Persamaan Diferensial Parsial dan Metode Numerik

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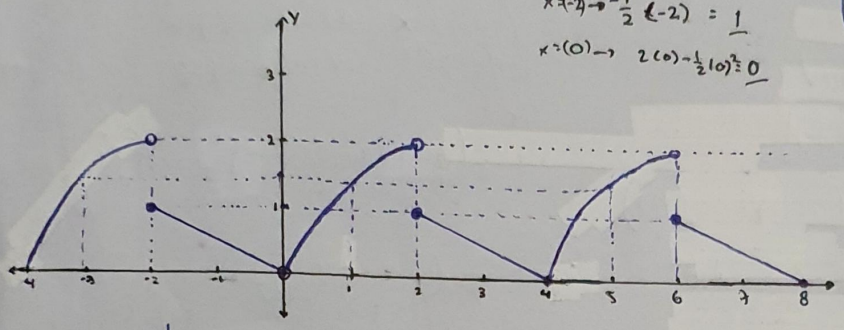
* Untuk m = 1 → tulis tangan ☺

NO.1

1.) $$f(x) = \begin{cases} -\frac{1}{2}x, & -2 \le x < 0; \\ 2x - \frac{1}{2}x^2, & 0 \le x < 2; \end{cases} \qquad f(x+4) = f(x)$$

a.) function graph sketch (3 periods)

$x=(2) \Rightarrow 2(2) - \frac{1}{2}(2) = 2$
$x=(0) \Rightarrow -\frac{1}{2}(0) = 0$
$x=(-2) \rightarrow -\frac{1}{2}(-2) = 1$
$x=(0) \rightarrow 2(0) - \frac{1}{2}(0)^2 = 0$



Repeat periodically
$f(x+4) = f(x)$

b.) $T = 4$, $L = \frac{T}{2} = 2$

Fourier series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right)$$

$$f(x) = \frac{11}{12} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{2}\right) + b_n \sin\left(\frac{n\pi x}{2}\right) \right)$$

$$f(x) = 1{,}833 + \sum_{n=1}^{\infty} \left(a_n \cdot \cos\left(\frac{n\pi x}{2}\right) + b_n \sin\left(\frac{n\pi x}{2}\right) \right)$$

Partial sum (order 1)

input a_0

$$S_1 = \frac{11}{12} - \frac{6}{\pi^2}\cos\left(\frac{\pi x}{2}\right) + \frac{(8+\pi^2)}{\pi^3}\sin\left(\frac{\pi x}{2}\right)$$

Fourier Series → $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{2}\right) + b_n \sin\left(\frac{n\pi x}{2}\right) \right)$$

c.) $e_m(x) = f(x) - S_m(x)$ Describe how the fourier series seems to be converging untuk range $-2 \le x < 2$

→ Deret fourier akan converging sesuai dengan $f(x)$ pada range $-2 < x < 2$, sedangkan pada titik $x = -2$, deret fourier akan converging ke nilai $y = 1{,}5$,

didapat dari = $$\frac{\lim_{x \to -2^-} f(x) + \lim_{x \to -2^+} f(x)}{2}$$

akan dijelaskan bagaimana cara dapatnya

* mengulang secara periodik → $f(x+4) = f(x)$

d.) plot $|e_m(x)| = |f(x) - S_m(x)|$ versus x for $0 \le x \le 2$ for several values of m

for m = 1

$$S_m = S_1 = a_0 + \left(a_1 \cos\left(\frac{\pi x}{2}\right) + b_1 \sin\left(\frac{\pi x}{2}\right) \right)$$

$$a_n = \frac{1}{L}\int_a^b f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$b_n = \frac{1}{L}\int_a^b f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$a_0 = \frac{1}{2}\left(\int_{-2}^{0} \left(-\frac{1}{2}x\right)\cos\left(\frac{0\pi x}{2}\right)dx + \int_0^2 \left(2x - \frac{x^2}{2}\right)\cos\left(\frac{0\pi x}{2}\right)dx \right)$$

$$a_0 = \frac{1}{2}\left(-\frac{1}{2}\int_{-2}^{0} x\,(1)\,dx + \int_0^2 \left(2x - \frac{x^2}{2}\right)(1)\,dx \right)$$

$$a_0 = \frac{1}{2}\left(-\frac{1}{2}\left[\frac{x^2}{2}\right]_{-2}^{0} + \left[x^2 - \frac{x^3}{6}\right]_0^2 \right)$$

$$a_0 = \frac{1}{2}\left(-\frac{1}{2}\left(-\frac{(-2)^2}{2}\right) + \left(2^2 - \frac{2^3}{6}\right) \right)$$

$$a_0 = \frac{1}{2}\left(1 + 4 - \frac{4}{3}\right) = \frac{1}{2}\left(\frac{15}{3} - \frac{4}{3}\right)$$

$$a_0 = \frac{1}{2}\left(\frac{11}{3}\right) = \frac{11}{6}$$

$$a_0 = \frac{11}{6} \approx 1{,}833$$

$$a_1 = \frac{1}{2}\left(\int_{-2}^{0}\left(-\frac{1}{2}x\cos\left(\frac{1\cdot\pi x}{2}\right)\right)dx + \int_{0}^{2}\left(\left(2x-\frac{1}{2}x^2\right)\cos\left(\frac{1\pi x}{2}\right)\right)dx\right)$$

$$a_1 = \frac{1}{2}\left(-\frac{1}{2}\int_{-2}^{0}\left(x\cos\left(\frac{\pi x}{2}\right)\right)dx + \int_{0}^{2}\left(\left(2x-\frac{1}{2}x^2\right)\cos\left(\frac{\pi x}{2}\right)\right)dx\right)$$

$$\int_{-2}^{0} x\cos\left(\frac{\pi x}{2}\right)dx = \frac{2x}{\pi}\sin\left(\frac{\pi x}{2}\right)\Big]_{-2}^{0} - \frac{2}{\pi}\int_{-2}^{0}\sin\left(\frac{\pi x}{2}\right)dx$$

$u = x \qquad dv = \cos\left(\frac{\pi x}{2}\right)dx$

$du = dx \qquad v = \sin\left(\frac{\pi x}{2}\right)\cdot\left(\frac{2}{\pi}\right)$

$$\int_{-2}^{0} x\cos\left(\frac{\pi x}{2}\right)dx = \frac{2x}{\pi}\sin\left(\frac{\pi x}{2}\right)\Big]_{-2}^{0} - \frac{2}{\pi}\left(\frac{-2}{\pi}\cos\left(\frac{\pi x}{2}\right)\Big]_{-2}^{0}\right)$$

$$\int_{-2}^{0} x\cos\left(\frac{\pi x}{2}\right)dx = \frac{2x}{\pi}\sin\left(\frac{\pi x}{2}\right)\Big]_{-2}^{0} + \frac{4}{\pi^2}\cos\left(\frac{\pi x}{2}\right)\Big]_{-2}^{0}$$

$$\int_{-2}^{0} x\cos\left(\frac{\pi x}{2}\right)dx = \frac{2}{\pi}\left(0\sin(0) - (-2)\sin(-\pi)\right) + \frac{4}{\pi^2}\left(\cos(0) - \cos(-\pi)\right)$$

$$\int_{-2}^{0} x\cos\left(\frac{\pi x}{2}\right)dx = 0 + \frac{4}{\pi^2}\left(1-(-1)\right) = \frac{4}{\pi^2}(2) = \frac{8}{\pi^2} \simeq 0{,}810$$

$$\int_{0}^{2}\left(\left(2x-\frac{1}{2}x^2\right)\cos\left(\frac{\pi x}{2}\right)\right)dx = \frac{2}{\pi}\left(2x-\frac{1}{2}x^2\right)\sin\left(\frac{\pi x}{2}\Big]_{0}^{2} - \frac{2}{\pi}\int_{0}^{2}(2-x)\sin\left(\frac{\pi x}{2}\right)dx\right) \quad (=0)$$

$u = 2x - \frac{1}{2}x^2 \qquad dv = \cos\left(\frac{\pi x}{2}\right)$

$du = (2-x)dx \qquad v = \sin\left(\frac{\pi x}{2}\right)\cdot\frac{2}{\pi}$

$$= -\frac{2}{\pi}\int_{0}^{2}(2-x)\sin\left(\frac{\pi x}{2}\right)dx$$

$u = 2-x \qquad dv = \sin\left(\frac{\pi x}{2}\right)dx$

$du = -dx \qquad v = -\frac{2}{\pi}\cos\left(\frac{\pi x}{2}\right)$

$$= -\frac{2}{\pi}\left(-\frac{2}{\pi}(2-x)\cos\left(\frac{\pi x}{2}\right)\Big]_{0}^{2} - \frac{2}{\pi}\int_{0}^{2}\cos\left(\frac{\pi x}{2}\right)dx\right) \quad (=0)$$

1 período

$$= \frac{4}{\pi^2}\left((2-x)\cos\left(\frac{\pi x}{2}\right)\Big]_{0}^{2}\right)$$

$$= \frac{4}{\pi^2}\left((2-2)\cos(\pi x) - (2)(\cos(0))\right)$$

$$= \frac{4}{\pi^2}(-2)$$

$$= \frac{-8}{\pi^2} \simeq -0{,}810$$

$$a_1 = \frac{1}{2}\left(-\frac{1}{2}\left(\frac{8}{\pi^2}\right) + -\left(\frac{8}{\pi^2}\right)\right)$$

$$a_1 = \frac{-2}{\pi^2} - \frac{4}{\pi^2} = \frac{-6}{\pi^2} \simeq -0{,}1608$$

$$b_1 = \frac{1}{2}\left(\int_{-2}^{0}\left(-\frac{1}{2}x\sin\left(\frac{\pi x}{2}\right)\right)dx + \int_{0}^{2}\left(\left(2x-\frac{1}{2}x^2\right)\sin\left(\frac{\pi x}{2}\right)\right)dx\right)$$

$$b_1 = \frac{1}{2}\left(-\frac{1}{2}\int_{-2}^{0} x\sin\left(\frac{\pi x}{2}\right)dx + \int_{0}^{2}\left(\left(2x-\frac{1}{2}x^2\right)\sin\left(\frac{\pi x}{2}\right)\right)dx\right)$$

$$\int_{-2}^{0} x\sin\left(\frac{\pi x}{2}\right)dx = \left(\frac{-2x}{\pi}\cos\left(\frac{\pi}{2}x\right)\right)\Big]_{-2}^{0} - \frac{-2}{\pi}\int_{-2}^{0}\cos\left(\frac{\pi}{2}x\right)dx \quad (=0)$$

$u = x \qquad dv = \sin\left(\frac{\pi x}{2}\right)dx$

$du = dx \qquad v = \frac{-2}{\pi}\cos\left(\frac{\pi}{2}x\right)$

$$= \frac{-2(0)}{\pi}\left(\cos\left(\frac{\pi}{2}\cdot 0\right)\right) - \frac{2}{\pi}(-2)\cos(-\pi)$$

$$= \frac{-4}{\pi}(-1)$$

$$= \frac{4}{\pi} = 1{,}273$$

$$\int_{0}^{2}\left(\left(2x-\frac{1}{2}x^2\right)\sin\left(\frac{\pi x}{2}\right)\right)dx$$

$u = 2x - \frac{1}{2}x^2 \qquad dv = \sin\left(\frac{\pi x}{2}\right)dx$

$du = 2-x \qquad v = -\frac{2}{\pi}\cos\left(\frac{\pi x}{2}\right)$

$$= \left(2x-\frac{1}{2}x^2\right)\left(-\frac{2}{\pi}\right)\left(\cos\frac{\pi x}{2}\right)\Big]_{0}^{2} + \frac{2}{\pi}\int_{0}^{2}(2-x)\cos\left(\frac{\pi x}{2}\right)dx$$

$$= \left(2\left(\frac{-2}{\pi}\right)(-1) - 0\right) + \frac{2}{\pi}\left(\frac{2}{\pi}(2-x)\sin\left(\frac{\pi x}{2}\right)\Big]_{0}^{2} - \frac{2}{\pi}\int_{0}^{2}\sin\left(\frac{\pi x}{2}\right)dx\right) \quad (\text{primer término} = 0)$$

$u = 2-x \qquad dv = \cos\left(\frac{\pi x}{2}\right)$

$du = -dx \qquad v = \frac{2}{\pi}\sin\left(\frac{\pi x}{2}\right)$

$$= \left(\frac{4}{\pi} + \frac{2}{\pi}\left(\frac{-2}{\pi}\cdot\frac{+2}{\pi}\cos\left(\frac{\pi x}{2}\right)\Big]_{0}^{2}\right)\right)$$

$$= \left(\frac{4}{\pi} + \frac{2}{\pi}\left(\frac{-4}{\pi^2}(-1-1)\right)\right)$$

$$= \left(\frac{4}{\pi} + \frac{2}{\pi}\left(\frac{8}{\pi^2}\right)\right) = \frac{4}{\pi} + \frac{16}{\pi^3} = \frac{16+4\pi^2}{\pi^3} \simeq 1{,}789$$

$$b_1 = \frac{1}{2}\left(\frac{-1}{2}\left(\frac{4}{\pi}\right) + \frac{16+4\pi^2}{\pi^3}\right) = \frac{1}{2}\left(\frac{-2}{\pi} + \frac{16+4\pi^2}{\pi^3}\right)$$

$$b_1 = \frac{-1}{\pi} + \frac{8+2\pi^2}{\pi^3} = \frac{8+2\pi^2-\pi^2}{\pi^3} = \frac{8+\pi^2}{\pi^3} \simeq 0{,}576$$

So, the $S_1 = \frac{11}{12} - \frac{6}{\pi^2}\cos\left(\frac{\pi x}{2}\right) + \frac{(8+\pi^2)}{\pi^3}\sin\left(\frac{\pi x}{2}\right)$ ↗ so $e_1(x) \leq$

The error will be

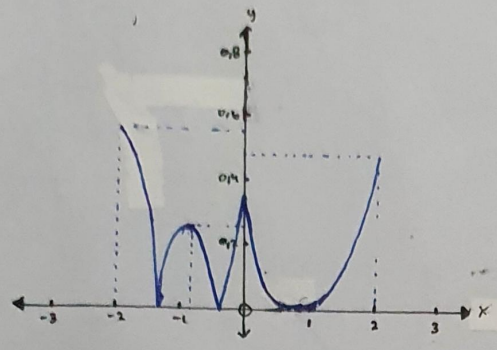
$\leftarrow |e_1(x)| = |f(x) - S_1(x)| = 0,524$ → the highest value how I get it, see ... (EXPLANATION)

↑ max

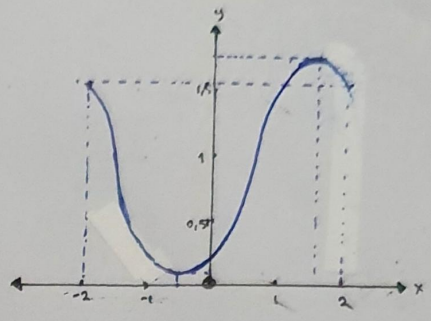
How the error graph looks like :

for $-2 \leq x \leq 2$ → then it will repeat periodically

1.) Grafik error, untuk e_1



2.) Grafik S_1



→ didapatkan dengan input $f(x)$, $S_1(x)$, dan $e_1(x)$ ke geogebra

e.) error untuk $m=1$ → $e_1 = 0,622$ → got from integrating $e_m(x)$ ($e_1(x)$) with trapezional rule $e_1 = \sqrt{\frac{1}{2L}\int_{-2}^{2}(f(x) - S_1(x))^2\,dx}$

untuk dapat error $\leq 0,01$ → $e_m \leq 0,01$, maka $m = 194$

↓ didapatkan dengan kode untuk menscan deret fourier.

saat $m=194$, $e_{194} = 0,009994377737544885$ → memenuhi syarat $e_m \leq 0,01$

↳ kalau pakainya mean squared error, kalau pakainya $|e_m(x)|$, maka :

berdasarkan mean squared error

agar $|e_m(x)| \leq 0,01$ → error max pada soal ini 0,5, what are a dwiana lim kiri dan lim kanan beda

↳ m = ga mungkin

untuk $m=2$, $m=4$, $m=8$, $m=16$, $m=32$, $m=64$, $m=128$, dan $m=194$ akan dibuat menggunakan matlab

(EXPLANATION)

$e_1(x) = 2(x) - \frac{1}{2}x^2 - \frac{11}{12} + \frac{6}{\pi^2}\cos\left(\frac{\pi x}{2}\right) - \frac{(8+\pi^2)}{\pi^3}\sin\left(\frac{\pi x}{2}\right)$

coba $e_1(-2) = 2(-2) - \frac{1}{2}(-2)^2 - \frac{11}{12} + \frac{6}{\pi^2}\cos\left(\frac{-2\pi}{2}\right) - \frac{8+\pi^2}{\pi^3}(\sin(\frac{-2\pi}{2})$

$e_1(-2) = 0,524$

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PR #1 PDPMN

No. 2 ! (n = 2 → dikerjakan manual, selebihnya matlab)

No. 3 !

