Examples:

1) $R = \{(1,2), (3,4), (1,1), (1,4)\}$

$R^{-1} = \{(2,1), (4,3), (1,1), (4,1)\}$

3) $S = \{(1,a), (2,b)\}$ $\quad S \subseteq \{1,2\} \times \{a,b\}$

$S^{-1} = \{(a,1), (b,2)\}$

2) R on $\mathbb{N}$

xRy iff $x<y$

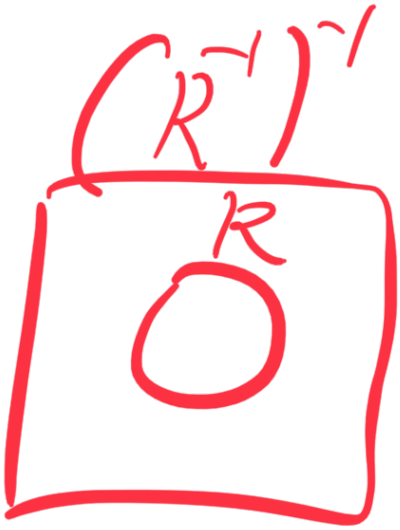
$(1,2) \in R$

R^{-1}

$aR^{-1}b$ iff $b<a$

$(2,1) \in R^{-1}$

Fact: $(R^{-1})^{-1} = R$



Lets $(a,b) \in R \Rightarrow$

$\Rightarrow (b,a) \in R^{-1} \Rightarrow$

$\Rightarrow (a,b) \in (R^{-1})^{-1}$

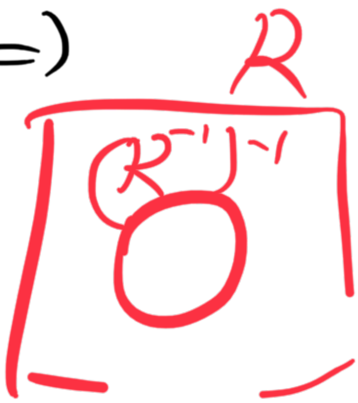
if $(a,b) \in R$ then $(a,b) \in (R^{-1})^{-1}$

We proved $R \subseteq (R^{-1})^{-1}$

if we can prove that $(R^{-1})^{-1} \subseteq R$

then $R = (R^{-1})^{-1}$

Lets $(x,y) \in (R^{-1})^{-1} \Rightarrow$



$\Rightarrow (y,x) \in R^{-1} \Rightarrow$

$\Rightarrow (x,y) \in R$

We proved that if $(x,y) \in (R^{-1})^{-1}$

then $(x,y) \in R$

$$(R^{-1})^{-1} \subseteq R$$

Now $(R^{-1})^{-1} \subseteq R$ and

$$R \subseteq (R^{-1})^{-1} \Rightarrow (R^{-1})^{-1} = R$$

Composition of relations

if $R \subseteq A \times B$ and $S \subseteq B \times C$

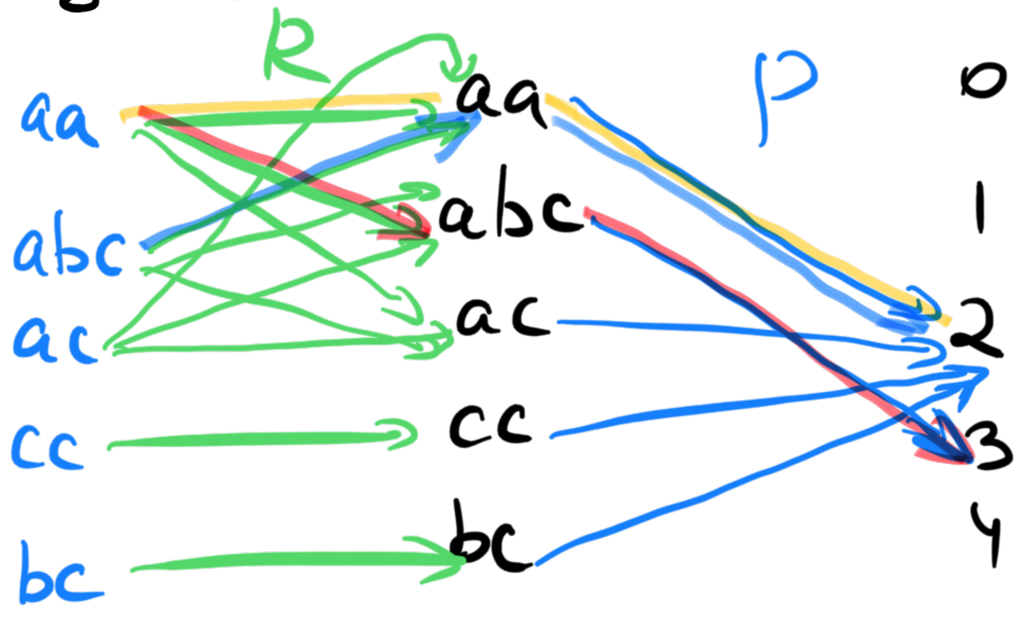
$RS = \{(x,y) : \exists\, b \in B,$ s.t. xRb and $bSy\}$

