CONVOLUTION AND ITS PROPERTIES

COE150

CONVOLUTION

A convolution is an integral that expresses the amount of overlap of one function when it is shifted over another function

convolution

impulse response

$*$

• multiplication

$$y[n] = x[n] * h[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$x[n] \rightarrow \boxed{\text{DSP}} \rightarrow y[n]$

$\delta[n] \rightarrow \boxed{} \rightarrow y[n]$

impulse

$h[n]$

unit impulse

$h[-(k-n)]$

shift sa signal

$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$

$h[n]$

$x[n]$

0

0 1 0 0

$y[0] = 1$

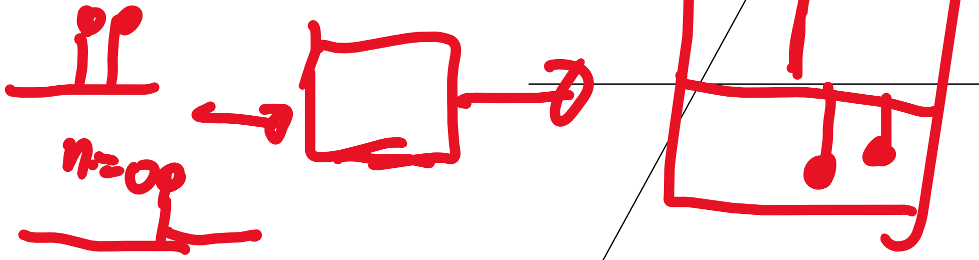
impulse response of a system

$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$

$x[n] \rightarrow \boxed{\text{system}} \rightarrow y[n]$

$\downarrow$

$\delta[n] \rightarrow \boxed{\ } \rightarrow \boxed{h[n]}$

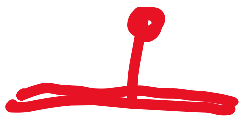


impulse response

$h[n]$

$y[n] = x[n] - 2x[n-1] + x[n-2]$

$x[n] = \delta[n]$



$y[n] = h[n]$ impulse response

$h[n] = \delta[n] - 2\delta[n-1] + \delta[n-2]$

$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$

$y[n] = x[n] * h[n]$

impulse response

CONVOLUTION PROPERTIES

1. Commutativity

$$x[n] * h[n] = h[n] * x[n]$$

2. Associativity

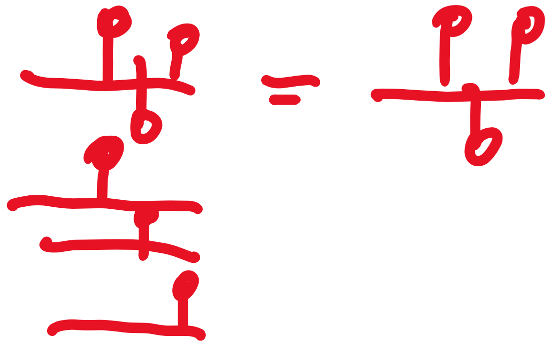
$$[x[n] * h_1[n]] * h_2[n] = x[n] * [h_1[n] * h_2[n]]$$

