Fuzzy Logic and Soft Computing (LTAT.02.005)

ASSIGNMENT 2

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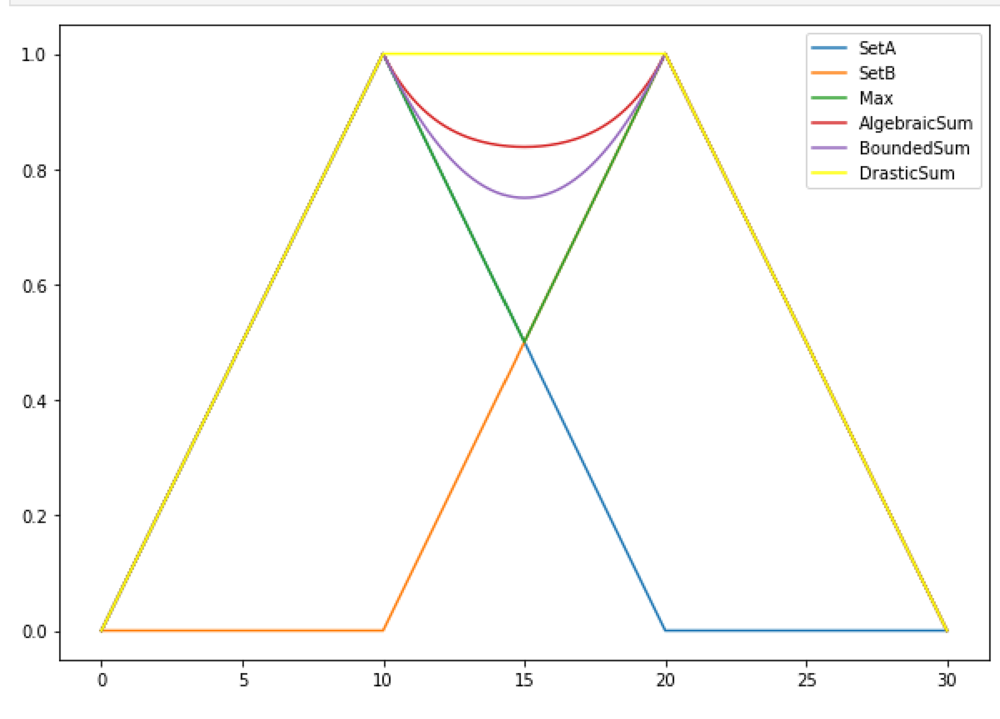
1. Consider two triangular fuzzy sets. Apply all the known t-conorms and plot the results to show the difference among the different t-conorms (2 points). Repeat considering two trapezoidal fuzzy sets (2 points). (hint: you can modify the t_norm Scilab function

In [45]:
```
import fuzzylogic
from fuzzylogic.classes import Domain
from fuzzylogic.functions import triangular,trapezoid, R, S
from fuzzylogic.hedges import minus
from matplotlib import pyplot as plt
import numpy as np
```