$f(x) = x^3 - 5x^2 + 5x + 1 \quad 0 < x < 3$

a.) g(x) dan h(x)

Saat :

$x = 0 \rightarrow f(0) = 0^3 - 5(0)^2 + 5(0) + 1 = 1$

$x = 3 \rightarrow f(3) = 3^3 - 5(3)^2 + 5(3) + 1 = -2$

$f(x) = 0 \rightarrow 0 = x^3 - 5x^2 + 5x + 1$

$0 = x((x-5)x + 5) + 1$

↳ Saat $x_1 = -0,17009$

$x_2 = 1,6889$ → hanya x_2 yang berada di range $0 < x < 3$

$x_3 = 3,4812$

$f'(x) = 3x^2 - 10x + 5$

$0 = 3x^2 - 10x + 5$

$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \frac{-(-10) \pm \sqrt{10^2 - 4(3)(5)}}{2(3)}$

$x_{1,2} = \frac{10 \pm \sqrt{40}}{6}$

$x_1 = \frac{5 + \sqrt{10}}{3} = 2,721$ → tembah

$x_2 = \frac{5 - \sqrt{10}}{3} = 0,612$ → pancak

ada di range $0 < x < 3$

→ Saat $x = 0,612$

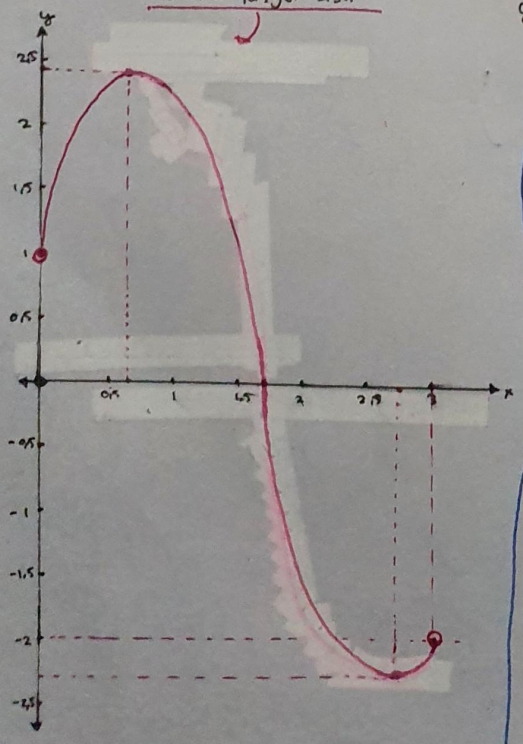
↳ $f(0,612) = (0,612)^3 - 5(0,612)^2 + 5 \cdot 0,612 + 1$

$f(0,612) = 2,416$

→ Saat $x = 2,721$ → $f(2,721) = (2,721)^3 - 5(2,721)^2 + 5(2,721) + 1$

$f(2,721) = -2,268$

Grafik fungsi asli



$g(x)$ → $f(x)$ fungsi genap

$h(x)$ → $f(x)$ fungsi ganjil

$0 < x < L$

$0 < x < 3 \rightarrow L = 3$

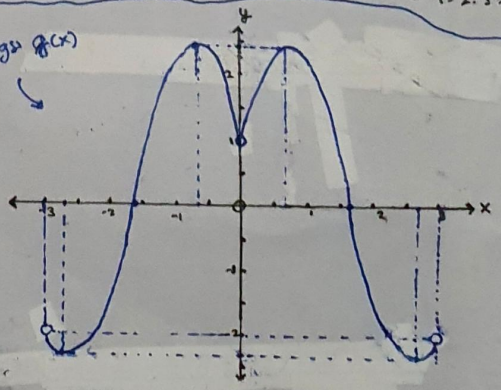
↓

So, new function will have Period of 2L

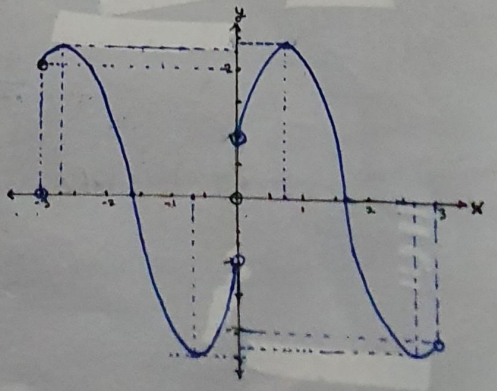
↳ $T = 2.3 = 6$

∴

grafik fungsi $g(x)$

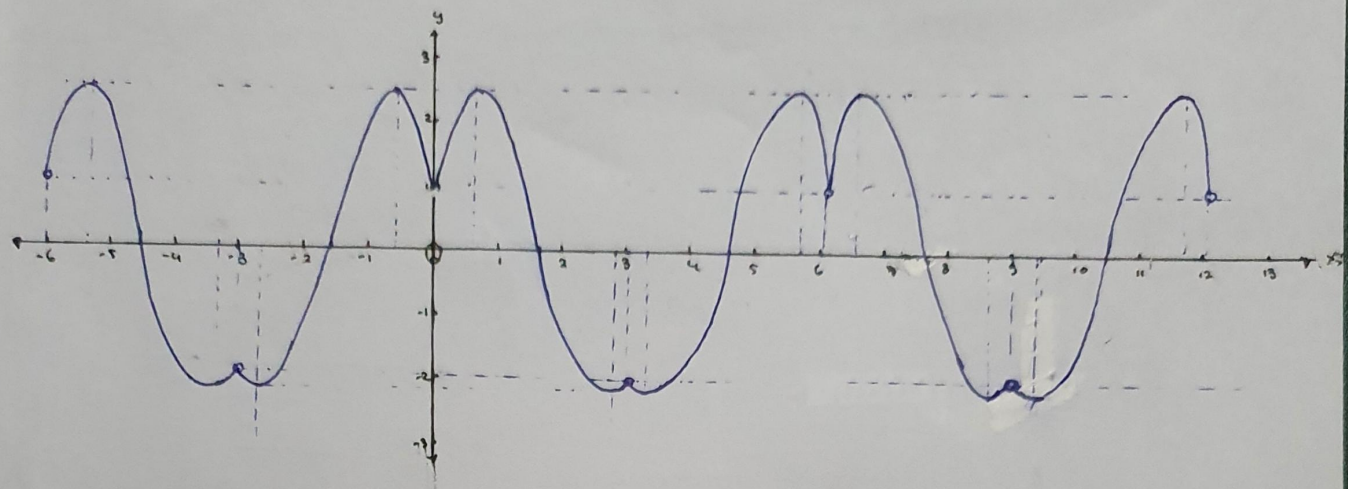


grafik fungsi $h(x)$

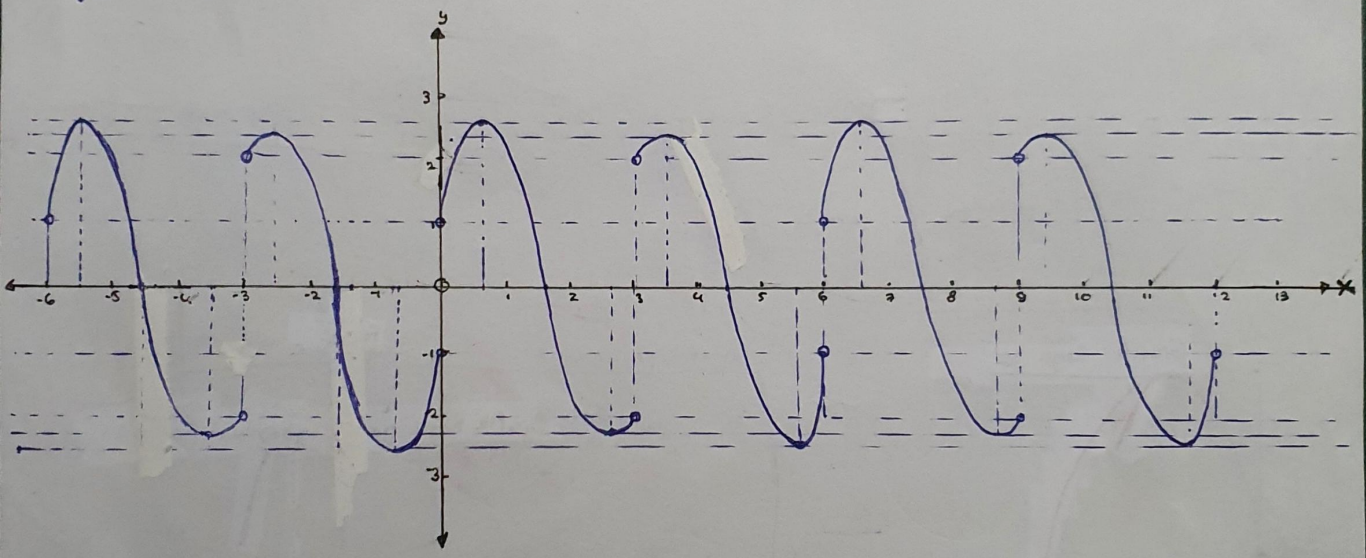


a.) Plot for $g(x)$ and $h(x)$ for 3 periods

(grafik untuk $g(x)$)



grafik untuk $h(x)$



b.) $f(x) = x^3 - 5x^2 + 5x + 1 \qquad 0 < x < 3$

untuk $g(x)$ (fungsi genap) $\quad g(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{2}\right) + b_n \sin\left(\frac{n\pi x}{2}\right) \right)$ ← karena ini membuat fungsi jadi ada sifat ganjil

$$g(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{2}\right) \right)$$

untuk $h(x)$ (fungsi ganjil) → $h(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{2}\right) + b_n \sin\left(\frac{n\pi x}{2}\right) \right)$ ← ini membuat fungsi punya sifat genap

$$h(x) = a_0 + \sum_{n=1}^{\infty} \left(b_n \sin\left(\frac{n\pi x}{2}\right) \right)$$

$a_n = \frac{1}{L}\int_a^b f(x)\cos\left(\frac{n\pi x}{L}\right)dx \quad b_n = \frac{1}{L}\int_a^b f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

now : calculate a_0

$a_0 = \frac{1}{L}\left[\int_0^3 (x^3-5x^2+5x+1)(\cos(0))dx\right]$

$a_0 = \frac{1}{3}\int_0^3 x^3-5x^2+5x+1\,dx$

$a_0 = \frac{1}{3}\left(\frac{x^4}{4} - \frac{5x^3}{3} + \frac{5}{2}x^2 + x\right]_{-3}^{3}$

$a_0 = \frac{1}{3}\left(\frac{3}{4}\right)$

$a_0 = \frac{1}{4}$

for $g(x)$

$a_1 = \frac{1}{3}\int_0^3 (x^3-5x^2+5x+1)\cos\left(\frac{\pi x}{3}\right)dx$

$a_2 = \frac{1}{3}\int_0^3 (x^3-5x^2+5x+1)\cos\left(\frac{2\pi x}{3}\right)dx$

for $h(x)$

$b_1 = \frac{1}{3}\int_0^3 (x^3-5x^2+5x+1)\sin\left(\frac{\pi x}{3}\right)dx$

$b_2 = \frac{1}{3}\int_0^3 (x^3-5x^2+5x+1)\sin\left(\frac{2\pi x}{3}\right)dx$

bentuk umum

$a_n = \frac{2}{3}\int_0^3 (x^3-5x^2+5x+1)\left(\cos\left(\frac{n\pi x}{3}\right)\right)dx$ ↷ $u = x^3-5x^2+5x+1 \quad dv = \cos\left(\frac{n\pi x}{3}\right)dx$

$du = (3x^2-10x+5)\,dx \quad v = \sin\left(\frac{n\pi x}{3}\right)\cdot\left(\frac{3}{n\pi}\right)$

$a_n = \frac{2}{3}\left(\frac{3}{n\pi}\left(x^3-5x^2+5x+1\right)\sin\left(\frac{n\pi x}{3}\right)\right]_0^3 - \frac{3}{n\pi}\int_0^3 (3x^2-10x+5)\sin\left(\frac{n\pi x}{3}\right)dx$

$a_n = \frac{2}{3}\left(\frac{3}{n\pi}\left(\left(x^3-5x^2+5x+1\right)\sin\left(\frac{n\pi x}{3}\right)\right]_0^3 - \int_0^3 (3x^2-10x+5)\sin\left(\frac{n\pi x}{3}\right)dx\right)$ ↷ $u = 3x^2-10x+5 \quad dv = \sin\left(\frac{n\pi x}{3}\right)dx$

$du = (6x-10)\,dx \quad v = -\frac{3}{n\pi}\cos\left(\frac{n\pi x}{3}\right)$

$a_n = \frac{2}{n\pi}\cdot\left(\left(x^3-5x^2+5x+1\right)\sin\left(\frac{n\pi x}{3}\right)\right]_0^3 - \left(\frac{-3}{n\pi}\left(\left(3x^2-10x+5\right)\cos\left(\frac{n\pi x}{3}\right)\right]_0^3 - \int_0^3 (6x-10)\cos\left(\frac{n\pi x}{3}\right)dx\right)$ ↷ $u = 6x-10 \quad du = 6dx \quad dv = \cos\left(\frac{n\pi x}{3}\right)dx \quad v = \frac{3}{n\pi}\sin\left(\frac{n\pi x}{3}\right)$

$a_n = \frac{2}{n\pi}\left(\left(x^3-5x^2+5x+1\right)\sin\left(\frac{n\pi x}{3}\right)\right]_0^3 + \left(\frac{3}{n\pi}\left(\left(3x^2-10x+5\right)\cos\left(\frac{n\pi x}{3}\right)\right]_0^3 - \left(\frac{3}{n\pi}\left((6x-10)\sin\left(\frac{n\pi x}{3}\right)\right]_0^3 - 6\int_0^3 \sin\left(\frac{n\pi x}{3}\right)dx\right)\right)$

$a_n = \frac{2}{n\pi}\left(\left(x^3-5x^2+5x+1\right)\sin\left(\frac{n\pi x}{3}\right)\right)_0^3 + \frac{6}{n^2\pi^2}\left(\left(3x^2-10x+5\right)\cos\left(\frac{n\pi x}{3}\right)\right]_0^3 - \frac{18}{n^3\pi^3}\left(\left(6x-10\right)\sin\left(\frac{n\pi x}{3}\right)\right]_0^3 + \frac{162}{n^4\pi^4}\left(\cos\left(\frac{n\pi x}{3}\right)\right]_0^3$

$a_n = \frac{2}{n\pi}\left(-2\left(\sin(n\pi)\right) + \left(\sin(0)\right)\right) + \frac{6}{n^2\pi^2}\left(2\cos(n\pi) - 5\cdot\cos(0)\right) - \frac{18}{n^3\pi^3}\left(8\sin(n\pi) - 0\sin(0)\right) + \frac{162}{n^4\pi^4}\left(\cos(n\pi) - \cos(0)\right)$

$a_n = \frac{2\left(-45\pi^2n^2 - 6\pi\left(\pi^2n^2+36\right)n\sin(\pi n) + 18\left(\pi^2n^2-27\right)\cos(\pi n)\right) + 486}{3\pi^4n^4}$ .... (1)

a_1, input $n = 1$ to ... (1) $\quad a_1 = \frac{2\left(-9\pi^2 - 6\pi\left(\pi^2+36\right)\sin(\pi) + 18\left(\pi^2-27\right)\cos(\pi)\right) + 486}{3\pi^4} = 2{,}3969$

$a_2 = \frac{2\left(-45\cdot2^2\pi^2 - 6\cdot2\pi\left(2^2\pi^2+36\right)\sin(2\pi) + 18\left(2^2\pi^2-27\right)\cos(2\pi)\right) + 486}{3\cdot2^4\pi^4} = -0{,}45595 = -\frac{9}{2\pi^2}$

→ $g(x)$?

$$S_{g2}(x) = a_0 + a_1 \cos\left(\frac{x\pi}{3}\right) + a_2 \cos\left(\frac{2\pi x}{3}\right)$$

$$S_{g2}(x) = \frac{1}{4} + \frac{2(9\pi^2 - 63\pi^2)}{3\pi^4}\cos\left(\frac{x\pi}{3}\right) - \frac{9}{2\pi^2}\cos\left(\frac{2\pi x}{3}\right)$$

$$S_{g2}(x) = 0{,}25 + 2{,}3969\cos\left(\frac{x\pi}{3}\right) - 0{,}4559\cos\left(\frac{2\pi x}{3}\right)$$

→ fourier series for $n=2$ for $g(x)$

untuk $h(x)$ → $a_0 = -28$ (sama)

$$b_n = \frac{2}{3}\int_0^3 (x^3 - 5x^2 + 5x + 1)\sin\left(\frac{\pi x n}{3}\right)dx$$

$u = x^3 - 5x^2 + 5x + 1$ $\quad dv = \sin\left(\frac{\pi n x}{3}\right)dx$

$du = (3x^2 - 10x + 5)dx$ $\quad v = -\frac{3}{\pi n}\cos\left(\frac{\pi n x}{3}\right)$

$$b_n = \frac{2}{3}\left(-\frac{3}{\pi n}\left((x^3 - 5x^2 + 5x + 1)\cos\left(\frac{\pi n x}{3}\right)\right]_0^3 - \int_0^3 (3x^2 - 10x + 5)\cos\left(\frac{\pi n x}{3}\right)dx\right)$$

$u = 3x^2 - 10x$

$du = (6x - 10)\,dx$

$dv = \cos\left(\frac{\pi n x}{3}\right)dx$

$v = \frac{3}{\pi n}\sin\left(\frac{n\pi x}{3}\right)$

$$b_n = \frac{-2}{\pi n}\left((x^3 - 5x^2 + 5x + 1)\cos\left(\frac{\pi n x}{3}\right)\right]_0^3 - \frac{3}{\pi n}\left((3x^2 - 10x + 5)\sin\left(\frac{\pi n x}{3}\right)\right]_0^3 - \int_0^3 (6x - 10)\sin\left(\frac{n x \pi}{3}\right)dx\Big)$$

$u = 6x - 10$

$du = 6dx$

$dv = \sin\left(\frac{\pi n x}{3}\right)dx$

$v = \frac{-3}{n\pi}\cos\left(\frac{\pi n x}{3}\right)$

$$b_n = \frac{-2}{\pi n}\left((x^3 - 5x^2 + 5x + 1)\cos\left(\frac{\pi n x}{3}\right)\right]_0^3 - \frac{3}{\pi n}\left((3x^2 - 10x + 5)\sin\left(\frac{\pi n x}{3}\right)\right]_0^3 + \frac{3}{\pi n}\left((6x - 10)\cos\left(\frac{\pi n x}{3}\right)\right]_0^3 - 6\int_0^3 \cos\left(\frac{\pi n x}{3}\right)dx\Big)$$

$$b_n = \frac{-2}{\pi n}\left((x^3 - 5x^2 + 5x + 1)\cos\left(\frac{\pi n x}{3}\right)\right]_0^3 + \frac{3}{\pi^2 n^2}\left((3x^2 - 10x + 5)\sin\left(\frac{\pi n x}{3}\right)\right]_0^3 + \frac{9}{n^3\pi^3}\left((6x - 10)\cos\left(\frac{\pi n x}{3}\right)\right]_0^3 - \frac{162}{n^4\pi^4}\left(\sin\left(\frac{\pi n x}{3}\right)\right]_0^3$$

$$b_n = \frac{-2}{\pi n}\left(-2\cos(\pi n) + \cos(0)\right) + \frac{3}{\pi^2 n^2}\left(2\sin(\pi n) - 5\sin(0)\right) + \frac{9}{n^3\pi^3}\left(8\cos(n\pi) - 10\cos(0)\right) - \frac{162}{n^4\pi^4}\left(\sin(\pi n) - \sin(0)\right)$$

$$b_n = \frac{2(\pi n(\pi^2 n^2 + 90) + 6(\pi^2 n^2 - 27)\sin(\pi n) + 2\pi n(\pi^2 n^2 + 36)\cos(\pi n))}{\pi^4 n^4} \quad \ldots (2)$$

↳ b_1 = input $n = 1$ to ... (2)

$$b_1 = \frac{2(54 - 3\pi^2)}{3\pi^3} \approx 0{,}5244$$

b_2 = input $n = 2$ to ... (2)

$$b_2 = \frac{3(27 + 2\pi^2)}{2\pi^3} = 2{,}2611$$

$h(x)$ → $S_{h2}(x) = \frac{2(54 - 3\pi^2)}{3\pi^3}\sin\left(\frac{\pi x}{3}\right) + \frac{3(27 + 2\pi^2)}{2\pi^3}\sin\left(\frac{2\pi x}{3}\right)$

