1. Let's determine the appropriate probability distributions for each scenario:

(a) The number of errored bits in a packet of 1024 bits.

- This scenario can be approximated by a binomial distribution. Each bit can be considered a trial with two possible outcomes (errored or not errored), and the probability of a bit being errored or not is independent of other bits.

(b) Time between packets arriving at a router.

- This scenario can be modeled by an exponential distribution. The time between events in a Poisson process (such as packet arrivals) follows an exponential distribution.

(c) A student has their reaction time tested for 100 stimuli. Count as "success" = reacting within 0.1 seconds. Count the number of successes.

- This scenario follows a binomial distribution. There are a fixed number of trials (100 stimuli), and each trial has two possible outcomes (success or failure) based on reacting within 0.1 seconds or not.

So, the appropriate probability distributions for each scenario are: (a) A: The variable is binomial (approximately). (b) B: The variable is Poisson (approximately). (c) A: The variable is binomial (approximately).

2. The probability of success (a battery lasting 10 hours or more) is given as p = 0.25, and you want to find the probability of exactly k = 2 successes in 5 trials.

Let's calculate the probabilities:

(a) Find the probability that exactly two of the batteries will last 10 hours or more:

You can use the binomial probability formula for this: P(X = k) = (n choose k) * p^k * (1 - p)^(n - k)

Where:

- n is the number of trials (5 batteries).
- k is the number of successes you want (2 batteries).
- p is the probability of success (0.25).
- (n choose k) is the binomial coefficient, which can be calculated as n! / (k!(n - k)!).

P(X = 2) = (5 choose 2) * (0.25)^2 * (0.75)^(5 - 2)

P(X = 2) = (10) * (0.0625) * (0.421875)

P(X = 2) ≈ 0.2637

(b) Find the expected number of batteries that will last 10 hours or more:

The expected number of successes (batteries lasting 10 hours or more) in a binomial distribution is given by: $E(X) = n * p$

$E(X) = 5 * 0.25 = 1.25$

So, the expected number of batteries that will last 10 hours or more is 1.25.

3. (a) Find $P(X \leq 0)$:
- This corresponds to $F(0)$, which is given as 0.41 in the CDF.
- $P(X \leq 0) = F(0) = 0.41$

(b) Find $P(X \leq 1)$:

- This corresponds to $F(1)$, which is given as 0.72 in the CDF.
- $P(X \leq 1) = F(1) = 0.72$

(c) What is the probability that exactly one error is detected, i.e., $P(X = 1)$?

- $P(X = 1)$ can be calculated as the difference between the probabilities of $X \leq 1$ and $X \leq 0$:
- $P(X = 1) = P(X \leq 1) - P(X \leq 0) = 0.72 - 0.41 = 0.31$

(d) What is the probability that more than one error is detected?

- This can be calculated as the complement of $P(X \leq 1)$, which means $1 - P(X \leq 1)$:
- $P(X > 1) = 1 - P(X \leq 1) = 1 - 0.72 = 0.28$

(e) What is $P(X \geq 2)$?

- $P(X \geq 2)$ can be calculated as the complement of $P(X \leq 1)$, which is the same as the result in part (d):
- $P(X \geq 2) = P(X > 1) = 0.28$

(f) What is the most probable number of errors to be detected?

- The most probable number of errors corresponds to the value of x where the CDF increases the most abruptly. In this case, it's where the CDF has the steepest increase, which is at $x = 2$.
- So, the most probable number of errors to be detected is 2.

To summarize: (a) $P(X \leq 0) = 0.41$ (b) $P(X \leq 1) = 0.72$ (c) $P(X = 1) = 0.31$ (d) $P(X > 1) = 0.28$ (e) $P(X \geq 2) = 0.28$ (f) Most probable number of errors = 2

4. (a) What is the probability that the second component you sample is defective, i.e., that your sample size is 2?

- In this case, you're looking for the probability that the first component is non-defective (1 - p) and the second component is defective (p).
- P(X = 2) = (1 - p) * p = (99/100) * (1/100) = 99/10,000.

(b) What is P(X ≤ 3)?

