Fourier series

fourier transform

discrete

FOURIER SERIES

EEE150

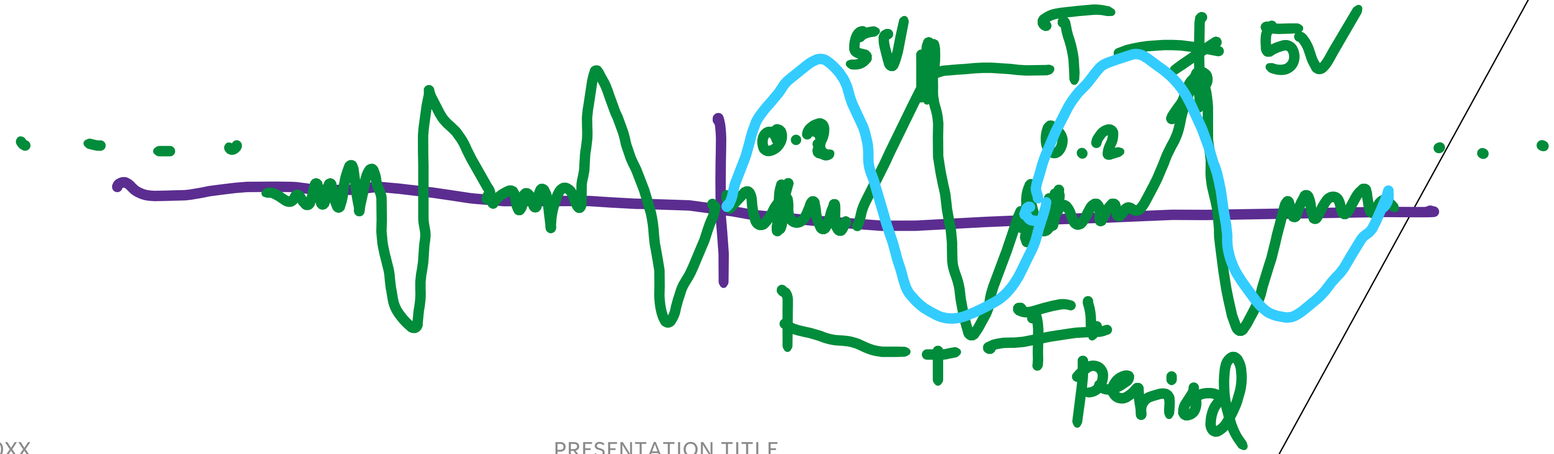
FOURIER SERIES

Every periodic continuous-time signal can be written as a sum of sinusoids.

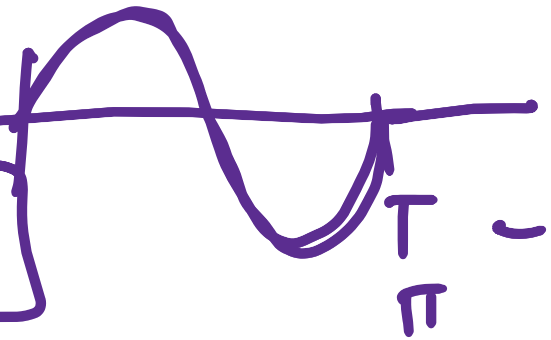
Periodic Signal x(t)

