Engr 123

<u>Typo's</u> A2) Q1, change Q to P
Q2 change trusts to agree's with.
Q3 add Bird(x)

L8
Rational + Irrational = Irrational
$\sqrt{2}$ is irrational
Cartesian product

In Engr 121, we said $\sqrt{2}$ is irrational.

How do we know?

$$\sqrt{2} \notin \mathbb{Q} \leftrightarrow \neg (\exists p \in \mathbb{Z})(\exists q \in \mathbb{Z} - \{0\})\left[\sqrt{2} = \frac{p}{q}\right]$$

First up, we'll prove the following

If x is rational and y is irrational, Then $x+y$ is irrational.

<u>Proof:</u> By contradiction

Assume x is rational, y is irrational $\Big\}$ premises

and $x+y$ is rational.

So $x = p/q$, y cannot be written as a fraction

and $x+y = r/s$

$$y = (x+y) - x = \frac{r}{s} - \frac{p}{q}$$

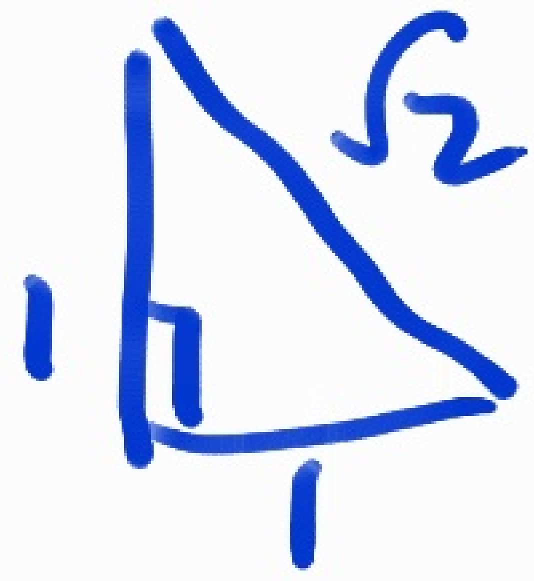
$$= \frac{qr - ps}{qs} \in \mathbb{Q}$$

But y isn't a fraction, $y \notin \mathbb{Q}$

This is a contradiction.

We will prove that the square-root of 2 is irrational

i.e. $\sqrt{2} \notin \mathbb{Q}$



Proof by contradiction

Assume $\sqrt{2} \in \mathbb{Q}$

$\sqrt{2} = p/q$ in it's lowest form (cancelled out common factors e.g. $\frac{1}{2}$ instead of $3/6$)