3. Distributivity of Addition

$$x[n] * (h_1[n] + h_2[n]) = (x[n] * h_1[n]) + (x[n] * h_2[n])$$

4. Identity Element

$$\delta[n] * h[n] = h[n]$$



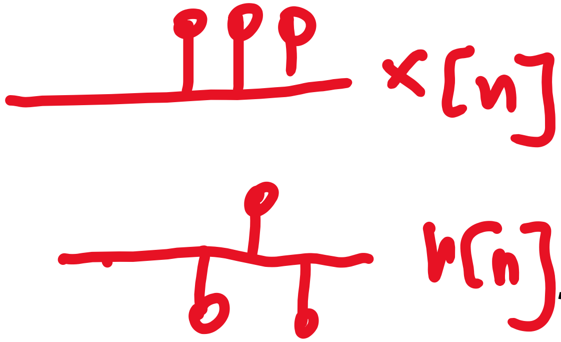
5. Delay

$$x[n] * \delta[n - n_0] = x[n - n_0]$$

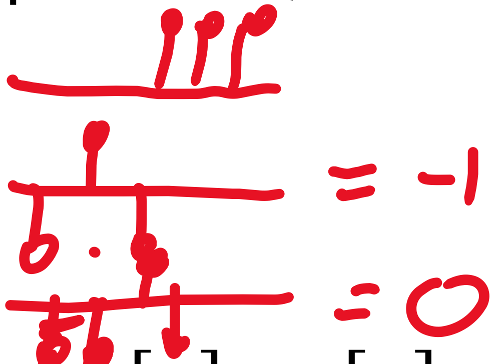
delay

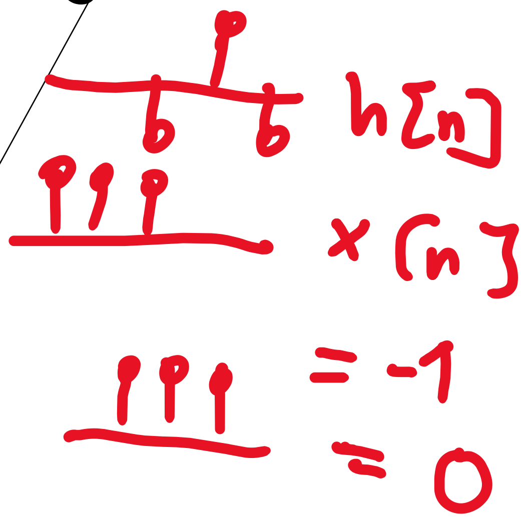
COMMUTATIVITY

Convolution is a commutative operation, meaning signals can be convolved in any order.



=





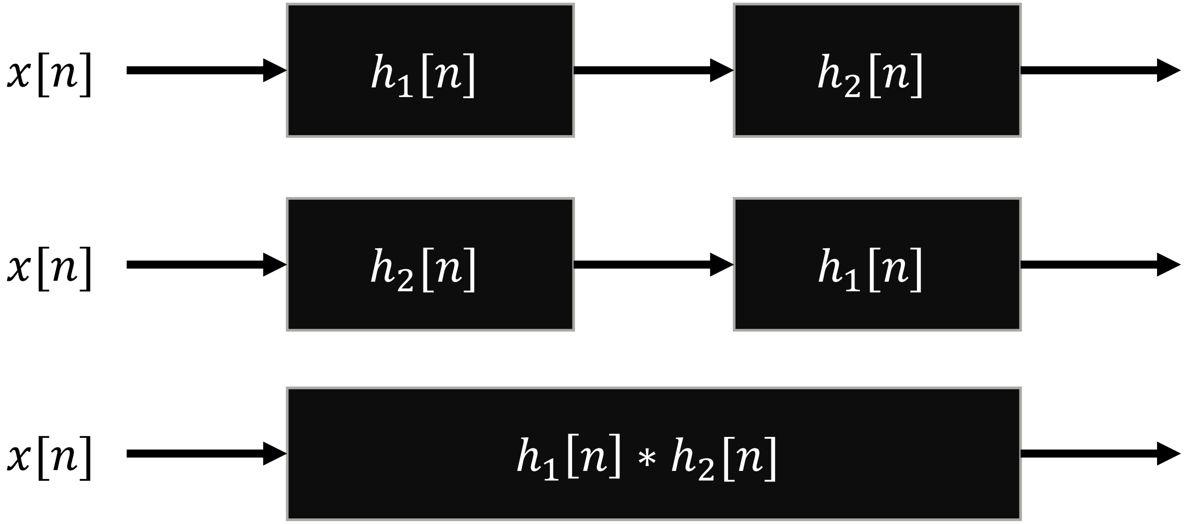
$$x[n] * h[n] = h[n] * x[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$$

ASSOCIATIVITY

Convolution is associative, meaning that convolution operations in series can be done in any order.