$$x(t+T) = x(t)$$

repeatitive



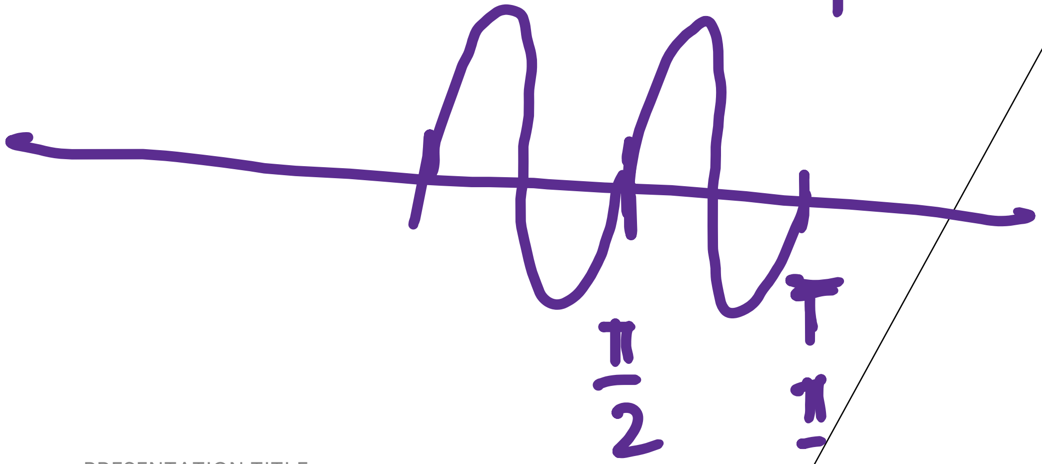
$w_0 = \frac{2\pi}{T}$ [cycles/second]



T - seconds

$$\cos w_0 t = \cos \frac{2\pi}{T} t$$

$$\cos 2w_0 t = \cos \frac{4\pi}{T}$$



FOURIER SERIES REPRESENTATION

e^{-jnw_0t}

$$x(t) = \sum_{-\infty}^{\infty} a_k e^{jkw_0t}$$

coeff.

euler

trigonometric

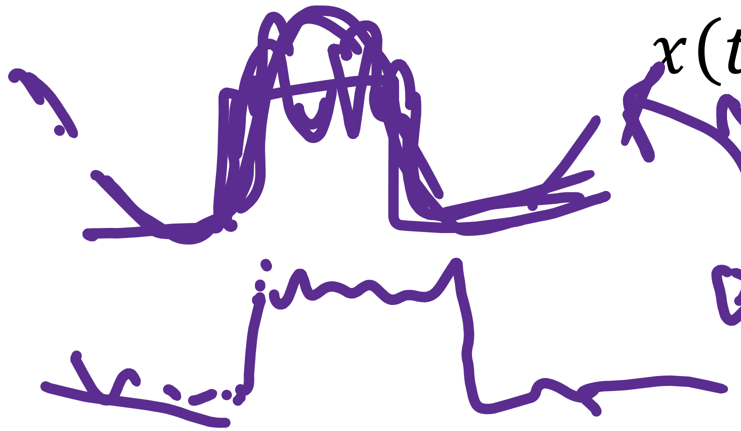
periodic signal

T-period

Compute a_k for a given x(t):

$$x(t)e^{-jnw_0t} = \left[\sum_{-\infty}^{\infty} a_k e^{jkw_0t}\right] e^{-jnw_0t}$$

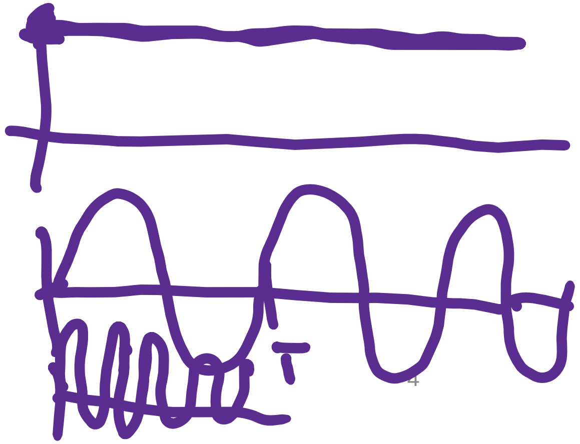
$$= \sum_{-\infty}^{\infty} a_k e^{j(k-n)w_0t}$$



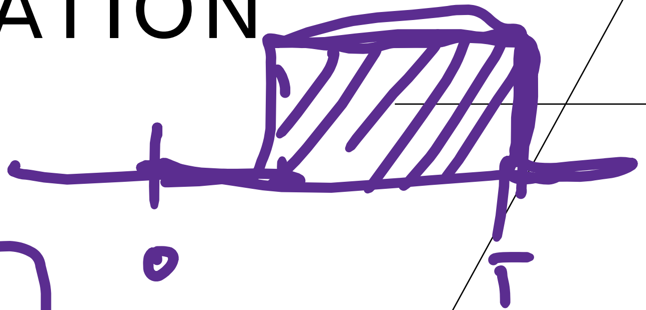
$a_0 =$

$a_1 =$

a_n.