$$\sqrt{2}\, q = p$$
$$2q^2 = p^2$$

$\rightarrow 2q^2 = p^2$

$\rightarrow p^2$ is even

$\rightarrow p$ is even

$\rightarrow p = 2n$

$\rightarrow 2q^2 = (2n)^2$

$\rightarrow 2q^2 = 4n^2$

$\rightarrow q^2 = 2n^2 = 2 \times$ some integer

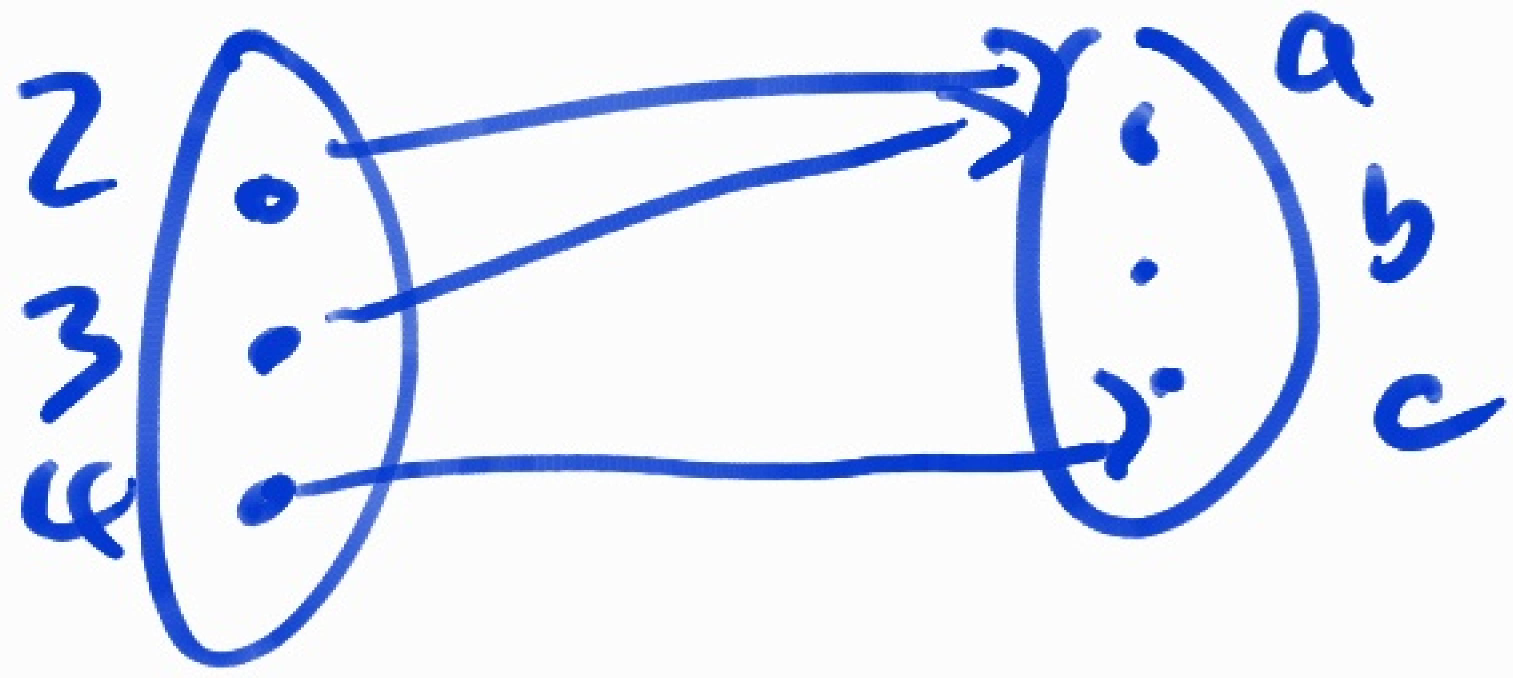
$\rightarrow q^2$ is even

$\rightarrow q$ is even

$\rightarrow q = 2m$

but p & q both have a common factor of 2.

Mappings



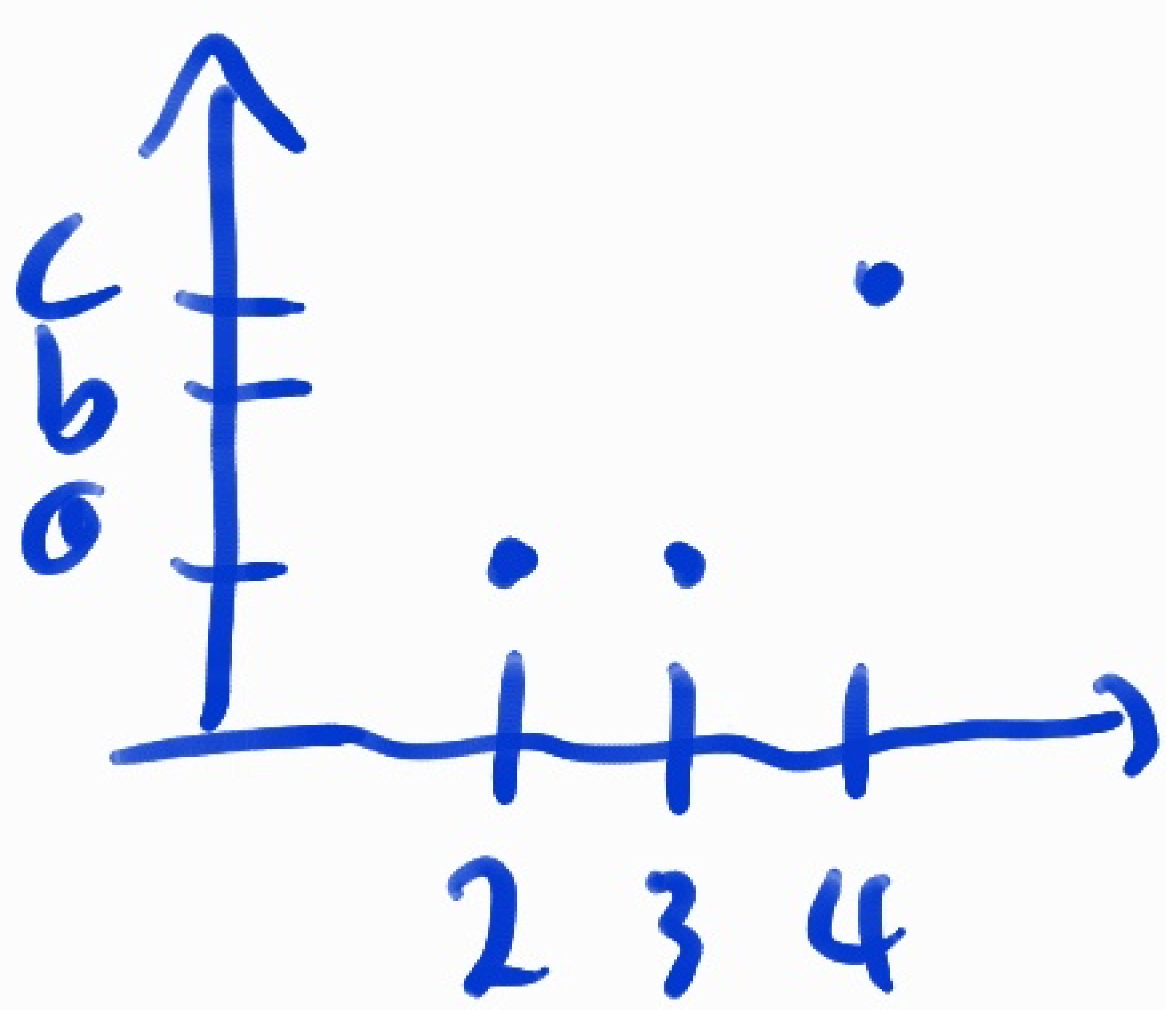
Cartesian product.

