Persamaan Diferensial Parsial

dan Metode Numerik

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* Untuk m = 1 → tulis tangan ^^

NO.1

1.) $f(x) = \begin{cases} -\frac{1}{2}x & , -2 \le x < 0; \\ 2x - \frac{1}{2}x^2 & , 0 \le x < 2; \end{cases}$ $\quad f(x+4) = f(x)$

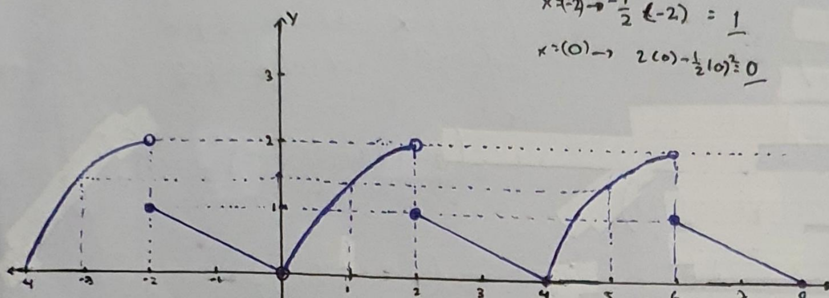
a.) function graph sketch (3 periods)

$x=(2) \rightarrow 2(2) - \frac{1}{2}(2)^2 = 2$

$x=(0) \rightarrow -\frac{1}{2}(0) = 0$

$x=(-2) \rightarrow -\frac{1}{2}(-2) = \underline{1}$

$x=(0) \rightarrow 2(0) - \frac{1}{2}(0)^2 = \underline{0}$



Repead periodically
$f(x+4) = f(x)$

Fourier Series → $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi x}{2}\right) + b_n \sin\left(\frac{n\pi x}{2}\right)\right)$

b.) $T = 4$, $L = \frac{T}{2} = 2$

Fourier series:

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right)$

$f(x) = \frac{11}{12} + \sum_{n=1}^{\infty}\left(a_n \cos\left(\frac{n\pi x}{2}\right) + b_n \sin\left(\frac{n\pi x}{2}\right)\right)$

$f(x) = 1{,}833 + \sum_{n=1}^{\infty}\left(a_n \cdot \cos\left(\frac{n\pi x}{2}\right) + b_n \sin\left(\frac{n\pi x}{2}\right)\right)$

input a_0

Partial sum (orde 1)

$S_1 = \frac{11}{12} - \frac{6}{\pi^2}\cos\left(\frac{\pi x}{2}\right) + \frac{(8+\pi^2)}{\pi^3}\sin\left(\frac{\pi x}{2}\right)$

akan dijelaskan bagaimana cara dapatnya

c.) $e_m(x) = f(x) - S_m(x)$ Describe how the fourier series seems to be converging untuk range $-2 \le x < 2$

→ Deret fourier akan converging sesuai dengan $f(x)$ pada range $-2 < x < 2$, sedangkan pada titik $x = -2$, deret fourier akan converging ke nilai $y = 1{,}5$,

$$\boxed{\text{didapat dari} = \frac{\lim_{x \to -2^-} f(x) + \lim_{x \to -2^+} f(x)}{2}}$$

* mengulang secara periodik → $f(x+4) = f(x)$

d.) plot $|e_m(x)| = |f(x) - S_m(x)|$ versus x for $0 \le x \le 2$ for several values of m

for m = 1

$S_m = S_1 = a_0 + \left(a_1 \cos\left(\frac{n\pi x}{2}\right) + b_1 \sin\left(\frac{n\pi x}{2}\right)\right)$

$a_n = \frac{1}{L}\int_a^b f(x)\cos\left(\frac{n\pi x}{L}\right)dx$

$b_n = \frac{1}{L}\int_a^b f(x)\sin\left(\frac{n\pi x}{L}\right)dx$

$a_0 = \frac{1}{2}\left(\int_{-2}^{0}\left(-\frac{1}{2}x \cos\left(\frac{n\pi x}{2}\right)\right)dx + \int_0^2\left(\left(2x - \frac{x^2}{2}\right)\cos\left(\frac{n\pi x}{2}\right)\right)dx\right)$

$a_0 = \frac{1}{2}\left(-\frac{1}{2}\int_{-2}^{0} x\,(1)\,dx + \int_0^2\left(2x - \frac{x^2}{2}\right)(1)\,dx\right)$

$a_0 = \frac{1}{2}\left(-\frac{1}{2}\left(\frac{x^2}{2}\right]_{-2}^{0} + \left(x^2 - \frac{x^3}{6}\right)\right]_0^2\right)$

$a_0 = \frac{1}{2}\left(-\frac{1}{2}\left(-\frac{(-2)^2}{2}\right) + \left(2^2 - \frac{2^3}{6}\right)\right)$

$a_0 = \frac{1}{2}\left(1 + 4 - \frac{4}{3}\right) = \frac{1}{2}\left(\frac{15}{3} - \frac{4}{3}\right)$

