Engr 123

L4
Nand/Nor
Predicates
Proofs

Back to Logic

$P \Leftrightarrow Q \equiv (P \to Q) \wedge (Q \to P)$

$P \to Q \equiv \neg P \vee Q$

$P \wedge Q \equiv \neg(\neg P \vee \neg Q)$

$P \text{ NAND } Q \equiv \neg(P \wedge Q)$

$P \text{ NOR } Q \equiv \neg(P \vee Q)$

$\neg P \equiv$ NAND

$P \wedge Q \equiv$

$P \vee Q \equiv$

Predicate Logic

A predicate is like a proposition, but it can have a variable

e.g. $7 \in \mathbb{N}$ is a proposition

$\frac{1}{2} \in \mathbb{N}$ " " "

$x \in \mathbb{N}$ is a predicate

$x \in \mathbb{N} \rightarrow x \in \mathbb{Z}$ $\mathbb{T}$

$x \in \mathbb{N} \rightarrow x+1 \in \mathbb{N}$ $\mathbb{T}$

$x \in \mathbb{N} \rightarrow x-1 \in \mathbb{N}$?$\mathbb{T}/\mathbb{F}$?

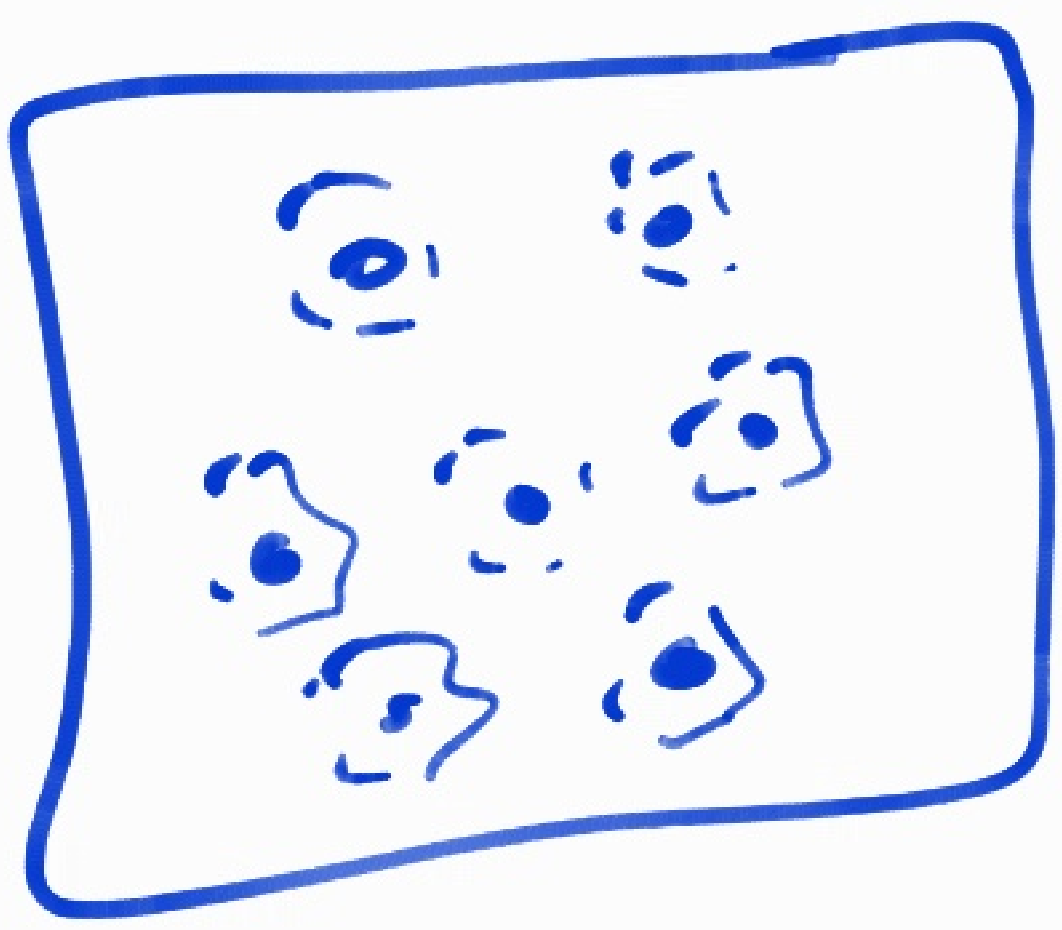
depends on x.