FOURIER SERIES REPRESENTATION



Integrate both sides:

$$\int_0^T x(t)e^{-jnw_0t}dt = \int_0^T \boxed{\sum_{-\infty}^{\infty} a_k} e^{j(k-n)w_0t}dt$$

$$= \sum_{-\infty}^{\infty} a_k \int_0^T e^{j(k-n)w_0t}dt$$

FOURIER SERIES REPRESENTATION

$$\int_0^T e^{j(k-n)w_0 t}\,dt = \int_0^T \cos(k-n)w_0\,dt + j\int_0^T \sin(k-n)w_0\,dt$$

$$\underbrace{\cos 0}_{1}\, w_0 dt + j \underbrace{\sin 0}_{0}\, w_0 dt$$

For $k = n$:

$$\int_0^T 1\,dt = T, \qquad \int_0^T 0\,dt = 0$$

FOURIER SERIES REPRESENTATION

For $k \neq n$:

$$\int_0^T \cos(integer)\, w_0 dt + j sin(integer)\, w_0 dt$$

Thus,

$$\int_0^T x(t)e^{-jnw_0t}dt = a_n T$$

$$\boxed{a_k = \frac{1}{T}\int_0^T x(t)e^{-jkw_0t}dt}$$

coefficient of Fourier series

FOURIER SERIES REPRESENTATION

When x(t) is real,

$$\boxed{x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jkw_0 t}}$$

trigonometric

real & imaginary & ~~exponential~~

FOURIER SERIES REPRESENTATION

When x(t) is real,

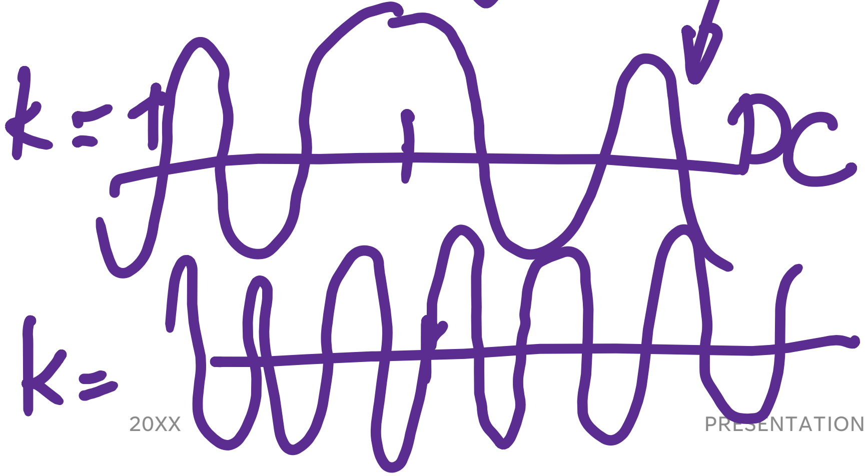
Shifting

$$x(t) = a_0 + \sum_{k=1}^{\infty} 2A_k \cos(\theta_k + k w_0 t)$$

$\phi = 0$

0.5

$$x(t) = a_0 + 2\sum_{k=1}^{\infty} B_k \cos(k w_0 t) + C_k \sin(k w_0 t)$$



DC

coefficients

Fourier series

magnitude of signal

FOURIER SERIES REPRESENTATION

Ex. $x(t) = 5 + 2\cos(w_0 t)$

$$a_k = \frac{1}{T}\int_0^T x(t)\, e^{-jkw_0t}$$