$a_0 = \frac{1}{2}\left(\frac{11}{3}\right) = \frac{11}{6}$

$a_0 = \frac{11}{6} \approx 1{,}833$

$$a_1 = \frac{1}{2}\left(\int_{-2}^{0}\left(-\frac{1}{2}x\cos\left(\frac{1 \cdot \pi x}{2}\right)\right)dx + \int_{0}^{2}\left(\left(2x-\frac{1}{2}x^2\right)\cos\left(\frac{1\pi x}{2}\right)\right)dx\right)$$

$$a_1 = \frac{1}{2}\left(-\frac{1}{2}\int_{-2}^{0}\left(x\cos\left(\frac{\pi x}{2}\right)\right)dx + \int_{0}^{2}\left(\left(2x-\frac{1}{2}x^2\right)\cos\left(\frac{\pi x}{2}\right)\right)dx\right)$$

$$\int_{-2}^{0} x\cos\left(\frac{\pi x}{2}\right)dx = \frac{2x}{\pi}\sin\left(\frac{\pi x}{2}\right)\Big]_{-2}^{0} - \frac{2}{\pi}\int_{-2}^{0}\sin\left(\frac{\pi x}{2}\right)dx$$

$u = x \qquad dv = \cos\left(\frac{\pi x}{2}\right)dx$

$du = dx \qquad v = \sin\left(\frac{\pi x}{2}\right)\left(\frac{2}{\pi}\right)$

$$\int_{-2}^{0}\left(x\cos\left(\frac{\pi x}{2}\right)\right)dx = \frac{2x}{\pi}\sin\left(\frac{\pi x}{2}\right)\Big]_{-2}^{0} - \frac{2}{\pi}\left(\frac{-2}{\pi}\cos\left(\frac{\pi x}{2}\right)\Big]_{-2}^{0}\right)$$

$$\int_{-2}^{0} x\cos\left(\frac{\pi x}{2}\right)dx = \frac{2x}{\pi}\sin\left(\frac{\pi x}{2}\right)\Big]_{-2}^{0} + \frac{4}{\pi^2}\cos\left(\frac{\pi x}{2}\right)\Big]_{-2}^{0}$$

$$\int_{-2}^{0} x\cos\left(\frac{\pi x}{2}\right)dx = \frac{2}{\pi}\left(0\sin(0) - (-2)\sin(-\pi)\right) + \frac{4}{\pi^2}\left(\cos(0) - \cos(-\pi)\right)$$

$$\int_{-2}^{0} x\cos\left(\frac{\pi x}{2}\right)dx = 0 + \frac{4}{\pi^2}\left(1-(-1)\right) = \frac{4}{\pi^2}(2) = \frac{8}{\pi^2} \simeq 0{,}810$$

$$\int_{0}^{2}\left(\left(2x-\frac{1}{2}x^2\right)\cos\left(\frac{\pi x}{2}\right)\right)dx = \underbrace{\cancel{\frac{2}{\pi}\left(2x-\frac{1}{2}x^2\right)\sin\left(\frac{\pi x}{2}\right)\Big]_{0}^{2}}}_{=0} - \frac{2}{\pi}\int_{0}^{2}(2-x)\sin\left(\frac{\pi x}{2}\right)dx$$

$u = 2x - \frac{1}{2}x^2 \qquad dv = \cos\left(\frac{\pi x}{2}\right)$

$du = (2-x)\,dx \qquad v = \sin\left(\frac{\pi x}{2}\right)\cdot\frac{2}{\pi}$

$$= -\frac{2}{\pi}\int_{0}^{2}(2-x)\sin\left(\frac{\pi x}{2}\right)dx$$

$u = 2-x \qquad dv = \sin\left(\frac{\pi x}{2}\right)dx$

$du = -dx \qquad v = -\frac{2}{\pi}\cos\left(\frac{\pi x}{2}\right)$

$$= -\frac{2}{\pi}\left(-\frac{2}{\pi}(2-x)\cos\left(\frac{\pi x}{2}\right)\Big]_{0}^{2} - \underbrace{\cancel{\frac{2}{\pi}\int_{0}^{2}\cos\left(\frac{\pi x}{2}\right)dx}}_{=0}\right)$$

1 periode

$$= \frac{4}{\pi^2}\left((2-x)\cos\left(\frac{\pi x}{2}\right)\Big]_{0}^{2}\right)$$

$$= \frac{4}{\pi^2}\left((2-2)\cos(\pi x) - (2)(\cos(0))\right)$$

$$= \frac{4}{\pi^2}(-2)$$

$$= \frac{-8}{\pi^2} \simeq -0{,}810$$

$$a_1 = \frac{1}{2}\left(-\frac{1}{2}\left(\frac{8}{\pi^2}\right) + -\left(\frac{8}{\pi^2}\right)\right)$$

$$a_1 = \frac{-2}{\pi^2} - \frac{4}{\pi^2} = \frac{-6}{\pi^2} \simeq -0{,}608$$

$$b_1 = \frac{1}{2}\left(\int_{-2}^{0}\left(-\frac{1}{2}x\sin\left(\frac{\pi x}{2}\right)\right)dx + \int_{0}^{2}\left(\left(2x-\frac{1}{2}x^2\right)\sin\left(\frac{\pi x}{2}\right)\right)dx\right)$$

$$b_1 = \frac{1}{2}\left(-\frac{1}{2}\int_{-2}^{0}x\sin\left(\frac{\pi x}{2}\right)dx + \int_{0}^{2}\left(\left(2x-\frac{1}{2}x^2\right)\sin\left(\frac{\pi x}{2}\right)\right)dx\right.$$

$$\int_{-2}^{0}x\sin\left(\frac{\pi x}{2}\right)dx = \left(-\frac{2x}{\pi}\cos\left(\frac{\pi}{2}x\right)\right)\Big]_{-2}^{0} - \underbrace{\cancel{-\frac{2}{\pi}\int_{-2}^{0}\cos\left(\frac{\pi}{2}x\right)dx}}_{=0}$$