Quantifiers (bounding)

for all x $\forall x$

all x have property P

$\forall x\, P(x)$

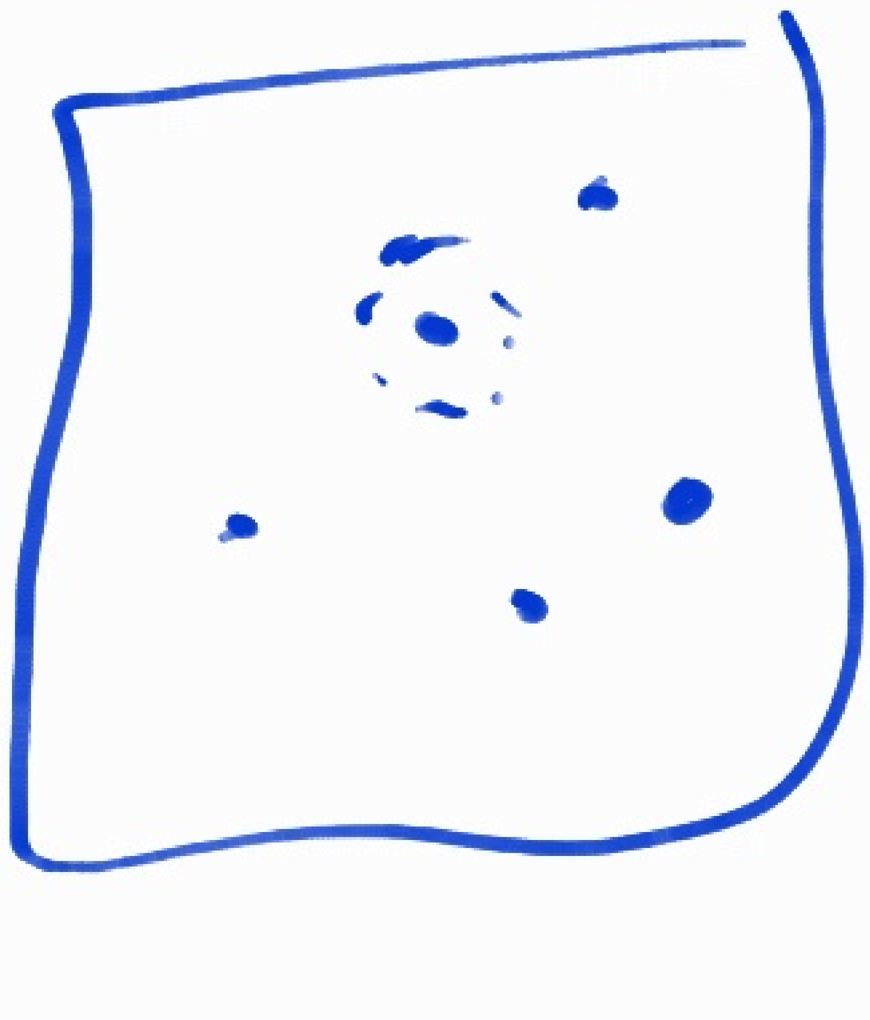


there exists an x $\exists x$

for some x

at least one x

$\exists x\, P(x)$

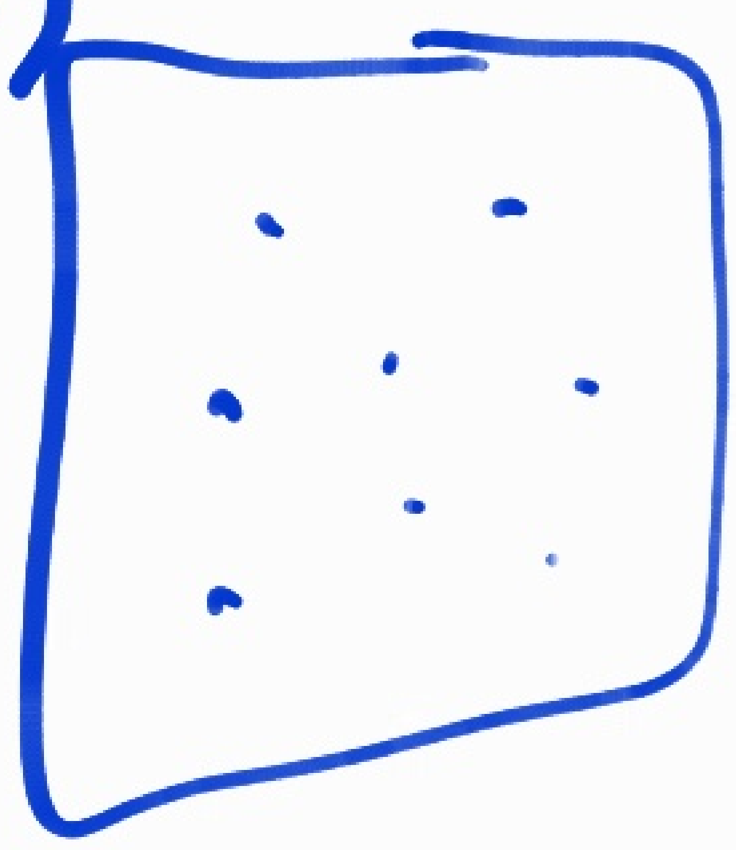


$$\forall x\, P(x) \equiv \neg \exists x\, \neg P(x)$$

$$\exists x\, P(x) \equiv \neg \forall x\, \neg P(x)$$

$$\neg \left[\forall x\, P(x) \right] \equiv \exists x\, \neg P(x)$$

$$\neg \left[\exists x\, P(x) \right] \equiv \forall x\, \neg P(x)$$



$\forall x$ is a bit too encompassing / powerful

