ENGR 123

Tanya Gvozdeva

Victoria University of Wellington

Lecturer Information

- I'm Tanya!
- My office location is Cotton Building, room CO362, and email address is tanya.gvozdeva@vuw.ac.nz
- Office hours: 10-11pm on lecture days (starting tomorrow)
- If you cannot make it to office hours email me!
- Zoom office hours: by appointment.

me!

Questions are awesome!

ASK QUESTIONS! BE BRAVE!

Relations

A binary relation R between sets A and B is $R \subseteq A \times B$

R is a set of pairs (a,b), s.t. $a \in A$, $b \in B$

Informally: a relation is a set of all pairs that are related somehow

Notation: $(a,b) \in R$
$aRb \Leftarrow$ a is related to b

Examples:

1) phone book is a relation R on people $\times$ phone numbers, where
aRb $\underbrace{\text{if and only if}}_{\text{iff}}$ a has phone number b

2) blood relations B is a relation on people $\times$ people, s.t.
xBy iff x is related to y by blood

3) friends in social media

look of an element

4) $S = \{(a,b) \mid a \in \mathbb{Z}, b \in \mathbb{R} \text{ and } a > b\}$

its property

$(2,1) \in S \qquad 2 S 1$

$(4,\pi) \in S$

$4 > \pi$

a relation R on A: is $R \subseteq A \times A$

Special relations:

1) the identity relation on A

$$id_A = \{(a,a) \mid a \in A\}$$

2) the empty relation from A to B

is just $\emptyset$

3) The universal relation is $A \times B$

Visualising relations:

1) tables

R on $\{1, 2, 3, 4\}$

$x R y$ $\underbrace{\text{if and only if}}_{\text{iff}}$ $x - y$ is divisible by 3

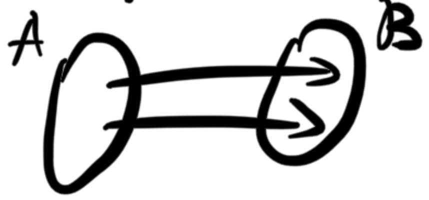
$R = \{(4,1), (1,4), (3,3), (1,1), (2,2)(4,4)\}$

	1	2	3	4	
1	X			X	(1,4)
2		X		.	(2,4)
3			X		
4	X			X	

(4,1)

2) directed graphs

$R \subseteq A \times B$

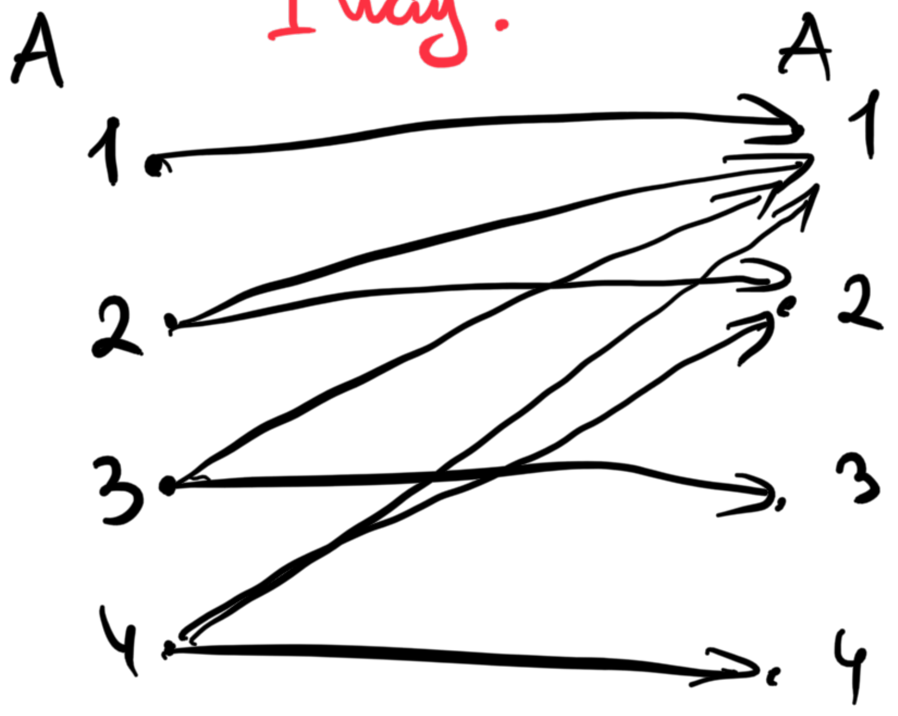


P on $\{1,2,3,4\}$

xPy if and only if x is divisible by y

$P = \{(1,1),(2,1),(3,1),(4,1),(4,2),(2,2),(3,3),(4,4)\}$ $\quad P \subseteq A \times A$ only one set

1 way.

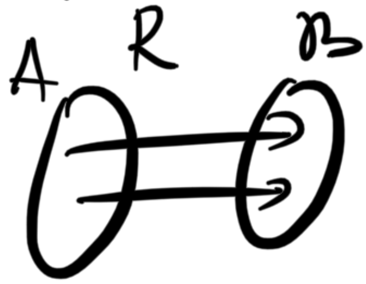


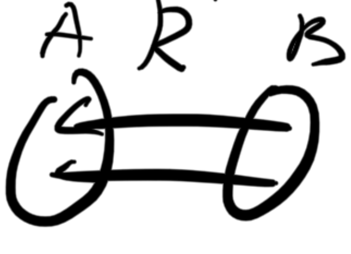
Operations on relations

The Inverse relation of $R \subseteq A \times B$

$$R^{-1} = \{(x,y) \mid (y,x) \in R\}$$

On a graph:





Examples:

1) $R = \{(1,2), (3,4), (1,1), (1,4)\}$

$R^{-1} = \{(2,1), (4,3), (1,1), (4,1)\}$

3) $S = \{(1,a), (2,b)\}$ $\quad S \subseteq \{1,2\} \times \{a,b\}$

$S^{-1} = \{(a,1), (b,2)\}$

2) R on $\mathbb{N}$

xRy iff $x<y$

$(1,2) \in R$

R^{-1}

$aR^{-1}b$ iff $b<a$

$(2,1) \in R^{-1}$

Fact: $(R^{-1})^{-1} = R$

Lets $(a,b) \in R \Rightarrow$

$\Rightarrow (b,a) \in R^{-1} \Rightarrow$

$\Rightarrow (a,b) \in (R^{-1})^{-1}$

if $(a,b) \in R$ then $(a,b) \in (R^{-1})^{-1}$

We proved $R \subseteq (R^{-1})^{-1}$

if we can prove that $(R^{-1})^{-1} \subseteq R$

then $R = (R^{-1})^{-1}$

Lets $(x,y) \in (R^{-1})^{-1} \Rightarrow$

$\Rightarrow (y,x) \in R^{-1} \Rightarrow$

$\Rightarrow (x,y) \in R$

We proved that if $(x,y) \in (R^{-1})^{-1}$ then $(x,y) \in R$

$$(R^{-1})^{-1} \subseteq R$$

Now $(R^{-1})^{-1} \subseteq R$ and $R \subseteq (R^{-1})^{-1} \Rightarrow (R^{-1})^{-1} = R$

Composition of relations