R is on $\mathbb{Z}$

xRy iff $x+y$ is even

$[0]_R = \{x \in \mathbb{Z}, x \text{ is even}\} = [2]_R = [-2]_R$

$= [4]_R$

$[1]_R = \{x \in \mathbb{Z} : x \text{ is odd}\}$

$[0]_R \cup [1]_R = \mathbb{Z}$

2) R is a relation on $\mathbb{Z}$

xRy iff $x \equiv y \pmod 5$

$x-y$ is div by 5

$[0]_R = \{x \in \mathbb{Z} : x \text{ is div by } 5\} =$

$= \{\ldots, -10, -5, 0, 5, 10, 15, \ldots\}$

$[1]_R = \{x \in \mathbb{Z}$: s.t. x is a multiple of $5+1$ (or the remainder is $1\} = \{1, 6, 11, 16, 21,$

$26, \ldots -4, -9, -14, \ldots\}$

$[2]_R = \{x \in \mathbb{Z} : x \text{ is a multiple of } 5+2\}$

$[3]_R = \{$ — " — " $+3\}$

$[4]_R = \{$ — " — " — .... $+4\}$

$[5]_R = [0]_R$

$[0]_R \cup [1]_R \cup [2]_R \cup [3]_R \cup [4]_R =$

$= \mathbb{Z}$

$[0]_R \cap [1]_R = \emptyset$

Partition

Def A partition of a set A is a collection of sets $C_1, C_2, \dots, C_n$ that satisfy 2 conditions:

1) $C_i \cap C_j = \emptyset$ for any $1 \leq i, j \leq n$
2) $C_1 \cup C_2 \cup C_3 \cup \dots \cup C_n = A$

Example: Partitions $\Leftrightarrow$ eq. classes.

1) A Partition of $\{1,2,3,4,5,6\} = A$

$$\underset{C_1}{\{1\}}, \underset{C_2}{\{2,3\}}, \underset{C_3}{\{4,5,6\}}$$

$C_1 \cap C_2 = \emptyset$ $\quad C_2 \cap C_3 = \emptyset$

$C_1 \cap C_3 = \emptyset$

$C_1 \cup C_2 \cup C_3 = A$

R is an eq. relation on A

C_1, C_2, C_3 are its eq. classes.

$\{1\} \Rightarrow 1R1$ $\qquad$ $\{2,3\} \Rightarrow 2R2, 3R3, 2R3, 3R2$

$\{4,5,6\} \Rightarrow 4R4, 5R5, 6R6, 4R5, 5R4, 4R6, 6R4, 5R6, 6R5$

2) R is eq. relation on $\{0,1,2,3,4,5,6\}$

xRy iff $x \equiv y \pmod 3$ $\qquad = C_2$

$[0]_R = \{0,3,6\} = C_1$ $\qquad$ $[1]_R = \{1,4\} = [4]_R$

$\parallel$

$[3]_R = [6]_R$ $\qquad$ $C_1 \cap C_2 = \emptyset$

$[2]_R = \{2,5\} = C_3$ $\qquad$ $C_1 \cap C_3 = \emptyset$

$C_2 \cap C_3 = \emptyset$

$C_1 \cup C_2 \cup C_3 = \{0,3,6\} \cup \{1,4\} \cup \{2,5\} =$

$= \{0,1,2,3,4,5,6\} = A$

C_1, C_2, C_3 is a partition of A

Fact : If you have a partition you can always make an eq. relation s.t. its eq. classes are sets in the partition.

If you have an eq. rel., then its eq. classes form a partition

Partial orders

Def: R is a relation on A. R is a partial order if

1) R is reflexive: $\forall a \in A \quad aRa$
2) R is antisym.: if aRb and bRa, then ~~aRa~~ $a=b$
3) R is transitive if aRb and bRc, then aRc

Notation: R, $\preccurlyeq$, $\leq$

R shows that non-equal element are different.

aRb iff $a=b$

Example: R is $\leq$ on $\mathbb{Z}$ is a partial order

1) reflexivity: ✓

Let $a \in \mathbb{Z}$ $\quad a \leq a \Rightarrow aRa$

2) antisym. ✓

$$\underbrace{aRb}_{a \leq b} \text{ and } \underbrace{bRa}_{b \leq a} \Rightarrow a=b$$

✓

3) Transitive:

$$\underbrace{aRb}_{a \leq b} \text{ and } \underbrace{bRc}_{b \leq c} \Rightarrow$$

$$\Rightarrow a \leq b \leq c \Rightarrow a \leq c \Rightarrow$$

$$\Rightarrow aRc$$

R is a partial order

Example A is a set of all subsets of S. Then $\subseteq$ is a partial order

elements of A are subsets of S

R is $\subseteq$ on A

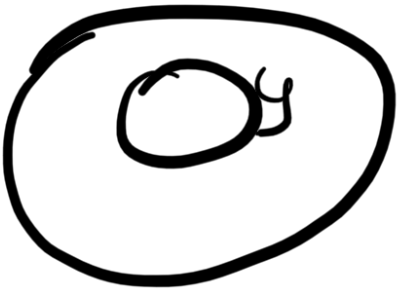
aRb iff $a \subseteq b$

Example $S = \{1,2,3\}$

$A = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\},$

$\rightarrow \{1,2,3\}\}$ $\{1\}$ and $\{3\}$ are incomp.

$\{1\} R \{1,2\} \Rightarrow \{1\} \subseteq \{1,2\}$



X

$y \subseteq X$

$X \subseteq X$

$a \subseteq b$ if any element of a is an element of b

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1) Refl. Let $a \in A$

$\rightarrow a \subseteq a \Rightarrow aRa$

2) Antisym: aRb and $bRa \Rightarrow$

$\Rightarrow a \subseteq b$ and $b \subseteq a$

$a = b$

3) transitive: aRb and bRc

$a \subseteq b$ $b \subseteq c$

R is a partial order on A.

$a \leq c$
aRc

Two elements x, y are comparable if aRb or bRa

x, y are incomparable if $x \not{R} y$ and $y \not{R} x$

Total order: is a partial order where any two element are comparable.

Examples: 1) $\leq$ on $\mathbb{Z}$

R is on $\mathbb{Z}$

aRb iff $a \geq b$

for any two elements you have $a \leq b$ or $b \leq a$

R is a total order.

2) R on $\mathbb{Z}_{\geq 1}$

$x R y$ if y is divisible by x

$3, 5$ 3 is not div. by 5

5 is not div by 3

$3 \not R 5$, $5 \not R 3$

not a total order.