$u = x \qquad dv = \sin\left(\frac{\pi x}{2}\right)dx$

$du = dx \qquad v = \frac{-2}{\pi}\cos\left(\frac{\pi}{2}x\right)$

$$= \frac{-2(0)}{\pi}\cos\left(\frac{\pi}{2}\cdot 0\right) - -\frac{2}{\pi}(-2)\cos(-\pi)$$

$$= \frac{-4}{\pi}(-1)$$

$$= \frac{4}{\pi} = 1{,}273$$

$$\int_{0}^{2}\left(\left(2x-\frac{1}{2}x^2\right)\sin\left(\frac{\pi x}{2}\right)\right)dx$$

$u = 2x - \frac{1}{2}x^2 \qquad dv = \sin\left(\frac{\pi x}{2}\right)dx$

$du = 2-x \qquad v = -\frac{2}{\pi}\cos\left(\frac{\pi x}{2}\right)$

$$= \left(2x-\frac{1}{2}x^2\right)\left(-\frac{2}{\pi}\right)\left(\cos\frac{\pi x}{2}\right)\Big]_{0}^{2} + \frac{2}{\pi}\int_{0}^{2}(2-x)\cos\left(\frac{\pi x}{2}\right)dx$$

$$= \left(2\left(-\frac{2}{\pi}\right)(-1) - 0\right) + \frac{2}{\pi}\left[\underbrace{\cancel{\frac{2}{\pi}(2-x)\sin\left(\frac{\pi x}{2}\right)}\Big]_{0}^{2}}_{=0} - \frac{2}{\pi}\int_{0}^{2}\sin\left(\frac{\pi x}{2}\right)dx\right)$$

$u = 2-x \qquad dv = \cos\left(\frac{\pi x}{2}\right)$

$du = -dx \qquad v = \frac{2}{\pi}\sin\left(\frac{\pi x}{2}\right)$

$$= \left(\frac{+4}{\pi} + \frac{2}{\pi}\left(-\frac{2}{\pi}\cdot +\frac{2}{\pi}\cos\left(\frac{\pi x}{2}\right)\Big]_{0}^{2}\right)\right)$$

$$= \left(\frac{+4}{\pi} + \frac{2}{\pi}\left(\frac{-4}{\pi^2}(-1-1)\right)\right)$$

$$= \left(\frac{+4}{\pi} + \frac{2}{\pi}\left(\frac{8}{\pi^2}\right)\right) = \frac{+4}{\pi} + \frac{16}{\pi^3} = \frac{16+4\pi^3}{\pi^3} \simeq 1{,}789$$

$$b_1 = \frac{1}{2}\left(-\frac{1}{2}\left(\frac{4}{\pi}\right) + \frac{16+4\pi^2}{\pi^3}\right) = \frac{1}{2}\left(-\frac{2}{\pi} + \frac{16+4\pi^2}{\pi^3}\right)$$

$$b_1 = \frac{-1}{\pi} + \frac{8+2\pi^2}{\pi^3} = \frac{8+2\pi^2-\pi^2}{\pi^3} = \frac{8+\pi^2}{\pi^3} \simeq 0{,}576$$

So, the $S_1 = \frac{11}{12} - \frac{6}{\pi^2}\cos\left(\frac{\pi x}{2}\right) + \frac{(8+\pi^2)}{\pi^3}\sin\left(\frac{\pi x}{2}\right)$ so $e_1(x) \leq$

The error will be

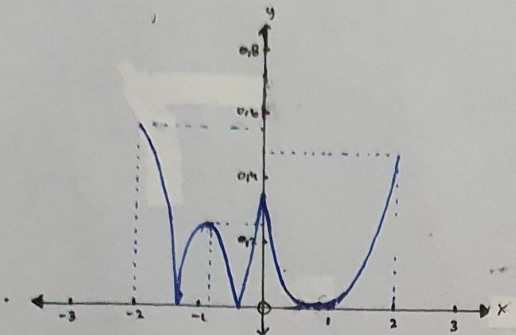
$|e_1(x)| = |f(x) - S_1(x)| = 0{,}524$ → the highest value how i get it, see ... (EXPLANATION)

max

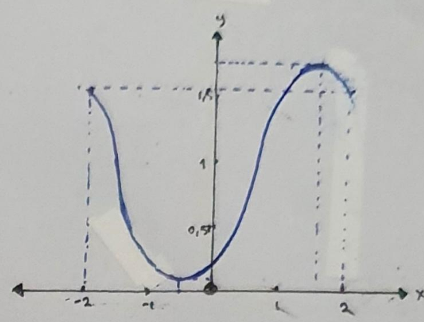
How the error graph looks like :

for $-2 \leq x \leq 2$ → then it will repeat periodically

1.) Grafik error, untuk e_1



2.) Grafik S_1



→ didapatkan dengan input $f(x)$, $S_1(x)$, dan $e_1(x)$ ke geogebra

e.) error untuk $m = 1$ → $e_1 = 0.622$ → got from integrating $e_m(x)$ ($e_1(x)$) with trapezional rule $e_1 = \sqrt{\frac{1}{2L}\int_{-2}^{2}(f(x) - S_1(x))^2\,dx}$

untuk dapat error $\leq 0{,}01$ → $e_m \leq 0{,}01$, maka $m = 194$

didapatkan dengan kode untuk mencari deret fourier.

