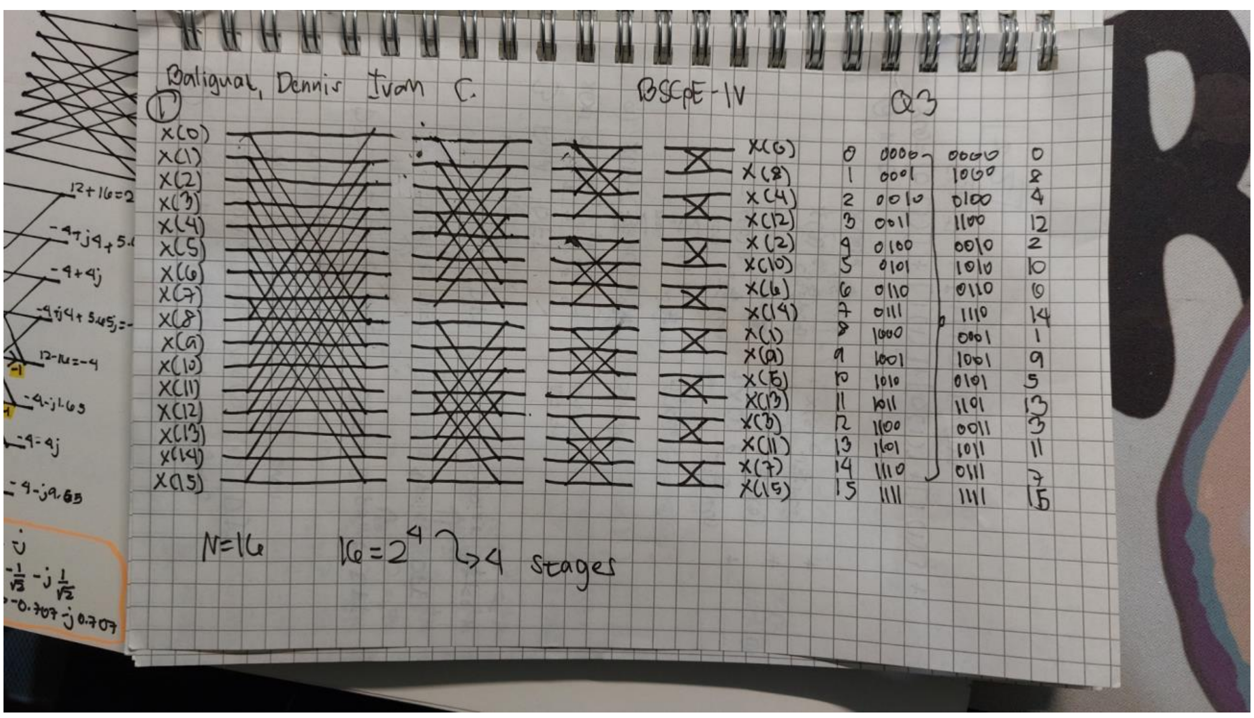
(60pts)

1. Draw the flow graph for DIF FFT where $n = 16$. (10)
2. Is it possible to create an algorithm for Decimation in Frequency DFT where n is even? Discuss briefly. (10)
3. Find the DFT of the signal $x[n] = \{0, 1, 0, 1, 1, -1, 0\}$: (20)
   a. Mathematically.
   b. Using the matrix representation.
4. Find the DFT of the signal $x[n] = \begin{cases} n, 0 \le n \le 3 \\ 0, otherwise \end{cases}$ using the DIT FFT. (10)
5. Find the signal x[n] using the IDFT of the $X[k] = \{3, 0, 0, 0, -3, 0, 0, 0\}$ where n=8. (10)



② Yes. Because DIF is just basically the reverse of DIT. And DIT is used for even number of samples.

DIT: 2-point DFT
$\Downarrow$
4-point DFT
$\Downarrow$
8-point DFT

DIF: 8-point DFT
$\Downarrow$
4-point DFT
$\Downarrow$
2-point DFT

3. Find DFT

$$\begin{array}{c} \\ x(n) \end{array}\begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ [0, & 1, & 0, & 1, & 1, & -1, & 0] \end{array}$$