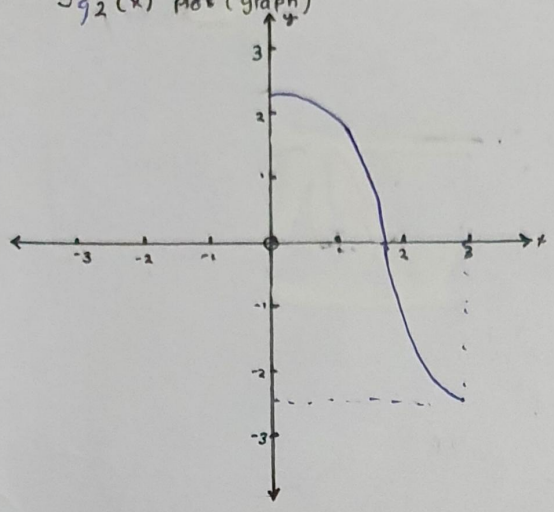
$S_{h2}(x) = 0{,}5244\sin\left(\frac{\pi x}{3}\right) + 2{,}2611\sin\left(\frac{2\pi x}{3}\right)$

→ fourier series for $h(x)$, $n = 2$

c.) for $n = 2$ (1 periode saja)

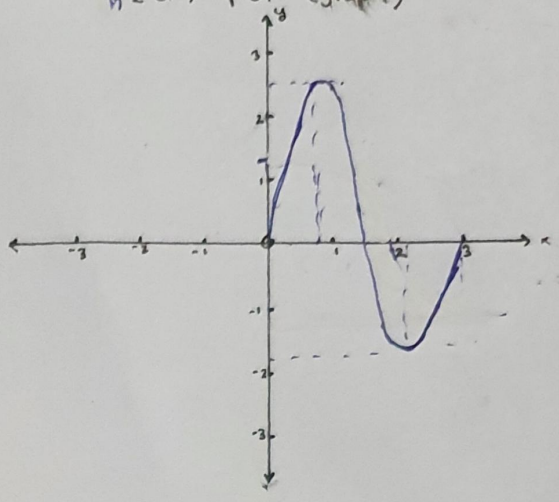
from $g(x)$

$S_{g2}(x)$ plot (graph)



from $h(x)$

$S_{h2}(x)$ plot (graph)



d.) Dependence on n of the maximum error on $[0, L]$

the more the order we use (semakin tinggi nilai n), e_n akan semakin kecil

$e_n = \sqrt{\frac{1}{2L}\int_a^b (f(x) - S_n(x))^2 \, dx}$ → tetapi nilai $e_n(x)$ tidak akan sebelah semakin dekat ke 0, jika seluruh bagian fungsi punya $\lim_{x \to c^+} f(x)$ dan $\lim_{x \to c^-} f(x)$ yang sama, maka $e_n(x)$ akan semakin mendekati 0, setiap nilai n bertambah.

Tetapi, jika $\lim_{x \to c^+} f(x)$ dan $\lim_{x \to c^-} f(x)$ pada suatu nilai c tidak sama, maka $e_n(x)$ akan semakin

$e_n(x) = \left| f(c) - \frac{\lim_{x \to c^+} f(x) + \lim_{x \to c^-} f(x)}{2} \right|$ dalam range $[0, L]$.

dengan alasan saat ada diskontinuitas dan limit kanan dan kiri berbeda, $S_n(x)$ pada nilai tersebut akan mendekati rerata dari nilai limit tersebut

$f(c)$ bisa dari fungsi kedua fungsi (fungsi di kiri $x=c$ atau di kanan $x=c$).

kalau ada fenomena overshoot Gibbs, maka error bisa tetap ada fluktuasi (tidak sepenuhnya menuju 0).

Hasil Kode No. 1

```
For N = 1:
a0 = 1.83333450
an 1 = -0.60792660    bn 1 = 0.57632111
Error (integral) = 0.62225431
Max error for one x value = 0.52459385

For N = 2:
a0 = 1.83333450
an 1 = -0.60792660    bn 1 = 0.57632111
an 2 = -0.10132001    bn 2 = -0.15915284
Error (integral) = 0.44634432
Max error for one x value = 0.57672616

For N = 4:
a0 = 1.83333450
an 1 = -0.60792660    bn 1 = 0.57632111
an 2 = -0.10132001    bn 2 = -0.15915284
an 3 = -0.06754695    bn 3 = 0.11565616
an 4 = -0.02532913    bn 4 = -0.07957327
Error (integral) = 0.26180558
Max error for one x value = 0.53450833

For N = 8:
a0 = 1.83333450
an 1 = -0.60792660    bn 1 = 0.57632111
an 2 = -0.10132001    bn 2 = -0.15915284
an 3 = -0.06754695    bn 3 = 0.11565616
an 4 = -0.02532913    bn 4 = -0.07957327
an 5 = -0.02431658    bn 5 = 0.06572083
an 6 = -0.01125674    bn 6 = -0.05304535
an 7 = -0.01240617    bn 7 = 0.04621772
an 8 = -0.00633140    bn 8 = -0.03978034
Error (integral) = 0.15233010
Max error for one x value = 0.51537372

For N = 16:
a0 = 1.83333450
an 1 = -0.60792660    bn 1 = 0.57632111
an 2 = -0.10132001    bn 2 = -0.15915284
an 3 = -0.06754695    bn 3 = 0.11565616
an 4 = -0.02532913    bn 4 = -0.07957327
an 5 = -0.02431658    bn 5 = 0.06572083
an 6 = -0.01125674    bn 6 = -0.05304535
an 7 = -0.01240617    bn 7 = 0.04621772
an 8 = -0.00633140    bn 8 = -0.03978034
an 9 = -0.00750477    bn 9 = 0.03571225
an 10 = -0.00405168    bn 10 = -0.03182049
an 11 = -0.00502369    bn 11 = 0.02911957
an 12 = -0.00281331    bn 12 = -0.02651323
an 13 = -0.00359670    bn 13 = 0.02458917
an 14 = -0.00206661    bn 14 = -0.02272173
an 15 = -0.00270140    bn 15 = 0.02128137
an 16 = -0.00158197    bn 16 = -0.01987758
Error (integral) = 0.08747174
```

```
Max error for one x value = 0.50706073

For N = 32:
a0 = 1.83333450
an 1 = -0.60792660    bn 1 = 0.57632111
an 2 = -0.10132001    bn 2 = -0.15915284
an 3 = -0.06754695    bn 3 = 0.11565616
an 4 = -0.02532913    bn 4 = -0.07957327
an 5 = -0.02431658    bn 5 = 0.06572083
an 6 = -0.01125674    bn 6 = -0.05304535
an 7 = -0.01240617    bn 7 = 0.04621772
an 8 = -0.00633140    bn 8 = -0.03978034
an 9 = -0.00750477    bn 9 = 0.03571225
an 10 = -0.00405168    bn 10 = -0.03182049
an 11 = -0.00502369    bn 11 = 0.02911957
an 12 = -0.00281331    bn 12 = -0.02651323
an 13 = -0.00359670    bn 13 = 0.02458917
an 14 = -0.00206661    bn 14 = -0.02272173
an 15 = -0.00270140    bn 15 = 0.02128137
an 16 = -0.00158197    bn 16 = -0.01987758
an 17 = -0.00210305    bn 17 = 0.01875879
an 18 = -0.00124971    bn 18 = -0.01766499
an 19 = -0.00168351    bn 19 = 0.01677083
an 20 = -0.00101204    bn 20 = -0.01589450
an 21 = -0.00137802    bn 21 = 0.01516343
an 22 = -0.00083619    bn 22 = -0.01444554
an 23 = -0.00114870    bn 23 = 0.01383662
an 24 = -0.00070245    bn 24 = -0.01323772
an 25 = -0.00097218    bn 25 = 0.01272267
an 26 = -0.00059836    bn 26 = -0.01221539
an 27 = -0.00083342    bn 27 = 0.01177402
an 28 = -0.00051577    bn 28 = -0.01133881
an 29 = -0.00072236    bn 29 = 0.01095634
an 30 = -0.00044914    bn 30 = -0.01057883
an 31 = -0.00063210    bn 31 = 0.01024417
an 32 = -0.00039461    bn 32 = -0.00991358
Error (integral) = 0.04942634
Max error for one x value = 0.50334569

For N = 64:
a0 = 1.83333450
an 1 = -0.60792660    bn 1 = 0.57632111
an 2 = -0.10132001    bn 2 = -0.15915284
an 3 = -0.06754695    bn 3 = 0.11565616
an 4 = -0.02532913    bn 4 = -0.07957327
an 5 = -0.02431658    bn 5 = 0.06572083
an 6 = -0.01125674    bn 6 = -0.05304535
an 7 = -0.01240617    bn 7 = 0.04621772
an 8 = -0.00633140    bn 8 = -0.03978034
an 9 = -0.00750477    bn 9 = 0.03571225
an 10 = -0.00405168    bn 10 = -0.03182049
an 11 = -0.00502369    bn 11 = 0.02911957
an 12 = -0.00281331    bn 12 = -0.02651323
an 13 = -0.00359670    bn 13 = 0.02458917
```

```
an 14 = -0.00206661    bn 14 = -0.02272173
an 15 = -0.00270140    bn 15 = 0.02128137
an 16 = -0.00158197    bn 16 = -0.01987758
an 17 = -0.00210305    bn 17 = 0.01875879
an 18 = -0.00124971    bn 18 = -0.01766499
an 19 = -0.00168351    bn 19 = 0.01677083
an 20 = -0.00101204    bn 20 = -0.01589450
an 21 = -0.00137802    bn 21 = 0.01516343
an 22 = -0.00083619    bn 22 = -0.01444554
an 23 = -0.00114870    bn 23 = 0.01383662
an 24 = -0.00070245    bn 24 = -0.01323772
an 25 = -0.00097218    bn 25 = 0.01272267
an 26 = -0.00059836    bn 26 = -0.01221539
an 27 = -0.00083342    bn 27 = 0.01177402
an 28 = -0.00051577    bn 28 = -0.01133881
an 29 = -0.00072236    bn 29 = 0.01095634
an 30 = -0.00044914    bn 30 = -0.01057883
an 31 = -0.00063210    bn 31 = 0.01024417
an 32 = -0.00039461    bn 32 = -0.00991358
an 33 = -0.00055774    bn 33 = 0.00961828
an 34 = -0.00034942    bn 34 = -0.00932635
an 35 = -0.00049576    bn 35 = 0.00906383
an 36 = -0.00031155    bn 36 = -0.00880413
an 37 = -0.00044356    bn 37 = 0.00856920
an 38 = -0.00027949    bn 38 = -0.00833666
an 39 = -0.00039918    bn 39 = 0.00812518
an 40 = -0.00025213    bn 40 = -0.00791573
an 41 = -0.00036114    bn 41 = 0.00772433
an 42 = -0.00022858    bn 42 = -0.00753469
an 43 = -0.00032828    bn 43 = 0.00736063
an 44 = -0.00020817    bn 44 = -0.00718809
an 45 = -0.00029971    bn 45 = 0.00702910
an 46 = -0.00019036    bn 46 = -0.00687145
an 47 = -0.00027470    bn 47 = 0.00672565
an 48 = -0.00017473    bn 48 = -0.00658101
an 49 = -0.00025269    bn 49 = 0.00644682
an 50 = -0.00016094    bn 50 = -0.00631365
an 51 = -0.00023322    bn 51 = 0.00618971
an 52 = -0.00014870    bn 52 = -0.00606668
an 53 = -0.00021591    bn 53 = 0.00595186
an 54 = -0.00013781    bn 54 = -0.00583786
an 55 = -0.00020046    bn 55 = 0.00573118
an 56 = -0.00012806    bn 56 = -0.00562522
an 57 = -0.00018660    bn 57 = 0.00552584
an 58 = -0.00011930    bn 58 = -0.00542711
an 59 = -0.00017413    bn 59 = 0.00533429
an 60 = -0.00011140    bn 60 = -0.00524206
an 61 = -0.00016287    bn 61 = 0.00515517
an 62 = -0.00010425    bn 62 = -0.00506881
an 63 = -0.00015266    bn 63 = 0.00498729
an 64 = -0.00009776    bn 64 = -0.00490625
Error (integral) = 0.02743974
Max error for one x value = 0.50160969
```

For N = 128:
a0 = 1.83333450

an 1 = -0.60792660	bn 1 = 0.57632111
an 2 = -0.10132001	bn 2 = -0.15915284
an 3 = -0.06754695	bn 3 = 0.11565616
an 4 = -0.02532913	bn 4 = -0.07957327
an 5 = -0.02431658	bn 5 = 0.06572083
an 6 = -0.01125674	bn 6 = -0.05304535
an 7 = -0.01240617	bn 7 = 0.04621772
an 8 = -0.00633140	bn 8 = -0.03978034
an 9 = -0.00750477	bn 9 = 0.03571225
an 10 = -0.00405168	bn 10 = -0.03182049
an 11 = -0.00502369	bn 11 = 0.02911957
an 12 = -0.00281331	bn 12 = -0.02651323
an 13 = -0.00359670	bn 13 = 0.02458917
an 14 = -0.00206661	bn 14 = -0.02272173
an 15 = -0.00270140	bn 15 = 0.02128137
an 16 = -0.00158197	bn 16 = -0.01987758
an 17 = -0.00210305	bn 17 = 0.01875879
an 18 = -0.00124971	bn 18 = -0.01766499
an 19 = -0.00168351	bn 19 = 0.01677083
an 20 = -0.00101204	bn 20 = -0.01589450
an 21 = -0.00137802	bn 21 = 0.01516343
an 22 = -0.00083619	bn 22 = -0.01444554
an 23 = -0.00114870	bn 23 = 0.01383662
an 24 = -0.00070245	bn 24 = -0.01323772
an 25 = -0.00097218	bn 25 = 0.01272267
an 26 = -0.00059836	bn 26 = -0.01221539
an 27 = -0.00083342	bn 27 = 0.01177402
an 28 = -0.00051577	bn 28 = -0.01133881
an 29 = -0.00072236	bn 29 = 0.01095634
an 30 = -0.00044914	bn 30 = -0.01057883
an 31 = -0.00063210	bn 31 = 0.01024417
an 32 = -0.00039461	bn 32 = -0.00991358
an 33 = -0.00055774	bn 33 = 0.00961828
an 34 = -0.00034942	bn 34 = -0.00932635
an 35 = -0.00049576	bn 35 = 0.00906383
an 36 = -0.00031155	bn 36 = -0.00880413
an 37 = -0.00044356	bn 37 = 0.00856920
an 38 = -0.00027949	bn 38 = -0.00833666
an 39 = -0.00039918	bn 39 = 0.00812518
an 40 = -0.00025213	bn 40 = -0.00791573
an 41 = -0.00036114	bn 41 = 0.00772433
an 42 = -0.00022858	bn 42 = -0.00753469
an 43 = -0.00032828	bn 43 = 0.00736063
an 44 = -0.00020817	bn 44 = -0.00718809
an 45 = -0.00029971	bn 45 = 0.00702910
an 46 = -0.00019036	bn 46 = -0.00687145
an 47 = -0.00027470	bn 47 = 0.00672565
an 48 = -0.00017473	bn 48 = -0.00658101
an 49 = -0.00025269	bn 49 = 0.00644682
an 50 = -0.00016094	bn 50 = -0.00631365
an 51 = -0.00023322	bn 51 = 0.00618971
an 52 = -0.00014870	bn 52 = -0.00606668