- You're looking for the probability that you find a defective component within the first three trials.
- P(X ≤ 3) = 1 - P(X > 3).
- P(X > 3) is the probability that you do not find a defective component in the first three trials, which is (1 - p)^3.
- P(X ≤ 3) = 1 - (1 - p)^3 = 1 - (99/100)^3.

(c) What is the expected number in your sample (the mean sample size)?

- The expected value (mean) of a geometric distribution is given by E(X) = 1/p.

E(X) = 1 / (1/100) = 100.

5.

P(X = x) = 970,299/100,000,000

We've already established that 1/p = 100,000,000/970,299. Now, we'll solve for x:

P(X = x) = (1 - p)^(x-1) * p

970,299/100,000,000 = (99/100)^(x-1) * (1/100)

Now, take the natural logarithm (ln) of both sides:

ln(970,299/100,000,000) = (x-1) * ln(99/100)

Now, solve for x:

x - 1 = ln(970,299/100,000,000) / ln(99/100)

x = 1 + (ln(970,299/100,000,000) / ln(99/100))

Using a calculator:

x ≈ 686.62

So, for P(X = x) to be approximately 0.00970299, the value of x is approximately 686.62.

5. Given that the arrival rate is 5 requests per minute, we have $\lambda = 5$.

(a) In one minute, what is the probability of receiving i. 2 requests?

- P(X = 2) in a Poisson distribution can be calculated as follows:
- $P(X = 2) = (e^{-\lambda} * \lambda^2) / 2!$, where e is the base of the natural logarithm.

$P(X = 2) = (e^{-5} * 5^2) / 2!$ $P(X = 2) \approx (0.00674 * 25) / 2$ $P(X = 2) \approx 0.1685$

ii. Fewer than 2 requests?

- $P(X < 2)$ is the cumulative probability for X. You can calculate it by adding the probabilities for X = 0 and X = 1:
- $P(X < 2) = P(X = 0) + P(X = 1)$

$P(X < 2) = (e^{-5} * 5^0) / 0! + (e^{-5} * 5^1) / 1!$

$P(X < 2) = (e^{-5} + 5e^{-5}) / (1 + 1)$

$P(X < 2) \approx (0.00674 + 0.0337) / 2$ $P(X < 2) \approx 0.0202$

iii. At least 4 requests?

- $P(X \geq 4)$ is the complement of $P(X < 4)$. You can calculate $P(X < 4)$ as shown above and then subtract it from 1.

$P(X \geq 4) = 1 - P(X < 4)$ $P(X \geq 4) = 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3)$

Calculate each of the probabilities and subtract from 1.

$P(X = 8) = (e^{-10} * 10^8) / 8!$

Using the value of e (approximately 2.71828):

$P(X = 8) \approx (2.71828^{-10} * 10^8) / (8 * 7 * 6 * 5 * 4 * 3 * 2 * 1)$

$P(X = 8) \approx (4.54147e-05 * 100000000) / 40320$

$P(X = 8) \approx 0.11259$

So, the probability that the server receives exactly 8 requests in two minutes is approximately 0.11259, or about 11.26%.

6. Expected number of damaged fridges in a 30-day month:

   ○ The rate λ is given as 2.5 fridges per day, so in a 30-day month, the expected number of damaged fridges is λ * 30 = 2.5 * 30 = 75 fridges.

   Expected cost of scrapping these fridges:

   ○ Each fridge costs $250 to be scrapped. So, the expected cost is 75 fridges * $250/fridge = $18,750.

   Variance of the number of damaged fridges in a 30-day month:

   ○ For a Poisson distribution, the variance is also λ. So, the variance of the number of damaged fridges is 2.5 * 30 = 75 fridges.

   Variance of the cost of scrapping these fridges:

   ○ Since each fridge costs $250 to be scrapped, and the number of fridges follows a Poisson distribution with a variance of 75, we can calculate the variance of the cost as follows:
     Variance of cost = (Number of fridges) * (Variance of cost per fridge) = 75 * (250^2) = 75 * 62500 = 4,687,500

   Standard deviation of the cost:

   ○ To find the standard deviation, take the square root of the variance:
     Standard deviation = √(Variance of cost) = √(4,687,500) ≈ $2,165.06

So, the mean cost of scrapping damaged fridges in a 30-day month is $18,750, and the standard deviation is approximately $2,165.06.