saat $m = 194$, $e_{194} = 0{,}0099943777 37544885$ → memenuhi syarat $e_m \leq 0{,}01$

berdasarkan mean squared error

→ kalau patokan mean squared error, kalau patokannya $|e_m(x)|$, maka

agar $|e_m(x)| \leq 0{,}01$

→ m = ga mungkin → error max pasti arah ke 0,5, that area dimana lim kiri dan lim kanan beda

untuk $m = 2$, $m = 4$, $m = 8$, $m = 16$, $m = 32$, $m = 64$, $m = 128$, dan $m = 194$ akan dibuat menggunakan matlab

(EXPLANATION)

$e_1(x) = 2(x) - \frac{1}{2}x^2 - \frac{11}{12} + \frac{6}{\pi^2}\cos\left(\frac{\pi x}{2}\right) - \frac{(8+\pi^2)}{\pi^3}\sin\left(\frac{\pi x}{2}\right)$

coba $e_1(-2) = 2(-2) - \frac{1}{2}(-2)^2 - \frac{11}{12} + \frac{6}{\pi^2}\cos\left(\frac{-2\pi}{2}\right) - \frac{8+\pi^2}{\pi^3}\left(\sin\left(\frac{-2\pi}{2}\right)\right)$

$e_1(-2) = 0{,}524$

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PR #1 PDPMN

No. 2 ! (n = 2 → dikerjakan manual, selebihnya matlab)

No. 30 !

$f(x) = x^3 - 5x^2 + 5x + 1 \qquad 0 < x < 3$

a.) → g(x) dan h(x)

Saat :

$x = 0 \rightarrow f(0) = 0^3 - 5(0)^2 + 5(0) + 1 = 1$

$x = 3 \rightarrow f(3) = 3^3 - 5(3)^2 + 5(3) + 1 = -2$

$f(x) = 0 \rightarrow$

$0 = x^3 - 5x^2 + 5x + 1$

$0 = x((x-5)x + 5) + 1$

↳ saat $x_1 = -0{,}17009$

$\boxed{x_2 = 1{,}6889}$ → hanya x_2 yang berada di range $0 < x < 3$

$x_3 = 3{,}4812$

$f'(x) = 3x^2 - 10x + 5$

$0 = 3x^2 - 10x + 5$

$x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x_{1,2} = \dfrac{-(-10) \pm \sqrt{10^2 - 4(3)(5)}}{2(3)}$

$x_{1,2} = \dfrac{10 \pm \sqrt{40}}{6}$

$x_1 = \dfrac{5 + \sqrt{10}}{3} = 2{,}721$ → lembah

$x_2 = \dfrac{5 - \sqrt{10}}{3} \approx 0{,}612$ → puncak

ada di range $0 < x < 3$

→ Saat $x = 0{,}612$

↳ $f(0{,}612) = (0{,}612)^3 - 5(0{,}612)^2 + 5 \cdot 0{,}612 + 1$

$f(0{,}612) = 2{,}416$

→ Saat $x = 2{,}721$ → $f(2{,}721) = (2{,}721)^3 - 5(2{,}721)^2 + 5(2{,}721) + 1$

$f(2{,}721) = -2{,}268$

$0 < x < L$

$0 < x < 3 \rightarrow L = 3$

↓

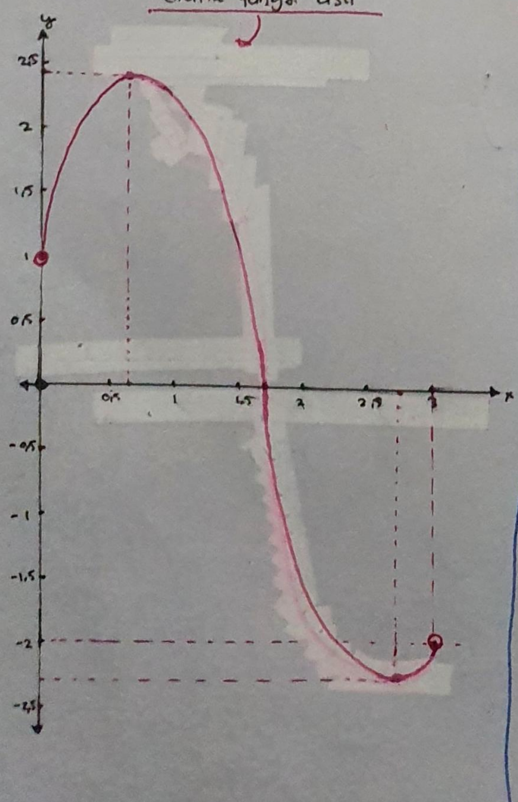
so, new function will have period of 2L

↳ $T = 2 \cdot 3 = 6$

$g(x) \rightarrow f(x)$ fungsi genap

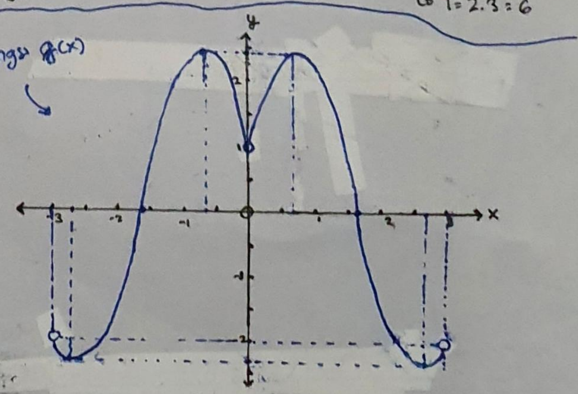
$h(x) \rightarrow f(x)$ fungsi ganjil

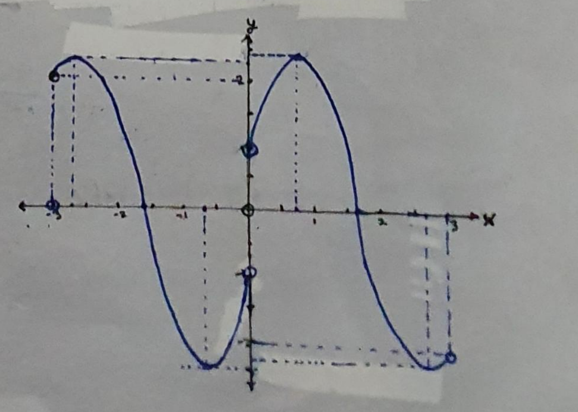
Grafik fungsi asli



∴

grafik fungsi g(x)