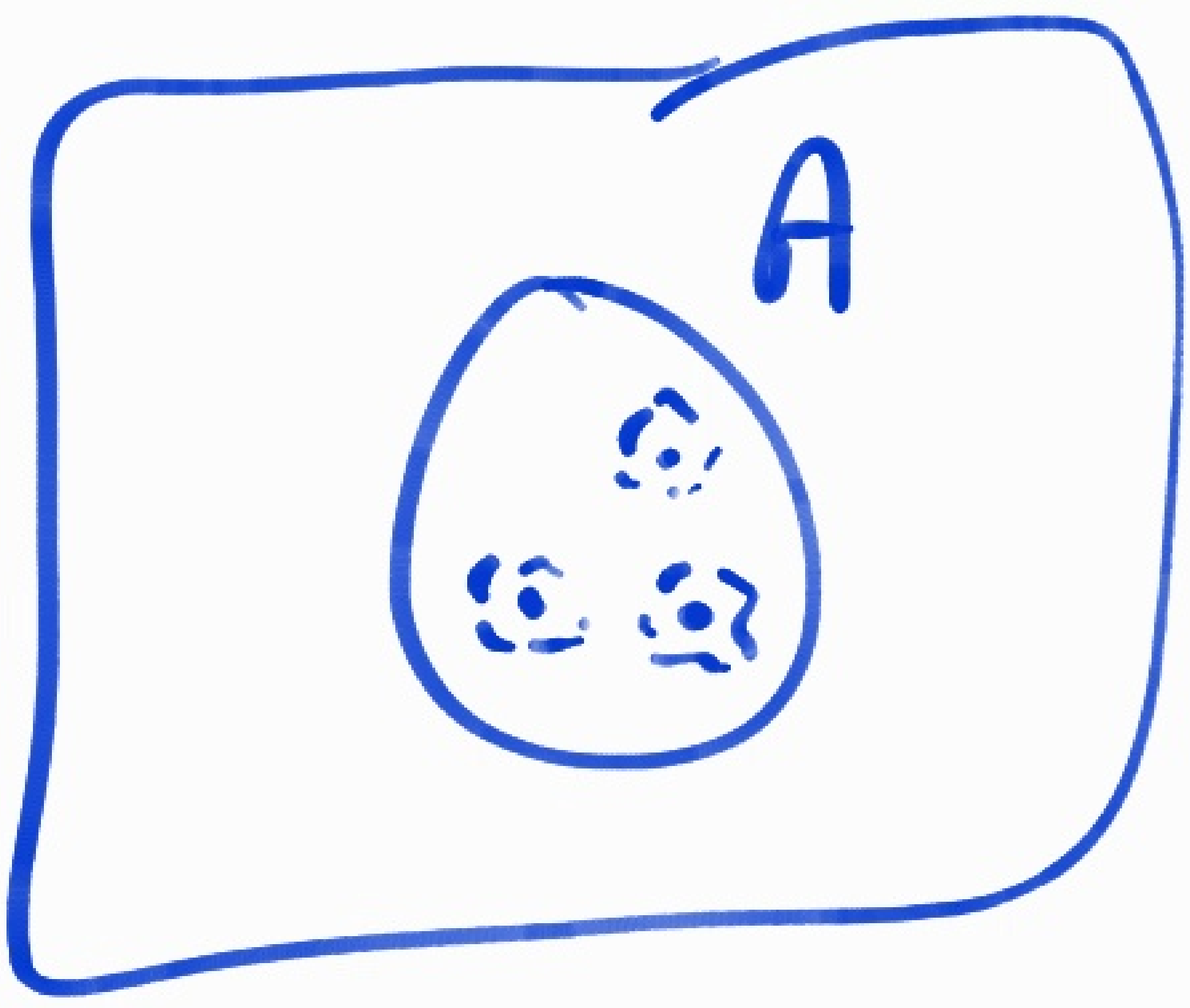
Bounded quantifiers

for all x in A

$$(\forall x \in A) P(x)$$

exists an x in A

$$(\exists x \in A) P(x)$$



$$\neg (\forall x \in A) P(x) \equiv (\exists x \in A)[\neg P(x)]$$

$$\neg (\exists x \in A) P(x) \equiv (\forall x \in A)[\neg P(x)]$$

$$(\forall x \in A) P(x) \equiv \forall x [x \in A \rightarrow P(x)]$$

$$(\exists x \in A) P(x) \equiv \exists x [x \in A \wedge P(x)]$$

$x \in \mathbb{N} \to x \in \mathbb{Z}$

$\forall x \in \mathbb{N} \, (x \in \mathbb{Z})$ true

$\exists x \in \mathbb{N} \, (x - 1 \notin \mathbb{N})$ true