Induction:

depends on n

Problem: Prove something for all integer $n \geq a$

Statement (n)

Ex. Prove $n \cdot (n+1)$ is even for all $n \in \mathbb{N}$

How to solve?

Base case ~~2~~ 2a Check your statement for the smallest value of n

Inductive hypothesis Assume your ~~the~~ statement is true for some arbitrary $n=k \geq a$, $k \in \mathbb{Z}$

Inductive step: Prove, knowing I.H. that the statement is true for $n=k+1$

Proof

Idea:

$$\frac{\text{Statement } k=a}{\text{true, base case}} \rightarrow \frac{\text{I.S.} + \text{I.H. } k=a}{a+1 \text{ is true.}} \rightarrow \frac{\text{I.S} + \text{I.H. } k=a+1}{a+2 \text{ is true.}} \rightarrow \ldots$$

Induction: 1) the first one is true

2) we know how to go from one to the next one

Example

1) $\boxed{1+2+\dots+n = \frac{n(n+1)}{2}}$ for any $n \in \mathbb{Z}^+$

Statement is true for any integer $n \geq 1$

Base step Check the formula works for $n=1$

LHS 1

RHS $\frac{1(1+1)}{2} = \frac{1 \cdot 2}{2} = 1$

RHS = LHS So the formula works for $n=1$

Inductive Hyp. Assume the formula works for $n=k$ where $k \geq 1$, $k \in \mathbb{Z}$

$$\boxed{1+2+\dots+k = \frac{k(k+1)}{2}}$$

is true (assumed)

Inductive Step: Prove the formula works for $n=k+1$

$$1+2+\dots+k+k+1 = \frac{\overbrace{(k+1)}^{n} \cdot \overbrace{(k+1+1)}^{n+1}}{2}$$

$$\underbrace{1+2+\ldots+k}+k+1 \overset{\text{I.H.}}{=} \frac{k(k+1)}{2}+k+1 =$$

$$=(k+1)\left(\frac{k}{2}+1\right)=\frac{(k+1)\cdot(k+2)}{2}$$

we proved the formula works for $n=k+1$

$$\frac{k\boxed{(k+1)}}{2}+\boxed{k+1}=$$

$$=(k+1)\left(\frac{k}{2}+1\right)=$$

$$=(k+1)\left(\frac{k}{2}+\frac{2}{2}\right)=$$

$$=(k+1)\cdot\left(\frac{k+2}{2}\right)$$

2) if $n \in \mathbb{Z}$ and $n > 4$ then

$$n^2 > n+16$$

Statement

1) Base: Prove statement is true for $n=5$

$n^2 = 5^2 = 25$

$n+16 = 5+16 = 21$

$25 > 21$

ineq. holds for $n=5$

2) Ind. H.

Assume the enequality holds for some $n=k$, where $k \in \mathbb{Z}$, $k>4$

You know $k^2 > k+16$ is true.

3) Ind. Step. We need to prove tha the eneq. works for $n=k+1$

$$(k+1)^2 > k+1+16 \Rightarrow \underline{(k+1)^2 > k+17}$$

want to prove

$$\underline{(k+1)^2} = \underline{k^2+2k+1} \overset{\text{I.H.}}{>} k+16+2k+1 =$$

$$= k+17+2k \underset{\text{since}}{>} \frac{k+17}{k>4,\ 2k>0}$$

$$(k+1)^2 > k+17$$

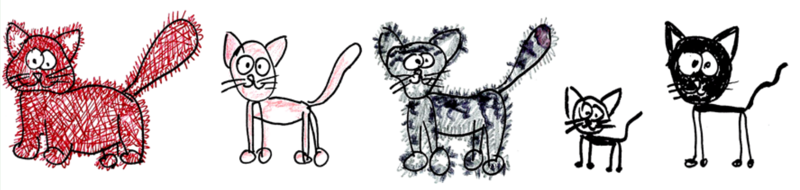
Example with algorithms: Bubble Sort

Bubble Sorting Algorithm

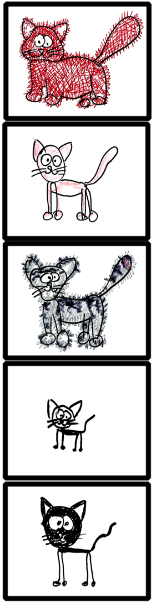
The algorithm sorts an array of elements by swapping adjacent elements if they are not in order. It stops when no swaps are required.

Example

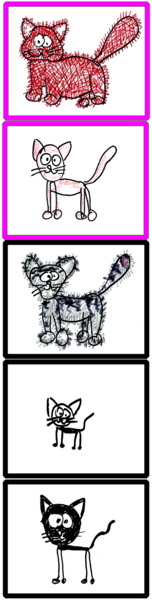
Sort the cats below in ascending order by weight.