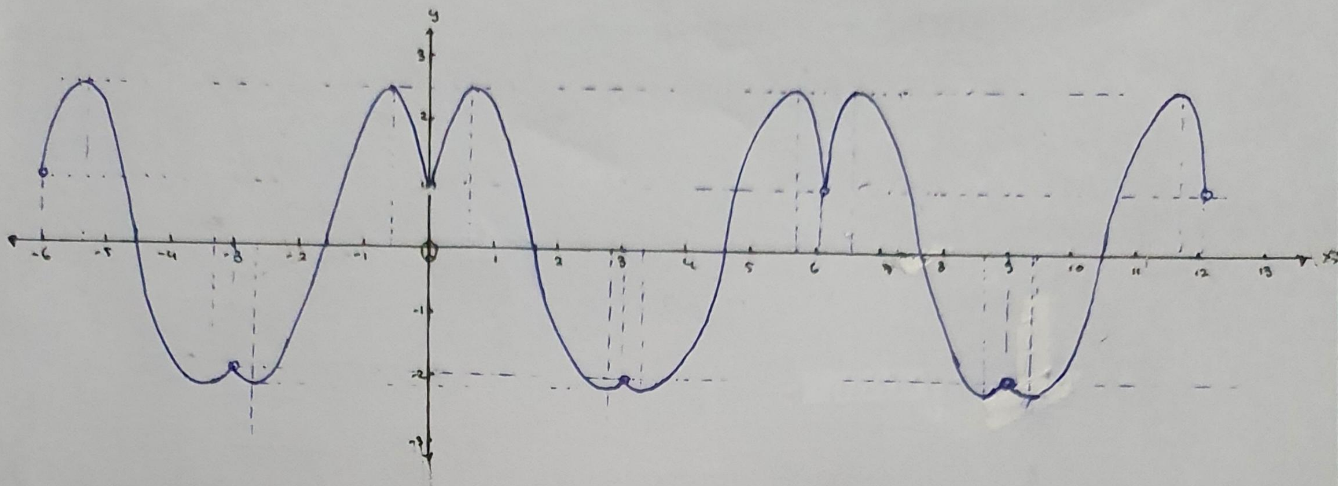




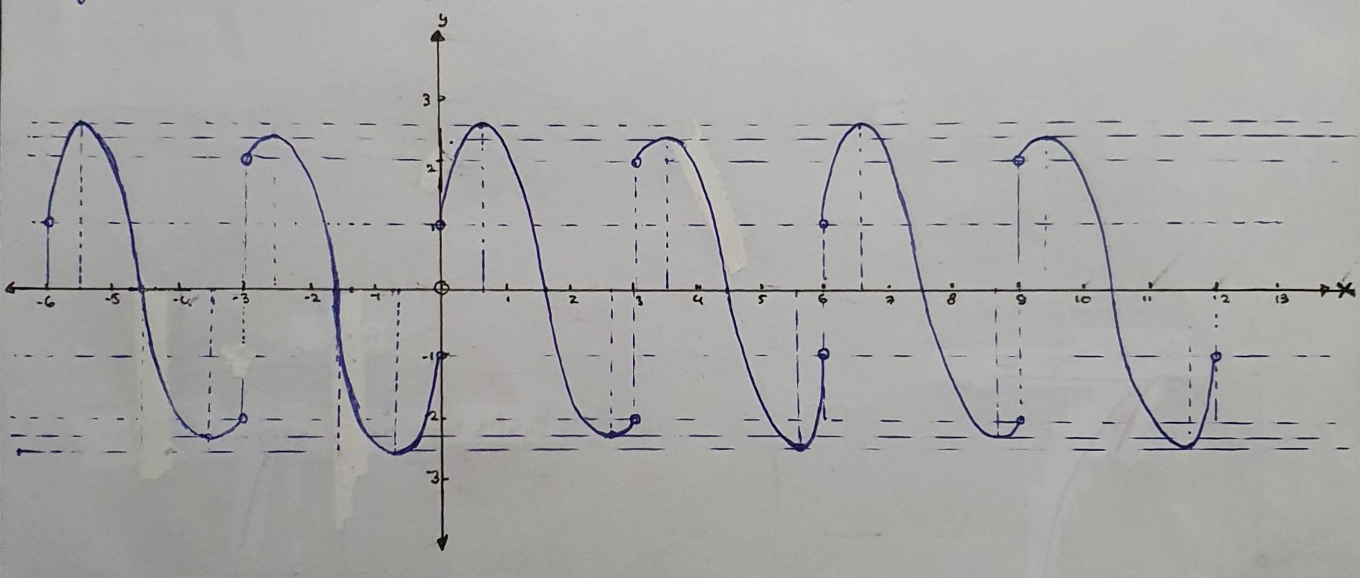
grafik fungsi h(x)