Example: R on $\overbrace{\{aa, abc, ac, cc, bc\}}^{A\ =}$

xRy iff the first letter is the same

P on $A \times \{0,1,2,3,4\}$

xPy iff y is the number of letters in x



R, P : R is $A \times \underline{A}$ $\quad \overset{P}{\underline{A} \times \{0,1,2,3,4\}}$

$RP = \{(aa,2), (aa,3), (abc,2), (abc,3),$

$(ac,2), (ac,3), (cc,2), (bc,2)\}$

$\{x : x \in R$ and $-1 \le x \le 1\}$

2) S, P are relations are on $[-1, 1]$

xSy iff $y = \sqrt{1-x^2} \Rightarrow x^2+y^2=1$

yPb iff $y = -b$

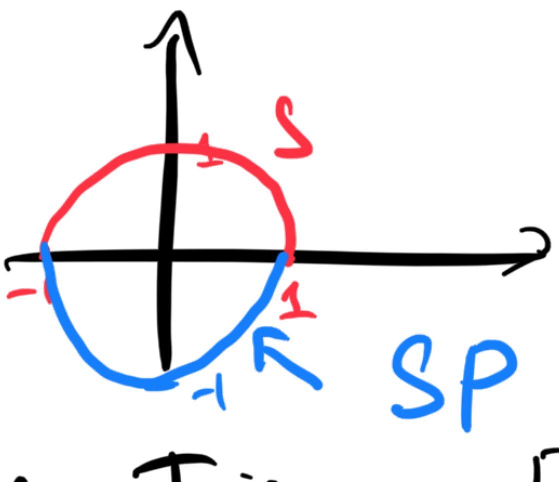
$xSPb$ if there is $y \in [-1,1]$ s.t

$\underline{xSy}$ and $\underline{yPb}$

$\downarrow$ $\Downarrow$

$y = \sqrt{1-x^2}$ $\quad$ $y = -b$ $\Rightarrow -b = \sqrt{1-x^2}$

$b = -\sqrt{1-x^2}$



is T is on $[-1, 1]$ $\quad xTy$ iff $x^2+y^2=1$

P will flip but you end up the same

Reflexive relations

Def Let R be a relation on A

R is reflexive if

for any element $a \in A$ you have aRa

Examples: 1) R on people in this room

xRy iff x has the same eyes colour

xRx because you have the same eyes colour as you

R is reflexive

2) P on all NZ cities

xPy iff there is an Air NZ flight (direct) from x to y

$x \not{P} x$ or $(x,x) \notin P$

there is no direct flight from city A to city A   P is not reflexive

3) S on $\{0, 1, 2, 3, 4\}$

xSy iff $x-y$ is divisible by 3

$x-x=0$ is divisible by 3

so xSx for any x

S is reflexive

Symmetric relations

Def: R is a lelation on A

R is symmetric if the following condition holds:

if xRy the yRx

Examples:

1) R on all people

xRy iff x and y are related by blood

R is sym. because

if x, y are blood relatives then y, x are all blood relatives

2) R on all Facebook users

xRy iff x follows y

R is not sym

if x follows y it does not mean that y follows x.

3) Q on $\{1,2,3,4\}$

xQy iff $x<y+1$

$1Q2$ $\quad 1<2+1$

but $2 \not Q 1$ because $2 \not< 1+1$

Q is not sym.