Engr 123

Chris   Alice   Bob   Dot

$L(x,y) = x$ is to the left of (or in the same position as) y

$L(\text{Chris}, \text{Bob})$ T

$L(\text{Chris}, \text{Chris})$ T
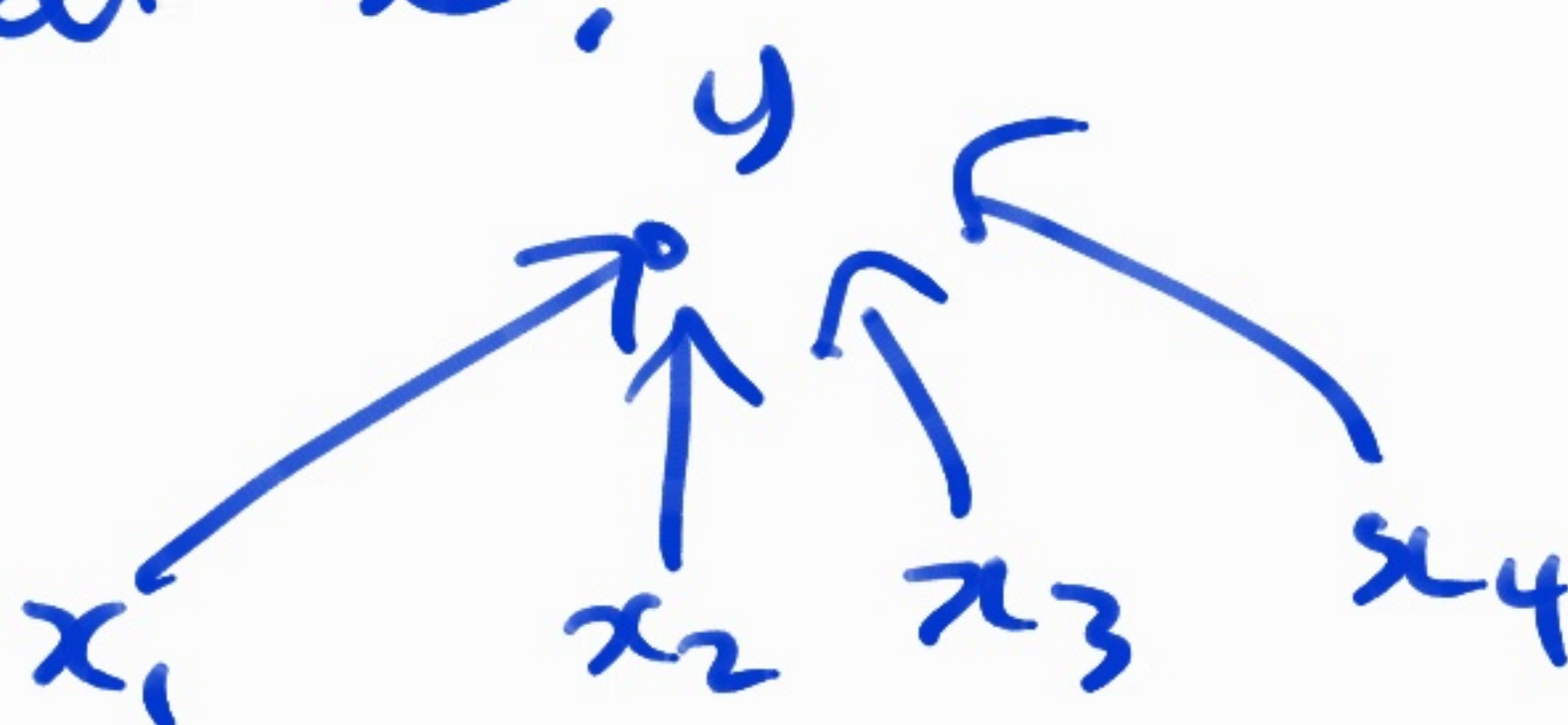
$L(\text{Dot}, \text{Alice})$ F

C A B D

$\forall x \; L(Chris, x) \quad T$

$\exists y \; L(y, Alice) \quad T$

y could be Chris
(or Alice)

$\exists y [\forall x [L(y, x)]] \quad T$

y could be Chris

Is there some y that works
for all x?

y

$x_1 \quad x_2 \quad x_3 \quad x_4$

$$\forall y [\exists x [L(x,y)]]$$

every y has some x

In $\mathbb{N}$
$$G(x,y) = x \geq y$$

$\forall x \, \forall y \; G(x,y)$ F $\neg \forall x \forall y G(x,y)$
$\forall x \, \exists y \; G(x,y)$ T $\equiv \exists x \exists y$
$\exists x \, \forall y \; G(x,y)$ F $y=x$ $\neg G(x,y)$
$\exists x \, \exists y \; G(x,y)$ T $x=2, y=1$

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Proofs (arguments)

Premiss 1 P_1
Premiss 2 P_2
$\vdots$
Premiss n P_n dividing line

Conclusion C

We want the conclusion to follow from the premisses.

$$[P_1 \wedge P_2 \wedge \ldots \wedge P_n] \rightarrow C$$

should be a tautology

we call such a proof valid.

How to use proofs

Fred is a fish
All fish swim in the ocean

Fred swims in the ocean

$Fish(x) =$ "x is a fish"

$Swim(y) =$ "y swims in the ocean"

$Fish(Fred)$

$\forall y (Fish(y) \rightarrow Swim(y))$

$Swim(Fred)$