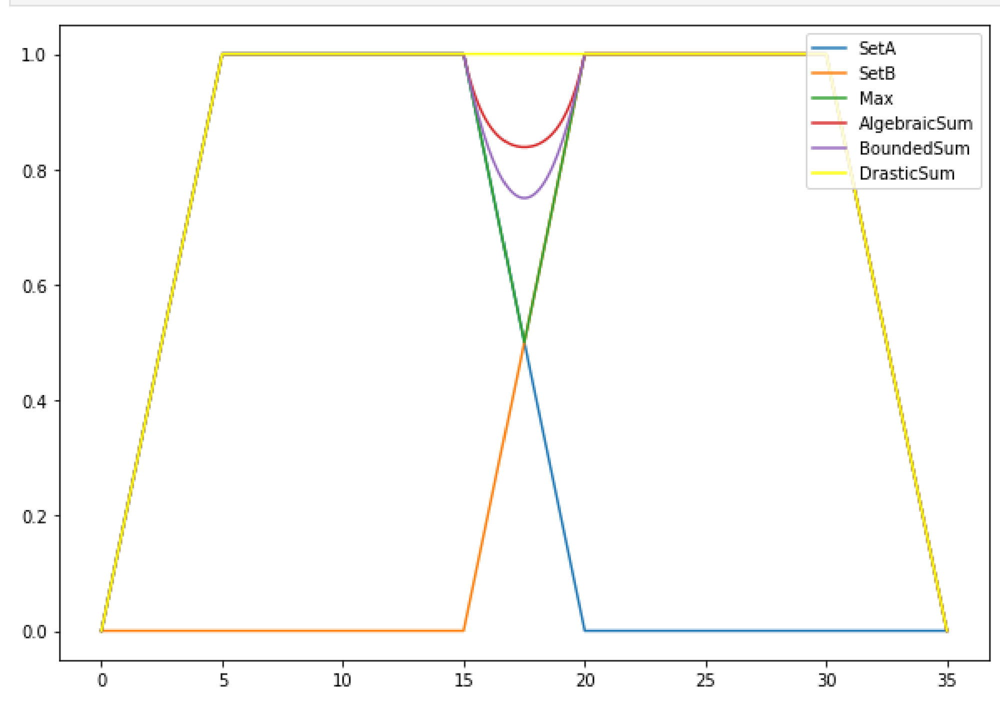
(A) : According to Lecture 5, there is 4 different types of t-conorms: \

- Max-operator: $max\{x, y\} = x \vee y$
- Algebraic sum: $x + y - xy$
- Bounded sum: $min\{1, x + y\}$
- Drastic sum: $\begin{cases} x, & if \quad y = 0 \\ y, & if \quad x = 0 \\ 1, & otherwise \end{cases}$

In [46]:
```
numbers = Domain("numbers", 0, 30, res=0.1)
numbers.triangle1 = triangular(0, 20)
numbers.triangle2 = triangular(10, 30)

# Max-operator
numbers.conorm1 = fuzzylogic.combinators.MAX(
    numbers.triangle1, numbers.triangle2)
# Algebraic sum
numbers.product = minus(fuzzylogic.combinators.multiply(
    numbers.triangle1, numbers.triangle2))
numbers.algebraicSum = numbers.triangle1 + numbers.triangle2 + numbers.product
# Bounded sum
numbers.boundedSum = numbers.triangle1+numbers.triangle2

triangle1_arr = numbers.triangle1.array()
triangle2_arr = numbers.triangle2.array()

# Drastic sum
drastic_sum = []
for x, y in zip(triangle1_arr, triangle2_arr):
    if x == 0:
        drastic_sum.append(y)
    elif y == 0:
        drastic_sum.append(x)
    else:
        drastic_sum.append(1)
drastic_sum = np.array(drastic_sum)

numbers.triangle1.plot()
numbers.triangle2.plot()
numbers.conorm1.plot()
numbers.algebraicSum.plot()
numbers.boundedSum.plot()
plt.plot(numbers.range, drastic_sum, c="yellow")


# plot everyhting

plt.legend(['SetA', 'SetB', 'Max', 'AlgebraicSum',
            'BoundedSum', 'DrasticSum'], loc="upper right")
plt.rcParams["figure.figsize"] = (10, 7)
plt.show()
```



