated data for the lifespans of populations of chips. They have failure probability density curves similar in shape to the one you have been working with. Make a labelled histogram for the smaller population (100 chips) found in the file BathTub100.txt using LibreOffice Calc or something similar, or use the Python code below. Note the resulting histogram is very noisy - a sample size of a 100 is actually very small. It would be hard to work out the shape of the curve in any detail from this data. Use Python to make a histogram of the larger data set, BathTub10000.txt. The results are much better! Just paste your histograms in as answers.

A note about Python: the vast majority of ENGR students will be using Python first term next year. Learning about Python is time well-spent.

Python code:

```
import numpy as np              (this line and next import the functions you need)
import matplotlib.pyplot as plt
fname='name.txt'                 (put the files in your home directory)
a=np.loadtxt(fname)              (loads the data)
a                                (lists  the data)
plt.hist(a, bins=[0,1,2,3,4,5,6,7,8,9,10])  (makes a histogram with 10 bins but you want 20 bins)
plt.show()                      (displays histogram)
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