a.) Plot for g(x) and h(x) for 3 periods

(grafik untuk g(x))



grafik untuk h(x)



b.) $f(x) = x^3 - 5x^2 + 5x + 1 \qquad 0 < x < 3$

karena ini membuat fungsi jadi ada sifat ganjil

untuk g(x) (fungsi genap) $g(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{2}\right) + \cancel{b_n \sin\left(\frac{n\pi x}{2}\right)} \right)$

$$g(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{2}\right) \right)$$

ini membuat fungsi punya sifat genap

untuk h(x) (fungsi ganjil) → $h(x) = \cancel{a_0} + \sum_{n=1}^{\infty} \left(\cancel{a_n \cos\left(\frac{n\pi x}{2}\right)} + b_n \sin\left(\frac{n\pi x}{2}\right) \right)$

$$h(x) = \cancel{a_0} + \sum_{n=1}^{\infty} \left(b_n \sin\left(\frac{n\pi x}{2}\right) \right)$$

now: calculate a_0

$$a_n = \frac{1}{L}\int_a^b f(x)\cos\left(\frac{n\pi x}{L}\right)dx \quad b_n = \frac{1}{L}\int_a^b f(x)\sin\left(\frac{n\pi x}{2}\right)dx$$

$$a_0 = \frac{1}{L}\left[\int_0^3 (x^3 - 5x^2 + 5x + 1)(\cos(0))dx\right]$$

$$a_0 = \frac{1}{3}\int_0^3 x^3 - 5x^2 + 5x + 1\, dx$$

$$a_0 = \frac{1}{3}\left(\frac{x^4}{4} - \frac{5x^3}{3} + \frac{5}{2}x^2 + x\Big]_{-3}^{3}\right)$$

$$a_0 = \frac{1}{3}\left(3/4\right)$$

$$a_0 = \frac{1}{4}$$

for $g(x)$

$$a_1 = \frac{1}{3}\int_0^3 (x^3 - 5x^2 + 5x + 1)\cos\left(\frac{\pi x}{3}\right)dx$$

$$a_2 = \frac{1}{3}\int_0^3 (x^3 - 5x^2 + 5x + 1)\cos\left(\frac{2\pi x}{3}\right)dx$$

for $h(x)$

$$b_1 = \frac{1}{3}\int_0^3 (x^3 - 5x^2 + 5x + 1)\sin\left(\frac{\pi x}{3}\right)dx$$

$$b_2 = \frac{1}{3}\int_0^3 (x^3 - 5x^2 + 5x + 1)\sin\left(\frac{2\pi x}{3}\right)dx$$

bentuk umum

$$a_n = \frac{2}{3}\int_0^3 \left((x^3 - 5x^2 + 5x + 1)\left(\cos\left(\frac{n\pi x}{3}\right)\right)\right)dx$$

$u = x^3 - 5x^2 + 5x + 1$ $\quad dv = \cos\left(\frac{n\pi x}{3}\right)dx$

$du = (3x^2 - 10x + 5)\,dx$ $\quad v = \sin\left(\frac{n\pi x}{3}\right)\cdot\left(\frac{3}{n\pi}\right)$

$$a_n = \frac{2}{3}\left(\frac{3}{n\pi}(x^3 - 5x^2 + 5x + 1)\sin\left(\frac{n\pi x}{3}\right)\Big]_0^3 - \frac{3}{n\pi}\int_0^3 (3x^2 - 10x + 5)\sin\left(\frac{n\pi x}{3}\right)dx\right)$$

$$a_n = \frac{2}{3}\left(\frac{3}{n\pi}\left(\left((x^3 - 5x^2 + 5x + 1)\sin\left(\frac{n\pi x}{3}\right)\right]_0^3 - \int_0^3 (3x^2 - 10x + 5)\sin\left(\frac{n\pi x}{3}\right)dx\right)\right)$$

$u = 3x^2 - 10x + 5$ $\quad dv = \sin\left(\frac{n\pi x}{3}\right)dx$

$du = (6x - 10)\,dx$ $\quad v = -\frac{3}{n\pi}\cos\left(\frac{n\pi x}{3}\right)$

$$a_n = \frac{2}{n\pi}\cdot\left((x^3 - 5x^2 + 5x + 1)\sin\left(\frac{n\pi x}{3}\right)\right]_0^3 - \left(\frac{-3}{n\pi}\left((3x^2 - 10x + 5)\cos\left(\frac{n\pi x}{3}\right)\right]_0^3 - \int_0^3 (6x - 10)\cos\left(\frac{n\pi x}{3}\right)dx\right)\right)$$

$u = 6x - 10$

$du\ 6dx$

$dv = \cos\left(\frac{n\pi x}{3}\right)dx$

$v = \frac{3}{n\pi}\sin\left(\frac{n\pi x}{3}\right)$

$$a_n = \frac{2}{n\pi}\left((x^3 - 5x^2 + 5x + 1)\sin\left(\frac{n\pi x}{3}\right)\right]_0^3 + \left(\frac{3}{n\pi}\left((3x^2 - 10x + 5)\cos\left(\frac{n\pi x}{3}\right)\right]_0^3 - \left(\frac{3}{n\pi}\left((6x - 10)\sin\left(\frac{n\pi x}{3}\right)\right]_0^3 - 6\int_0^3 \sin\left(\frac{n\pi x}{3}\right)dx\right)\right)\right)$$

