Hasse diagram:

R is a partial order on A.

1) Vertices are all elements of A

2) if aRb and there is no c s.t. aRc and cRb, then draw an arrow from a to b
   a is below b.

3) no loops.

no horisontal lines in Hasse diagramm. Everything goes from bottom to top.

Example R on $A = \{1,2,3,4\}$

xRy iff $x \leq y$

$\{(1,1),(1,2),(1,3),(1,4),(2,2),(2,3),(2,4),(3,3),(3,4),(4,4)\} = R$

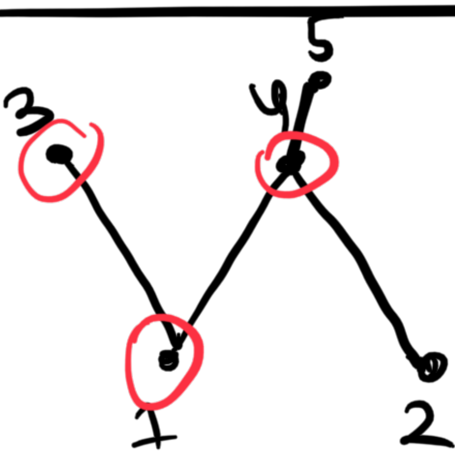
1 and 2

$1Rc$ and $cR2$

$c \neq 1,2$

no arrow

Just the line because the direction is build in the diagram. you move from bottom to the top.



$R: \{(1,1),(2,2),(3,3),(4,4),(5,5),(1,3),(1,4),(2,4),(4,5),(1,5),(2,5)\}$

$2 \not R\, 3$

$1R4$ and $4R5 \Rightarrow 1R5$

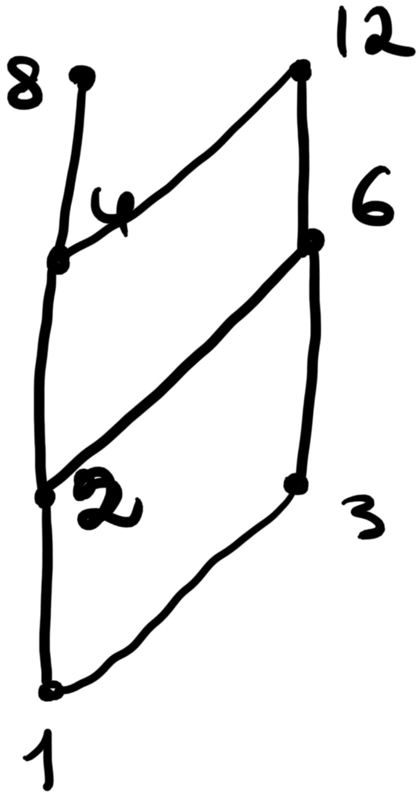
Example R on $A = \{1, 2, 3, 4, 6, 8, 12\}$

xRy iff y is divisible by x

R is a pat. order.

Do we have x. s.t. there is no $y \neq x$, s.t. yRx

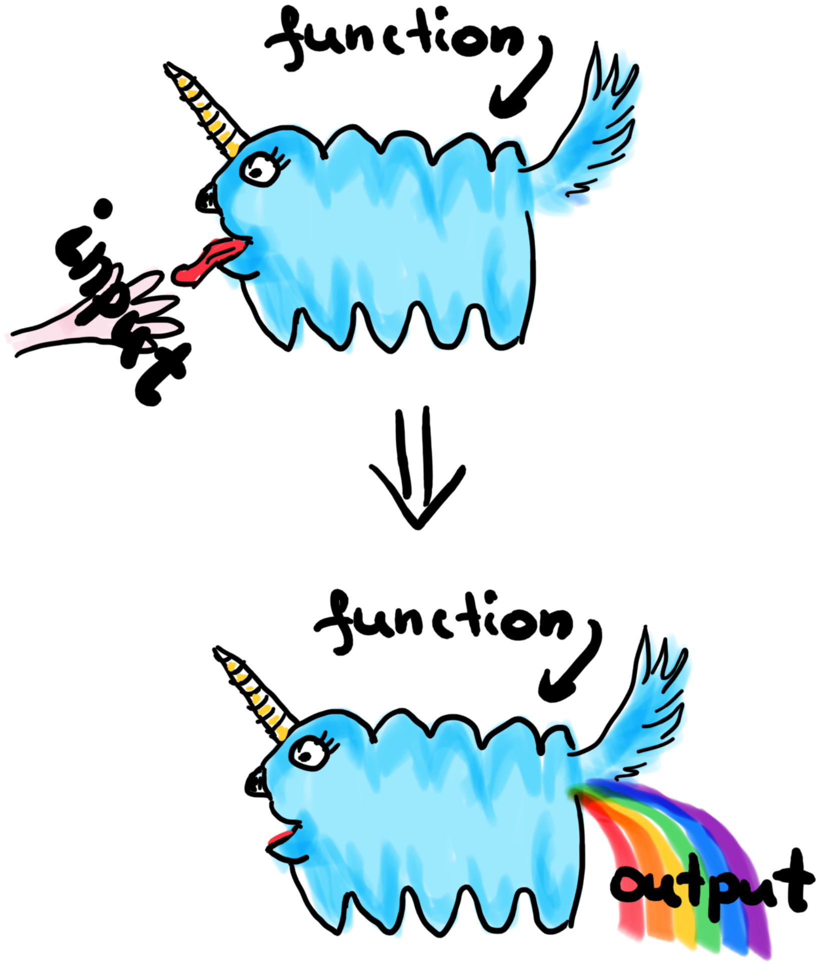
$y \not R 1$ is not there



$2 \not R 3$

$3 \not R 2$

Functions



Informal:

Function is a rule that takes an input and gives you one output

(input, output)

they are related by this function

Def: R is a relation from A to B

the domain of R

$\operatorname{dom} R = \{ a \in A \mid \text{there is } b \in B \ aRb \}$

R on $\mathbb{Z}$

$R = \{(1,1),(2,3)\}$ $\operatorname{dom} R = \{1,2\}$

The range of R is

$\operatorname{ran} R = \{ b \in B \mid \text{there is } a \in A \ aRb \}$

$\operatorname{ran} R = \{1, 3\}$

Intuition: $\underbrace{\text{all}}_{\text{The domain is}}$ used inputs from A

the range is all used outputs from B

$\mathbb{R} \times \mathbb{R} = \{ (x,y) : x, y \in \mathbb{R} \}$

$A \times B = \{ (x,y) : x \in A, y \in B \}$

Example

1) $R_2 = \{ (x,y) \in \mathbb{R} \times \mathbb{R} : y = x^2 \}$

$\operatorname{dom}(R_2) = \mathbb{R}$

$\operatorname{range}(R_2) = \{ y \mid y \in \mathbb{R} \text{ and } y \geq 0 \}$

Let c is the output you want

$c \geq 0$ then input is $\pm\sqrt{c}$

2) $R_{-1} = \{(x,y) \in \mathbb{R} \times \mathbb{R} : y = \frac{1}{x}\}$

$\text{dom}(R_{-1}) = R^+ \cup R^-$

$\text{range}(R_{-1}) = R^+ \cup R^-$

0 is not possible as for any $x \neq 0$

$\frac{1}{x} \neq 0$

if $y \neq 0$ the output then

$x = \frac{1}{y}$ is the input in this case that gives you y