A set of ordered pairs

Let A and B be sets

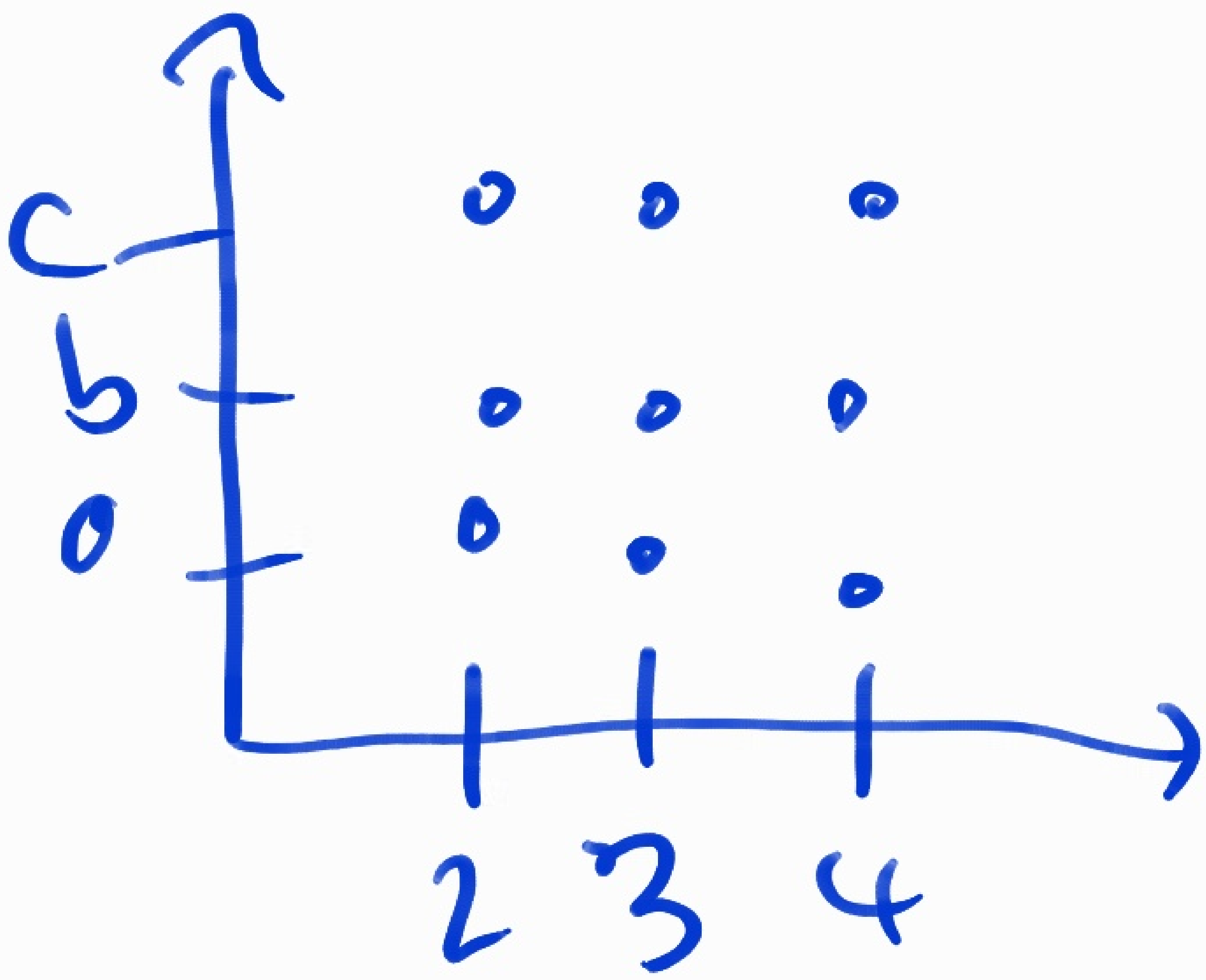
e.g. $A = \{2, 3, 4\}$ $B = \{a, b, c\}$