In [47]:
```
numbers = Domain("numbers", 0, 35, res=0.1)
numbers.trapezoid1 = trapezoid(0, 5, 15, 20)
numbers.trapezoid2 = trapezoid(15, 20, 30, 35)

# max-operator
numbers.conorm1 = fuzzylogic.combinators.MAX(numbers.trapezoid1,numbers.trapezoid2)
# algebraic sum
numbers.product = minus(fuzzylogic.combinators.multiply(numbers.trapezoid1,numbers.trapezoid2))
numbers.algebraicSum = numbers.trapezoid1 + numbers.trapezoid2 + numbers.product
# bounded sum
numbers.boundedSum = numbers.trapezoid1+numbers.trapezoid2

trapezoid1_arr = numbers.trapezoid1.array()
trapezoid2_arr = numbers.trapezoid2.array()

drastic_sum = []
for x,y in zip(trapezoid1_arr, trapezoid2_arr):
    if x == 0:
        drastic_sum.append(y)
    elif y == 0:
        drastic_sum.append(x)
    else:
        drastic_sum.append(1)
drastic_sum = np.array(drastic_sum)

numbers.trapezoid1.plot()
numbers.trapezoid2.plot()
numbers.conorm1.plot()
numbers.algebraicSum.plot()
numbers.boundedSum.plot()
plt.plot(numbers.range, drastic_sum,c = "yellow")


# plot everyhting

plt.legend(['SetA','SetB','Max','AlgebraicSum','BoundedSum','DrasticSum'],loc="upper right")
plt.rcParams["figure.figsize"] = (10,7)
plt.show()
```



1. Consider the linguistic variable “damage state” (e.g. referred to a bridge or a building).\ Choose an orthogonal term set (2 points). Apply some linguistic modifiers and check whether the term set is still orthogonal (2 points).

(A) : Let X be a linguistic variable, labelled as `damage state` of building[1], with $X = [0, 100]$. The term set $T(damage) = \{$ No damage/none, Slight damage/minor, Moderate damage/moderate, Severe damage/major, Complte damage/collapse $\}$. Every term of fuzzy set are shown as following below:

$M(none) = \{(x, \mu_{none}(x)) \mid x \in [0, 1)\}$,

$$\mu_{none}(x) = \begin{cases} 0, & x > 0 \\ 1, & x = 0 \end{cases}$$

$M(minor) = \{(x, \mu_{minor}(x)) \mid x \in (0, 30)\}$,

$$\mu_{minor}(x) = \begin{cases} 0, & x > 35 \\ -(x - 35) \times 0.2, & 30 < x \leq 35 \\ 1, & 0 < x \leq 30 \end{cases}$$

$M(moderate) = \{(x, \mu_{moderate}(x)) \mid x \in [30, 60)\}$,

$$\mu_{moderate}(x) = \begin{cases} 0, & 25 < x \geq 65 \\ ((x - 25) \times 0.2), & 25 \leq x < 30 \\ -((x - 65) \times 0.2), & 60 \leq x < 65 \\ 1, & 30 \leq x < 60 \end{cases}$$

$M(major) = \{(x, \mu_{major}(x)) \mid x \in [60, 90)\}$,

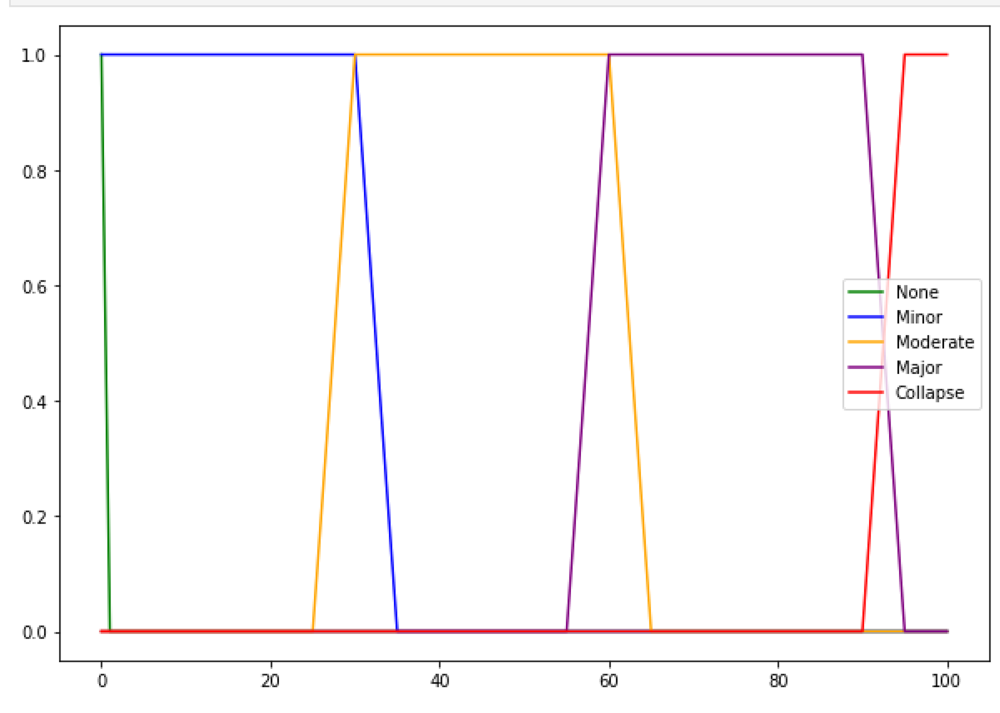
$$\mu_{major}(x) = \begin{cases} 0, & 55 < x \geq 95 \\ ((x - 55) \times 0.2), & 55 \leq x < 90 \\ -((x - 95) \times 0.2), & 60 \leq x < 95 \\ 1, & 60 \leq x < 90 \end{cases}$$

