There is some revision for number 1.a, 1.b, and 1.c as I miswrote pi into 12, therefore , here is the right one (please) look at the next page

$$C_n = \frac{1}{L}\left[\frac{-L}{(1+n)\pi}\cos\left((1+n)\frac{\pi x}{L}\right) - \frac{L}{(1-n)\pi}\cos\left((1-n)\frac{\pi x}{L}\right)\right]_0^L$$

↳ pretty hard to solve, so use calculator such as wolfram

$$C_n = \frac{2\cos(\pi n)+2}{\pi - \pi n^2} = \frac{2}{\pi}\left(\frac{\cos(\pi n)+1}{1-n^2}\right),$$ where $n \neq 1$ because it will resulting invalid value, divergent value

So ⇒ $C_n = \begin{cases} 0 & \text{for } n = \text{odd } (\cos\pi = -1) \rightarrow \text{so } (-2+2) = 0 \\ \frac{4}{\pi(1-n^2)} & \text{for } n = \text{even } (\cos(2\pi) = 1) \rightarrow \text{so } (2+2) = 4 \end{cases}$

$$C_0 = \frac{4}{\pi(1-0^2)} = \frac{4}{\pi}$$

$$\longrightarrow U(x,t) = \frac{2}{\pi} + \frac{2}{\pi}\sum_{n=2}^{\infty}\left(\frac{\cos(n\pi)+1}{1-n^2} \cdot \cos\left(\frac{n\pi x}{L}\right) \cdot e^{-\left(\frac{n\pi}{L}\right)^2 t\alpha^2}\right)$$

input L now

a.)

$$\therefore\ \boxed{U(x,t) = \frac{2}{\pi} + \frac{2}{\pi}\sum_{n=2}^{\infty}\frac{\cos(n\pi)+1}{(1-n^2)}\cos\left(\frac{n\pi x}{L}\right)e^{-\alpha^2\left(\frac{n\pi}{L}\right)^2 t}} \longrightarrow \cos(n\pi) = (-1)^n$$

so

$$\boxed{U(x,t) = \frac{2}{\pi} + \frac{2}{\pi}\sum_{n=2}^{\infty}\left(\left(\frac{(-1)^n+1}{1-n^2}\right)\cos\left(\frac{n\pi x}{L}\right)e^{-\left(\frac{\alpha n\pi}{L}\right)^2 t}\right)}$$

for $t = \infty$ try $\lim_{t\to\infty}\frac{\cos(n\pi)+1}{1-n^2}\cos\left(\frac{n\pi x}{L}\right) \cdot e^{-\left(\frac{n\pi}{L}\right)^2 t\alpha^2}$

↓ equal

$$\lim_{t\to\infty}\frac{\left(\frac{\cos(n\pi)+1}{1-n^2}\right)\cdot\cos\left(\frac{n\pi x}{L}\right)}{e^{\left(\frac{n\pi}{40}\right)^2\infty\alpha^2}} = 0$$

so $U(x,\infty) = \frac{2}{\pi} + 0 = \frac{2}{12}$ ⟶

b) ∴ the temperature at $t = \infty$ is $U(x,\infty) = \frac{2}{\pi}$ (steady)

(unit unknown)

c.)

$\alpha^2 = 1$

$L = 40$ ⟶

c.)

$$\therefore)\ U(x,t) = \frac{2}{\pi} + \frac{2}{\pi}\sum_{n=2}^{\infty}\left(\left(\frac{\cos(\pi n)+1}{1-n^2}\right)\cos\left(\frac{n\pi x}{40}\right)\cdot e^{-\left(\frac{\pi n}{40}\right)^2 t}\right)$$

↓

$$U(x,t) = \frac{2}{\pi} + \frac{2}{\pi}\sum_{n=2}^{\infty}\left(\left(\frac{(-1)^n+1}{1-n^2}\right)\cos\left(\frac{n\pi x}{40}\right)e^{-\left(\frac{n\pi}{40}\right)^2 t}\right)$$