an 53 = -0.00021591 bn 53 = 0.00595186
an 54 = -0.00013781 bn 54 = -0.00583786
an 55 = -0.00020046 bn 55 = 0.00573118
an 56 = -0.00012806 bn 56 = -0.00562522
an 57 = -0.00018660 bn 57 = 0.00552584
an 58 = -0.00011930 bn 58 = -0.00542711
an 59 = -0.00017413 bn 59 = 0.00533429
an 60 = -0.00011140 bn 60 = -0.00524206
an 61 = -0.00016287 bn 61 = 0.00515517
an 62 = -0.00010425 bn 62 = -0.00506881
an 63 = -0.00015266 bn 63 = 0.00498729
an 64 = -0.00009776 bn 64 = -0.00490625
an 65 = -0.00014338 bn 65 = 0.00482962
an 66 = -0.00009186 bn 66 = -0.00475342
an 67 = -0.00013491 bn 67 = 0.00468124
an 68 = -0.00008646 bn 68 = -0.00460946
an 69 = -0.00012718 bn 69 = 0.00454134
an 70 = -0.00008152 bn 70 = -0.00447359
an 71 = -0.00012008 bn 71 = 0.00440921
an 72 = -0.00007699 bn 72 = -0.00434516
an 73 = -0.00011357 bn 73 = 0.00428420
an 74 = -0.00007282 bn 74 = -0.00422356
an 75 = -0.00010756 bn 75 = 0.00416575
an 76 = -0.00006898 bn 76 = -0.00410824
an 77 = -0.00010202 bn 77 = 0.00405335
an 78 = -0.00006542 bn 78 = -0.00399872
an 79 = -0.00009689 bn 79 = 0.00394652
an 80 = -0.00006213 bn 80 = -0.00389457
an 81 = -0.00009214 bn 81 = 0.00384487
an 82 = -0.00005908 bn 82 = -0.00379540
an 83 = -0.00008773 bn 83 = 0.00374802
an 84 = -0.00005624 bn 84 = -0.00370085
an 85 = -0.00008362 bn 85 = 0.00365562
an 86 = -0.00005360 bn 86 = -0.00361060
an 87 = -0.00007980 bn 87 = 0.00356738
an 88 = -0.00005114 bn 88 = -0.00352434
an 89 = -0.00007623 bn 89 = 0.00348300
an 90 = -0.00004884 bn 90 = -0.00344183
an 91 = -0.00007289 bn 91 = 0.00340224
an 92 = -0.00004668 bn 92 = -0.00336281
an 93 = -0.00006977 bn 93 = 0.00332486
an 94 = -0.00004467 bn 94 = -0.00328706
an 95 = -0.00006684 bn 95 = 0.00325065
an 96 = -0.00004277 bn 96 = -0.00321438
an 97 = -0.00006409 bn 97 = 0.00317941
an 98 = -0.00004100 bn 98 = -0.00314457
an 99 = -0.00006150 bn 99 = 0.00311096
an 100 = -0.00003932 bn 100 = -0.00307747
an 101 = -0.00005907 bn 101 = 0.00304513
an 102 = -0.00003775 bn 102 = -0.00301292
an 103 = -0.00005678 bn 103 = 0.00298178
an 104 = -0.00003626 bn 104 = -0.00295076
an 105 = -0.00005461 bn 105 = 0.00292076
an 106 = -0.00003486 bn 106 = -0.00289087

```
an 107 = -0.00005257    bn 107 = 0.00286194
an 108 = -0.00003354    bn 108 = -0.00283311
an 109 = -0.00005064    bn 109 = 0.00280519
an 110 = -0.00003228    bn 110 = -0.00277737
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an 113 = -0.00004708    bn 113 = 0.00269750
an 114 = -0.00002997    bn 114 = -0.00267153
an 115 = -0.00004544    bn 115 = 0.00264635
an 116 = -0.00002890    bn 116 = -0.00262124
an 117 = -0.00004388    bn 117 = 0.00259687
an 118 = -0.00002789    bn 118 = -0.00257257
an 119 = -0.00004240    bn 119 = 0.00254898
an 120 = -0.00002692    bn 120 = -0.00252545
an 121 = -0.00004099    bn 121 = 0.00250260
an 122 = -0.00002601    bn 122 = -0.00247981
an 123 = -0.00003965    bn 123 = 0.00245765
an 124 = -0.00002513    bn 124 = -0.00243556
an 125 = -0.00003837    bn 125 = 0.00241408
an 126 = -0.00002430    bn 126 = -0.00239265
an 127 = -0.00003715    bn 127 = 0.00237180
an 128 = -0.00002351    bn 128 = -0.00235101
Error (integral) = 0.01488569
Max error for one x value = 0.50075999

For N = 194:
a0 = 1.83333450
an 1 = -0.60792660    bn 1 = 0.57632111
an 2 = -0.10132001    bn 2 = -0.15915284
an 3 = -0.06754695    bn 3 = 0.11565616
an 4 = -0.02532913    bn 4 = -0.07957327
an 5 = -0.02431658    bn 5 = 0.06572083
an 6 = -0.01125674    bn 6 = -0.05304535
an 7 = -0.01240617    bn 7 = 0.04621772
an 8 = -0.00633140    bn 8 = -0.03978034
an 9 = -0.00750477    bn 9 = 0.03571225
an 10 = -0.00405168    bn 10 = -0.03182049
an 11 = -0.00502369    bn 11 = 0.02911957
an 12 = -0.00281331    bn 12 = -0.02651323
an 13 = -0.00359670    bn 13 = 0.02458917
an 14 = -0.00206661    bn 14 = -0.02272173
an 15 = -0.00270140    bn 15 = 0.02128137
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an 17 = -0.00210305    bn 17 = 0.01875879
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an 19 = -0.00168351    bn 19 = 0.01677083
an 20 = -0.00101204    bn 20 = -0.01589450
an 21 = -0.00137802    bn 21 = 0.01516343
an 22 = -0.00083619    bn 22 = -0.01444554
an 23 = -0.00114870    bn 23 = 0.01383662
an 24 = -0.00070245    bn 24 = -0.01323772
an 25 = -0.00097218    bn 25 = 0.01272267
an 26 = -0.00059836    bn 26 = -0.01221539
an 27 = -0.00083342    bn 27 = 0.01177402
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an 28 = -0.00051577	bn 28 = -0.01133881
an 29 = -0.00072236	bn 29 = 0.01095634
an 30 = -0.00044914	bn 30 = -0.01057883
an 31 = -0.00063210	bn 31 = 0.01024417
an 32 = -0.00039461	bn 32 = -0.00991358
an 33 = -0.00055774	bn 33 = 0.00961828
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an 37 = -0.00044356	bn 37 = 0.00856920
an 38 = -0.00027949	bn 38 = -0.00833666
an 39 = -0.00039918	bn 39 = 0.00812518
an 40 = -0.00025213	bn 40 = -0.00791573
an 41 = -0.00036114	bn 41 = 0.00772433
an 42 = -0.00022858	bn 42 = -0.00753469
an 43 = -0.00032828	bn 43 = 0.00736063
an 44 = -0.00020817	bn 44 = -0.00718809
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an 51 = -0.00023322	bn 51 = 0.00618971
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an 53 = -0.00021591	bn 53 = 0.00595186
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an 55 = -0.00020046	bn 55 = 0.00573118
an 56 = -0.00012806	bn 56 = -0.00562522
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an 59 = -0.00017413	bn 59 = 0.00533429
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an 61 = -0.00016287	bn 61 = 0.00515517
an 62 = -0.00010425	bn 62 = -0.00506881
an 63 = -0.00015266	bn 63 = 0.00498729
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an 65 = -0.00014338	bn 65 = 0.00482962
an 66 = -0.00009186	bn 66 = -0.00475342
an 67 = -0.00013491	bn 67 = 0.00468124
an 68 = -0.00008646	bn 68 = -0.00460946
an 69 = -0.00012718	bn 69 = 0.00454134
an 70 = -0.00008152	bn 70 = -0.00447359
an 71 = -0.00012008	bn 71 = 0.00440921
an 72 = -0.00007699	bn 72 = -0.00434516
an 73 = -0.00011357	bn 73 = 0.00428420
an 74 = -0.00007282	bn 74 = -0.00422356
an 75 = -0.00010756	bn 75 = 0.00416575
an 76 = -0.00006898	bn 76 = -0.00410824
an 77 = -0.00010202	bn 77 = 0.00405335
an 78 = -0.00006542	bn 78 = -0.00399872
an 79 = -0.00009689	bn 79 = 0.00394652
an 80 = -0.00006213	bn 80 = -0.00389457
an 81 = -0.00009214	bn 81 = 0.00384487

an 82 = -0.00005908 bn 82 = -0.00379540
an 83 = -0.00008773 bn 83 = 0.00374802
an 84 = -0.00005624 bn 84 = -0.00370085
an 85 = -0.00008362 bn 85 = 0.00365562
an 86 = -0.00005360 bn 86 = -0.00361060
an 87 = -0.00007980 bn 87 = 0.00356738
an 88 = -0.00005114 bn 88 = -0.00352434
an 89 = -0.00007623 bn 89 = 0.00348300
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an 91 = -0.00007289 bn 91 = 0.00340224
an 92 = -0.00004668 bn 92 = -0.00336281
an 93 = -0.00006977 bn 93 = 0.00332486
an 94 = -0.00004467 bn 94 = -0.00328706
an 95 = -0.00006684 bn 95 = 0.00325065
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an 97 = -0.00006409 bn 97 = 0.00317941
an 98 = -0.00004100 bn 98 = -0.00314457
an 99 = -0.00006150 bn 99 = 0.00311096
an 100 = -0.00003932 bn 100 = -0.00307747
an 101 = -0.00005907 bn 101 = 0.00304513
an 102 = -0.00003775 bn 102 = -0.00301292
an 103 = -0.00005678 bn 103 = 0.00298178
an 104 = -0.00003626 bn 104 = -0.00295076
an 105 = -0.00005461 bn 105 = 0.00292076
an 106 = -0.00003486 bn 106 = -0.00289087
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an 110 = -0.00003228 bn 110 = -0.00277737
an 111 = -0.00004881 bn 111 = 0.00275042
an 112 = -0.00003109 bn 112 = -0.00272355
an 113 = -0.00004708 bn 113 = 0.00269750
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an 119 = -0.00004240 bn 119 = 0.00254898
an 120 = -0.00002692 bn 120 = -0.00252545
an 121 = -0.00004099 bn 121 = 0.00250260
an 122 = -0.00002601 bn 122 = -0.00247981
an 123 = -0.00003965 bn 123 = 0.00245765
an 124 = -0.00002513 bn 124 = -0.00243556
an 125 = -0.00003837 bn 125 = 0.00241408
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an 127 = -0.00003715 bn 127 = 0.00237180
an 128 = -0.00002351 bn 128 = -0.00235101
an 129 = -0.00003599 bn 129 = 0.00233077
an 130 = -0.00002275 bn 130 = -0.00231058
an 131 = -0.00003488 bn 131 = 0.00229092
an 132 = -0.00002203 bn 132 = -0.00227131
an 133 = -0.00003383 bn 133 = 0.00225221
an 134 = -0.00002134 bn 134 = -0.00223315
an 135 = -0.00003281 bn 135 = 0.00221457

an 136 = -0.00002068 bn 136 = -0.00219604
an 137 = -0.00003185 bn 137 = 0.00217797
an 138 = -0.00002004 bn 138 = -0.00215994
an 139 = -0.00003092 bn 139 = 0.00214235
an 140 = -0.00001944 bn 140 = -0.00212481
an 141 = -0.00003003 bn 141 = 0.00210768
an 142 = -0.00001886 bn 142 = -0.00209060
an 143 = -0.00002918 bn 143 = 0.00207392
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an 153 = -0.00002541 bn 153 = 0.00191745
an 154 = -0.00001583 bn 154 = -0.00190277
an 155 = -0.00002475 bn 155 = 0.00188840
an 156 = -0.00001539 bn 156 = -0.00187407
an 157 = -0.00002410 bn 157 = 0.00186004
an 158 = -0.00001497 bn 158 = -0.00184604
an 159 = -0.00002349 bn 159 = 0.00183233
an 160 = -0.00001457 bn 160 = -0.00181865
an 161 = -0.00002289 bn 161 = 0.00180525
an 162 = -0.00001418 bn 162 = -0.00179187
an 163 = -0.00002232 bn 163 = 0.00177877
an 164 = -0.00001380 bn 164 = -0.00176570
an 165 = -0.00002176 bn 165 = 0.00175289
an 166 = -0.00001344 bn 166 = -0.00174010
an 167 = -0.00002123 bn 167 = 0.00172756
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an 169 = -0.00002072 bn 169 = 0.00170278
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an 171 = -0.00002022 bn 171 = 0.00167853
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an 173 = -0.00001974 bn 173 = 0.00165478
an 174 = -0.00001210 bn 174 = -0.00164304
an 175 = -0.00001928 bn 175 = 0.00163152
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an 177 = -0.00001883 bn 177 = 0.00160873
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an 179 = -0.00001840 bn 179 = 0.00158640
an 180 = -0.00001122 bn 180 = -0.00157535
an 181 = -0.00001798 bn 181 = 0.00156451
an 182 = -0.00001094 bn 182 = -0.00155368
an 183 = -0.00001757 bn 183 = 0.00154304
an 184 = -0.00001067 bn 184 = -0.00153242
an 185 = -0.00001718 bn 185 = 0.00152199
an 186 = -0.00001042 bn 186 = -0.00151157
an 187 = -0.00001680 bn 187 = 0.00150134
an 188 = -0.00001016 bn 188 = -0.00149111
an 189 = -0.00001643 bn 189 = 0.00148107

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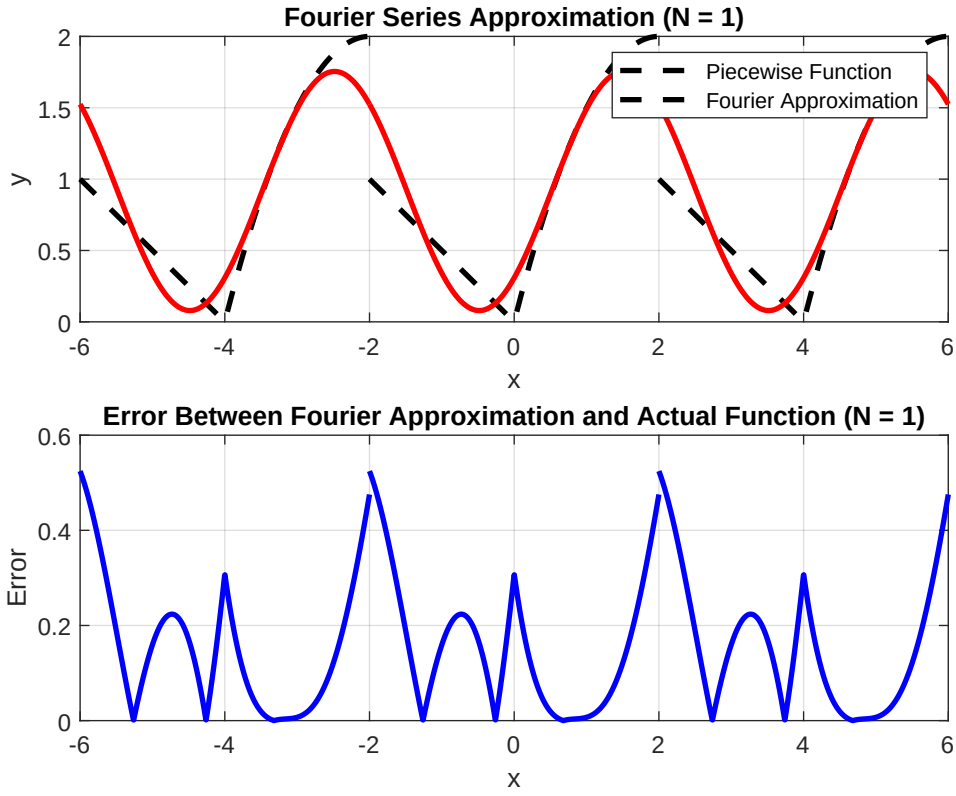
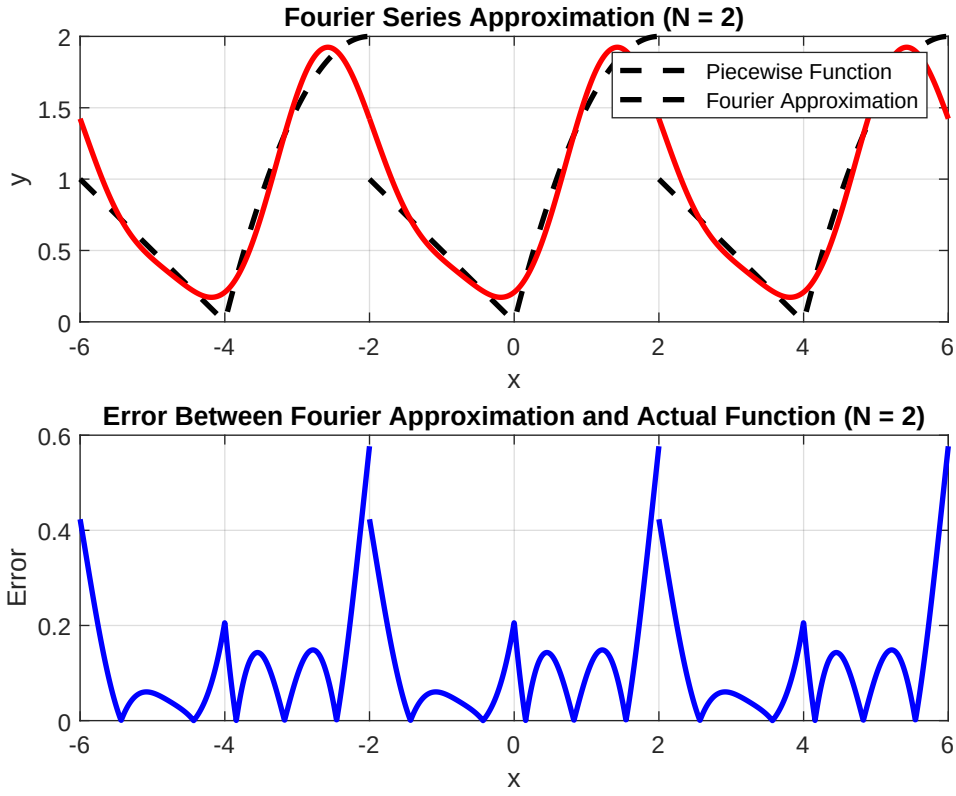
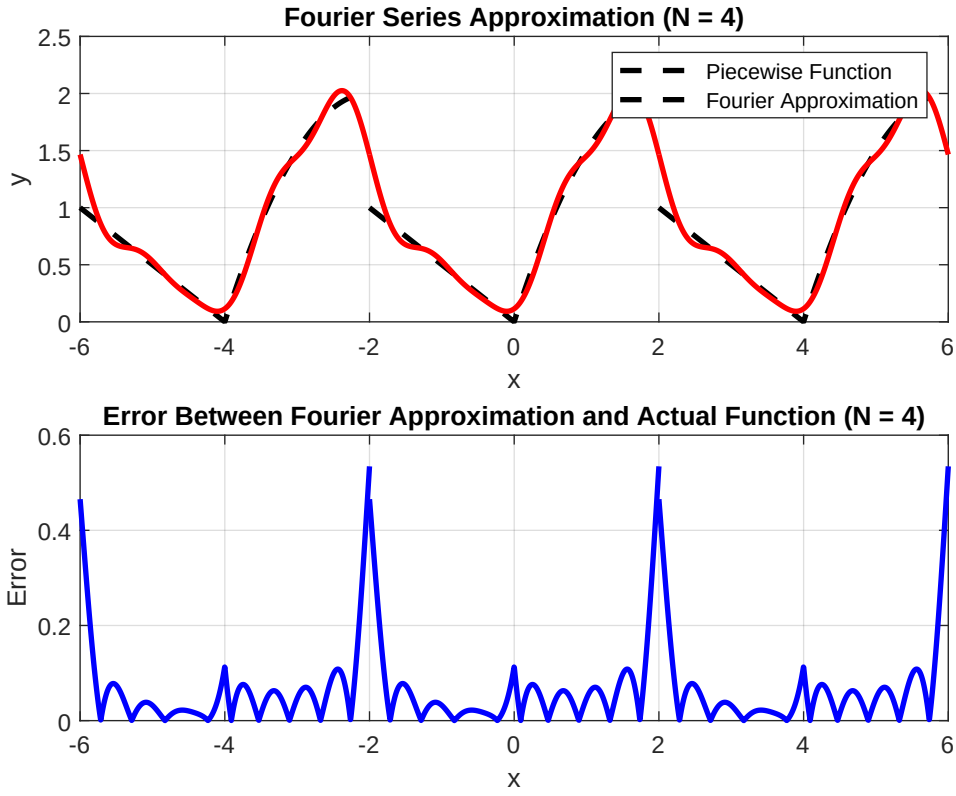
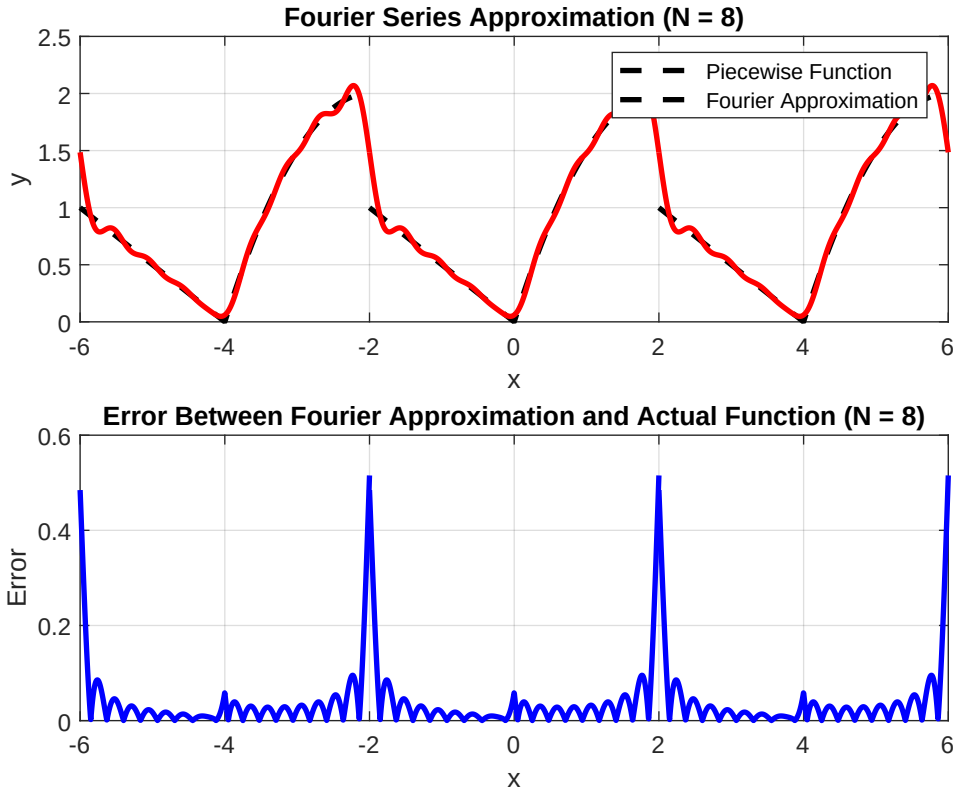
an 190 = -0.00000992    bn 190 = -0.00147104
an 191 = -0.00001608    bn 191 = 0.00146117
an 192 = -0.00000969    bn 192 = -0.00145133
an 193 = -0.00001573    bn 193 = 0.00144164
an 194 = -0.00000946    bn 194 = -0.00143197
Error (integral) = 0.00999438
Max error for one x value = 0.50046051
```