$M(collapse) = \{(x, \mu_{collapse}(x)) \mid x \in [90, 100]\}$,

$$\mu_{collapse}(x) = \begin{cases} 0, & 85 < x \\ (x - 85) \times 0.2, & 85 \leq x < 90 \\ 1, & x \geq 90 \end{cases}$$

[1] Post-earthquake assessment of building damage degree using LiDAR data and imagery

In [48]:
```
building = Domain("damages", 0, 100, res=0.1)

building.none = S(0,1)
building.minor = S(30,35)
building.moderate = trapezoid(25,30,60,65)
building.major = trapezoid(55,60,90,95)
building.collapse = R(90,95)

plt.plot(building.range, building.none.array(),c = "green")
plt.plot(building.range, building.minor.array(),c = "blue")
plt.plot(building.range, building.moderate.array(),c = "orange")
plt.plot(building.range, building.major.array(),c = "purple")
plt.plot(building.range, building.collapse.array(),c = "red")

plt.legend(['None','Minor','Moderate','Major','Collapse'],loc="right")
plt.rcParams["figure.figsize"] = (10,7)
plt.show()
```



We have a term set $T(damage)$, and to check this linguistic variable is orthogonal or not, we can use : $\sum_{i=1}^{n} \mu_{t_i}(x) = 1, \forall x \in X$.

In [55]:
```
none_arr = building.none.array()
minor_arr = building.minor.array()
moderate_arr = building.moderate.array()
major_arr = building.major.array()
collapse_arr = building.collapse.array()

is_orth = True
for arr in zip(none_arr, minor_arr, moderate_arr, major_arr, collapse_arr):
    if sum(arr) != 1:
        print("it is not orthogonal")
        print(arr, sum(arr))
        is_orth = False
        break
if is_orth:
    print("it is orthogonal")
```

```
it is orthogonal
```

For Linguistic modifiers, we can use Concentration and dilation to each fuzzy set, shown as following below:

