№6

$n = 200$ людей

10 - без болезни

181 - 1-кран

9 - 2-крана

$H_0: \xi \sim Bi(2),\ m=2$

$H_1: \overline{H_0};$

$\tilde{p}_1 = \frac{10}{200};\ \tilde{p}_2 = \frac{181}{200};\ \tilde{p}_3 = \frac{9}{200}$

$Bi\ (m=2) \Rightarrow\ p_B = C_x^2\, p^x (1-p)^{2-x};\ C_x^n = \frac{n!}{(n-x)!\,x!}$

$p_1 = (1-p)^2;\ p_2 = 2p(1-p);\ p_3 = p^2$

ОМП:

$L(p) = (1-p)^{2\cdot 10} \cdot (2p(1-p))^{181} \cdot p^{2\cdot 9} = (1-p)^{20} \cdot (2p(1-p))^{181} \cdot p^{18} \to max$

$\ln L(p) = 181 \ln 2 + 199 \ln p + 201 \ln(1-p) \to max$

$\frac{\partial \ln L}{\partial p} = 0 + \frac{199}{p} - \frac{201}{1-p} = 0$

$\frac{199 - 199p - 201p}{p(1-p)} = 0;\quad 199 - 400p = 0$

$p = \frac{199}{400}$

Считаем дельту: $\Delta = \sum_1^3 \frac{(m_i - 200\, p_i(\vec{\theta}))^2}{200 \cdot p_i(\vec{\theta})}$

$$\Delta = \frac{\left(10 - 200\left(1 - \frac{199}{400}\right)^2\right)^2}{200\left(1 - \frac{199}{400}\right)^2} + \frac{\left(181 - 200 \cdot 2 \frac{199}{400} \cdot \frac{201}{400}\right)^2}{200 \cdot 2 \cdot \frac{199}{400} \cdot \frac{201}{400}} +$$

$$+ \frac{\left(9 - \left(\frac{199}{400}\right)^2 \cdot 200\right)^2}{200 \cdot \left(\frac{199}{400}\right)^2} = \frac{\left(10 - \frac{40401}{800}\right)^2}{\frac{40401}{800}} + \frac{\left(181 - \frac{39999}{400}\right)^2}{\frac{39999}{400}} +$$

$$+ \frac{\left(9 - \frac{39601}{400}\right)^2}{\frac{39601}{800}} = 131.23$$

$$\Delta = \chi^2(s) = \chi^2(m-1-s) = \chi^2(3-1) = \chi^2(2)$$

$$\text{p-value} = P(\Delta \geq \hat{\Delta} | H_0) = \int_{131}^{+\infty} \frac{1}{2\Gamma(1)} x^0 e^{-\frac{x}{2}} dx =$$

$$= \int_{131}^{+\infty} \frac{1}{2} e^{-\frac{x}{2}} dx = 3{,}5 \cdot 10^{-29} << 0{,}05 \Rightarrow H_0 \text{ отвергаем}$$

№2

$n = 100$

	−	=	+
I	25	50	25
II	52	41	7

H_0: однородность

H_1: $\bar{H}_0$

$$p_1 = \frac{77}{200} \quad p_2 = \frac{91}{200} \quad p_3 = \frac{32}{200}$$

$$\Delta_i = \sum_j \frac{(n_{ij} - n_i p_j)^2}{n_i p_j}$$

$$\Delta_1 = \frac{\left(25 - 100 \cdot \frac{77}{200}\right)^2}{100 \cdot \frac{77}{200}} + \frac{\left(50 - 100 \cdot \frac{91}{200}\right)^2}{100 \cdot \frac{91}{200}} +$$

$$+ \frac{\left(25 - 100 \cdot \frac{32}{200}\right)^2}{32/200 \cdot 100}$$

$$\Delta_1 = \frac{\left(25 - \frac{77}{2}\right)^2}{\frac{77}{2}} + \frac{\left(50 - \frac{91}{2}\right)^2}{\frac{91}{2}} + \frac{\left(25 - 16\right)^2}{16} =$$

$$= 10,241$$

$$\Delta_2 = \frac{\left(52 - \frac{77}{2}\right)^2}{\frac{77}{2}} + \frac{\left(41 - \frac{91}{2}\right)^2}{\frac{91}{2}} + \frac{(7-16)^2}{16} =$$

$$= 10,241$$

$$\Delta = 20,482 \sim \chi^2((3-1)(2-1)) = \chi^2(2)$$

$$p\text{-value} = P(\Delta \geq \tilde{\Delta} \mid H_0) = \int_{20,482}^{+\infty} \frac{1}{2\Gamma(1)} x^0 e^{-\frac{x}{2}} dx =$$

$$= \int_{20,482}^{+\infty} \frac{1}{2} e^{-\frac{x}{2}} dx = 0,0000359 = 3,59 \cdot 10^{-5} \ll$$