$$[x[n] * h_1[n]] * h_2[n] = x[n] * [h_1[n] * h_2[n]]$$



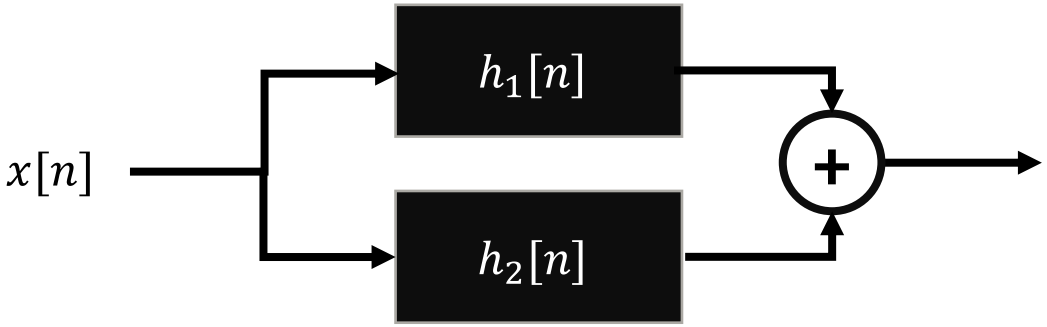
$y[n] = x[n] * (h_1[n] * h_2[n])$

$y[n] = x[n] * (h_2[n] * h_1[n])$

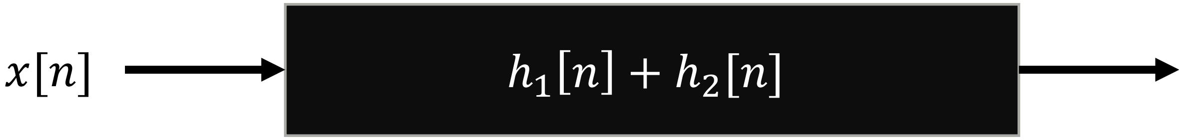
$y[n] = x[n] * (h_1[n] * h_2[n])$

DISTRIBUTIVITY OF ADDITION

$$x[n] * (h_1[n] + h_2[n]) = (x[n] * h_1[n]) + (x[n] * h_2[n])$$



$$y[n] = (x[n] * h_1[n]) + (x[n] * h_2[n])$$



$$y[n] = x[n] * (h_1[n] + h_2[n])$$

DIRECT METHOD

$x[n] \to \boxed{h[n]} \to y[n]$

$h[n]$	1	$\underline{2}$	-1	0
$x[n]$	1	$\underline{-1}$	2	

$h[n] = [1 \ \underset{\uparrow}{2} \ -1]$ impulse response

$x[n] = [\underset{-1}{1} \ \ \underset{\uparrow}{-1} \ \ \underset{1}{2}]$

$N_{OUTPUT} = N_1 + N_2 - 1$

$3 + 3 - 1$

$h[n] = x[n+1] + 2x[n] - x[n-1]$

$y[n] =$

$n = -3 \quad y[-3] = 0$

$n = -2 \quad y[-2] = x[-1] + 2x[-2] - x[-3] = 1 + 0 + 0 = 1$

$n = -1 \quad -1 + 2(1) - 0 = 1$

$n = 0$

$y[0] = 2 + 2(-1) - 1$

$= -1$

$n = 1$

$y[1] = 0 + 2(2) - (-1)$

$= 5$

$n = 2$

$y[2] = 0 + 2(0) - 2$

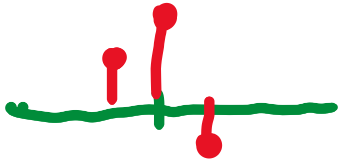
$= -2$

$y[n] = \{1, 1, \underline{-1}, 5, -2\}$

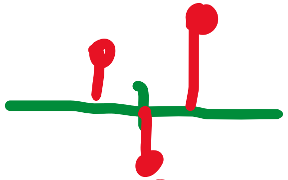
FLIP AND SLIDE ONE SIGNAL

$h[n] = [1 \;\; 2 \;\; -1]$

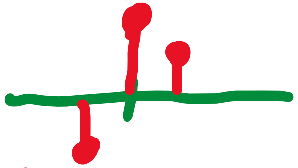
$x[n] = [1 \;\; -1 \;\; 2]$



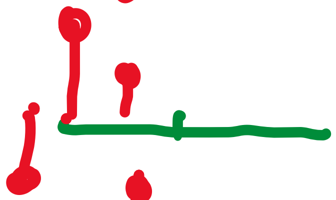
$h[k]$



$x[k]$

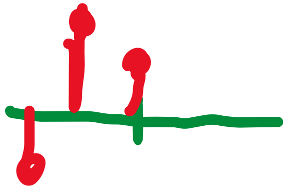


$h[-k]$ flip



$h[-2-k]$

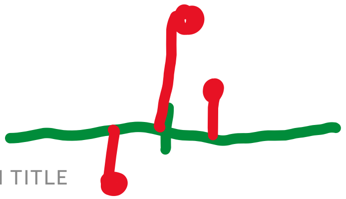
$1(1) = 1$



$h[-1-k]$

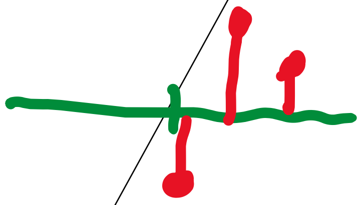
$0 + 2(1) + 1(-1)$

$= 1$



$h[-k]$