$N = 7$
$N-1 = 6$

a) $X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$; $0 \leq k \leq N-1$

$$= \sum_{n=0}^{6} x(n) e^{-j\frac{2\pi}{7}kn}, \quad 0 \leq k \leq 6$$

$$= x(0) e^{-j\frac{2\pi}{7}k(0)} + x(1) e^{-j\frac{2\pi}{7}k(1)} +$$
$$x(2) e^{-j\frac{2\pi}{7}k(2)} + x(3) e^{-j\frac{2\pi}{7}k(3)} +$$
$$x(4) e^{-j\frac{2\pi}{7}k(4)} + x(5) e^{-j\frac{2\pi}{7}k(5)} +$$
$$x(6) e^{-j\frac{2\pi}{7}k(6)}$$

@ $k = 0$

$X(0) = [0(1)] + [1(1)] + [0(1)] + [1(1)] + [1(1)] + [-1(1)] + [0(1)]$

$\boxed{= 2}$

$x(n) = \{0, 1, 2, 3, 4, 5, 6, 7\}$

$X(k) = \{28, -4+j9.05j, -4+4j, -4+1.65j, -4, -4-1.6j, -4-$

$W_N = e^{-\frac{2\pi j}{N}}$ $\quad W_7 = e^{-\frac{2\pi j}{7}}$

Twiddle factor

$W_7^0 = e^{-\frac{2\pi j}{7}(0)} = 1$

$W_7^1 = e^{-\frac{2\pi j}{7}(1)} = 0.62 - 0.78j = W_7^8 = W_7^{15}$

$W_7^2 = e^{-\frac{2\pi j}{7}(2)} = -0.22 - 0.97j = W_7^9 = W_7^{16}$

$W_7^3 = e^{-\frac{2\pi j}{7}(3)} = -0.90 - 0.43j = W_7^{10} = W_7^{17}$

$W_7^4 = e^{-\frac{2\pi j}{7}(4)} = -0.90 + 0.43j = W_7^{11} = W_7^{18}$

$W_7^5 = e^{-\frac{2\pi j}{7}(5)} = -0.22 + 0.97j = W_7^{12} = W_7^{19}$

$W_7^6 = e^{-\frac{2\pi j}{7}(6)} = 0.62 + 0.78j = W_7^{13} = W_7^{20}$

$W_7^7 = e^{-\frac{2\pi j}{7}(7)} = 1 = W_7^{14} = W_7^{21}$

@ $K=1$

$X(1) = [0(W_7^0)] + [1(W_7^1)] + [0(W_7^2)] +$
$[1(W_7^3)] + [1(W_7^4)] + [-1(W_7^5)] +$
$[0(W_7^6)]$
$\boxed{= -0.956 - 1.757j}$

@ $K=2$

$X(2) = [0(W_7^0)] + [1(W_7^2)] + [0(W_7^4)] +$
$[1(W_7^6)] + [1(W_7^8)] + [-1(W_7^{10})] +$
$[0(W_7^{12})]$
$\boxed{= 1.925 - 0.541j}$

0, 1, 0, 1, 1, -1, 0

@ $K=3$

$$x(3) = [0(W_7^0)] + [1(W_7^3)] + [0(W_7^6)] + [1(W_7^9)] + [1(W_7^{12})] + [-1(W_7^{15})] + [0(W_7^{18})]$$

$$\boxed{= -1.97 + 0.348j}$$

@ $K=4$

$$x(4) = [0(W_7^0)] + [1(W_7^4)] + [0(W_7^8)] + [1(W_7^{12})] + [1(W_7^{16})] + [-1(W_7^{20})] + [0(W_7^{24})]$$

$$\boxed{= -1.97 - 0.348j}$$

@ $K=5$

$$x(5) = [0(W_7^0)] + [1(W_7^5)] + [0(W_7^{10})] + [1(W_7^{15})] + [1(W_7^{20})] + [-1(W_7^{25})] + [0(W_7)^{25}]$$

$$\boxed{= 1.925 + 0.541j}$$

@ $K=6$

$$x(6) = [0(W_7^0)] + [1(W_7^6)] + [0(W_7^{12})] + [1(W_7^{18})] + [1(W_7^{24})] + [-1(W_7^{30})] + [0(W_7^{36})]$$

$$\boxed{= -0.954 + 1.757j}$$

$$\boxed{X(K) = \{2, -0.956 - 1.757j, 1.925 - 0.541j, -1.97 + 0.348j, -1.97 - 0.348j, 1.925 + 0.541j, -0.954 + 1.757j}$$

