IEL – protokol k projektu

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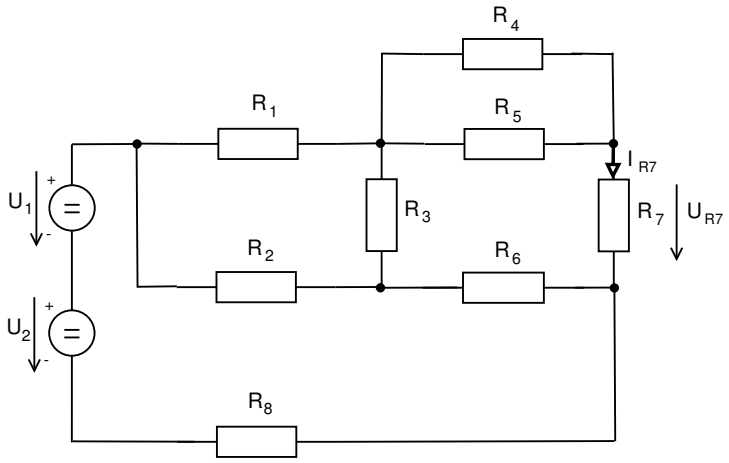
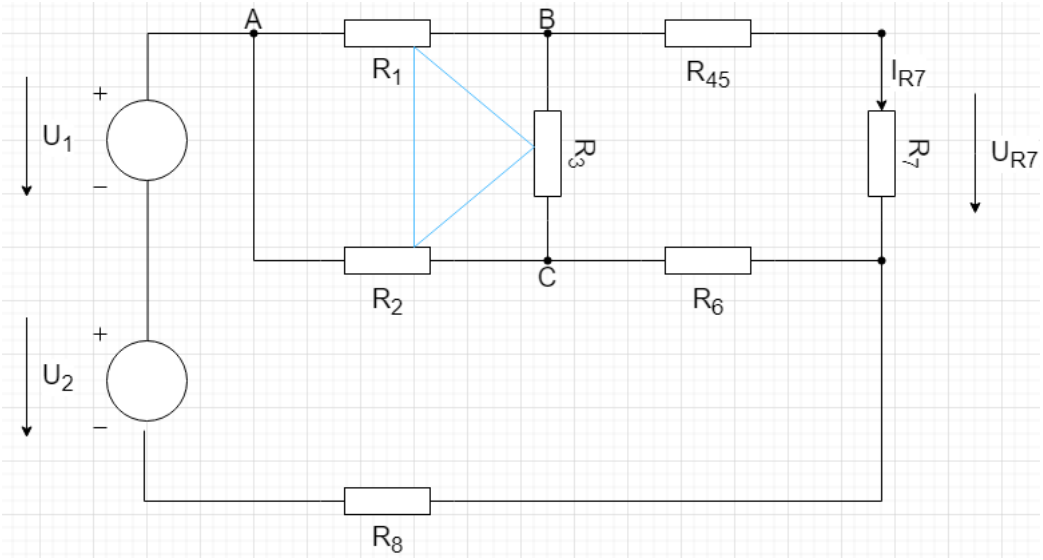
Příklad 1

Stanovte napětí U_{R7} a proud I_{R7}. Použijte metodu postupného zjednodušování obvodu.

sk.	U_1 [V]	U_2 [V]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [Ω]	R_6 [Ω]	R_7 [Ω]	R_8 [Ω]
C	100	80	450	810	190	220	220	720	260	180