euler $\frac{e^{i\theta} + e^{-i\theta}}{2} = \cos\theta$

$$x(t) = 5 + 2\left[\frac{1}{2}\left(e^{jw_0t} + e^{-jw_0t}\right)\right]$$

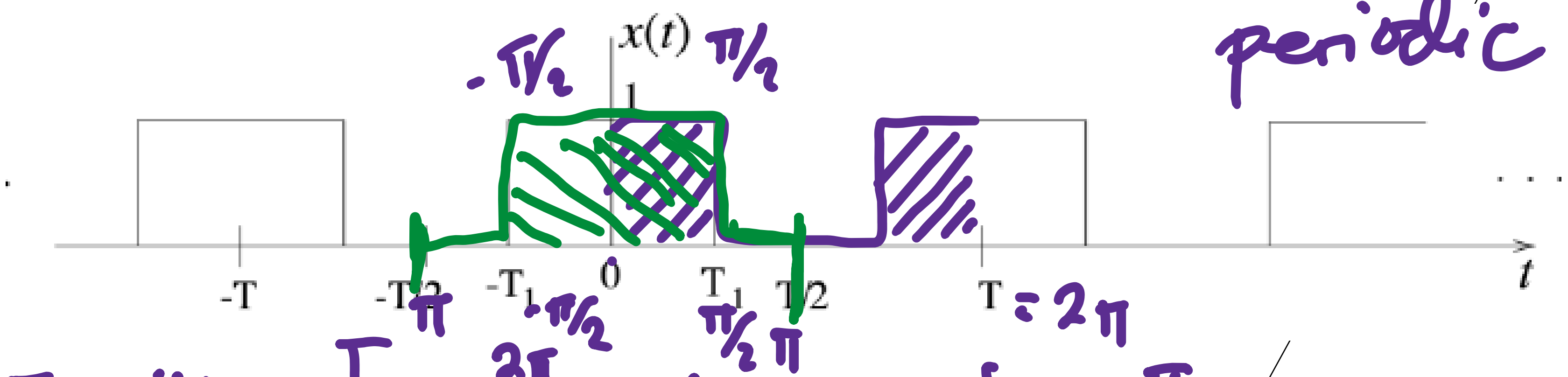
$$x(t) = 5 + 1e^{+jw_0t} + 1e^{-jw_0t}$$

$$a_0 = 5 \quad a_1 = 1 \quad a_{-1} = 1$$

FOURIER SERIES REPRESENTATION

Ex.

periodic



$T = 2\pi \qquad \omega_0 = \frac{T}{2\pi} = \frac{2\pi}{2\pi} = 1$

$a_0 = \frac{1}{T}\int_0^T x(t)e^{-j\underbrace{k\omega_0}_{=0} t}\,dt$

$= \frac{1}{2\pi}\int_0^T x(t)\,e^0\,dt$

$= \frac{1}{2\pi}\int_0^T x(t)\,dt$

$= \frac{1}{2\pi}\int_{-\pi}^{\pi} x(t)\,dt \quad = \cancel{\frac{1}{2\pi}[\pi]}$

$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} 1\,dt$

$= \frac{1}{2\pi}\, t\Big|_{-\pi/2}^{+\pi/2} = \frac{1}{2\pi}\left[\pi/2 - (-\pi/2)\right]$

$a_0 = \frac{1}{2}$

$$a_k = \frac{1}{T}\int_{-T/2}^{+T/2} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} x(t)\, e^{-jk(1)t}\, dt = \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{-jkt}\, dt$$

$$= \frac{1}{2\pi}\frac{-1}{jk}\int_{-\pi/2}^{\pi/2} e^{-jkt}\, -jk\, dt$$

$$= \frac{-1}{2\pi jk}\left[e^{-jkt}\right]_{-\pi/2}^{\pi/2}$$

$$= -\frac{1}{2\pi jk}\left[e^{-jk\pi/2} - e^{-jk(-\frac{\pi}{2})}\right]$$

$$a_k = -\frac{1(2)}{2\pi jk}\left[\frac{e^{-jk\pi/2} - e^{-jk(-\frac{\pi}{2})}}{2}\right]$$