$$\ll 0,05$$

H_0 отвергаем

№8

		2	3	4	5
$n = 300$	1 поток	33	43	80	144
	2 поток	39	35	72	154

H_0: гипотеза однородности

H_1: $\overline{H_0}$

$$p_1 = \frac{72}{600}; \quad p_2 = \frac{78}{600}; \quad p_3 = \frac{152}{600}; \quad p_4 = \frac{298}{600}$$

$$\Delta_1 = \frac{(33-36)^2}{36} + \frac{(43-39)^2}{39} + \frac{(80-76)^2}{76} +$$

$$+ \frac{(144-149)^2}{149} = 1,03857$$

$$\Delta_2 = \frac{(39-36)^2}{36} + \frac{(35-39)^2}{39} + \frac{(72-76)^2}{76} +$$

$$+ \frac{(154-149)^2}{149} = 1,038$$

$$\Delta = 2,076 \sim \chi^2(3)$$

$$\text{p-value} = P(\Delta \geq \tilde{\Delta} \mid H_0) = \int_{2,076}^{+\infty} \frac{1}{2^{3/2}\Gamma\left(\frac{3}{2}\right)} \sqrt{x} \cdot$$

$$e^{-\frac{x}{2}} dx = \int_{2,076}^{+\infty} \frac{1}{\sqrt{2\pi}} \sqrt{x}\, e^{-\frac{x}{2}} dx = 0,5587 >> 0,05$$

$\Rightarrow H_0$ не отвергаем

№9

	0	1	2	3	4	5	6	7	8	9
I	5	8	6	12	14	18	11	6	13	7

а) проверить

$H_0: \xi \sim R$

$H_1: \overline{H_0}$

$$\Delta = \frac{(5-10)^2}{10} + \ldots + \frac{(7-10)^2}{10} = 2,5 + 0,4 +$$

$$+ 1,6 + 0,4 + 1,6 + 6,4 + 0,1 + 3,6 + 0,9 + 0,9 =$$

$$= 16,94$$

$$\Delta = \chi^2(9)$$

$$\text{p-value} \int_{16,4}^{+\infty} \frac{1}{2^{\frac{9}{2}}\Gamma\left(\frac{9}{2}\right)} x^{\frac{7}{2}} e^{-\frac{x}{2}} dx = \int_{16,4}^{+\infty} \frac{1}{105\sqrt{2\pi}} x^{\frac{7}{2}} e^{-\frac{x}{2}} dx =$$

$= 0,0586 > 0,05$ — нет оснований отвергать H_0

Колмогоров:

$\tilde{\Delta} = \sqrt{n} \sup |F(x) - \tilde{F}(x)| \to K(x)$

равномерное распр:

$\tilde{\Delta} = \max\limits_{i} \left(|\tilde{F}(x_i - 0) - F(x_i)|, |F(x_i + 0) - F(x_i)| \right)$

$\tilde{\Delta} = 1,59 \approx 1,6$

$K(x) = P(\sqrt{n} \Delta < x) = 1 + 2 \sum\limits_{k=1}^{\infty} (-1)^k e^{-2k^2x^2}$

$p\text{-value} = 1 - \left(1 + 2 \sum\limits_{k=1}^{\infty} (-1)^k e^{-2k^2}\right) = 0,01193 52 \approx$

$\approx 0,012 < 0,05$ — H_0 отвергается

б) $H_0: \xi \sim N(a, \sigma^2)$; $H_1: \overline{H_0}$

$N: p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-a)^2}{2\sigma^2}}$

$p_1 = \int\limits_{-\infty}^{0,5} p(x)dx, \ \ldots \ p_{10} = \int\limits_{8,5}^{+\infty} p(x) dx$

$L = p_1^5 \cdot \ldots \cdot p_{10}^7 \to \max$

$\Rightarrow \tilde{a} = 4{,}81$; $\tilde{b} = 2{,}69$

$$\tilde{\Delta} = \sum_{1}^{k} \frac{n_i - n p_i(\tilde{a}, \tilde{b})}{n p_i(\tilde{a}, \tilde{b})} = 9{,}82$$

$$\Delta \sim \chi^2(10-2-1) = \chi^2(7)$$

$$P(\Delta \geq \tilde{\Delta} \mid H_0) = \int_{9{,}82}^{+\infty} \frac{1}{2^{7/2}\, \Gamma\left(\frac{7}{2}\right)} x^{\frac{5}{2}} e^{-\frac{x}{2}}\, dx =$$

$$= 0{,}199 \geq \alpha$$

нет оснований отвергать H_0

Колмогоров:

$\tilde{\Delta} = 0{,}862$; $p\text{-value} = 0{,}99316 > \alpha$ –

– нет оснований

отвергать H_1