№1

$H_0: p_0(x) = \begin{cases} 1, & x\in(0,1) \\ 0, & x\notin(0,1)\end{cases}$ $\quad H_1: p_1(x) = \begin{cases} \frac{e^{1-x}}{e-1}, & x\in(0,1)\\ 0, & x\notin(0,1)\end{cases}$

a) $n=1$; $l(x) = \frac{L_1}{L_0} = \frac{e^{1-x}}{e-1} \geq C$, $e^{-x} \geq \tilde{C}$

$G: x \leq B$ крит. обл.

$P(x\in G | H_0) = \alpha$

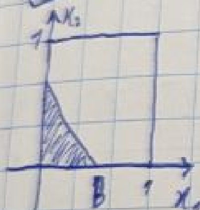
$\int_0^B 1\cdot dx = \alpha$; $B=\alpha$, $G: x\leq\alpha$

$W = P(x\in G|H_1) = \int_0^\alpha \frac{e^{1-x}}{e-1}dx = \frac{e}{e-1}(1-e^{-\alpha})$

$\alpha_2 = 1-W$, $\alpha_1 = \alpha$

b) $n=2$; $l = \frac{L_1}{L_0} = \frac{e^{-x_1+1}}{e-1}\cdot\frac{e^{-x_2+1}}{e-1} \geq C$; $e^{-x_1-x_2}\geq \tilde C$

$G: x_1+x_2 \leq B$; $P(\vec x; G|H_0) = \alpha$



$\iint_G 1\cdot 1\cdot dx_1\, dx_2 = \alpha$; $\frac{B^2}{2}=\alpha$; $B=\sqrt{2\alpha}$

$G: x_1+x_2 \leq \sqrt{2\alpha}$ – крит. обл.

$W = P(\vec x\in G|H_1) = \frac{e^2}{(e-1)}\iint_G e^{-x_1-x_2}dx_1\, dx_2 =$

$= \frac{e^2}{(e-1)^2}\int_0^B dx_1 \int_0^{B-x_1} e^{-x_1}\cdot e^{-x_2}dx_2 = \frac{e^2}{(e-1)}\int_0^B e^{-x_1}(1-e^{-(B-x_1)})dx_1 =$

$= \frac{e^2}{(e-1)^2}(1-e^{-B}-Be^{-B})$

c) Дос. по n $\quad l = \frac{L_1}{L_0} = \prod_{i=1}^{n} p_1(x)$; $\ln l = \sum_{i=1}^{n} \ln p_1(x_i)$

$G: l \geq C \Rightarrow \eta = \ln p_1(\xi) = \ln \frac{e^{1-\xi}}{e-1} = \ln \frac{e}{e-1} - \xi$

$P(\ln l \geq \ln C \mid H_0) = \alpha$

$$\frac{\sum \eta_i - nM[\eta]}{\sqrt{nD[\eta]}} \sim N(0,1)$$

$H_0: M[\eta] = M[\ln \frac{e}{e-1} - \xi] = \ln \frac{e}{e-1} - \frac{1}{2}$

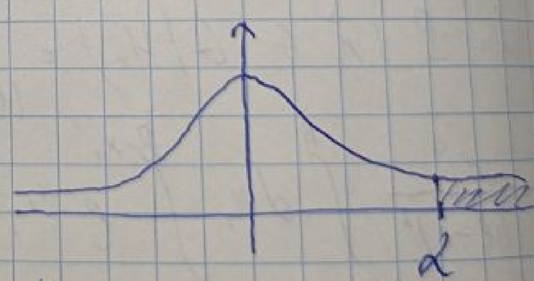
$M[\xi] = \int_0^1 x p_0(x)\,dx = \frac{1}{2} \qquad (p_0(x) = 1)$