$CON(M(None)) = M(None)^2$

$DIL(M(Minor)) = M(Minor)^{0.5}$

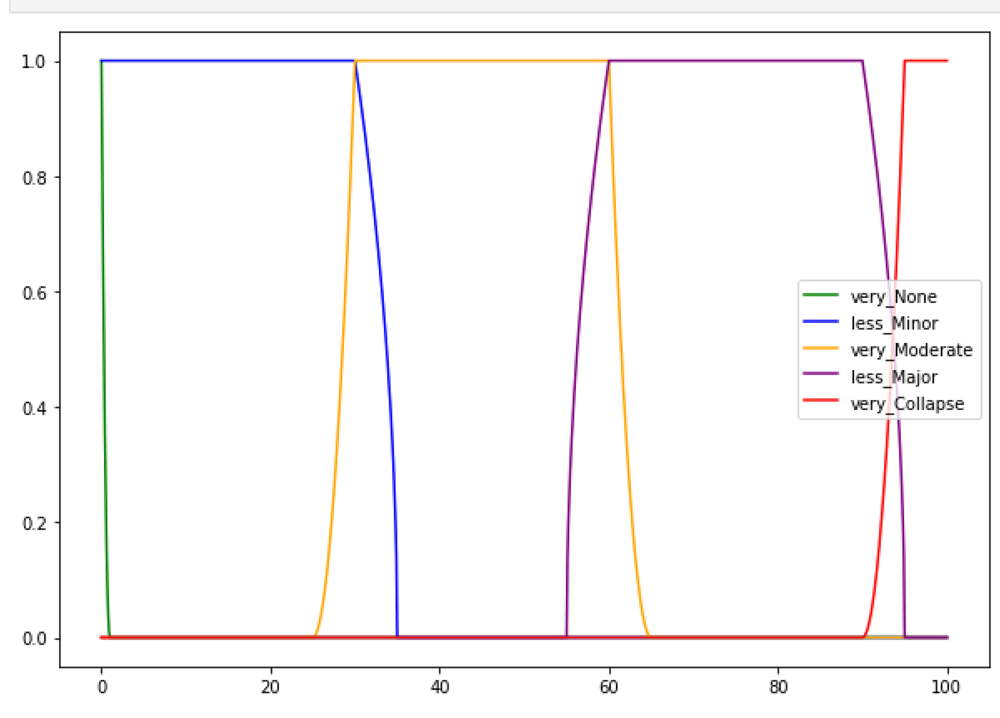
$CON(M(Moderate)) = M(Moderate)^2$

$DIL(M(Major)) = M(Major)^{0.5}$

$CON(M(Collapse)) = M(Collapse)^2$

In [50]:
```
none_arr = building.none.array()**2
minor_arr = building.minor.array()**0.5
moderate_arr = building.moderate.array()**2
major_arr = building.major.array()**0.5
collapse_arr = building.collapse.array()**2
```

In [51]:
```
plt.plot(building.range, none_arr, c="green")
plt.plot(building.range, minor_arr, c="blue")
plt.plot(building.range, moderate_arr, c="orange")
plt.plot(building.range, major_arr, c="purple")
plt.plot(building.range, collapse_arr, c="red")

plt.legend(['very None', 'less Minor', 'very Moderate',
            'less Major', 'very Collapse'], loc="right")
plt.rcParams["figure.figsize"] = (10, 7)
plt.show()
```



In [52]:
```
is_it = True
for arr in zip(none_arr,minor_arr,moderate_arr,major_arr,collapse_arr):
    if sum(arr) != 1:
        print("it is not orthogonal")
        print(arr, sum(arr))
        is_it = False
        break
if is_it:
    print("it is orthogonal")
```

```
it is not orthogonal
(1.0, 1.0, 0.0, 0.0, 0.0) 2.0
```

1. Provide an example (not present in the study material or textbooks) of generalized modus ponens (as a guideline, see Example page 41 Lecture 5) (5 points).

According to Generalized modus ponens, we can have :

- **Premise:** Tonight the moon is very bright.
- **Implication:** When the moon is bright, then it is romantic.
- **Conclusion:** Tonight the moon is very romantic.