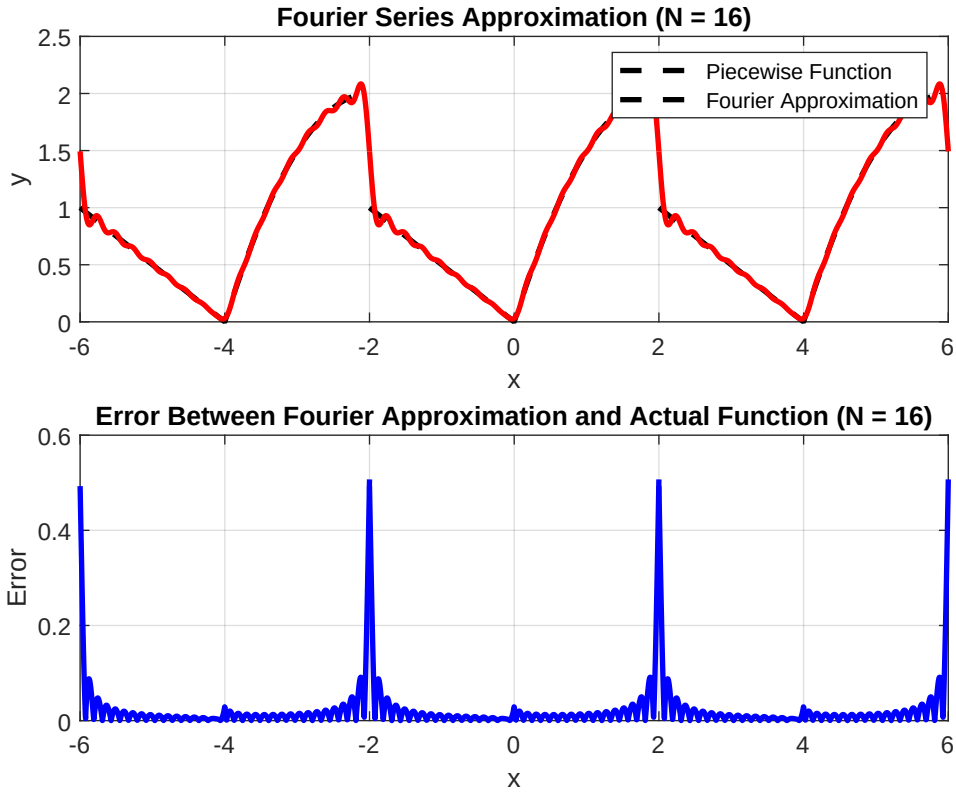

Fourier Series Approximation (N = 1)
Piecewise Function
Fourier Approximation
y
x
2
1.5
1
0.5
0
-6
-4
-2
0
2
4
6

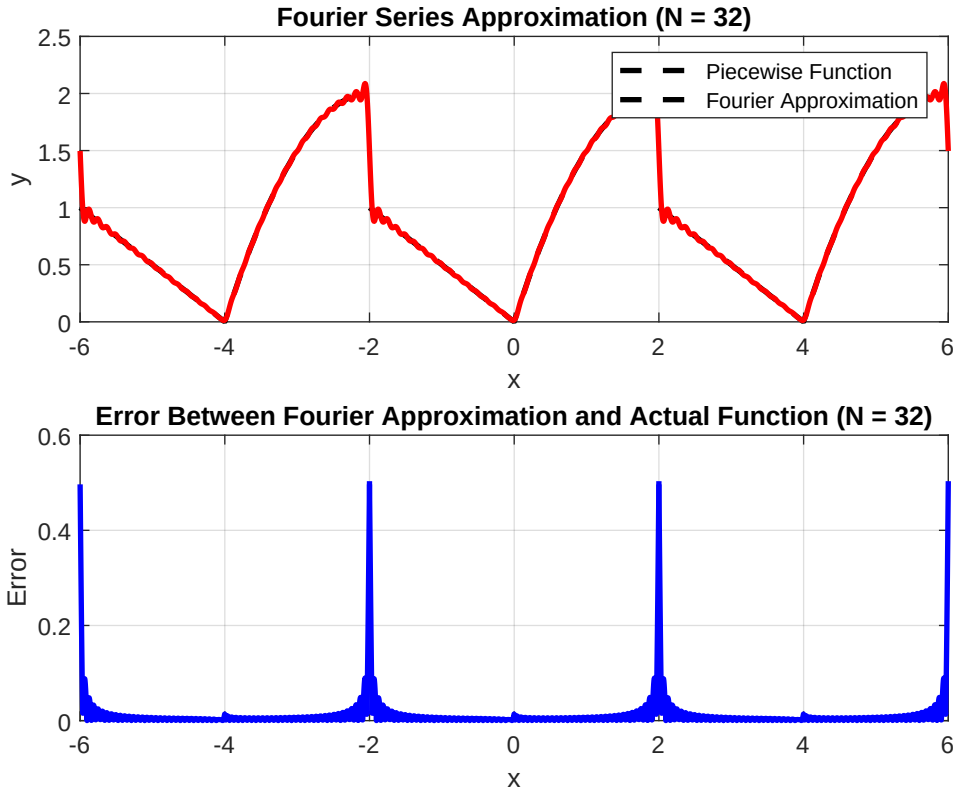
Error Between Fourier Approximation and Actual Function (N = 1)
Error
x
0.6
0.4
0.2
0
-6
-4
-2
0
2
4
6

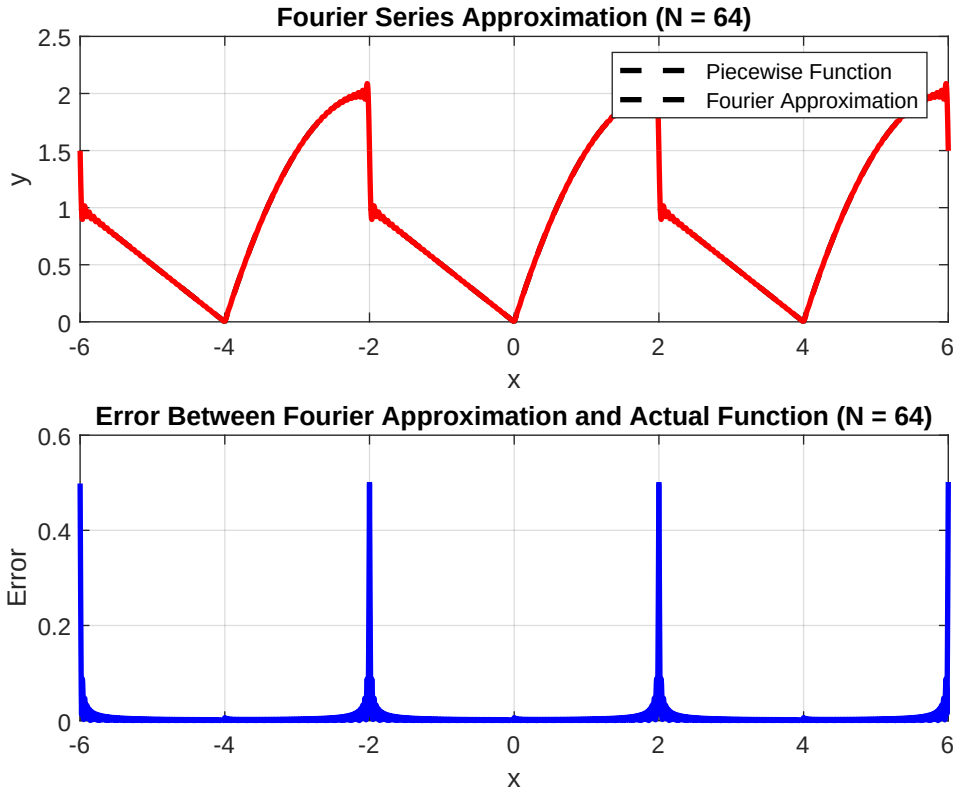

Fourier Series Approximation (N = 2)
Piecewise Function
Fourier Approximation
y
x
2
1.5
1
0.5
0
-6
-4
-2
0
2
4
6
Error Between Fourier Approximation and Actual Function (N = 2)
Error
x
0.6
0.4
0.2
0

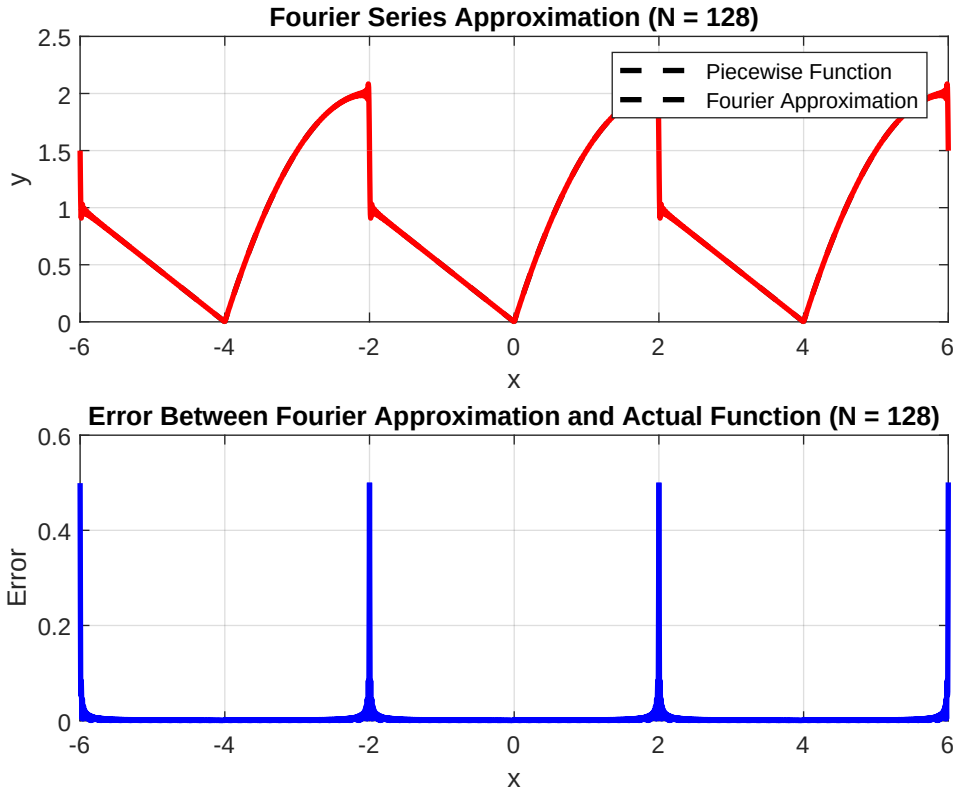

Fourier Series Approximation (N = 4)
Piecewise Function
Fourier Approximation
y
x
2.5
2
1.5
1
0.5
0
-6
-4
-2
0
2
4
6
Error Between Fourier Approximation and Actual Function (N = 4)
Error
x
0.6
0.4
0.2
0

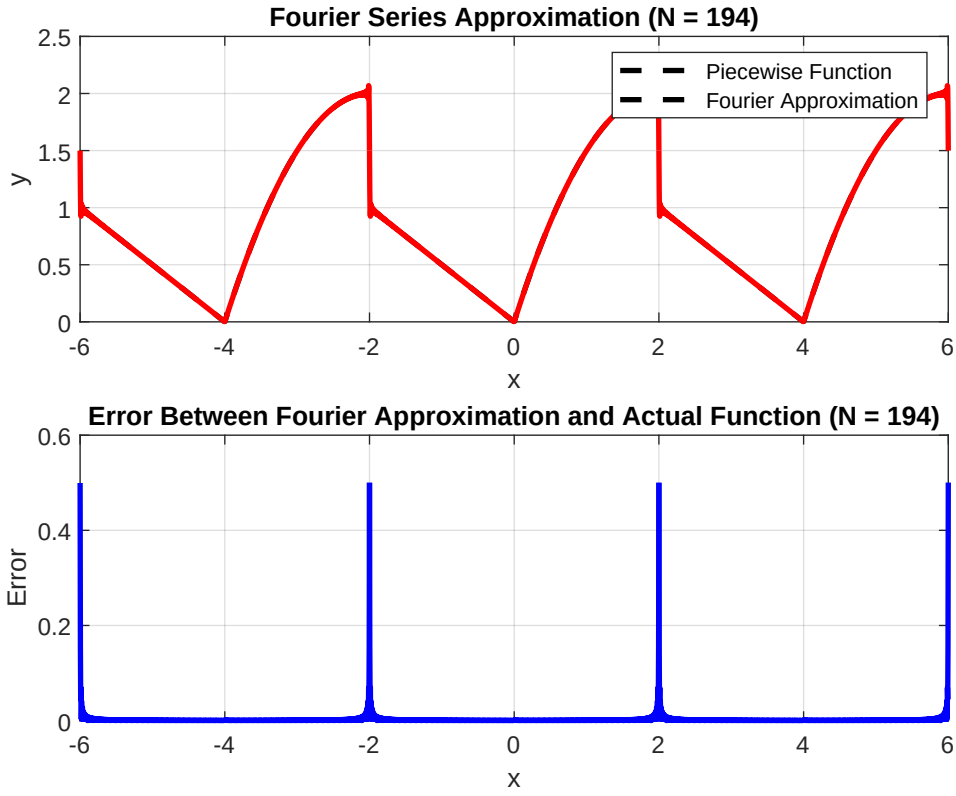

Fourier Series Approximation (N = 8)
Piecewise Function
Fourier Approximation
y
x
2.5
2
1.5
1
0.5
0
-6
-4
-2
0
2
4
6
Error Between Fourier Approximation and Actual Function (N = 8)
Error
x
0.6
0.4
0.2
0


Fourier Series Approximation (N = 16)
Piecewise Function
Fourier Approximation
y
x
2.5
2
1.5
1
0.5
0
-6
-4
-2
0
2
4
6
Error Between Fourier Approximation and Actual Function (N = 16)
Error
x
0.6
0.4
0.2
0


Fourier Series Approximation (N = 32)
Piecewise Function
Fourier Approximation
y
x
2.5
2
1.5
1
0.5
0
-6
-4
-2
0
2
4
6
Error Between Fourier Approximation and Actual Function (N = 32)
Error
x
0.6
0.4
0.2
0


Fourier Series Approximation (N = 64)
Piecewise Function
Fourier Approximation
y
x
2.5
2
1.5
1
0.5
0
-6
-4
-2
0
2
4
6
Error Between Fourier Approximation and Actual Function (N = 64)
Error
x
0.6
0.4
0.2
0


Fourier Series Approximation (N = 128)
Piecewise Function
Fourier Approximation
y
x
2.5
2
1.5
1
0.5
0
-6
-4
-2
0
2
4
6
Error Between Fourier Approximation and Actual Function (N = 128)
Error
x
0.6
0.4
0.2
0


Fourier Series Approximation (N = 194)
Piecewise Function
Fourier Approximation
y
x
2.5
2
1.5
1
0.5
0
-6
-4
-2
0
2
4
6
Error Between Fourier Approximation and Actual Function (N = 194)
Error
x
0.6
0.4
0.2
0

