Engr 123

L 7

Divisibility

Proof strategies

if and only if $\Leftrightarrow$ $\equiv$

$\downarrow$

n is even $\underline{\text{iff}}$ $\exists k \in \mathbb{Z} \, (n = 2k)$

n is odd $\underline{\text{iff}}$ $\exists k \in \mathbb{Z} \, (n = 2k+1)$

<u>Question</u> which of these numbers are even? Do check!

$10 = 2 \times 5$ ✓

$9 = 2 \times 4 + 1$ ×

$8 = 2 \times 4$ ✓

$-8 = 2 \times -4$ ✓

$7 = 2 \times 3 + 1$ ×

$-7 = 2 \times -4 + 1$ ×

$0 = 2 \times 0$

$10 + 8 = 2 \times 5 + 2 \times 4$
$= 2(5+4)$

$10 - 8 = 2(5-4)$

$10 \times 9 = 2 \times 5 \times 9$
$= 2(5 \times 9)$

$9 \times 7 = (2 \times 4 + 1)(2 \times 3 + 1)$

$10^2 = (2 \times 5)^2$
$= 2^2 \times 5^2$
$= 2(2 \times 5^2)$

Prove: If x and y are even

Then $x - y$ is even

First strategy: Direct

Assume the premisses are true

x is even $\equiv \exists k \in \mathbb{Z} (x = 2k)$

y is even $\equiv \exists j \in \mathbb{Z} (x = 2j)$

Look at $x - y$

$$x - y = 2k - 2j$$
$$= 2(k - j)$$

Because $k, j \in \mathbb{Z}$, I know $k - j \in \mathbb{Z}$

Prove if x is odd then x^2 is odd

Assume

$$x \text{ is odd} \equiv \exists k \in \mathbb{Z}\,(x = 2k+1)$$

$$x^2 = (2k+1)^2$$
$$= (2k+1)(2k+1)$$
$$= 2k \times 2k + 2k \times 1 + 1 \times 2k + 1 \times 1$$
$$= 4k^2 + 4k + 1$$

$$x^2 = 2(2k^2 + 2k) + 1$$

As 2 and k are integers, and we are multiplying and adding, $2k^2 + 2k \in \mathbb{Z}$

Prove if x^2 is odd

then x is odd

A direct proof looks impossible

Proof by contrapositive

Assume the conclusion is false

i.e. x is not odd

$\rightarrow x$ is even $\equiv \exists n \in \mathbb{Z} (x = 2n)$

$$x^2 = (2n)^2$$
$$= 4n^2$$
$$= 2(2n^2)$$

$2, n \in \mathbb{Z}$ and we are multiplying, so

$2n^2 \in \mathbb{Z}$, $\quad x^2$ is not odd

$$[P_1 \wedge \dots \wedge P_n] \to C$$

Direct	Contrapositive	Contradiction
P_1	$\neg C$	P_1
$\vdots$	$\vdots$	$\vdots$
P_n		P_n
$\vdots$		$\neg C$
		$\vdots$
$\overline{\quad C \quad}$	$\overline{\neg P_1 \vee \dots \vee \neg P_n}$	$\overline{R \wedge \neg R}$