$x(n) = \{0, 1, 2, 3, 4, 5, 6, 7\}$
$X(k) = \{28, -4+j9.05j, ...$

ger

3. $x[n] = \{0, 1, 0, 1, 1, -1, 0\}$

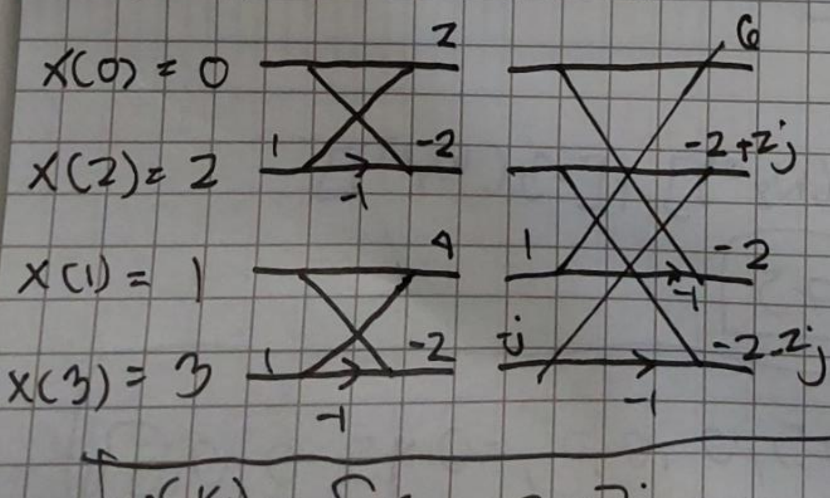
b)
$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_7^1 & W_7^2 & W_7^3 & W_7^4 & W_7^5 & W_7^6 \\ 1 & W_7^2 & W_7^4 & W_7^6 & W_7^8 & W_7^{10} & W_7^{12} \\ 1 & W_7^3 & W_7^6 & W_7^9 & W_7^{12} & W_7^{15} & W_7^{18} \\ 1 & W_7^4 & W_7^8 & W_7^{12} & W_7^{16} & W_7^{20} & W_7^{24} \\ 1 & W_7^5 & W_7^{10} & W_7^{15} & W_7^{20} & W_7^{25} & W_7^{30} \\ 1 & W_7^6 & W_7^{12} & W_7^{18} & W_7^{24} & W_7^{30} & W_7^{36} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} 2 \\ -0.956 - 1.757j \\ 1.925 - 0.541j \\ -1.97 + 0.348j \\ -1.97 - 0.348j \\ 1.925 + 0.541j \\ -0.956 + 1.757j \end{bmatrix}}$$

4. $x[n] = \begin{cases} n, & 0 \le n \le 3 \\ 0, & \text{elsewhere} \end{cases}$
$= \{0, 1, 2, 3\}$

$N = 4$
$4 = 2^2 \rightarrow 2$ stages

x(0)	00	=>	00	x(0)
x(1)	01		10	x(2)
x(2)	10		01	x(1)
x(3)	11		11	x(3)



$$\boxed{X(k) = \{\underline{6}, \underline{-2+2j}, \underline{-2}, \underline{-2-2j}\}}$$

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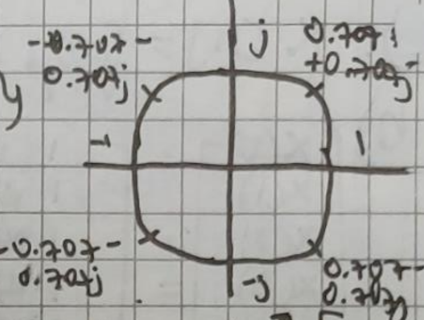
5. $X[K] = \{3, 0, 0, 0, -3, 0, 0, 0\}$

$x(n) = \frac{1}{8} W[K]$



$$= \frac{1}{8} \begin{bmatrix} W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8 & W_8^0 \\ W_8^0 & W_8^{-1} & W_8^{-2} & \longrightarrow & & & W_8^{-8} & \\ W_8^0 & W_8^2 & & & & & & \\ W_8^0 & & & & & & & \\ W_8^0 & & & & & & & \\ W_8^0 & & & & & & & \\ W_8^0 & & & & & & & \\ W_8^0 & W_8^{-8} & & & & & W_8^{-56} & \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 0 \\ 0 \\ -3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

@ $K = 0$

$x(0) = \frac{1}{8}\left[[3(W_8^0)] + [-3(W_8^4)]\right]$

$= \frac{1}{8}[(3) + (-3)(-1)]$

$\boxed{= 0.75}$

@ $K = 4$

$x(4) = \frac{1}{8}\left[3(W_8^0)] + [-3(W_8^{16})\right]$

$\boxed{= -0.75}$

$$\boxed{x(n) = \{0.75, 0, 0, 0, -0.75, 0, 0, 0\}}$$