Hasil Kode No. 2

```
For N = 1:
a0 = 0.25200651
an 1 = 2.39286379    bn 1 = 0.52444071
Error cos (integral) = 5.66295330
Max error for one x value (cos)= 2.64485847
Error sin (integral) = 5.77996634
Max error for one x value (sin)= 2.44132211

For N = 2:
a0 = 0.25200651
an 1 = 2.39286379    bn 1 = 0.52444071
an 2 = -0.45193230    bn 2 = 2.26111051
Error cos (integral) = 5.54623257
Max error for one x value (cos)= 2.59278958
Error sin (integral) = 5.89180645
Max error for one x value (sin)= 2.63922469

For N = 4:
a0 = 0.25200651
an 1 = 2.39286379    bn 1 = 0.52444071
an 2 = -0.45193230    bn 2 = 2.26111051
an 3 = -0.39470733    bn 3 = -0.16918881
an 4 = -0.10997331    bn 4 = 0.64072563
Error cos (integral) = 5.25951332
Max error for one x value (cos)= 2.33327953
Error sin (integral) = 5.45837886
Max error for one x value (sin)= 2.67471287

For N = 8:
a0 = 0.25200651
an 1 = 2.39286379    bn 1 = 0.52444071
an 2 = -0.45193230    bn 2 = 2.26111051
an 3 = -0.39470733    bn 3 = -0.16918881
an 4 = -0.10997331    bn 4 = 0.64072563
an 5 = -0.16357882    bn 5 = -0.11800928
an 6 = -0.04664757    bn 6 = 0.36666830
an 7 = -0.08807907    bn 7 = -0.08752395
an 8 = -0.02448356    bn 8 = 0.25911640
Error cos (integral) = 5.01890448
Max error for one x value (cos)= 2.48060612
Error sin (integral) = 5.27803727
Max error for one x value (sin)= 2.63016364

For N = 16:
a0 = 0.25200651
an 1 = 2.39286379    bn 1 = 0.52444071
an 2 = -0.45193230    bn 2 = 2.26111051
an 3 = -0.39470733    bn 3 = -0.16918881
an 4 = -0.10997331    bn 4 = 0.64072563
an 5 = -0.16357882    bn 5 = -0.11800928
an 6 = -0.04664757    bn 6 = 0.36666830
an 7 = -0.08807907    bn 7 = -0.08752395
an 8 = -0.02448356    bn 8 = 0.25911640
an 9 = -0.05552599    bn 9 = -0.06909564
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an 10 = -0.01422479    bn 10 = 0.20140395
an 11 = -0.03871797    bn 11 = -0.05694449
an 12 = -0.00865212    bn 12 = 0.16516433
an 13 = -0.02895050    bn 13 = -0.04837406
an 14 = -0.00529198    bn 14 = 0.14018257
an 15 = -0.02278488    bn 15 = -0.04201859
an 16 = -0.00311112    bn 16 = 0.12186698
Error cos (integral) = 4.94393856
Max error for one x value (cos)= 2.43113538
Error sin (integral) = 5.13186918
Max error for one x value (sin)= 2.59895467

For N = 32:
a0 = 0.25200651
an 1 = 2.39286379    bn 1 = 0.52444071
an 2 = -0.45193230    bn 2 = 2.26111051
an 3 = -0.39470733    bn 3 = -0.16918881
an 4 = -0.10997331    bn 4 = 0.64072563
an 5 = -0.16357882    bn 5 = -0.11800928
an 6 = -0.04664757    bn 6 = 0.36666830
an 7 = -0.08807907    bn 7 = -0.08752395
an 8 = -0.02448356    bn 8 = 0.25911640
an 9 = -0.05552599    bn 9 = -0.06909564
an 10 = -0.01422479    bn 10 = 0.20140395
an 11 = -0.03871797    bn 11 = -0.05694449
an 12 = -0.00865212    bn 12 = 0.16516433
an 13 = -0.02895050    bn 13 = -0.04837406
an 14 = -0.00529198    bn 14 = 0.14018257
an 15 = -0.02278488    bn 15 = -0.04201859
an 16 = -0.00311112    bn 16 = 0.12186698
an 17 = -0.01864823    bn 17 = -0.03712269
an 18 = -0.00161592    bn 18 = 0.10783838
an 19 = -0.01574001    bn 19 = -0.03323732
an 20 = -0.00054642    bn 20 = 0.09673618
an 21 = -0.01361843    bn 21 = -0.03007964
an 22 = 0.00024489    bn 22 = 0.08772387
an 23 = -0.01202363    bn 23 = -0.02746298
an 24 = 0.00084675    bn 24 = 0.08025780
an 25 = -0.01079475    bn 25 = -0.02525926
an 26 = 0.00131514    bn 26 = 0.07396879
an 27 = -0.00982791    bn 27 = -0.02337779
an 28 = 0.00168679    bn 28 = 0.06859710
an 29 = -0.00905362    bn 29 = -0.02175255
an 30 = 0.00198662    bn 30 = 0.06395452
an 31 = -0.00842398    bn 31 = -0.02033439
an 32 = 0.00223201    bn 32 = 0.05990121
Error cos (integral) = 4.92413637
Max error for one x value (cos)= 2.41153010
Error sin (integral) = 5.03357795
Max error for one x value (sin)= 2.50927255

For N = 64:
a0 = 0.25200651
an 1 = 2.39286379    bn 1 = 0.52444071
```

```
an 2 = -0.45193230    bn 2 = 2.26111051
an 3 = -0.39470733    bn 3 = -0.16918881
an 4 = -0.10997331    bn 4 = 0.64072563
an 5 = -0.16357882    bn 5 = -0.11800928
an 6 = -0.04664757    bn 6 = 0.36666830
an 7 = -0.08807907    bn 7 = -0.08752395
an 8 = -0.02448356    bn 8 = 0.25911640
an 9 = -0.05552599    bn 9 = -0.06909564
an 10 = -0.01422479    bn 10 = 0.20140395
an 11 = -0.03871797    bn 11 = -0.05694449
an 12 = -0.00865212    bn 12 = 0.16516433
an 13 = -0.02895050    bn 13 = -0.04837406
an 14 = -0.00529198    bn 14 = 0.14018257
an 15 = -0.02278488    bn 15 = -0.04201859
an 16 = -0.00311112    bn 16 = 0.12186698
an 17 = -0.01864823    bn 17 = -0.03712269
an 18 = -0.00161592    bn 18 = 0.10783838
an 19 = -0.01574001    bn 19 = -0.03323732
an 20 = -0.00054642    bn 20 = 0.09673618
an 21 = -0.01361843    bn 21 = -0.03007964
an 22 = 0.00024489    bn 22 = 0.08772387
an 23 = -0.01202363    bn 23 = -0.02746298
an 24 = 0.00084675    bn 24 = 0.08025780
an 25 = -0.01079475    bn 25 = -0.02525926
an 26 = 0.00131514    bn 26 = 0.07396879
an 27 = -0.00982791    bn 27 = -0.02337779
an 28 = 0.00168679    bn 28 = 0.06859710
an 29 = -0.00905362    bn 29 = -0.02175255
an 30 = 0.00198662    bn 30 = 0.06395452
an 31 = -0.00842398    bn 31 = -0.02033439
an 32 = 0.00223201    bn 32 = 0.05990121
an 33 = -0.00790509    bn 33 = -0.01908592
an 34 = 0.00243539    bn 34 = 0.05633111
an 35 = -0.00747243    bn 35 = -0.01797826
an 36 = 0.00260582    bn 36 = 0.05316222
an 37 = -0.00710791    bn 37 = -0.01698869
an 38 = 0.00275005    bn 38 = 0.05033018
an 39 = -0.00679794    bn 39 = -0.01609916
an 40 = 0.00287320    bn 40 = 0.04778374
an 41 = -0.00653215    bn 41 = -0.01529508
an 42 = 0.00297918    bn 42 = 0.04548156
an 43 = -0.00630255    bn 43 = -0.01456459
an 44 = 0.00307104    bn 44 = 0.04338992
an 45 = -0.00610284    bn 45 = -0.01389791
an 46 = 0.00315118    bn 46 = 0.04148108
an 47 = -0.00592805    bn 47 = -0.01328692
an 48 = 0.00322151    bn 48 = 0.03973195
an 49 = -0.00577421    bn 49 = -0.01272482
an 50 = 0.00328357    bn 50 = 0.03812319
an 51 = -0.00563809    bn 51 = -0.01220589
an 52 = 0.00333862    bn 52 = 0.03663847
an 53 = -0.00551708    bn 53 = -0.01172523
an 54 = 0.00338766    bn 54 = 0.03526389
an 55 = -0.00540902    bn 55 = -0.01127870
```

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an 56 = 0.00343154    bn 56 = 0.03398758
an 57 = -0.00531213    bn 57 = -0.01086271
an 58 = 0.00347096    bn 58 = 0.03279928
an 59 = -0.00522491    bn 59 = -0.01047415
an 60 = 0.00350651    bn 60 = 0.03169016
an 61 = -0.00514613    bn 61 = -0.01011033
an 62 = 0.00353867    bn 62 = 0.03065249
an 63 = -0.00507472    bn 63 = -0.00976890
an 64 = 0.00356787    bn 64 = 0.02967954
Error cos (integral) = 4.91840442
Max error for one x value (cos)= 2.41701425
Error sin (integral) = 4.97642305
Max error for one x value (sin)= 2.43735252

For N = 128:
a0 = 0.25200651
an 1 = 2.39286379    bn 1 = 0.52444071
an 2 = -0.45193230    bn 2 = 2.26111051
an 3 = -0.39470733    bn 3 = -0.16918881
an 4 = -0.10997331    bn 4 = 0.64072563
an 5 = -0.16357882    bn 5 = -0.11800928
an 6 = -0.04664757    bn 6 = 0.36666830
an 7 = -0.08807907    bn 7 = -0.08752395
an 8 = -0.02448356    bn 8 = 0.25911640
an 9 = -0.05552599    bn 9 = -0.06909564
an 10 = -0.01422479    bn 10 = 0.20140395
an 11 = -0.03871797    bn 11 = -0.05694449
an 12 = -0.00865212    bn 12 = 0.16516433
an 13 = -0.02895050    bn 13 = -0.04837406
an 14 = -0.00529198    bn 14 = 0.14018257
an 15 = -0.02278488    bn 15 = -0.04201859
an 16 = -0.00311112    bn 16 = 0.12186698
an 17 = -0.01864823    bn 17 = -0.03712269
an 18 = -0.00161592    bn 18 = 0.10783838
an 19 = -0.01574001    bn 19 = -0.03323732
an 20 = -0.00054642    bn 20 = 0.09673618
an 21 = -0.01361843    bn 21 = -0.03007964
an 22 = 0.00024489    bn 22 = 0.08772387
an 23 = -0.01202363    bn 23 = -0.02746298
an 24 = 0.00084675    bn 24 = 0.08025780
an 25 = -0.01079475    bn 25 = -0.02525926
an 26 = 0.00131514    bn 26 = 0.07396879
an 27 = -0.00982791    bn 27 = -0.02337779
an 28 = 0.00168679    bn 28 = 0.06859710
an 29 = -0.00905362    bn 29 = -0.02175255
an 30 = 0.00198662    bn 30 = 0.06395452
an 31 = -0.00842398    bn 31 = -0.02033439
an 32 = 0.00223201    bn 32 = 0.05990121
an 33 = -0.00790509    bn 33 = -0.01908592
an 34 = 0.00243539    bn 34 = 0.05633111
an 35 = -0.00747243    bn 35 = -0.01797826
an 36 = 0.00260582    bn 36 = 0.05316222
an 37 = -0.00710791    bn 37 = -0.01698869
an 38 = 0.00275005    bn 38 = 0.05033018
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an 39 = -0.00679794 bn 39 = -0.01609916
an 40 = 0.00287320 bn 40 = 0.04778374
an 41 = -0.00653215 bn 41 = -0.01529508
an 42 = 0.00297918 bn 42 = 0.04548156
an 43 = -0.00630255 bn 43 = -0.01456459
an 44 = 0.00307104 bn 44 = 0.04338992
an 45 = -0.00610284 bn 45 = -0.01389791
an 46 = 0.00315118 bn 46 = 0.04148108
an 47 = -0.00592805 bn 47 = -0.01328692
an 48 = 0.00322151 bn 48 = 0.03973195
an 49 = -0.00577421 bn 49 = -0.01272482
an 50 = 0.00328357 bn 50 = 0.03812319
an 51 = -0.00563809 bn 51 = -0.01220589
an 52 = 0.00333862 bn 52 = 0.03663847
an 53 = -0.00551708 bn 53 = -0.01172523
an 54 = 0.00338766 bn 54 = 0.03526389
an 55 = -0.00540902 bn 55 = -0.01127870
an 56 = 0.00343154 bn 56 = 0.03398758
an 57 = -0.00531213 bn 57 = -0.01086271
an 58 = 0.00347096 bn 58 = 0.03279928
an 59 = -0.00522491 bn 59 = -0.01047415
an 60 = 0.00350651 bn 60 = 0.03169016
an 61 = -0.00514613 bn 61 = -0.01011033
an 62 = 0.00353867 bn 62 = 0.03065249
an 63 = -0.00507472 bn 63 = -0.00976890
an 64 = 0.00356787 bn 64 = 0.02967954
an 65 = -0.00500981 bn 65 = -0.00944780
an 66 = 0.00359445 bn 66 = 0.02876540
an 67 = -0.00495061 bn 67 = -0.00914521
an 68 = 0.00361872 bn 68 = 0.02790485
an 69 = -0.00489649 bn 69 = -0.00885952
an 70 = 0.00364095 bn 70 = 0.02709328
an 71 = -0.00484687 bn 71 = -0.00858930
an 72 = 0.00366134 bn 72 = 0.02632659
an 73 = -0.00480127 bn 73 = -0.00833328
an 74 = 0.00368011 bn 74 = 0.02560112
an 75 = -0.00475927 bn 75 = -0.00809033
an 76 = 0.00369742 bn 76 = 0.02491361
an 77 = -0.00472050 bn 77 = -0.00785944
an 78 = 0.00371341 bn 78 = 0.02426112
an 79 = -0.00468463 bn 79 = -0.00763967
an 80 = 0.00372822 bn 80 = 0.02364102
an 81 = -0.00465139 bn 81 = -0.00743022
an 82 = 0.00374196 bn 82 = 0.02305093
an 83 = -0.00462052 bn 83 = -0.00723033
an 84 = 0.00375473 bn 84 = 0.02248870
an 85 = -0.00459180 bn 85 = -0.00703934
an 86 = 0.00376662 bn 86 = 0.02195238
an 87 = -0.00456504 bn 87 = -0.00685662
an 88 = 0.00377771 bn 88 = 0.02144020
an 89 = -0.00454006 bn 89 = -0.00668162
an 90 = 0.00378807 bn 90 = 0.02095054
an 91 = -0.00451671 bn 91 = -0.00651382
an 92 = 0.00379776 bn 92 = 0.02048193

