Antisymmetric relations

Def. A relation is antisymmetric if the following condition holds:

if xRy and yRx then $x=y$

Examples

1) R on $\mathbb{Z}$

xRy iff $x \leq y$

$1 \leq 2$ but $2 \leq 1$

Let $\dfrac{xRy}{x \leq y}$ and $\dfrac{yRx}{y \leq x}$ $\Rightarrow$

$\Rightarrow x=y$ It is antisym.

Refl. $x \in A \Rightarrow xRx$

Sym. $\dfrac{xRy}{yRx}$

Ant. sym

xRy and $yRx \rightarrow x=y$

if A then B

A	B	A ⇒ B
T	T	T
T	F	F
F	F	T
F	T	T

2) P on $\mathbb{N}$:

xPy iff x divides y

Suppose xPy and yPx $\Rightarrow$

x divides y — y divides x

$\Rightarrow x = y$

for 0 it is not true.

but $0 \not P 0$, so we have no problems with 0. The antisym.

3) S on the set of subsets of $\{1,2,3,4\}$

xSy iff $x \subseteq y$

Let xSy and ySx $\Rightarrow$

$x \subseteq y$ — $y \subseteq x$

$\Rightarrow x = y$

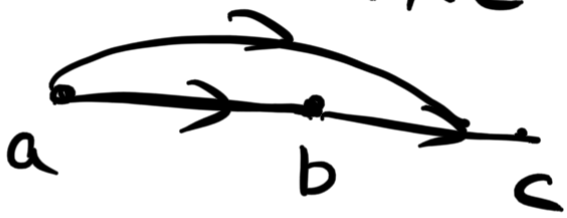
S is antisym.

$\{1,2,3,4\}$: $\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\} \ldots$

Transitive relations

Def Relation R on A is transitive if the following holds:

if aRb and bRc then aRc



Examples

1) P on $\mathbb{Z}$

xPy iff $x-y$ is divisible by 3

$1P4$ $\quad 4P7$

Suppose aPb and bPc

$a-b$ is div. by 3 $\quad$ $b-c$ is div. by 3

$a-c = a-b+b-c = \underbrace{(a-b)}_{\text{div by 3}} + \underbrace{(b-c)}_{\text{div by 3}} = \text{div by 3}$

aPc $\quad$ ($a-b+b$ is 0)

2) R on $\mathbb{N}$

xRy iff $x+y$ is odd

if you want to show the property do the proof

if you want to show the property does not work find a

an example

2 R 1 and 1 R 4 but

2 $\not R$ 4

R is not transitive.

3) S on all people

x R y iff x and y are related by blood

mum S child and child S dad

but mum $\not S$ dad

not trasitive.

Equivalence Relations

Def: R on A is eq. relation if

1) R is reflexive
2) R is symmetric
3) R is transitive

Notation $R, \sim, \equiv, \approx$

Example some sort of colour blindness on all colours

xRy if this person cannot distinguish them

Examples: R is on all VUW students

xRy iff x's and y's surnames start with the same letter