Hence, we can define a membership function when moon is going bright, the universe $X = [0, 100]$. Consider the magnitude of moon, we transfer into percentage, we can have membership function:

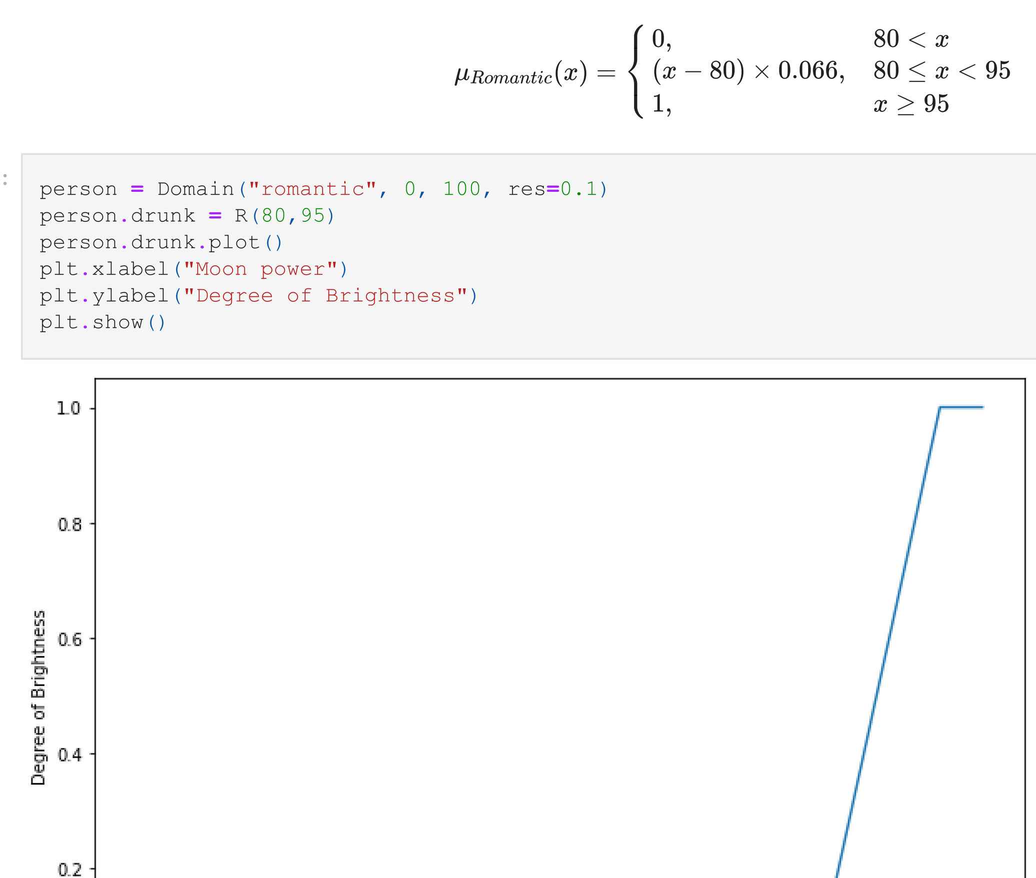
$$\mu_{brightness}(x) = \begin{cases} 0, & 45 < x \\ (x - 45) \times 0.03, & 45 \leq x < 75 \\ 1, & x \geq 75 \end{cases}$$

In [53]:
```
driver = Domain("brightness", 0, 100, res=0.1)
driver.slownes = R(45,75)
driver.slownes.plot()
plt.xlabel("Magnitude")
plt.ylabel("Degree of Brightness")
plt.show()
```

And we have fuzzy set `degree of romantic`, and it's membership function shown as :

$$\mu_{Romantic}(x) = \begin{cases} 0, & 80 < x \\ (x - 80) \times 0.066, & 80 \leq x < 95 \\ 1, & x \geq 95 \end{cases}$$

In [54]:
```
person = Domain("romantic", 0, 100, res=0.1)
person.drunk = R(80,95)
person.drunk.plot()
plt.xlabel("Moon power")
plt.ylabel("Degree of Brightness")
plt.show()
```



Finally, we can use minimum t-norm to get the membership function of the implication by : $\mu_{B^*}(y) = sup_x min[\mu_{A^*}(x), \mu_R(x, y)]$, $\mu_R(x, y)$ is the membership function of the implication.