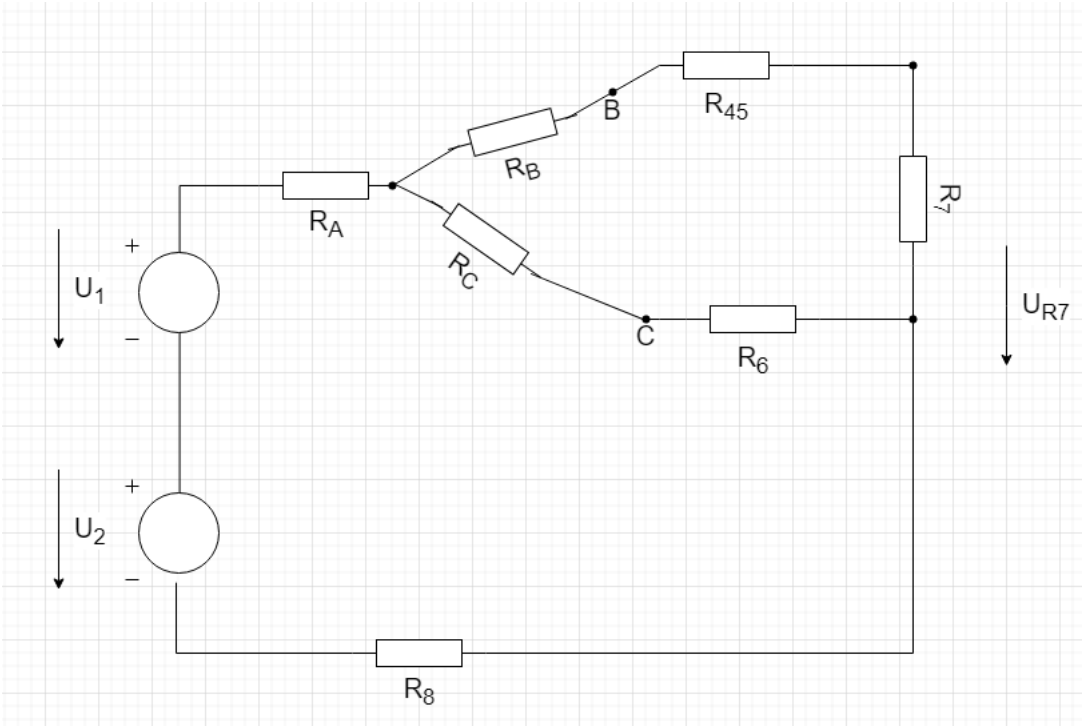
Výpočet R_{ekv}



Obrázek 1: $R_4 + R_5$

$$R_{45} = \frac{R_4 * R_5}{R_4 + R_5}$$

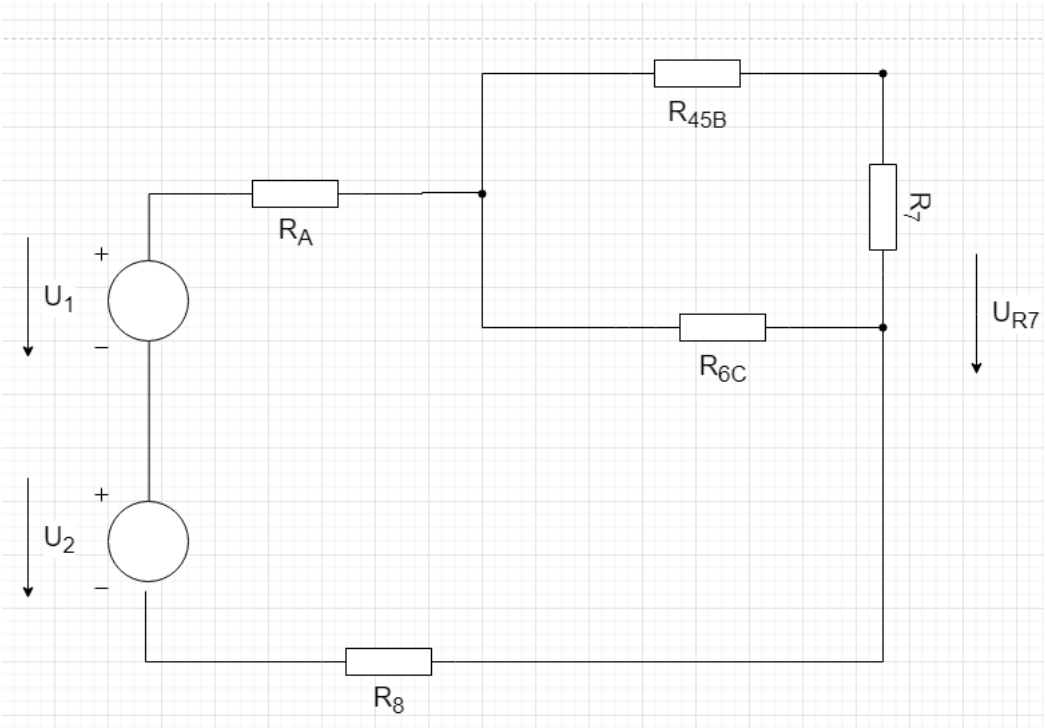
$$R_{45} = \frac{220\Omega * 220\Omega}{220\Omega + 220\Omega} = \frac{48400\Omega}{440\Omega} = 110\Omega$$



Obrázek 2: *Trojuhelnik hvezda*

$$R_A = \frac{R_1 * R_2}{R_1 + R_2 + R_3}$$
$$R_B = \frac{R_1 * R_3}{R_1 + R_2 + R_3}$$
$$R_C = \frac{R_3 * R_2}{R_1 + R_2 + R_3}$$

$$R_A = \frac{450\Omega * 810\Omega}{450\Omega + 810\Omega + 190\Omega} = \frac{364500\Omega}{1450\Omega} = 251.3793\Omega$$
$$R_B = \frac{450\Omega * 190\Omega}{450\Omega + 810\Omega + 190\Omega} = \frac{85500\Omega}{1450\Omega} = 58.9655\Omega$$
$$R_C = \frac{190\Omega * 810\Omega}{450\Omega + 810\Omega + 190\Omega} = \frac{153900\Omega}{1450\Omega} = 106.1379\Omega$$



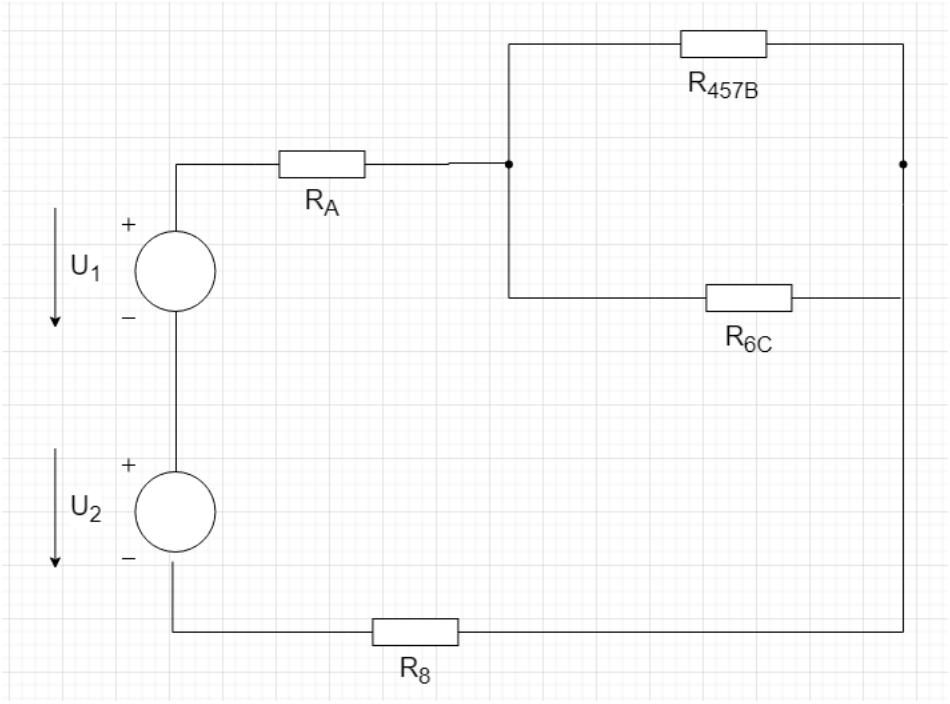
Obrázek 3: *Seriove zapojeni* $R_{45} + R_B$ *a* $R_6 + R_C$

$$R_{45B} = R_{45} + R_B$$

$$R_{6C} = R_6 + R_C$$

$$R_{45B} = 110\Omega + 58.9655\Omega = 168.9655\Omega$$

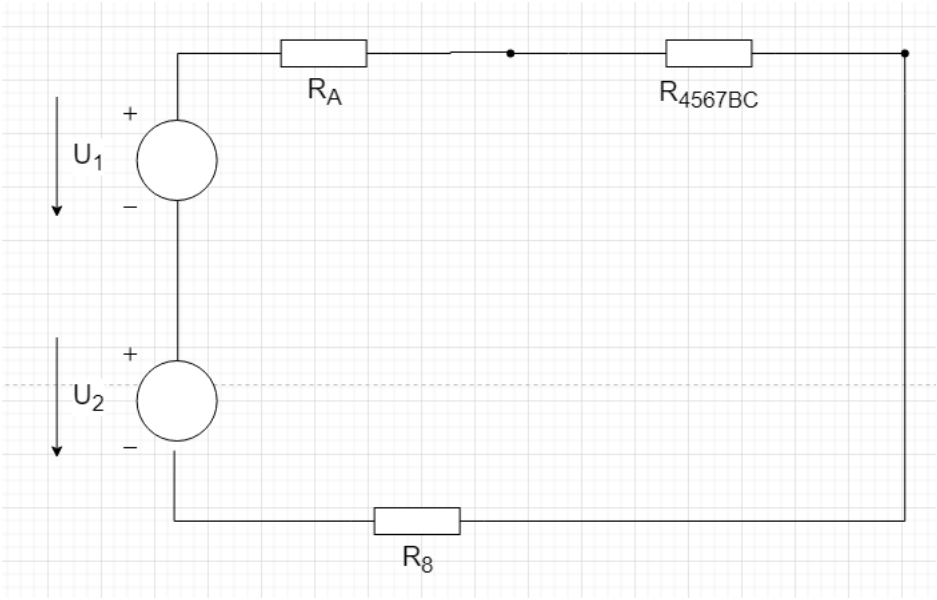
$$R_{6C} = 720\Omega + 106.1379\Omega = 826.1379\Omega$$