$= \{4, 3, 2\}$



$$A \times B = \{(a,b) : a \in A \text{ and } b \in B\}$$

e.g. $A \times B = \{(2,a),(3,a),(4,a),$
$(2,b),(3,b),(4,b),$
$(2,c),(3,c),(4,c)\}$

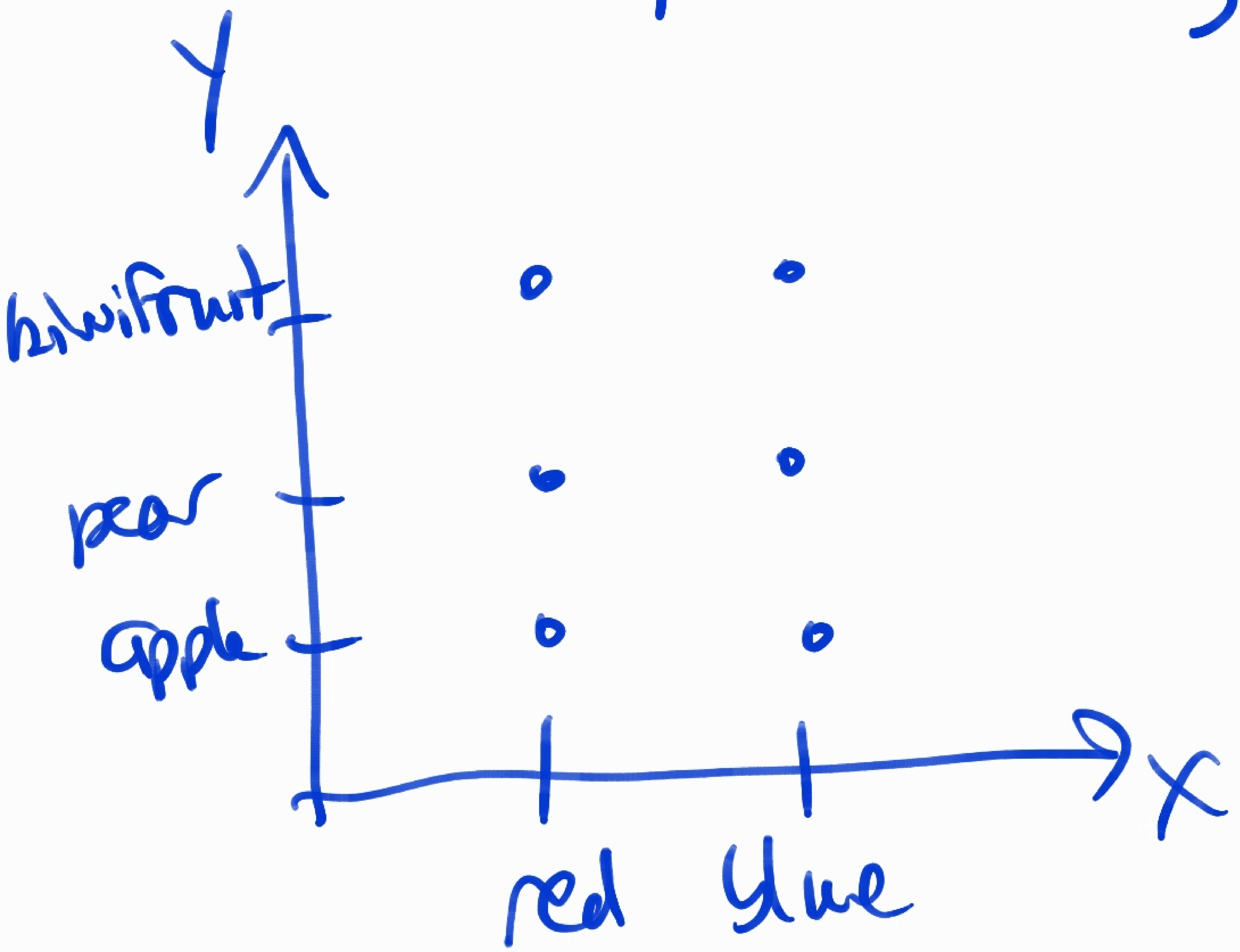


Suppose $X = \{\text{red}, \text{blue}\}$

$Y = \{\text{apple}, \text{pear}, \text{kiwifruit}\}$

$$X \times Y = \{(x,y) : x \in X \text{ and } y \in Y\}$$

$X \times Y = \{(\text{red}, \text{apple}), (\text{red}, \text{pear}), (\text{red}, \text{kiwi fruit}), (\text{blue}, \text{apple}), (\text{blue}, \text{pear}), (\text{blue}, \text{kiwifruit})\}$



Subsets