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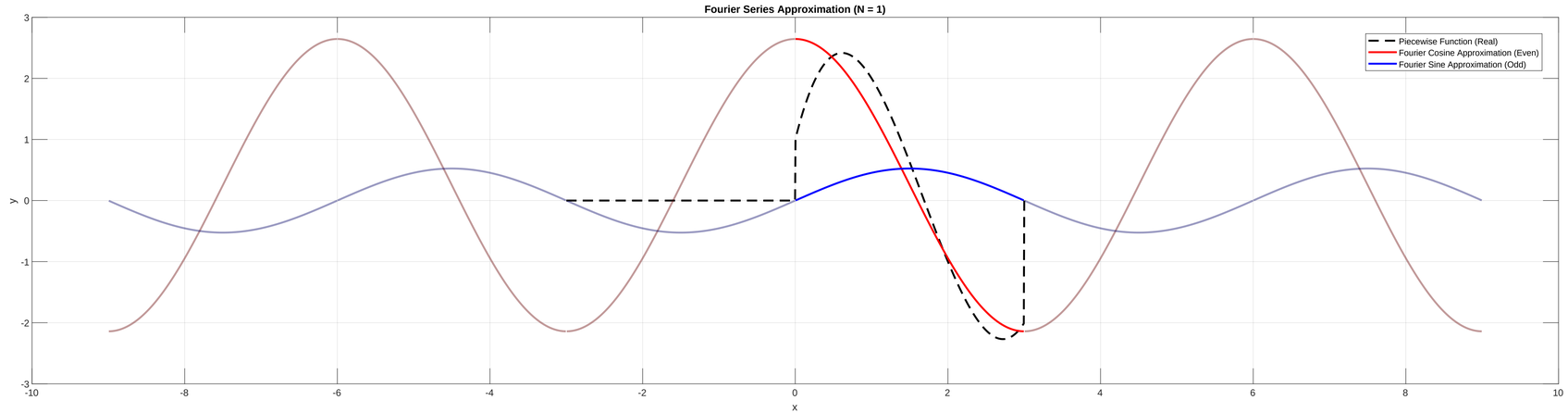
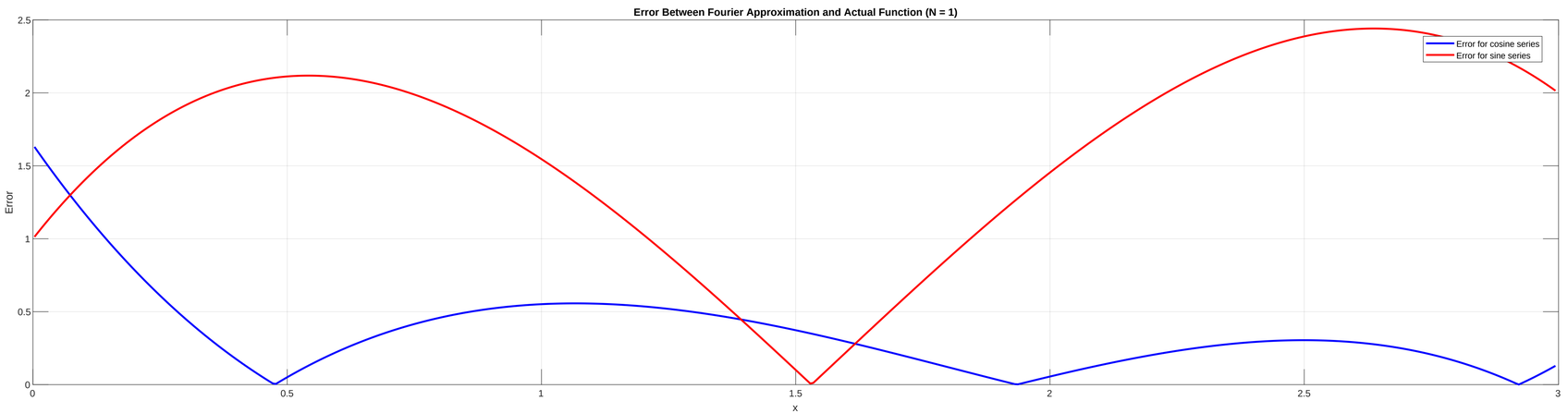
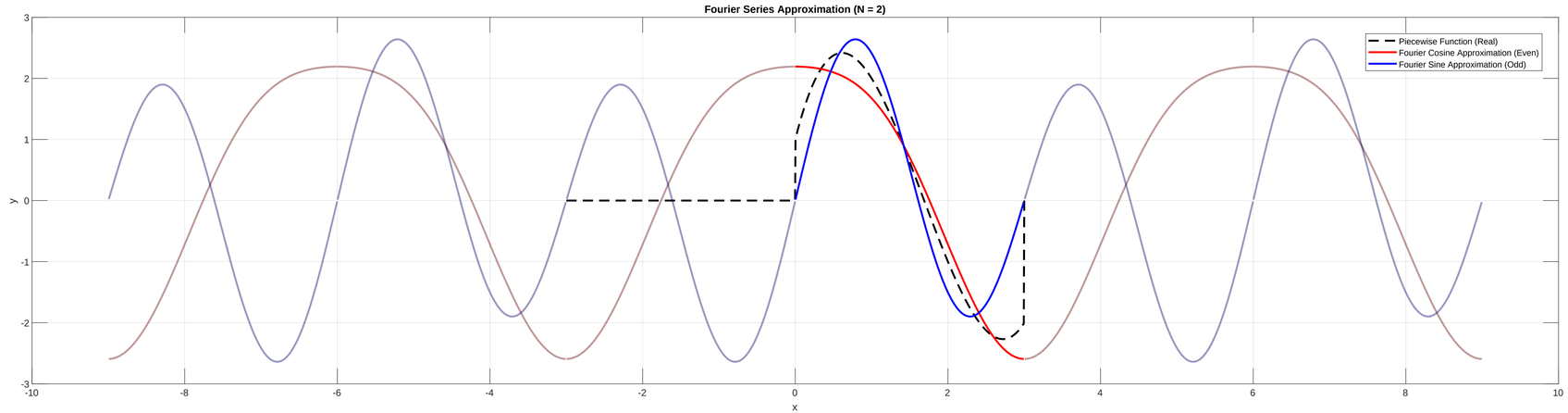
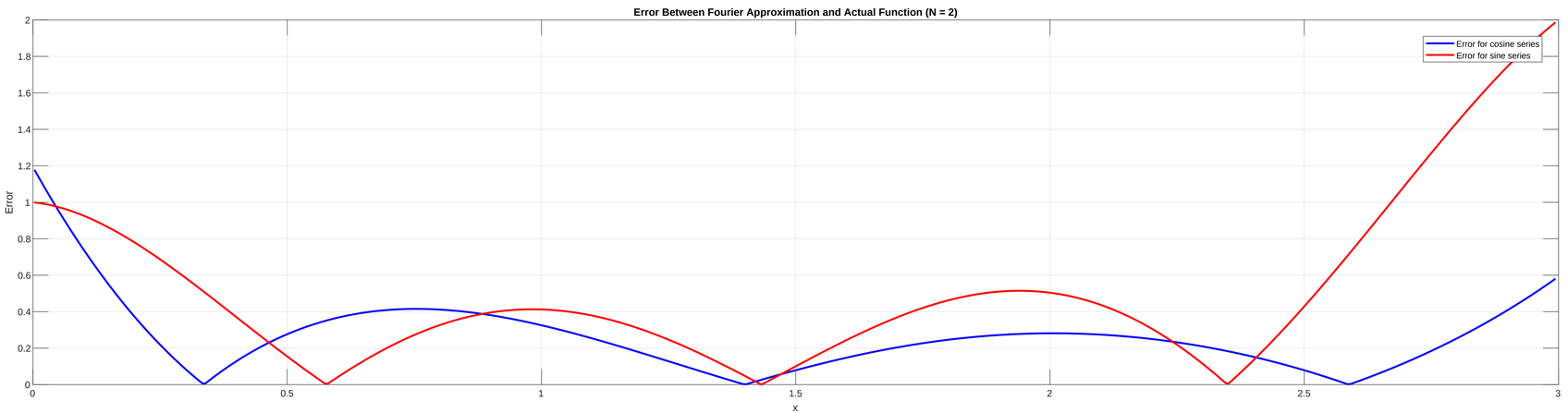
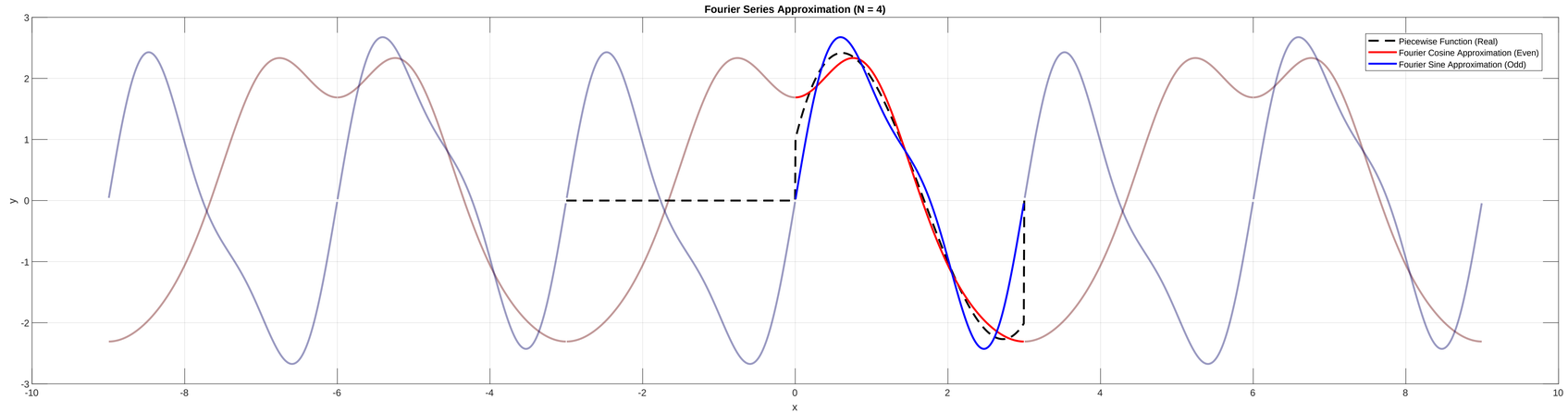
an 93 = -0.00449485    bn 93 = -0.00635278
an 94 = 0.00380685    bn 94 = 0.02003303
an 95 = -0.00447436    bn 95 = -0.00619804
an 96 = 0.00381537    bn 96 = 0.01960259
an 97 = -0.00445512    bn 97 = -0.00604924
an 98 = 0.00382337    bn 98 = 0.01918949
an 99 = -0.00443703    bn 99 = -0.00590600
an 100 = 0.00383090    bn 100 = 0.01879268
an 101 = -0.00442000    bn 101 = -0.00576800
an 102 = 0.00383799    bn 102 = 0.01841121
an 103 = -0.00440396    bn 103 = -0.00563493
an 104 = 0.00384468    bn 104 = 0.01804417
an 105 = -0.00438883    bn 105 = -0.00550650
an 106 = 0.00385100    bn 106 = 0.01769077
an 107 = -0.00437453    bn 107 = -0.00538246
an 108 = 0.00385696    bn 108 = 0.01735023
an 109 = -0.00436102    bn 109 = -0.00526257
an 110 = 0.00386261    bn 110 = 0.01702185
an 111 = -0.00434822    bn 111 = -0.00514659
an 112 = 0.00386796    bn 112 = 0.01670498
an 113 = -0.00433611    bn 113 = -0.00503433
an 114 = 0.00387302    bn 114 = 0.01639901
an 115 = -0.00432461    bn 115 = -0.00492558
an 116 = 0.00387783    bn 116 = 0.01610339
an 117 = -0.00431370    bn 117 = -0.00482017
an 118 = 0.00388240    bn 118 = 0.01581757
an 119 = -0.00430334    bn 119 = -0.00471792
an 120 = 0.00388674    bn 120 = 0.01554107
an 121 = -0.00429348    bn 121 = -0.00461868
an 122 = 0.00389087    bn 122 = 0.01527343
an 123 = -0.00428411    bn 123 = -0.00452231
an 124 = 0.00389481    bn 124 = 0.01501423
an 125 = -0.00427517    bn 125 = -0.00442866
an 126 = 0.00389856    bn 126 = 0.01476305
an 127 = -0.00426666    bn 127 = -0.00433760
an 128 = 0.00390213    bn 128 = 0.01451952
Error cos (integral) = 4.91677889
Max error for one x value (cos)= 2.41840151
Error sin (integral) = 4.94059795
Max error for one x value (sin)= 2.42489035
```

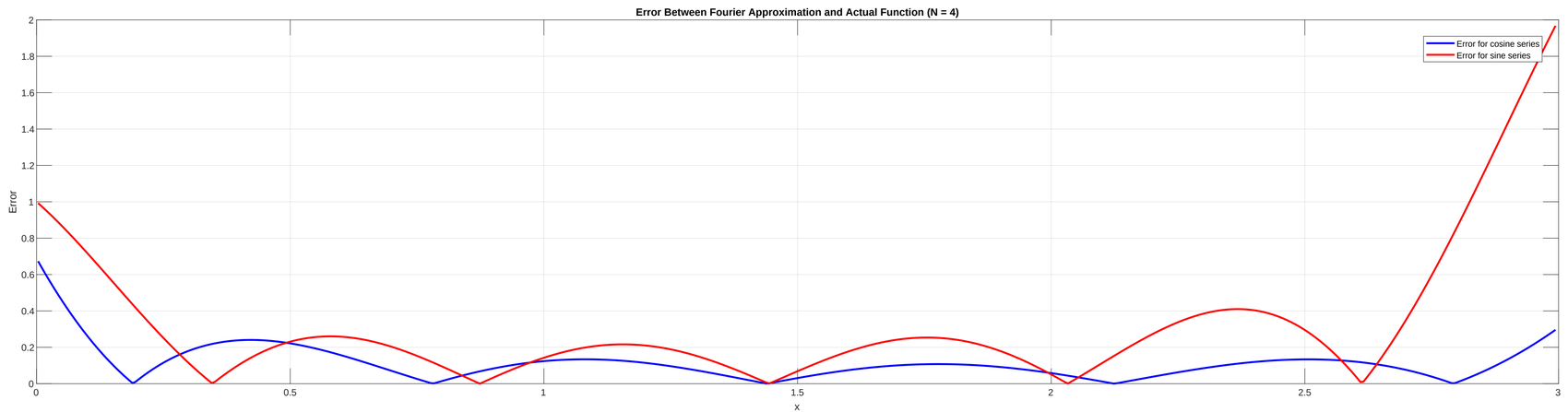

Fourier Series Approximation (N = 1)
Piecewise Function (Real)
Fourier Cosine Approximation (Even)
Fourier Sine Approximation (Odd)
y
x
3
2
1
0
-1
-2
-3
-10
-8
-6
-4
-2
0
2
4
6
8
10

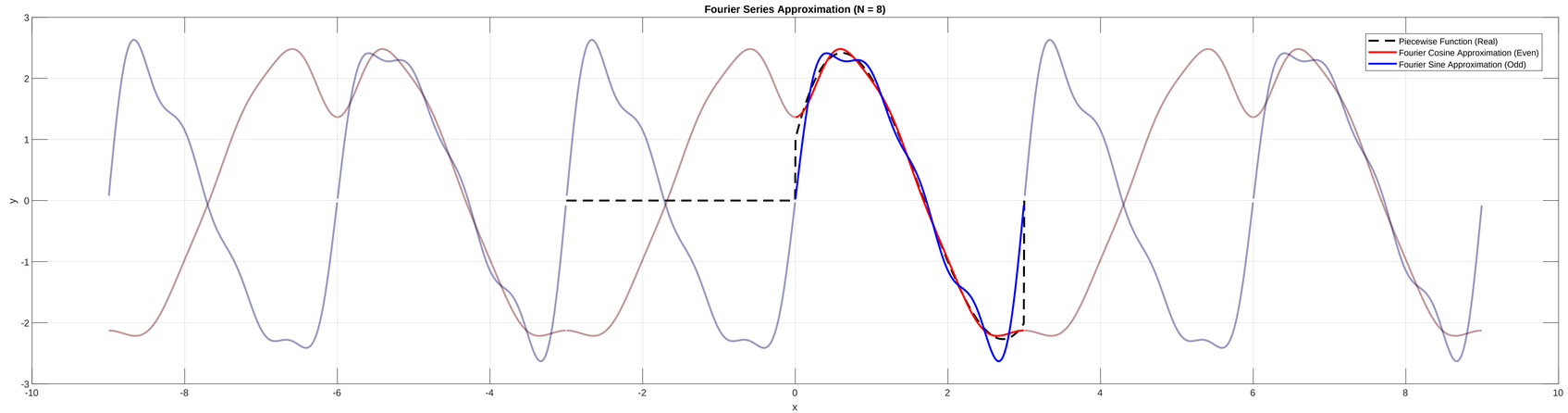

Error Between Fourier Approximation and Actual Function (N = 1)
Error for cosine series
Error for sine series
Error
x
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3

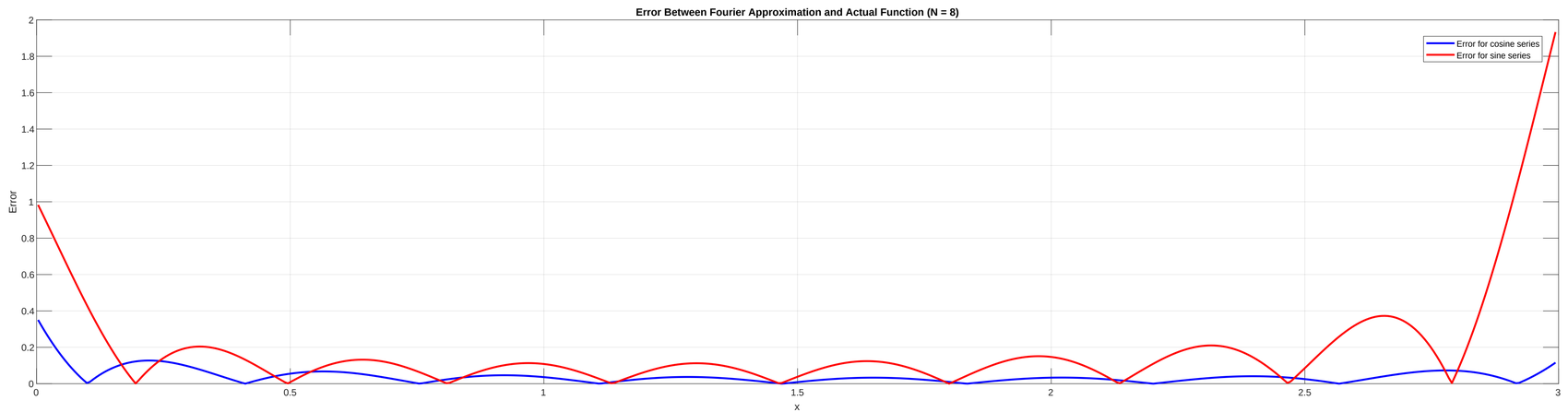

Fourier Series Approximation (N = 2)
Piecewise Function (Real)
Fourier Cosine Approximation (Even)
Fourier Sine Approximation (Odd)
y
x
3
2
1
0
-1
-2
-3
-10
-8
-6
-4
-2
0
2
4
6
8
10

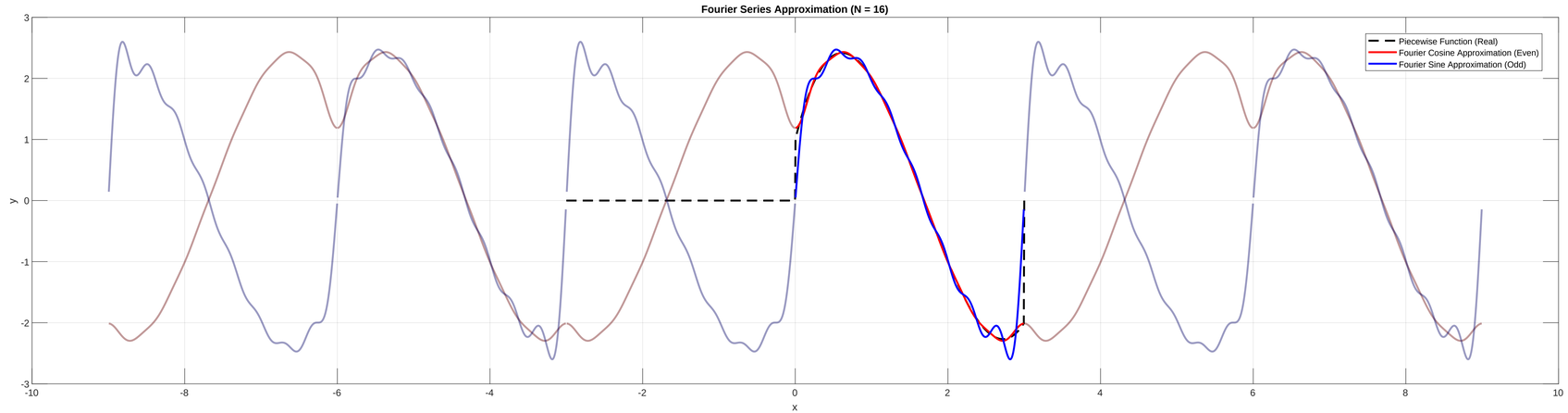

Error Between Fourier Approximation and Actual Function (N = 2)
Error for cosine series
Error for sine series
Error
x
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3

