

Engr 123

A2 Q5, Base 2

$$\begin{array}{r} 10 \\ \times 11 \\ \hline 10 \\ + 100 \\ \hline 110 \end{array}$$

$$1 \times 2 + 0 \times 1$$

$$1 \times 2 + 1 \times 1$$

L6  
Arguments

Proofs

Uniqueness

Base 10

27

$\times 35$

$13^3 5$

$+ 8^2 10$

945

Q4 d, e

(d) combine A, B, C

(e) possible sol<sup>n</sup>s



From yesterday

Fred is a fish

All fish swim in the ocean

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Fred swims in the ocean

$\text{Fish}(x) = "x \text{ is a fish}"$

$\text{Swim}(y) = "y \text{ swims in the ocean}"$

} p.c.t

$\text{Fish}(\text{Fred})$

$\forall y (\text{Fish}(y) \rightarrow \text{Swim}(y))$

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$\text{Swim}(\text{Fred})$

Valid

$\text{Fish}(\text{Fred})$

$\forall y (\text{Swim}(y) \rightarrow \text{Fish}(y))$

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$\text{Swim}(\text{Fred})$

Invalid

Shup to pg 4



## Uniqueness

Existential quantifier  $\exists$

$$\exists x P(x)$$

there is at least one  $x$   
satisfying  $P$

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Want to say, there exists  
a unique  $x$  satisfying  $P$

$$\exists x (P(x) \wedge \forall y [P(y) \rightarrow y=x])$$

$$\exists x (P(x) \wedge \neg \exists y [P(y) \wedge y \neq x])$$



Fish(Fred)

(i)

$$\forall y [\neg \text{Swim}(y) \rightarrow \neg \text{Fish}(y)] \quad (ii)$$

Swim(Fred)

Valid

$$(ii) \equiv \forall y [\text{Fish}(y) \rightarrow \text{Swim}(y)]$$

$$P \rightarrow Q \equiv \neg Q \rightarrow \neg P$$

$$[(P \rightarrow Q) \wedge (Q \rightarrow R)] \rightarrow [P \rightarrow R] \equiv \top$$

$\top$  is a tautology

$$P \rightarrow Q$$

$$Q \rightarrow R$$

$$\hline P \rightarrow R$$

transitivity



P

$$\hline P \rightarrow R$$

R

modus ponens



$$\frac{P}{\neg R \rightarrow \neg P}$$

R

modus tollens

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Number theory ( $\mathbb{Z}$ )



Divisibility

$n$  is even iff  $\exists k \in \mathbb{Z} (n = 2k)$