Def. A relation f from A to B is a function if

1) $A = \text{dom}(f)$ ← each element from A is related to an element in B

2) if $(a,b) \in R$ and $(a,c) \in R$ then $b = c$

each element of A is related to exactly 1 element of

Notation: if $f \subseteq A \times B$ we ~~[illegible]~~ write $f : \underbrace{A}_{\text{domain}} \to B$

instead $\underline{(a,b) \in f}$ or $\underline{afb}$

write $f(a) = b$

Example R on people

1) xRy iff x is a parent of y

Is it a function?

1) not true because there are people who have no children.

2) not true one person can have several children

R is not a function

2) S on $\mathbb{R}$

xSy iff $x^2+y^2=1$

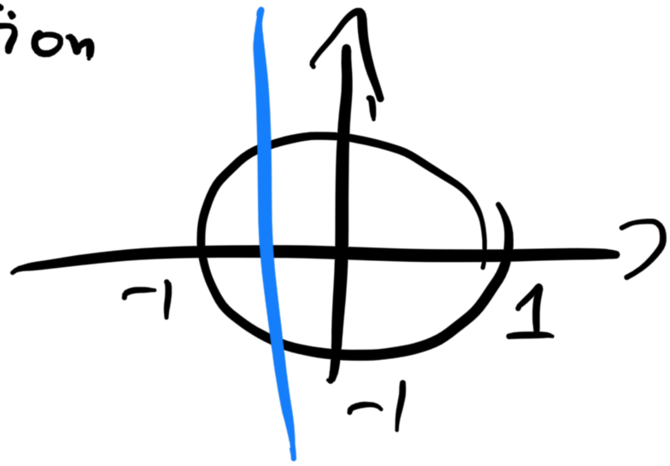
1) for any $x \in \mathbb{R}$ you can find $y \in \mathbb{R}$, s.t. xSy.

if $x=2$ $\quad 2^2+y^2=1$

$y^2=-3$

no real y's, s.t $y^2=-3$

not a function

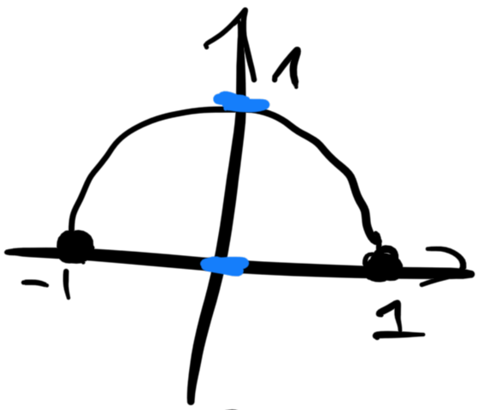


$[-1,1]$

3) S from $\cancel{\mathbb{R}}$ to $\mathbb{R}^+ \cup \{0\}$

xSy if $x^2+y^2=1$

S is a function



1) for any $x \in [-1,1]$ you can find y, s.t. xRy

$\sqrt{1-x^2}$

2) for any $x \in [-1,1]$ there is only one output

$$f: A \to B$$

domain

codomain

Example $f: \mathbb{N} \to \mathbb{R}$ codomain $f(x) = 5$

$\operatorname{ran}(f) = \{5\}$

$g: \mathbb{R}^+ \to \mathbb{Z}$

dom. codom.

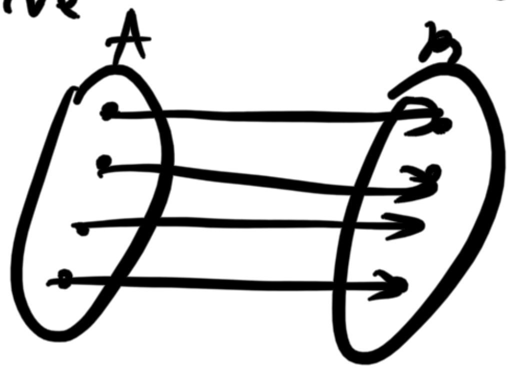
Def Let $f: A \to B$ be a function

f is surjective (onto) for $\forall b \in B$ $\exists a \in A$, s.t. $f(a) = b$

the same as codomain = range

f is injective (one-to-one) if for any $a, c \in A$, $a \neq c$, $f(a) \neq f(c)$

f is bijective if f is surjective and injective at the same time



Examples

dom codom

1) $f: \mathbb{R} \to \mathbb{R} \; : x \to 5x^4$

surjection. for any $y \in \mathbb{R}$ there is $x \in \mathbb{R}$, s.t. $y = 5x^4$

e.g., $y = -10$ then no x, s.t.

$$-10 = 5x^4$$

not a surjection

2) injection: for any $x_1, x_2 \in \mathbb{R}$

$$f(x_1) \neq f(x_2)$$

nope, $x_1 = 1$, $x_2 = -1$ and

$$f(x_1) = f(x_2)$$

not a bijetion.

Example 1) $f: \mathbb{Z} \to \mathbb{Z} : x \to 3x$

$f(x) = 3x$

1) surjection.

for any $y \in \mathbb{Z}$, $\exists x \in \mathbb{Z}$

s.t. $y = 3x$

no, if $y = 2$ then $x = \frac{2}{3} \notin \mathbb{Z}$

$2 = 3x$

✓ 2) injection $\forall \underline{x_1, x_2} \in \mathbb{Z}$

$x_1 \neq x_2$

$f(x_1) \neq f(x_2)$

if $x_1 \neq x_2$ then $\underset{f(x_1)}{\underset{\shortparallel}{3x_1}} \neq \underset{f(x_2)}{\underset{\shortparallel}{3x_2}}$

2) $f: \mathbb{R} \to \mathbb{R} : x \to 4x - 10$

✓ surjection: For any $y \in \mathbb{R}$

you shoul be able to find $x \in \mathbb{R}$

s.t. $y = 4x - 10$

$\Downarrow$

$x = \frac{y+10}{4} \in \mathbb{R}$

✓ injection: $\forall x_1, x_2 \in \mathbb{R}$ $x_1 \neq x_2$

you should have $f(x_1) \neq f(x_2)$

if $x_1 \neq x_2$ then $\underset{\substack{\uparrow \\ f(x_2)}}{4x_1 - 10} \neq \underset{\substack{\uparrow \\ f(x_2)}}{4x_2 - 10}$

Inverse

f^{-1} = Inverse relation. ← it should be a function

Sometimes f^{-1} is not a function

Theorem: f^{-1} is a function iff f is a bijection

Examples 1) $f: \mathbb{Z} \to \mathbb{Z} : x \to 3x$

f^{-1} is not a function

2) [redacted]

3) $f: \mathbb{R} \to \mathbb{R} : x \to 4x - 10$

f^{-1} is a function