Obrázek 4: *Zjednoduseni* R_{45B} *s* R_7

$$R_{457B} = R_{45B} + R_7$$

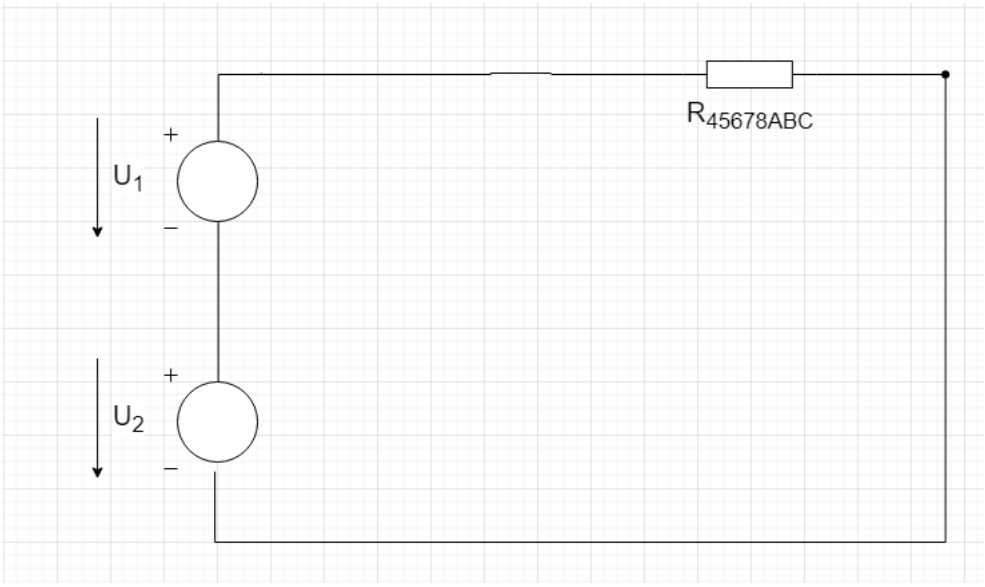
$$R_{457B} = 168.9655\Omega + 260\Omega = 428.9655\Omega$$



Obrázek 5: *Paralelne zapojene R_{457B} a R_{6C}*

$$R_{4576BC} = \frac{R_{457B} * R_{6C}}{R_{457B} + R_{6C}}$$

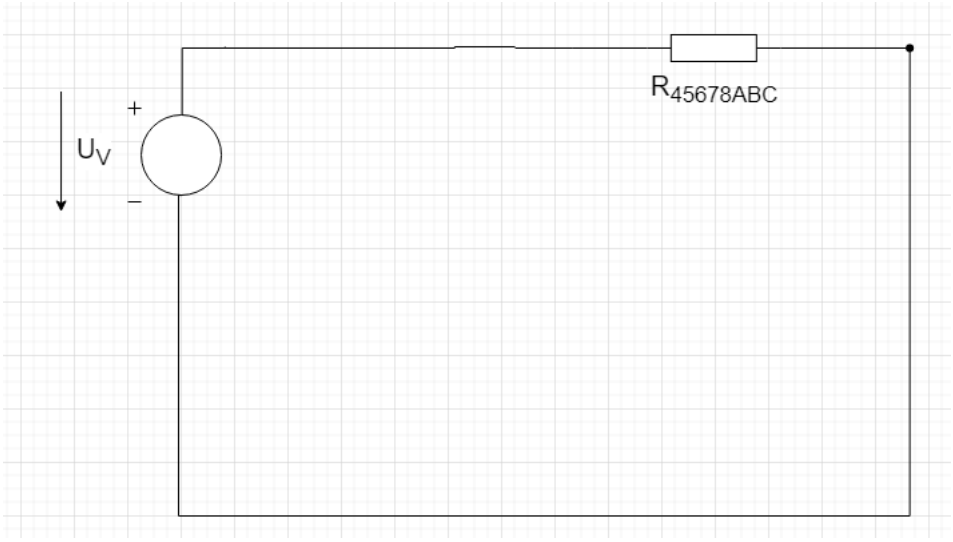
$$R_{4576BC} = \frac{428.9655\Omega * 826.1379\Omega}{428.9655\Omega + 826.1379\Omega} = \frac{354384,6573\Omega}{1255.1034\Omega} = 282.3549\Omega$$



Obrázek 6: *Zjednoduseni do R_{ekv}*

$$R_{ekv} = R_{45678ABC} = R_A + R_{4567BC} + R_8$$

$$R_{ekv} = R_{45678ABC} = 251.3793\Omega + 282.3549\Omega + 180\Omega = 713.7342\Omega$$



Obrázek 7: Vyhledávání U a I

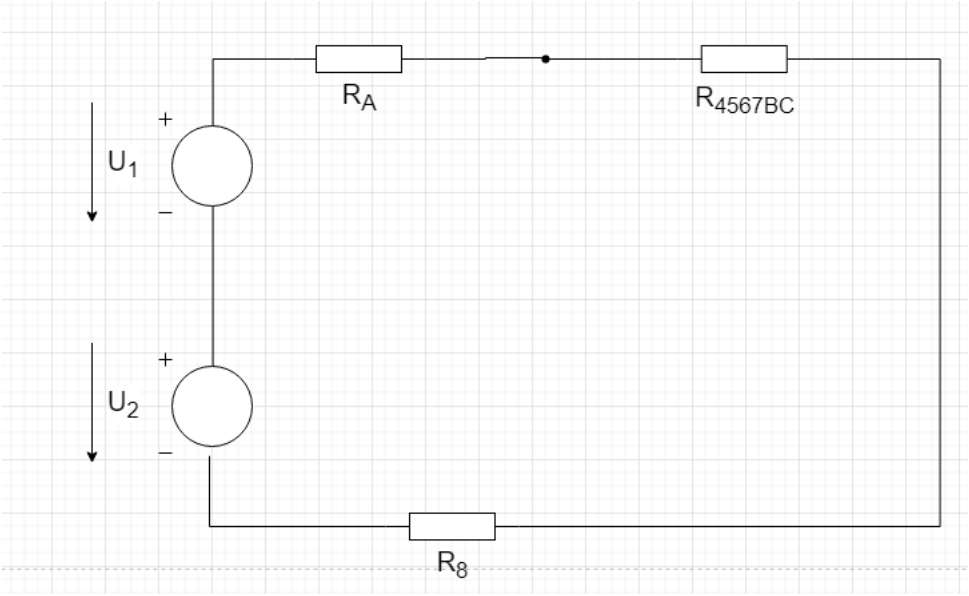
S R_{ekv} nyní můžeme vypočítat celkový proud v obvodu Ohmovým zákonem: $I = \frac{U}{R_{ekv}}$