$$a_n = \frac{2}{n\pi}\left((x^3 - 5x^2 + 5x + 1)\sin\left(\frac{n\pi x}{3}\right)\Big]_0^3 + \frac{6}{n\pi^2}\left((3x^2 - 10x + 5)\cos\left(\frac{n\pi x}{3}\right)\right]_0^3\right) - \frac{18}{n^3\pi^3}\left((6x - 10)\sin\left(\frac{n\pi x}{3}\right)\right]_0^3 - \frac{162}{n^4\pi^4}\left(\cos\left(\frac{n\pi x}{3}\right)\right]_0^3$$

$$a_n = \frac{2}{n\pi}\left(-2(\sin(n\pi)) - 1(\sin(0))\right) + \frac{6}{n^2\pi^2}\left(2\cos(\pi n) - 5\cdot\cos(0)\right) - \frac{18}{n^3\pi^3}\left(8\sin(n\pi) - 10\sin(0)\right) - \frac{324}{n^4\pi^4}\left(\cos(\pi n) - \cos(0)\right)$$

$$a_n = \frac{2\left(-45\pi^2 n^2 - 6\pi(\pi^2 n^2 + 36)n\sin(\pi n) + 18(\pi^2 n^2 - 27)\cos(\pi n)\right) + 486)}{3\pi^4 n^4} \quad \ldots (1)$$

$a_1 \to$ input $n = 1$ to ...(1)

$$a_1 = \frac{2\left(45\pi^2 - 6\pi(\pi^2 + 36)\sin(\pi) + 18(\pi^2 - 27)\cos(\pi)\right) + 486)}{3\pi^4} = 2{,}3969$$

$$a_1 = \left(\frac{972 - 63\pi^2}{3\pi^4}\right)2$$

$$a_2 = \frac{2\left(45\cdot 2^2\pi^2 - 6\cdot 2\cdot\pi(2^2\pi^2 + 36)\sin(2\pi) + 18(2^2\pi^2 - 27)\cos(2\pi) + 486\right)}{3\cdot 2^4\pi^4}$$

$$a_2 = -0{,}45595 = -\frac{9}{2\pi^2}$$

$\rightarrow g(x) \Rightarrow$

$$S_{g2}(x) = a_0 + a_1 \cos\left(\frac{x\pi}{3}\right) + a_2 \cos\left(\frac{2\pi x}{3}\right)$$

$$S_{g2}(x) = \frac{1}{4} + \frac{2(972 - 63\pi^2)}{3\pi^4} \cos\left(\frac{x\pi}{3}\right) - \frac{9}{2\pi^2} \cos\left(\frac{2\pi x}{3}\right)$$

$$S_{g2}(x) = 0{,}25 + 2{,}3969 \cos\left(\frac{x\pi}{3}\right) - 0{,}45595 \cos\left(\frac{2\pi x}{3}\right)$$

$\rightarrow$ fourier series for $n = 2$ for $g(x)$

untuk $h(x) \rightarrow a_0 = -28$ (sama)

$$b_n = \frac{2}{3} \int_0^3 (x^3 - 5x^2 + 5x + 1) \sin\left(\frac{\pi x n}{3}\right) dx$$

$u = x^3 - 5x^2 + 5x + 1$ $\quad dv = \sin\left(\frac{\pi n x}{3}\right) dx$

$du = (3x^2 - 10x + 5)\, dx$ $\quad v = -\frac{3}{\pi n} \cos\left(\frac{\pi n x}{3}\right)$

$$b_n = \frac{2}{3} \left[-\frac{3}{\pi n} \left((x^3 - 5x^2 + 5x + 1) \cos\left(\frac{\pi n x}{3}\right) \right]_0^3 - \int_0^3 (3x^2 - 10x + 5) \cos\left(\frac{\pi n x}{3}\right) dx \right)\right)$$

$u = 3x^2 - 10x$

$du = (6x - 10)\, dx$

$dv = \cos\left(\frac{\pi n x}{3}\right) dx$

$v = \frac{3}{\pi n} \sin\left(\frac{n\pi x}{3}\right)$

$$b_n = \frac{-2}{\pi n} \left((x^3 - 5x^2 + 5x + 1) \cos\left(\frac{\pi n x}{3}\right) \right]_0^3 - \frac{3}{\pi n} \left((3x^2 - 10x + 5) \sin\left(\frac{\pi n x}{3}\right) \right]_0^3 - \int_0^3 (6x - 10) \sin\left(\frac{n x \pi}{3}\right) dx \right)\right)$$

$u = 6x - 10$

$du = 6dx$

$dv = \sin\left(\frac{\pi n x}{3}\right) dx$

$v = \frac{-3}{n\pi} \cos\left(\frac{\pi n x}{3}\right)$

$$b_n = \frac{-2}{\pi n} \left((x^3 - 5x^2 + 5x + 1) \cos\left(\frac{\pi n x}{3}\right) \right]_0^3 - \frac{3}{\pi n} \left((3x^2 - 10x + 5) \sin\left(\frac{\pi n x}{3}\right) \right]_0^3 + \frac{3}{\pi n} \left((6x - 10) \cos\left(\frac{\pi n x}{3}\right) \right]_0^3 - 6 \int_0^3 \cos\left(\frac{n\pi x}{3}\right) dx \right)\right)$$