$D[\eta] = D[\ln \frac{e}{e-1} - \xi] = D[\xi] = \frac{1}{12}$

$D[\xi] = \frac{(b-a)^2}{12} = \frac{1}{12}$

$$P(\ln l \geq \ln C) = P\left(\frac{\sum \eta_i - nM[\eta]}{\sqrt{nD[\eta]}} \geq \frac{\ln C - nM[\eta]}{\sqrt{nD[\eta]}}\right) = \alpha$$

$$\frac{\ln C - n(\ln \frac{e}{e-1} - \frac{1}{2})}{\sqrt{\frac{n}{12}}} = u_{1-\alpha}$$



$$\ln C = n\left(\ln \frac{e}{e-1} - \frac{1}{2}\right) + u_{1-\alpha}\sqrt{\frac{n}{12}}$$

$$\ln l = \sum_{i=1}^{n} \ln\left(\frac{e}{e-1} e^{-x_i}\right) = n \ln \frac{e}{e-1} - \sum_{i=1}^{n} x_i$$

$G: \ln l \geq \ln C$; $\quad -\sum_{i=1}^{n} x_i \geq -\frac{n}{2} + u_{1-\alpha}\sqrt{\frac{n}{12}}$

$G: \bar{x} \leq \frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}}$ — крит. обл.

$P_i(x_i)$

$W = P(\vec{x} \in G|H_1) = P(\bar{x} \le \frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}}|H_1)$; $\alpha_1 = \alpha$, $\alpha_2 = 1-W$

$\frac{\bar{x} - M[\xi]}{\sqrt{D[\xi]}}\sqrt{n} \rightsquigarrow N(0,1)$ $\qquad W = P(\bar{x} \le \frac{1}{2} - \frac{u_{1-\alpha}}{\sqrt{12n}}|H_1)$

$H_1: M[\xi] = \int_0^1 x \frac{e}{e-1} e^{-x} dx = \frac{e}{e-1}\left[-xe^{-x}\Big|_0^1 + \int_0^1 e^{-x} dx\right] =$

$= \frac{e}{e-1}\left[-e^{-1} + 1 - e^{-1}\right] = \frac{e-2}{e-1}$ $\qquad \cancel{\frac{2e^2-3e+5 - e^2+4e-4}{(e-1)^2}}$

$M[\xi^2] = \int_0^1 x^2 \frac{e}{e-1} e^{-x} dx = \frac{2e-5}{e-1}$, $D[\xi] = \frac{e^2-3e+1}{(e-1)^2}$

$W = P(\bar{x} \le A|H_1) = P\left(\frac{\bar{x} - M[\xi]}{\sqrt{D[\xi]}}\sqrt{n} \le \frac{A - M[\xi]}{\sqrt{D[\xi]}}\sqrt{n}\right) =$

$= \int_{-\infty}^{\frac{A-M[\xi]}{\sqrt{D[\xi]}}\sqrt{n}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$

d) $G: x_{min} < c$ $\qquad F_{min}(x) = 1 - (1-F(x))^n$

$H_0: F_0(x) = \begin{cases} 0, & x < 0 \\ x, & x \in [0,1] \\ 1, & x > 1 \end{cases}$

$P(x_{min} \le c|H_0) = F_{min}(c) = \alpha$

$1 - (1-c)^n = \alpha \Rightarrow c = 1 - \sqrt[n]{1-\alpha}$ $\qquad \alpha_1 = \alpha$

$W = P(x_{min} \le c|H_1)$

$H_1: F(x) = \int_0^x \frac{e}{e-1} e^{-t} dt = \frac{e}{e-1}(1 - e^{-x})$ $\qquad (0,1)$

$W = F_{min}(c) \overset{?}{=} 1 - (1 - \frac{e}{e-1}(1-e^{-c}))^n = 1 - (1 - \frac{e}{e-1}(1 - e^{\sqrt[n]{1-\alpha} - 1}))^n$

$\alpha_2 = 1 - W = \left(1 - \frac{c}{c-1}\left(1 - c^{\sqrt[4]{1-\alpha}} - 1\right)\right)^4$