$$U_V = U_1 + U_2$$

$$U_V = 100\text{V} + 80\text{V} = 180\text{V}$$

$$I = \frac{180\text{V}}{713.7342\Omega} = 0.2521\text{A}$$

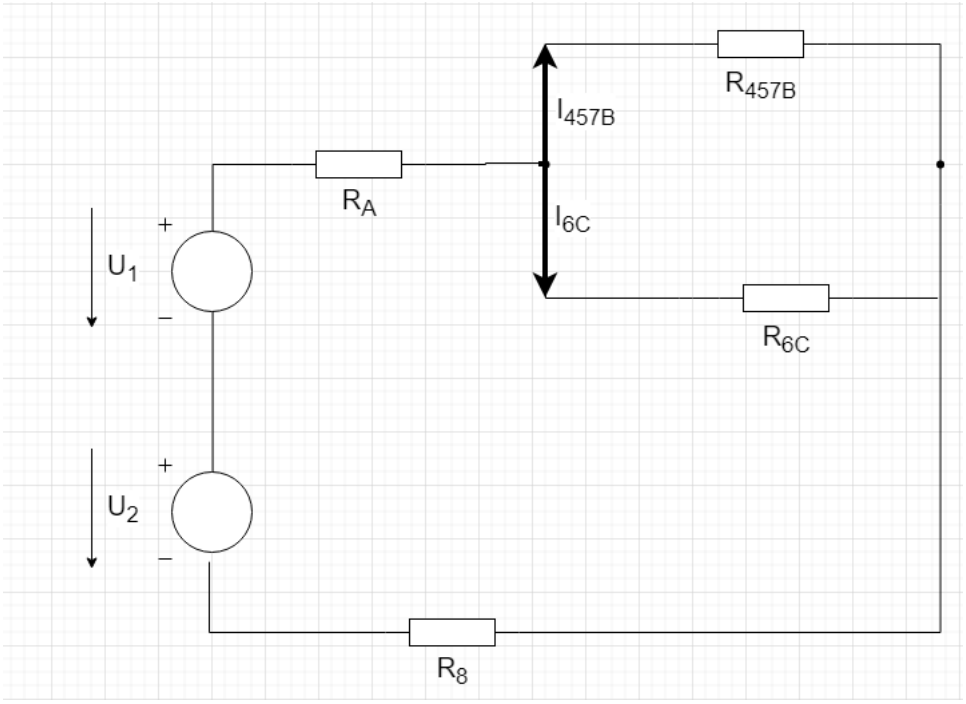
Výpočet U_{R7}



Rozložíme zpětně obvod

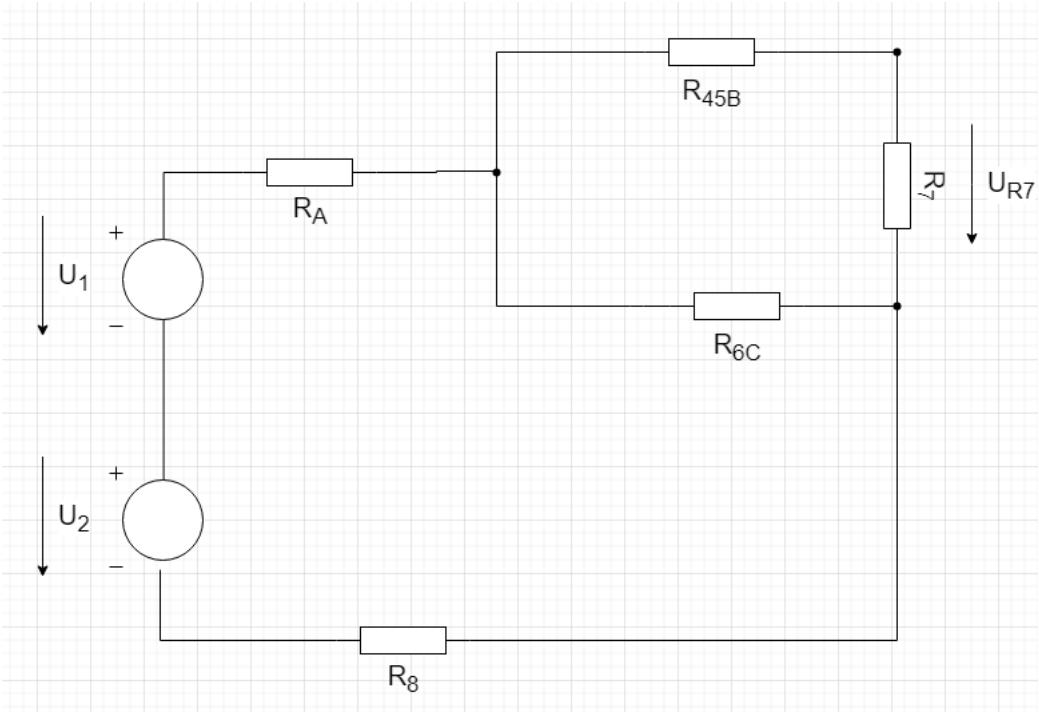
$$U_{R4576BC} = I * R_{4576BC}$$

$$U_{R4576BC} = 0.2521\text{A} * 282.3549\Omega = 71.1816\text{V}$$



$$I_{457B} = \frac{U_{R4576BC}}{R_{457B}}$$

$$I_{457B} = \frac{71.1816\text{V}}{428.9655\Omega} = 0.1659\text{A}$$



$$I_{457B} = I_{R7} = 0.1659\text{A}$$

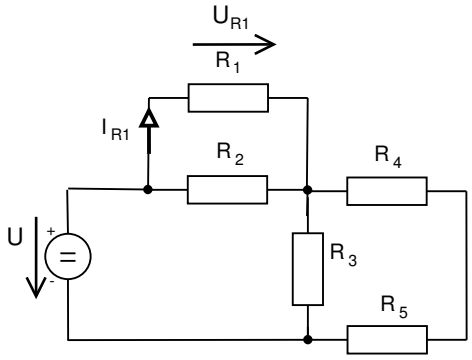
$$U_{R7} = I_{R7} * R_7$$

$$U_{R7} = 0.1659\text{A} * 260\Omega = 43.1438\text{V}$$

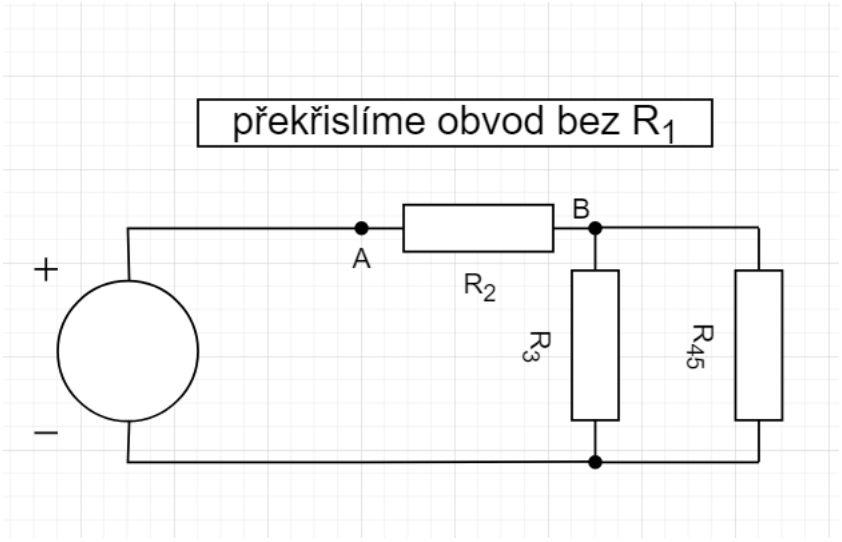
Příklad 2

Stanovte napětí U_{R1} a proud I_{R1}. Použijte metodu Théveninovy věty.

sk.	U [V]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [Ω]
G	180	250	315	615	180	460



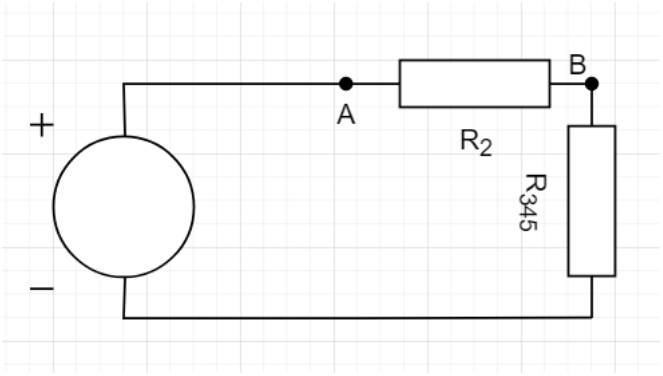
Výpočet Re