Example with algorithms: Bubble Sort



Example with algorithms: Bubble Sort



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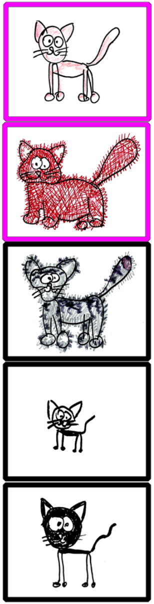

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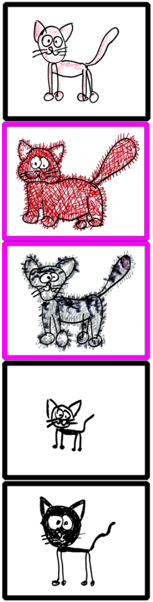

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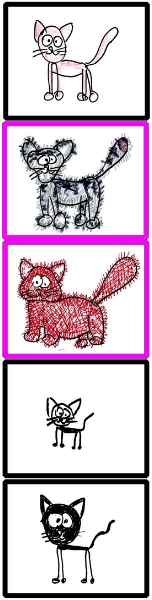

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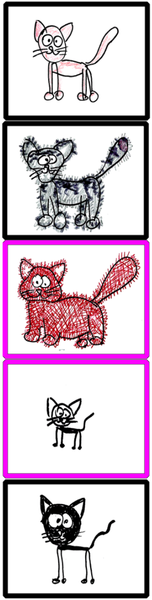

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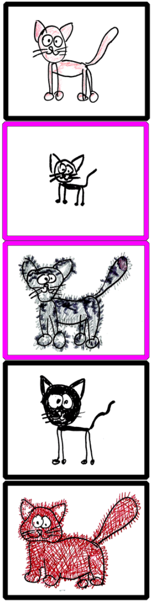

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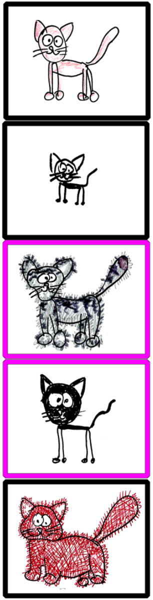

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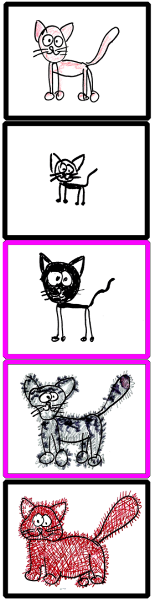

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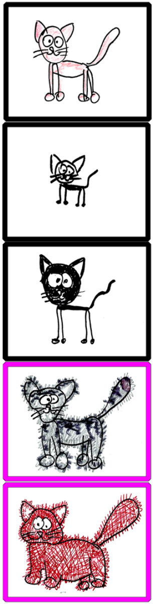

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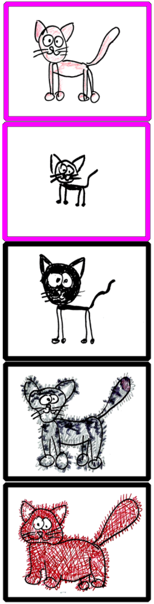

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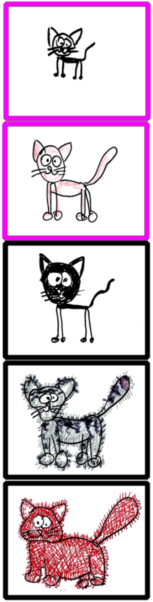

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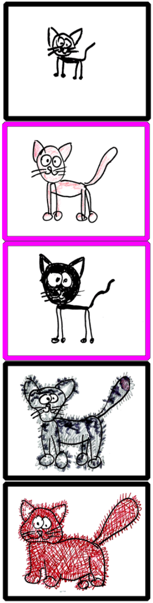

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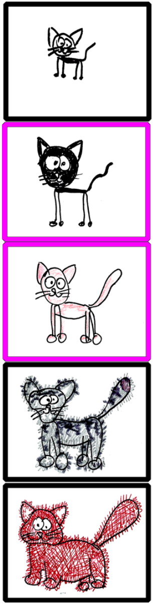

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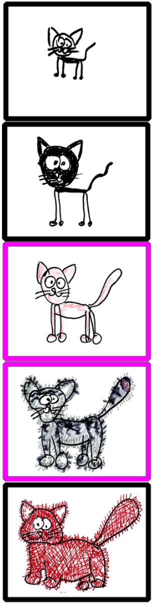

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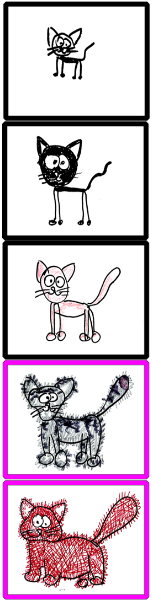

Statement

4) Bubble-sort:

Bubble-sort works for all lists with n elements, where $n \geq 1, n \in \mathbb{Z}$

Base: Show B-S works for lists with 1 element

All good!!

Inductive hyp.

Assume B-S work for any list with k elements, $k \in \mathbb{Z}, k \geq 1$

Inductive step: Prove B-S works for $n = k+1$ elements

Run B-S once, the heviest element will sink and you have k unsorted elements

we know that by I.H. B-S can sort it