№12	1	2	3	4
H_0	1/4	1/4	1/6	1/3
H_1	1/4	1/4	1/4	1/4

$p_0(x) = \frac{1}{3}\delta(x-4) + \frac{1}{6}\delta(x-3) + \frac{1}{4}\delta(x-2) + \frac{1}{4}\delta(x-1)$

$p_1(x) = \frac{1}{4}\delta(x-4) + \frac{1}{4}\delta(x-3) + \frac{1}{4}\delta(x-2) + \frac{1}{4}\delta(x-1)$

$n=2: \quad l = \frac{L_1}{L_0} = \frac{\prod \frac{1}{4}\delta(x-4) + \frac{1}{4}\delta(x-3) + \frac{1}{4}\delta(x-2) + \frac{1}{4}\delta(x-1)}{\prod \frac{1}{3}\delta(x-4) + \frac{1}{6}\delta(x-3) + \frac{1}{4}\delta(x-2) + \frac{1}{4}\delta(x-1)}$

$l \geq c$

$P(l \geq c \mid H_0) = \alpha = 0{,}2$

H_0:

II \ I	1	2	3	4
1	1/16	1/16	1/24	1/12
2	1/16	1/16	1/24	1/12
3	1/24	1/24	1/36	1/18
4	1/12	1/12	1/18	1/9

l:

II \ I	1	2	3	4
1	1	1	3/2	3/4
2	1	1	3/2	3/4
3	3/2	3/2	9/4	9/8
4	3/4	3/4	9/8	9/16

H_1: все значения $\frac{1}{16}$

выбираем l :

c	9/16	3/4	1	9/8	3/2	9/4
α_1	1	8/9	5/9	11/36	7/36	1/36
W	1	15/16	11/16	7/16	5/16	1/16

$\alpha_1 = \sum c,\ t \geq c,\ H_0$

$\alpha^* H_0 \quad \sum$ при $t \geq c$

$W \to \max$ при $\alpha_1 < 0,2 \Rightarrow W = 5/6$

$c = \frac{3}{2}$

№14

$x_n \sim N(a, 2)$; $y_m \sim N(b, 1)$ $\quad \alpha = 0,05$

$x = \{-3,11;\ -6,10;\ 2,92\}$ $\quad H_0: a = b$

$y = \{-2,29;\ -2,31\}$ $\quad H_1: a > b;\ a < b;\ a \neq b$

$$\Delta = \frac{(\bar{x} - \bar{y})\sqrt{\frac{nm}{n+m}}}{\sqrt{\frac{S_x^2(n-1) + S_y^2(m-1)}{n+m-2}}} \sim t(n+m-2)$$

$$\tilde{\Delta} = \frac{(-1,536 + 2,6)\sqrt{\frac{6}{5}}}{\sqrt{\frac{S_x^2(2) + S_y^2}{3}}} \sim t(3)$$

$$\Updownarrow$$

$$\hat{\Delta} = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\hat{\sigma}_x^2}{n_x} + \frac{\hat{\sigma}_y^2}{n_y}}}$$

$\tilde{\Delta} = 0,313$

1) $a > b$

$$p\text{-value} = P(\Delta \geq |\tilde{\Delta}|) = \int_{0,313}^{+\infty} \frac{2}{\sqrt{\pi^3}\sqrt{\pi}\sqrt{3^2}} \left(1 + \frac{y^2}{3}\right)^2 dy = 0,387 > 0,05$$

$\Rightarrow$ нет осн. отв. H_0

2) $a < b$

$$p\text{-value} = P(\Delta \leq -|\hat{\Delta}|) = \int_{-\infty}^{-0{,}313} t(s)\,ds = 0{,}387 > 0{,}05$$

$\Rightarrow$ нет осн. отв…

3) $a \neq b$

$$p\text{-value} = P(|\Delta| \geq |\hat{\Delta}|) = 2\int_{-\infty}^{-0{,}313} = 0{,}774 > 0{,}05$$

$\Rightarrow$ нет осн. отвергн…