Obrázek 8: Seriove zapojeni R_4 *a* R_5

$$R_{45} = R_4 + R_5$$

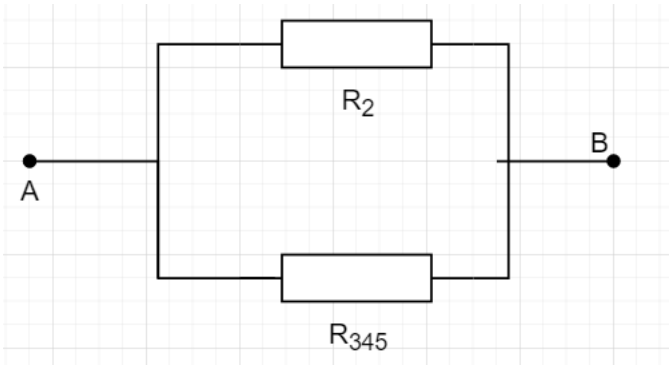
$$R_{45} = 180\Omega + 460\Omega = 640\Omega$$



Obrázek 9: Paralelne zapojeni R_{45} *a* R_3

$$R_{345} = \frac{R_{45} * R_3}{R_{45} + R_3}$$

$$R_{345} = \frac{640\Omega * 615\Omega}{640\Omega + 615\Omega} = \frac{393600\Omega}{1255\Omega} = 313.6254\Omega$$

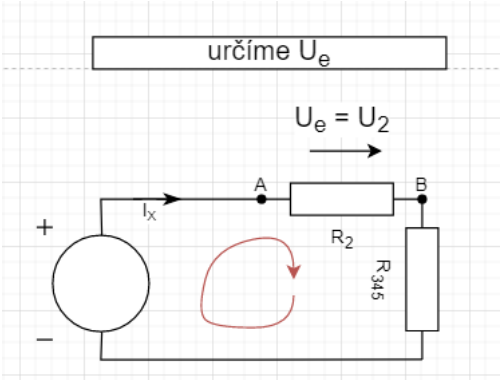


Obrázek 10: Paralelne zapojeni R_{345} *a* R_2

$$R_e = R_{2345} = \frac{R_{345} * R_2}{R_{345} + R_2}$$

$$R_e = \frac{313.6254\Omega * 315\Omega}{313.6254\Omega + 315\Omega} = \frac{98792.001\Omega}{628.6254\Omega} = 157.1556\Omega$$

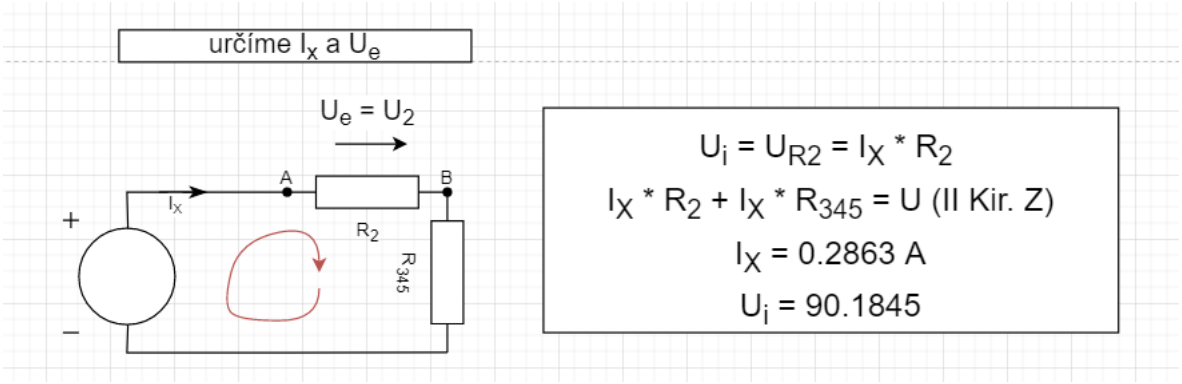
Výpočet Ue



Vypočítáme pomocí napětoví děliče

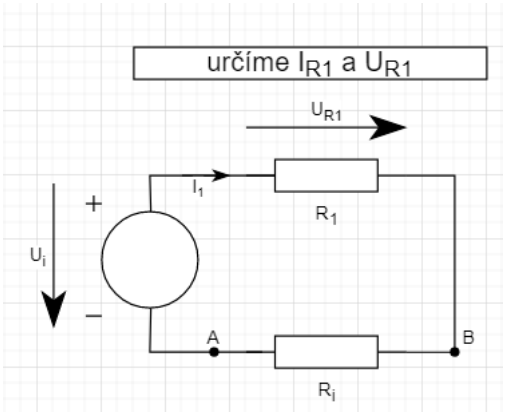
$$U_e = U * \frac{R_2}{R_2 + R_{345}}$$

$$U_e = 180\text{V} * \frac{315\Omega}{315\Omega + 313.6254\Omega} = 180\text{V} * \frac{315\Omega}{628.6254\Omega} = 90.1968\text{V}$$