$$b_n = \frac{-2}{\pi n} \left((x^3 - 5x^2 + 5x + 1) \cos\left(\frac{\pi n x}{3}\right) \right]_0^3 \Bigg) + \frac{3}{\pi^2 n^2} \left((3x^2 - 10x + 5) \sin\left(\frac{\pi n x}{3}\right) \right]_0^3 \Bigg) + \frac{9}{n^3 \pi^3} \left((6x - 10) \cos\left(\frac{n\pi x}{3}\right) \right]_0^3 \Bigg) - \frac{162}{n^4 \pi^4} \left(\sin\left(\frac{\pi n x}{3}\right) \right]_0^3 \Bigg)$$

$$b_n = \frac{-2}{\pi n} \left(-2 \cos(\pi n) + \cos(0) \right) + \frac{3}{\pi^2 n^2} \left(2 \sin(\pi n) - 5 \sin(0) \right) + \frac{9}{n^3 \pi^3} \left(8 \cos(n\pi) - 10 \cos(0) \right) - \frac{162}{n^4 \pi^4} \left(\sin(\pi n) - \sin(0) \right)$$

$$b_n = \frac{2(\pi n(\pi^2 n^2 + 90) + 6(\pi^2 n^2 - 27) \sin(\pi n) + 2\pi n (\pi^2 n^2 + 36) \cos(\pi n))}{\pi^4 n^4} \quad \text{.... (2)}$$

$\Rightarrow b_1 =$ input $n = 1$ to ... (2) $\qquad b_2 =$ input $n = 2$ to ... (2)

$$b_1 = \frac{2(54 - 3\pi^2)}{3\pi^3} = 0{,}5244 \qquad b_2 = \frac{3(27 + 2\pi^2)}{2\pi^3} = 2{,}2611$$

$$h(x) \rightarrow S_{h2}(x) = \frac{2(54 - 3\pi^2)}{3\pi^3} \sin\left(\frac{\pi x}{3}\right) + \frac{3(27 + 2\pi^2)}{2\pi^3} \sin\left(\frac{2\pi x}{3}\right)$$

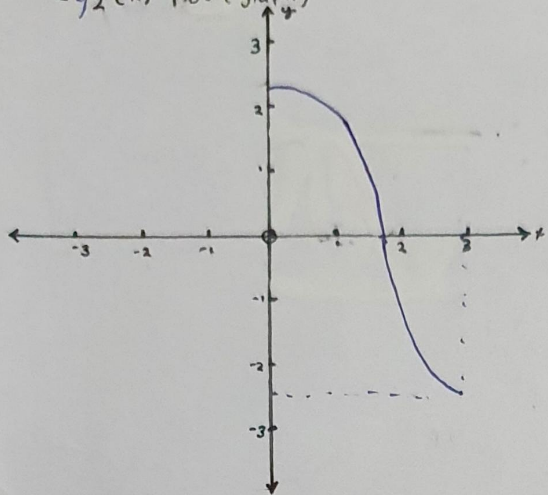
$$S_{h2}(x) = 0{,}5244 \sin\left(\frac{\pi x}{3}\right) + 2{,}2611 \sin\left(\frac{2\pi x}{3}\right)$$

$\rightarrow$ fourier series for $h(x)$, $n = 2$

c.) for $n = 2$ (1 periode saja)

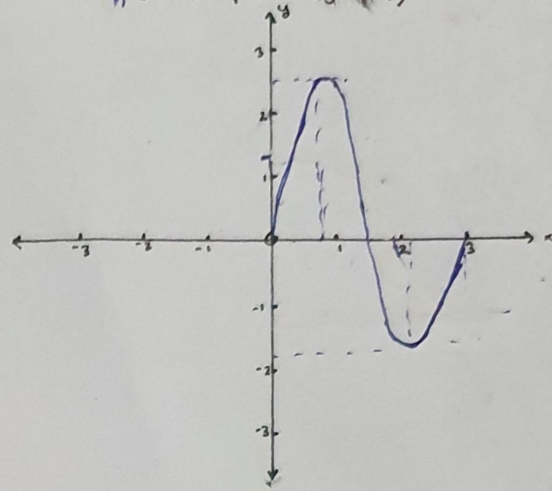
from $g(x)$

$S_{g2}(x)$ plot (graph)



from $h(x)$

$S_{h2}(x)$ plot (graph)



d.) Dependence on n of the maximum error on $[0, L]$

the more the order we use (semakin tinggi nilai n), e_n akan semakin kecil

$$e_n = \sqrt{\frac{1}{2L}\int_a^b (f(x) - S_n(x))^2\,dx}$$ → tetapi nilai $e_n(x)$ tidak akan selalu semakin dekat ke 0, jika seluruh ~~bagian~~ fungsi punya $\lim_{x \to c^+} f(x)$ dan $\lim_{x \to c^-} f(x)$ yang sama, maka $e_n(x)$ akan semakin mendekati 0, setiap nilai n bertambah. Tetapi, jika $\lim_{x \to c^+} f(x)$ dan $\lim_{x \to c^-} f(x)$ pada suatu nilai c tidak sama, maka $e_n(x)$ akan semakin

$$e_n(x) = \left| f(c) - \frac{\lim_{x \to c^+} f(x) + \lim_{x \to c^-} f(x)}{2} \right|$$ dalam range $[0, L]$.

dengan alasan saat ada diskontinuitas dan limit kanan dan kiri berbeda, $S_n(x)$ pada nilai tersebut akan mendekati rerata dari nilai limit tersebut

$f(c)$ bisa dari ~~tinjauan~~ kedua fungsi (fungsi di kiri $x=c$ atau di kanan $x=c$).

kalau ada fenomena overshoot Gibbs, maka error bisa tetap ada fluktuasi (tidak sepenuhnya menuju 0).