Baliguat, Dennis Ivan

1. $x[n]\ \{ \overset{-5}{3}\ \ \overset{-4}{0}\ \ \overset{-3}{0}\ \ \overset{-2}{0}\ \ \overset{-1}{0}\ \ \underset{\uparrow}{\overset{0}{6}}\ \ \overset{1}{1}\ \ \overset{2}{-4} \}$

$x[n] \rightleftharpoons X[z]$

$X[z] = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=-5}^{2} x[n] z^{-n}$

$= x(-5) z^{+5} + \cancel{x(-4)} z^{+4} + \cancel{x(-3)} z^{+3} + \cancel{x(-2)} z^{+2} +$
$\cancel{x(-1)} z^{+1} + x(0) z^{0} + x(1) z^{-1} + x(2) z^{-2}$

$= x(-5) z^{5} + x(0) + x(1) z^{-1} + x(2) z^{-2}$

$x(z) = 3z^{5} + 6 + z^{-1} + (-4z^{-2})$

$$\boxed{= 3z^{5} + 6 + \frac{1}{z} - \frac{4}{z^{2}}}$$

ROC = entire z-plane except at $z=0$ and $z=\infty$

2. $x[n] = n^{2} u[n]$ $\quad u(n) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$ $\quad \sum_{n=1}^{\infty} (n^{2} z^{-1})^{n} - 1$

$X[z] = \sum_{n=0}^{\infty} n^{2} u[n] z^{-n}$ $\quad \cancel{= \sum_{n=0}^{\infty} \frac{n^{2}}{z^{n}}}$

$= \sum_{n=0}^{\infty} n^{2} z^{-n} = (n^{2} z^{-1})^{n}$ $\cancel{- \sum_{n=0}^{\infty} \frac{n^{2}}{z^{3}} = \sum_{n=1}^{\infty} \frac{n^{2}}{z^{n}} - 1}$

~~ROC~~ @ $n=1$

$= \frac{1}{z} - 1$

$= \frac{1-z}{z}$

$S_{\infty} = \frac{\text{first term}}{1 - \text{common term}}$

common ratio

$= n^{2} z^{-1}$

$= \frac{n^{2}}{z}$

$= \dfrac{\frac{1-z}{z}}{1 - \frac{n^{2}}{z}}$ $\boxed{= \frac{1-z}{z^{2}(z-n^{2})}}$

$\boxed{\text{ROC} = |z| > 1}$

3. $x[n] = \begin{cases} \frac{1}{3}^n, & n \geq 0 \\ \left(\frac{1}{2}\right)^{-n}, & n < 0 \end{cases}$

$$x[z] = \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} + \sum_{n=-\infty}^{-1} \left(\frac{1}{2}\right)^{-n} z^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3z}\right)^n + \sum_{n=-\infty}^{-1} \left(\frac{z}{2}\right)^{-n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{3z}\right)^n + \sum_{R=1}^{\infty} \left(\frac{z}{2}\right)^R$$

$$= \left[\frac{1}{1-\left(\frac{1}{3z}\right)}\right] + \left[-1 + \frac{1}{1-\frac{z}{2}}\right]$$

$ROC = \left|\frac{1}{3z}\right| < 1$ $\quad ROC = \left|\frac{z}{2}\right| < 1$

$|z| > \frac{1}{3}$ $\quad = |z| < 2$

$$= \frac{1}{1-\frac{1}{3z}} + \frac{\frac{1}{2}z}{\left(1-\frac{z}{2}\right)}, \quad \boxed{ROC = \frac{1}{3} < |z| < 2}$$

$$= \frac{1}{\left(1-\frac{1}{3}z^{-1}\right)} + \frac{\frac{1}{2}z}{\left(1-\frac{z}{2}\right)}$$

$$\boxed{x(z) = \frac{\frac{5}{3}z}{\left(z-\frac{1}{3}\right)(2-z)}}$$

2.

a. $x[n] = b^{|n|}, \quad b>0$

$x[n] = b^{n}u[n] + b^{-n}u[-n]$

3. $x[n] = b^{|n|}, \; b>0$

$= b^{n}u[n] + b^{-n}u[-n-1]$

$= x_1[n] + x_2[n]$

4.

$x_1[n] = b^{n}u[n]$

$X_1(z) = \dfrac{z}{z-b} \quad \boxed{ROC = |z|>b}$

5.

$x_2[n] = b^{-n}u[-n-1]$

$= \left(\dfrac{1}{b}\right)^{n}u[-n-1]$

6.

$X_2[z] = \dfrac{-z}{z-\frac{1}{b}} \quad \boxed{ROC = |z| < \frac{1}{b}}$

7. $X[z] = X_1(z) + X_2(z)$

$= \dfrac{z}{z-b} - \dfrac{z}{z-\frac{1}{b}}, \quad ROC = b<|z|<\dfrac{1}{b}$

8.

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