Obrázek 11: Druhy zpusob reseni

Výpočet U_{R1} *a* I_{R1}



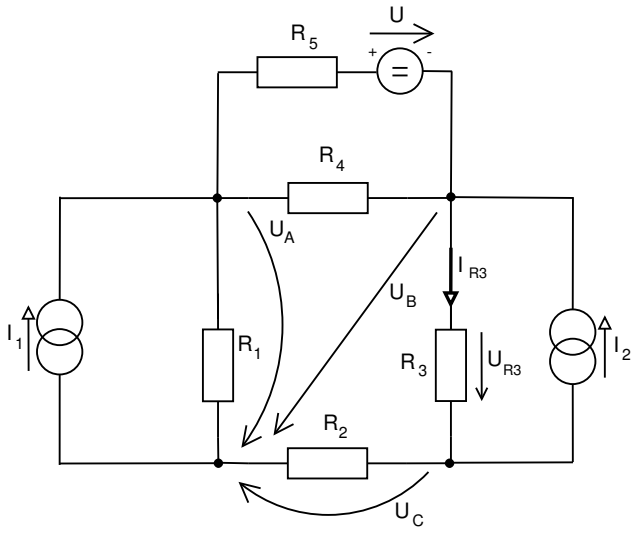
$$I_{R1} = \frac{U_i}{R_1 + R_e} = \frac{90.1968\text{V}}{250\Omega + 157.1556\Omega} = \frac{90.1968\text{V}}{407.1556\Omega} = 0.2215290665288651\text{A}$$

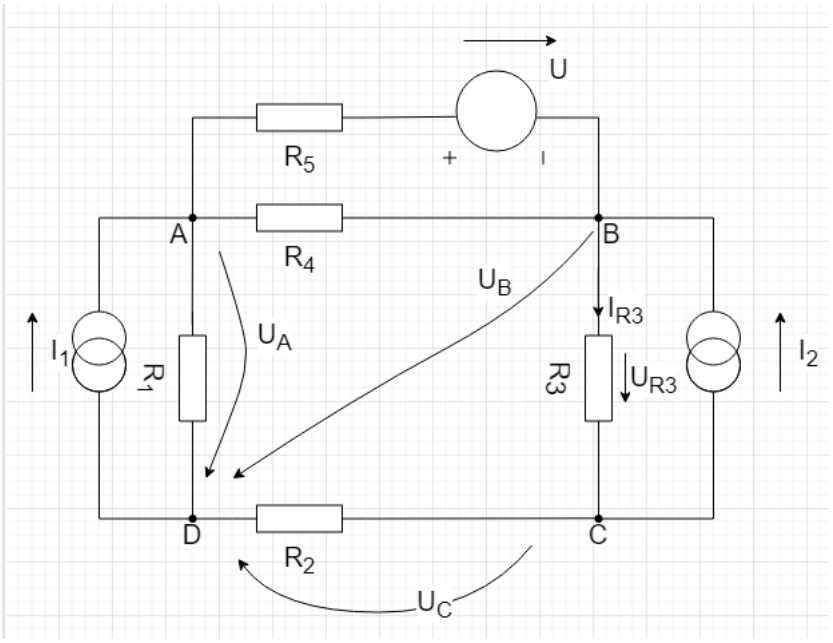
$$U_{R1} = R_1 * I_{R1} = 250\Omega * 0.2215290665288651\text{A} = 55.3822\text{V}$$

Příklad 3

Stanovte napětí U_{R3} a proud I_{R3}. Použijte metodu uzlových napětí (U_A, U_B, U_C).

sk.	U [V]	I_1 [A]	I_2 [A]	R_1 [Ω]	R_2 [Ω]	R_3 [Ω]	R_4 [Ω]	R_5 [Ω]
A	120	0.9	0.7	53	49	65	39	32





$$\phi D = 0$$

$$I_1 - I_{R1} - I_{R4} - I_{R5} = 0$$
$$I_{R5} + I_{R4} + I_2 - I_{R3} = 0$$
$$I_{R3} - I_2 - I_{R2} = 0$$

$$I_{R1} = \frac{(\phi D + \phi A)}{R_1} = \frac{\phi A}{R_1}$$
$$I_{R2} = \frac{(\phi C - \phi D)}{R_2} = \frac{\phi C}{R_2}$$
$$I_{R3} = \frac{(\phi B - \phi C)}{R_3}$$
$$I_{R4} = \frac{(\phi A - \phi B)}{R_4}$$
$$I_{R5} = \frac{(\phi A - \phi B - U)}{R_5}$$

$$I_1 - \frac{\phi A}{R_1} - \frac{\phi A - \phi B}{R_4} - \frac{(\phi A - \phi B - U)}{R_5} = 0$$
$$I_2 + \frac{\phi A - \phi B}{R_4} + \frac{(\phi A - \phi B - U)}{R_5} - \frac{(\phi B - \phi C)}{R_3} = 0$$
$$\frac{(\phi B - \phi C)}{R_3} - I_2 - \frac{\phi C}{R_2} = 0$$

$$I_1 - \frac{\phi A}{R_1} - \frac{\phi A}{R_4} + \frac{\phi B}{R_4} - \frac{\phi A}{R_5} + \frac{\phi B}{R_5} + \frac{U}{R_5} = 0$$
$$I_2 + \frac{\phi A}{R_4} - \frac{\phi B}{R_4} + \frac{\phi A}{R_5} - \frac{\phi B}{R_5} - \frac{U}{R_5} - \frac{\phi B}{R_3} + \frac{\phi C}{R_3} = 0$$
$$\frac{\phi B}{R_3} - \frac{\phi C}{R_3} - I_2 - \frac{\phi C}{R_2} = 0$$

$$-\phi A * (\frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_5}) + \phi B * (\frac{1}{R_4} + \frac{1}{R_5}) = -I_1 - \frac{U}{R_5}$$
$$\phi A * (\frac{1}{R_4} + \frac{1}{R_5}) - \phi B * (\frac{1}{R_4} + \frac{1}{R_5} + \frac{1}{R_3}) + \phi C * (\frac{1}{R_3}) = -I_2 + \frac{U}{R_5}$$
$$\phi B * (\frac{1}{R_3}) - \phi C * (\frac{1}{R_3} + \frac{1}{R_2}) = I_2$$