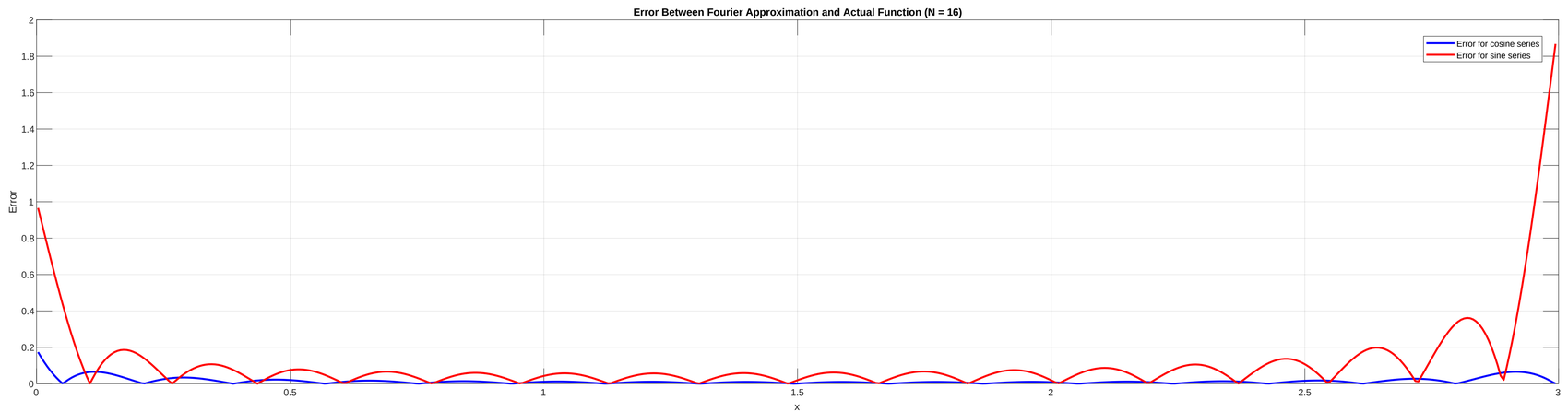

Fourier Series Approximation (N = 4)
Piecewise Function (Real)
Fourier Cosine Approximation (Even)
Fourier Sine Approximation (Odd)
y
x
3
2
1
0
-1
-2
-3
-10
-8
-6
-4
-2
0
2
4
6
8
10

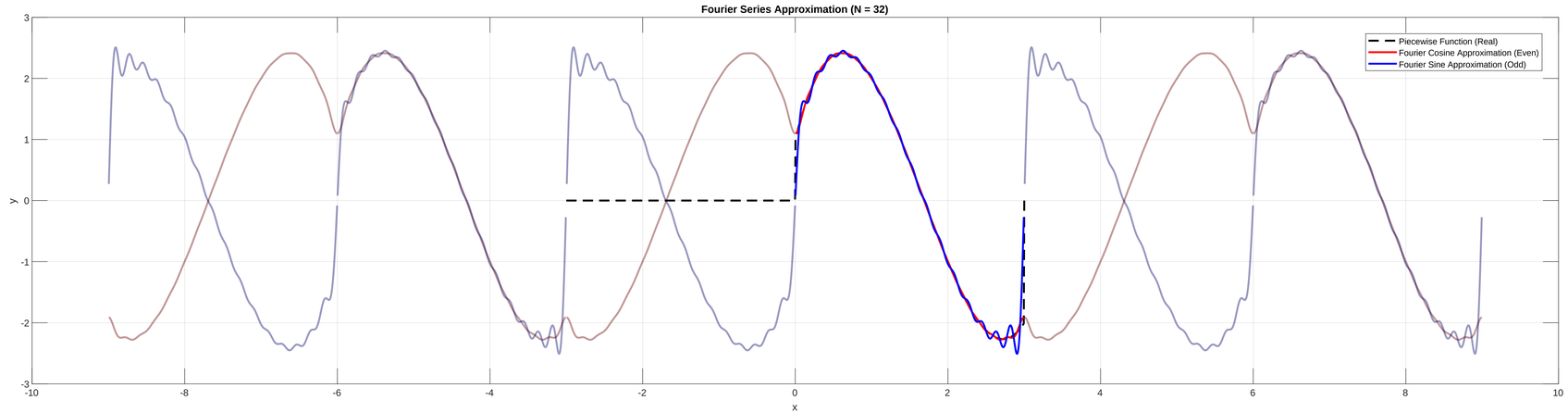

Error Between Fourier Approximation and Actual Function (N = 4)
Error for cosine series
Error for sine series
Error
x
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3

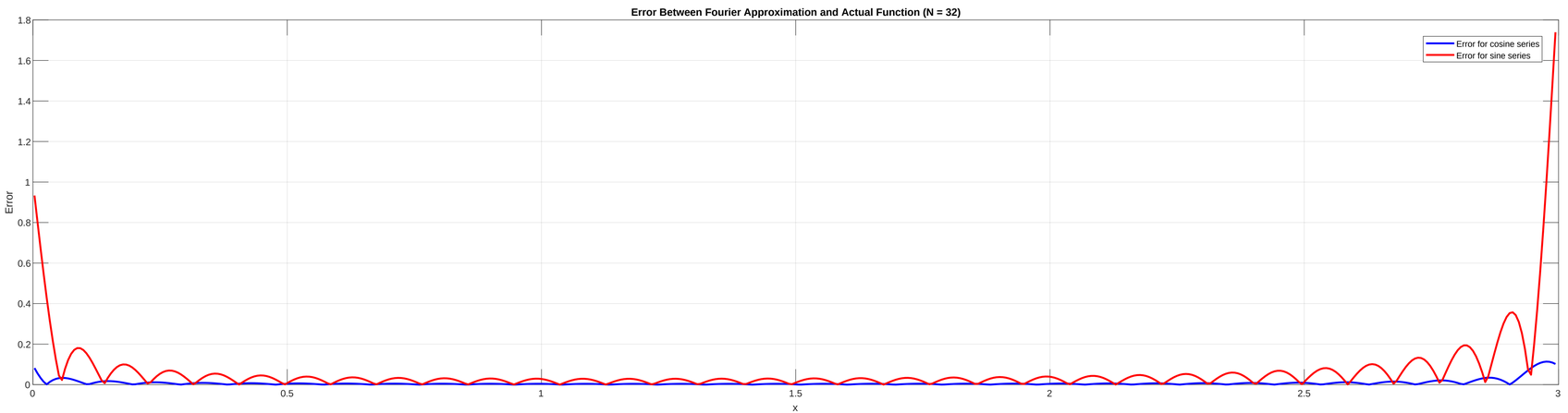

Fourier Series Approximation (N = 8)
Piecewise Function (Real)
Fourier Cosine Approximation (Even)
Fourier Sine Approximation (Odd)
y
x
3
2
1
0
-1
-2
-3
-10
-8
-6
-4
-2
0
2
4
6
8
10

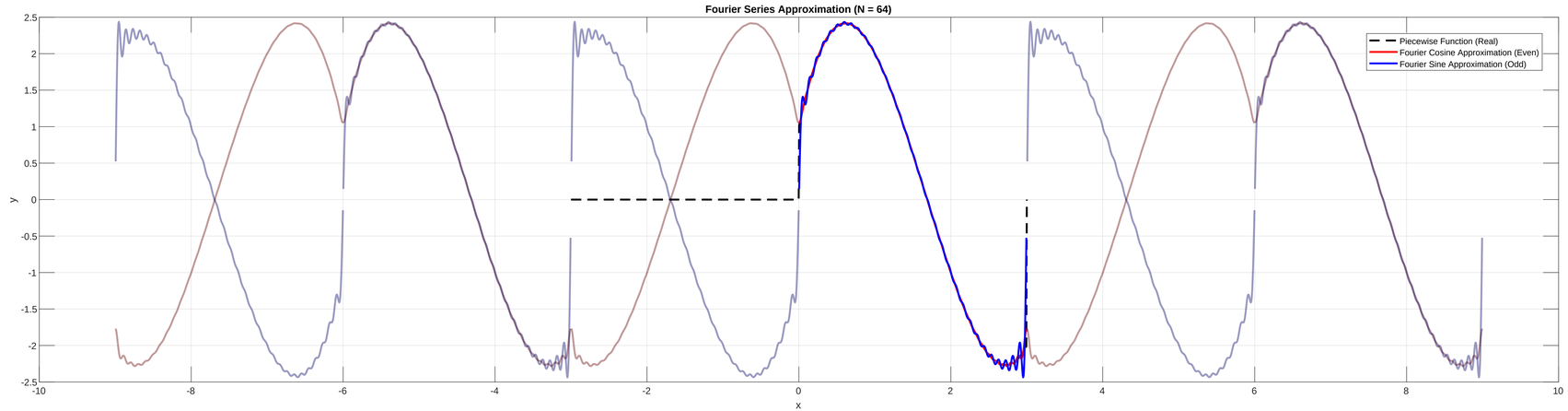

Error Between Fourier Approximation and Actual Function (N = 8)
Error for cosine series
Error for sine series
Error
x
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3

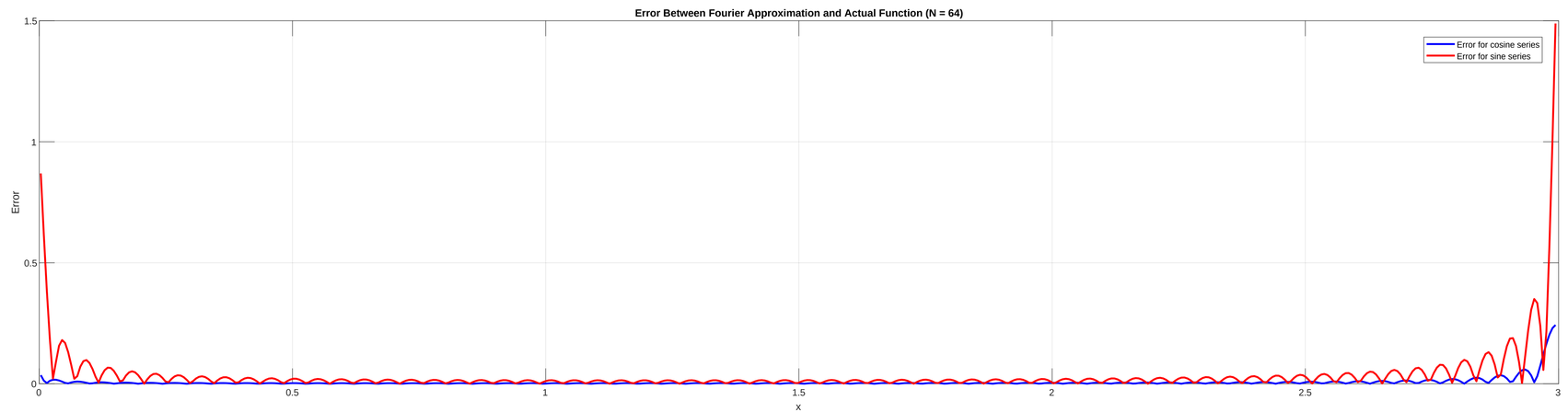

Fourier Series Approximation (N = 16)
Piecewise Function (Real)
Fourier Cosine Approximation (Even)
Fourier Sine Approximation (Odd)
y
x
3
2
1
0
-1
-2
-3
-10
-8
-6
-4
-2
0
2
4
6
8
10

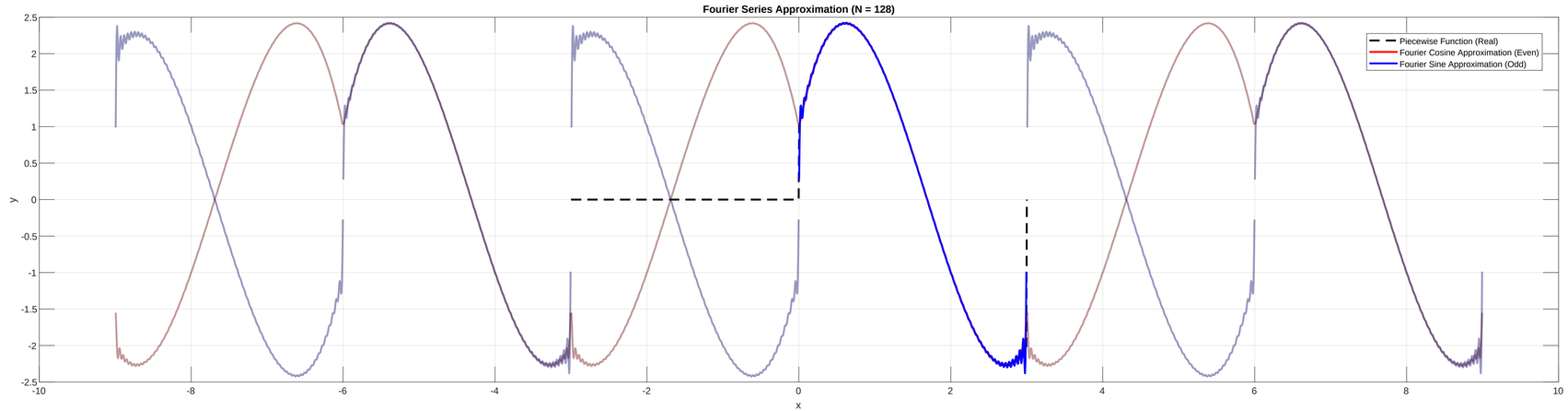

Error Between Fourier Approximation and Actual Function (N = 16)
Error for cosine series
Error for sine series
Error
x
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3

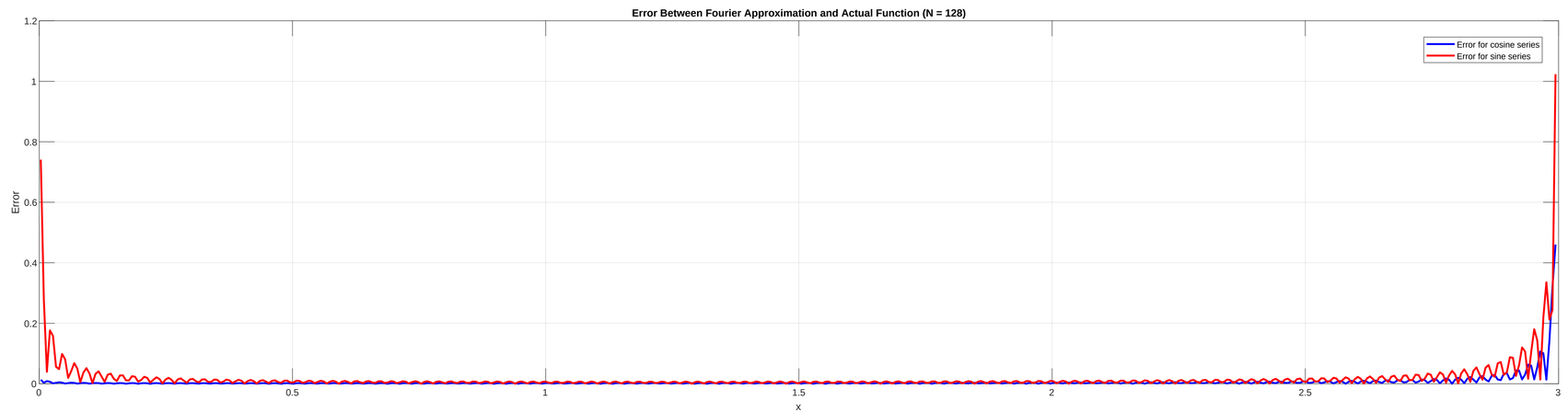

Fourier Series Approximation (N = 32)
Piecewise Function (Real)
Fourier Cosine Approximation (Even)
Fourier Sine Approximation (Odd)
y
x
3
2
1
0
-1
-2
-3
-10
-8
-6
-4
-2
0
2
4
6
8
10

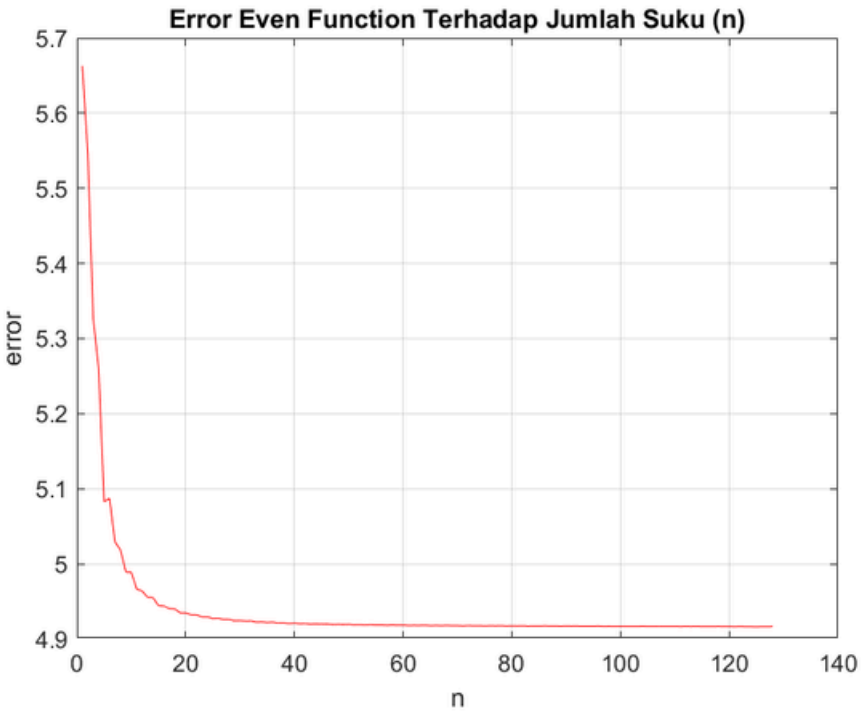

Error Between Fourier Approximation and Actual Function (N = 32)
Error for cosine series
Error for sine series
Error
x
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3

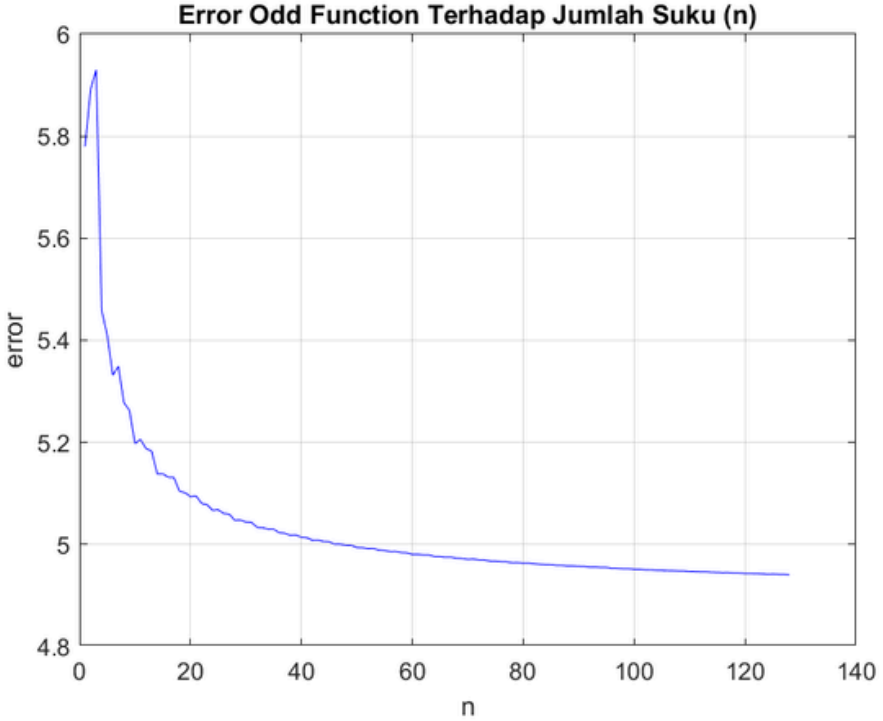

Fourier Series Approximation (N = 64)
Piecewise Function (Real)
Fourier Cosine Approximation (Even)
Fourier Sine Approximation (Odd)
y
x
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-10
-8
-6
-4
-2
0
2
4
6
8
10


Error Between Fourier Approximation and Actual Function (N = 64)
Error for cosine series
Error for sine series
Error
x
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3


Fourier Series Approximation (N = 128)
Piecewise Function (Real)
Fourier Cosine Approximation (Even)
Fourier Sine Approximation (Odd)
y
x
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
-10
-8
-6
-4
-2
0
2
4
6
8
10


Error Between Fourier Approximation and Actual Function (N = 128)
Error for cosine series
Error for sine series
Error
x
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
2.5
3


Error Even Function Terhadap Jumlah Suku (n)
error
n
5.7
5.6
5.5
5.4
5.3
5.2
5.1
5
4.9
0
20
40
60
80
100
120
140


Error Odd Function Terhadap Jumlah Suku (n)
error
n
6
5.8
5.6
5.4
5.2
5
4.8
0
20
40
60
80
100
120
140