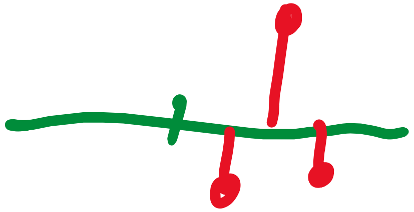
$= 1(-1) + (-1)(2) + 2(1) = -1$



$h[1-k]$

$= -1(-1) + 2(2) + 0(1)$

$= 5$



$h[2-k]$

$= 2(-1) + 0(2) + 0(-1)$

$= -2$

$y[n] = \{1, 1, \underline{-1}, 5, -2\}$

CONVOLUTION SUM

$h[n] = \{1\ \underline{\underline{2}}\ -1\}$

$x[n] = \{1\ -1\ 2\}$

1	$\underline{\underline{2}}$	-1		
1	$\underline{\underline{-1}}$	2		
1	2	-1		
	-1	-2	1	
		2	4	-2
1	1	$\underline{\underline{-1}}$	5	-2

CONVOLUTION ARRAY

$h[n] = \{1 \;\; 2 \;\; -1\}$

$x[n] = \{1 \;\; -1 \;\; 2\}$

		$x[n]$	
$h[n]$	1	-1	2
1	1	-1	2
2	2	-2	4
-1	-1	1	-2

1 1 -1 5 -2

$y[n] = [1 \;\; 1 \;\; -1 \;\; 5 \;\; -2]$

MATRIX BY VECTOR

$h[n] = 1 \quad \underline{2} \quad -1$

$x[n] = 1 \quad -1 \quad 2$

$y(n) = ?$

$y[n] = h[n] * x[n]$

$$y[n] = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 1 \\ 0 & -1 & 2 \\ 0 & 0 & -1 \end{bmatrix} * \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 5 \\ -2 \end{bmatrix}$$

$$\frac{\text{number of column 1st matrix}}{} = \text{no. of rows in the second matrix}$$

3

$$= \begin{bmatrix} 1(1)+0(-1)+0(2) \\ 2(1)+1(-1)+0(2) \\ -1(1)+2(-1)+1(2) \\ 0(1)+-1(-1)+2(2) \\ 0(1)+0(-1)+(-1)(2) \end{bmatrix}$$

ACTIVITY

Friday may 6: before 10:30AM

Perform the convolution of the signal below:

x[n]=[1 3 0 2 -1]

h[n]=[1 3 -2]

may 6 : Quiz #1

Using the:

a) Direct Method

b) Graphical Method

c) Convolution Sum

d) Convolution Array

e) Matrix by Vector

LABORATORY ACTIVITY

Perform in MATLAB/SCILAB the five methods of convolution.

Use your own sets of h[n] and x[n].

may 11, 12 MN.