$$-\phi A * (\frac{1}{53\Omega} + \frac{1}{39\Omega} + \frac{1}{32\Omega}) + \phi B * (\frac{1}{39\Omega} + \frac{1}{32\Omega}) = -0.9\text{A} - \frac{120\text{V}}{32\Omega}$$
$$\phi A * (\frac{1}{39\Omega} + \frac{1}{32\Omega}) - \phi B * (\frac{1}{39\Omega} + \frac{1}{32\Omega} + \frac{1}{65\Omega}) + \phi C * (\frac{1}{65\Omega}) = -0.7\text{A} + \frac{120\text{V}}{32\Omega}$$
$$\phi B * (\frac{1}{65\Omega}) - \phi C * (\frac{1}{65\Omega} + \frac{1}{49\Omega}) = 0.7\text{A}$$

$$-\phi A * (0.0757) + \phi B * (0.0568) = -4.65$$

$$\phi A * (0.0568) - \phi B * (0.0722) + \phi C * (0.0153) = 3.05$$

$$\phi B * (0.0153) - \phi C * (0.0357) = 0.7$$

$$A = \begin{bmatrix} -0.0757 & 0.0568 & 0 \\ 0.0568 & -0.0722 & 0.0153 \\ 0 & 0.0153 & -0.0357 \end{bmatrix}$$

$$I = \begin{bmatrix} -4.65 \\ 3.05 \\ 0.7 \end{bmatrix}$$

$$x = A^{-1} * I$$

```
octave:2> A = [-0.0757 0.0568 0; 0.0568 -0.0722 0.0153; 0 0.0153 -0.0357];
octave:3> I = [-4.65; 3.05; 0.7];
octave:4> x = A^(-1) * I
x =

   65.9577
    6.0387
  -17.0198
```

$$\phi A = 65.9577\text{V}$$
$$\phi B = 6.0387\text{V}$$
$$\phi C = -17.0198\text{V}$$

$$I_{R3} = \frac{(\phi B - \phi C)}{R_3} = \frac{(6.0387\text{V} + 17.0198\text{V})}{65\Omega} = 0.3547\text{A}$$

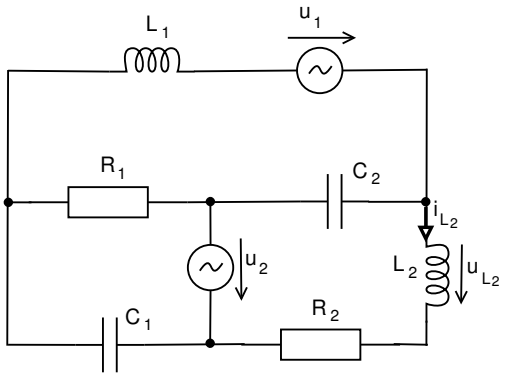
$$U_{R3} = I_{R3} * R_3 = 0.3547\text{A} * 65\Omega = 23.0587\text{V}$$

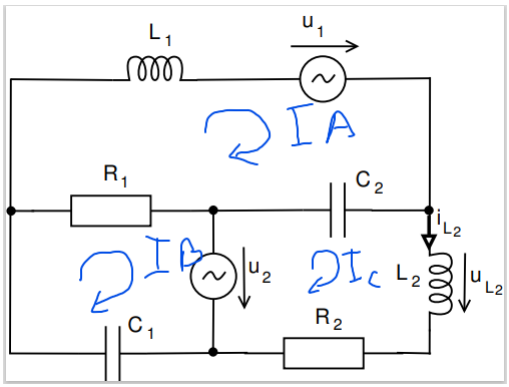
Příklad 4

Pro napájecí napětí platí: $u_1 = U_1 \cdot \sin(2\pi ft)$, $u_2 = U_2 \cdot \sin(2\pi ft)$.
Ve vztahu pro napětí $u_{L_2} = U_{L_2} \cdot \sin(2\pi ft + \varphi_{L_2})$ určete $|U_{L_2}|$ a φ_{L_2}. Použijte metodu smyčkových proudů.

Pozn: Pomocné směry šipek napájecích zdrojů platí pro speciální časový okamžik ($t = \frac{\pi}{2\omega}$).

sk.	U_1 [V]	U_2 [V]	R_1 [Ω]	R_2 [Ω]	L_1 [mH]	L_2 [mH]	C_1 [µF]	C_2 [µF]	f [Hz]
C	3	4	10	13	220	70	230	85	75





Impedance pro cívku a kondenzátor:

$$\omega = 2\pi f$$
$$Z_C = \frac{-j}{\omega C}$$
$$Z_L = j\omega L$$

$$I_A : U_{L1} + U_1 + U_{C2} + U_{R1} = 0$$
$$I_B : U_{R1} + U_2 + U_{C1} = 0$$
$$I_C : U_{C2} + U_{L2} + U_{R2} - U_2 = 0$$

$$
\begin{aligned}
&I_A : I_A * (Z_{L1} + Z_{C2} + R1) - I_B * R_1 - I_C * Z_{C2} = -U_1 \\
&I_B : -I_A * (R_1) + I_B * (R_1 + Z_{C1}) + 0 = -U_2 \\
&I_C : -I_A * Z_{C2} + 0 + I_C * (Z_{C2} + Z_{L2} + R2) = U_2
\end{aligned}
$$

Matice pro proudové smyčky:

$$
\begin{pmatrix} Z_{L1} + Z_{C2} + R_1 & -R_1 & -Z_{C2} \\ -R_1 & R_1 + Z_{C1} & 0 \\ -Z_{C2} & 0 & Z_{C2} + R_2 + Z_{L2} \end{pmatrix} * \begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix} = \begin{pmatrix} -U_1 \\ -U_2 \\ U_2 \end{pmatrix}
$$

$$
\begin{aligned}
\omega &= 2\pi f \\
Z_{C1} &= \frac{-j}{\omega C_1} \\
Z_{C2} &= \frac{-j}{\omega C_2} \\
Z_{L1} &= j\omega L_1 \\
Z_{L2} &= j\omega L_2
\end{aligned}
$$

$$
\begin{pmatrix} 10 + 78.707j & -10 & 24.9655j \\ -10 & 10 - 9.2264j & 0 \\ 24.9655j & 0 & 13 + 8.0212j \end{pmatrix} * \begin{pmatrix} I_A \\ I_B \\ I_C \end{pmatrix} = \begin{pmatrix} -3 \\ -4 \\ 4 \end{pmatrix}
$$

$$
\begin{aligned}
I_A &= (-0.171241 + 0.035563j)\text{A} \\
I_B &= (-0.3263 - 0.2655j)\text{A} \\
I_C &= (0.419275 + 0.070155j)\text{A}
\end{aligned}
$$

Výpočet

$$
\begin{aligned}
I_C &= I_{L2} \\
U_{L2} &= I_{L2} * Z_{L2}
\end{aligned}
$$

$$
\varphi_{L_2} = arctan(\frac{imag(U_{L2})}{real(U_{L2})}) * \frac{\pi}{180} + \pi
$$

$$
|U_{L2}| = \sqrt{real(U_{L2})^2 + imag(U_{L2})^2}
$$

$$I_C = I_{L2} = (0.419275 + 0.070155j)\text{A}$$

$$U_{L2} = (0.419275 + 0.070155j)\text{A} * 32.9867j = (-2.3142 + 13.8305j)\text{V}$$

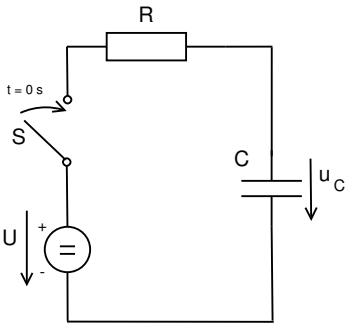
$$\varphi_{L_2} = (arctan(\frac{13.8305}{-2.3142}) * \frac{\pi}{180} + \pi) * \frac{180}{\pi} = 178.59499^\circ$$

$$|U_{L2}| = \sqrt{(-2.3142)^2 + 13.8305^2} = 14.02278\text{V} = 14.023\text{V}$$

Příklad 5

V obvodu na obrázku níže v čase $t = 0$ [s] sepne spínač S. Sestavte diferenciální rovnici popisující chování obvodu na obrázku, dále ji upravte dosazením hodnot parametrů. Vypočítejte analytické řešení $u_C = f(t)$. Proveďte kontrolu výpočtu dosazením do sestavené diferenciální rovnice.

sk.	U [V]	R [Ω]	C [F]	$u_C(0)$ [V]
G	10	50	25	7



Sestavení diferenciální rovnice

Sestavíme rovnici pro napětí na kondenzátoru U'$_C$:

$$U'_C = \frac{i}{C}$$

Napětí na kondenzátoru si můžeme vyjádřit pomocí 2. Kirchhoffova zákona:

$$\begin{aligned} U &= U_R + U_C \\ U_C &= U - U_R \\ i &= \frac{U_R}{R} \end{aligned}$$

Vzniklou diferenciální rovnici upravíme:

$$\begin{aligned} U'_C &= \frac{U_R}{R * C} \\ U'_C &= \frac{U - U_C}{R * C} \end{aligned}$$

Obyčejní diff. rovnice:

$$\begin{aligned} U'_C + \frac{U - U_C}{R * C} &= 0 \\ R * C * U'_C + U - U_C &= 0 \\ 1250 * U'_C + 10 - U_C &= 0 \\ 1250 * U'_C - U_C &= -10 \end{aligned}$$

$$U_C(t) = K(t) * e^{\lambda * t}$$

$$\begin{aligned} 1250\lambda - 1 &= 0 \\ 1250\lambda &= 1 \\ \lambda &= \frac{1}{1250} = 0.0008 \end{aligned}$$

$$\begin{aligned} U_C(t) &= K(t) * e^{\lambda * t} \\ U_C(t) &= K(t) * e^{0.0008 * t} \\ U_C(t)' &= K(t)' * e^{0.0008 * t} + 0.0008 * K(t) * e^{0.0008 * t} \end{aligned}$$

Dosadíme do naší diferenciální rovnice:

$$\begin{aligned} 1250(K(t)' * e^{0.0008 * t} + 0.0008 * K(t) * e^{0.0008 * t}) - K(t) * e^{0.0008 * t} &= -10 \\ 1250K(t)' * e^{0.0008 * t} + K(t) * e^{0.0008 * t} - K(t) * e^{0.0008 * t} &= -10 \\ 1250K(t)' * e^{0.0008 * t} &= -10 \\ K(t)' * e^{0.0008 * t} &= \frac{-10}{1250} \\ K(t)' * e^{0.0008 * t} &= \frac{-1}{125} \\ K(t)' &= \frac{-1}{125} * e^{-0.0008 * t} \end{aligned}$$

Máme $K(t)$. Musíme ještě zintegrovat:

$$\begin{aligned} K(t) &= \int \frac{-1}{125} * e^{-0.0008 * t} dt \\ K(t) &= \frac{10}{e^{0.0008 * t}} + C \end{aligned}$$

Nyní už máme co potřebujeme. Dosadíme do analytické rovnice:

$$\begin{aligned} U_C(t) &= K(t) * e^{\lambda * t} \\ U_C(t) &= (\frac{10}{e^{0.0008 * t}} + C) * e^{0.0008 * t} \\ U_C(t) &= 10 + C * e^{0.0008 * t} \end{aligned}$$

Vypočítáme C dle počáteční podmínky v čase $t = 0$:

$$\begin{aligned} U_c(0) &= 10 + C * e^{0.0008 * 0} \\ 7 &= 10 + C \\ C &= -3 \end{aligned}$$

Konečná rovnice má tento tvar:

$$U_C(t) = 10 + (-3) * e^{0.0008 * t}$$

Kontrola

$$U_C(t)' = (\frac{-1}{125} * e^{-0.0008*t}) * e^{0.0008*t} + 0.0008 * (\frac{10}{e^{0.0008*t}} - 3) * e^{0.0008*t}$$
$$U_C(t) = 10 + (-3) * e^{0.0008*t}$$

$$1250 * U_C' - U_C = -10$$
$$1250 * ((\frac{-1}{125} * e^{-0.0008*t}) * e^{0.0008*t} + 0.0008 * (\frac{10}{e^{0.0008*t}} - 3) * e^{0.0008*t}) - (10 + (-3) * e^{0.0008*t}) = -10$$
$$1250 * (\frac{-1}{125} + \frac{1}{125} - \frac{3}{1250} * e^{0.0008*t}) - (10 + (-3) * e^{0.0008*t}) = -10$$
$$1250 * (-\frac{3}{1250} * e^{0.0008*t}) - (10 + (-3) * e^{0.0008*t}) = -10$$
$$-3 * e^{0.0008*t} - 10 + 3 * e^{0.0008*t} = -10$$
$$-10 = -10$$

Shrnutí výsledků

Příklad	**Skupina**	**Výsledky**	
1	C	$U_{R7} = 43.1438\text{V}$	$I_{R7} = 0.1659\text{A}$
2	G	$U_{R1} = 55.3822\text{V}$	$I_{R1} = 0.2215\text{A}$
3	A	$U_{R3} = 23.0587\text{V}$	$I_{R3} = 0.3547\text{A}$
4	C	$\lvert U_{L_2}\rvert = 14.023\text{V}$	$\varphi_{L_2} = 178.59499°$
5	G	$u_C = 10 + (-3) * e^{0.0008*t}$	