$$a_k = \frac{1}{\pi k}\left[\frac{e^{-jk\pi/2} - e^{jk\pi/2}}{2j}\right]$$

$$a_k = \frac{1}{\pi k}\sin k\pi/2$$

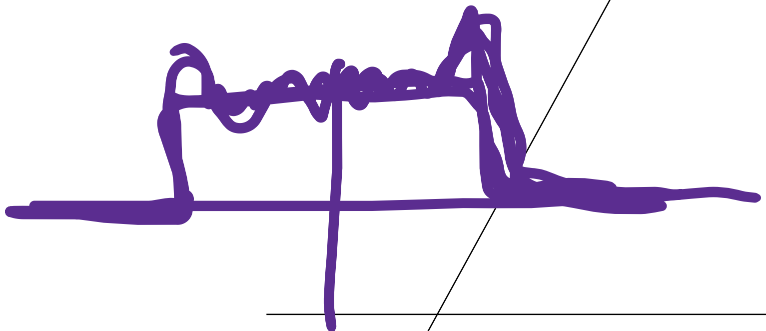
$$\boxed{a_k = \frac{\sin k\pi/2}{\pi k}}$$

$\{a_k\}$

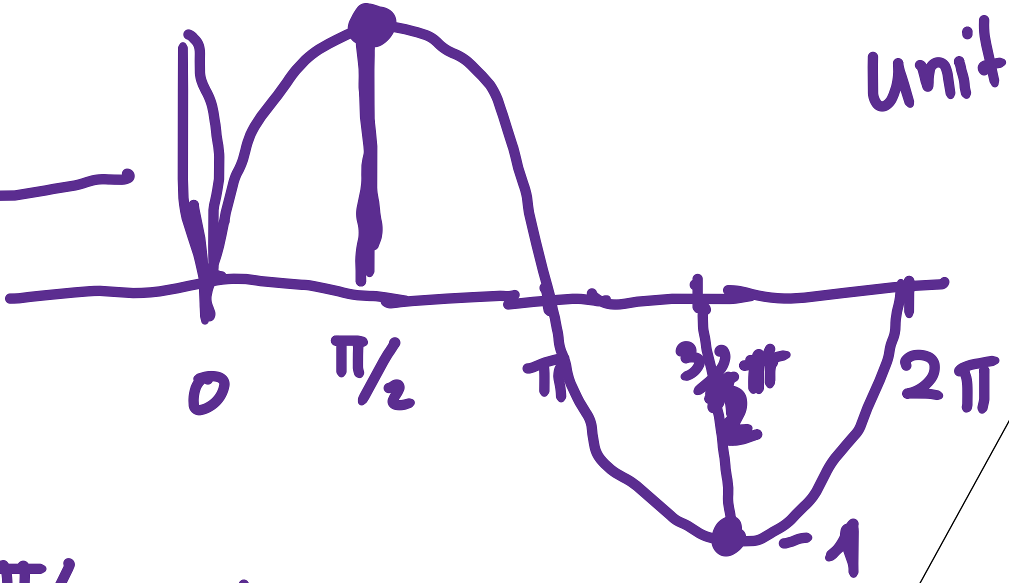
$a_0 = \frac{1}{2}$

$$\boxed{a_k = \frac{\sin k\pi/2}{\pi k}}$$

$\text{sinc}\, x = \frac{\sin x}{x}$



unit magnitu



~~$a_k =$~~

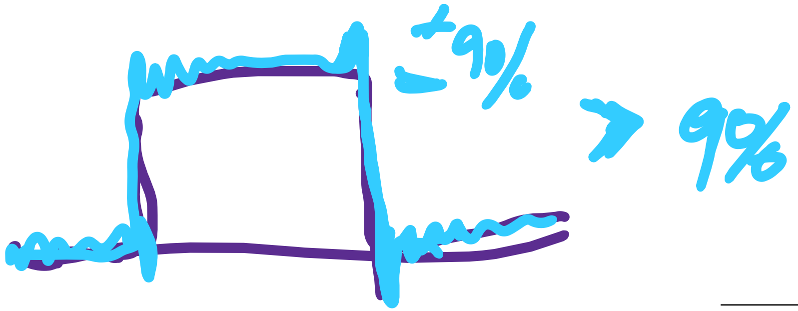
$k=1;\quad a_1 = \frac{\sin \pi/2}{\pi} = \frac{1}{\pi}$

$k=3;\quad a_3 = \frac{\sin 3(\pi/2)}{3\pi} = \frac{-1}{3\pi}$

$k=2;\quad a_2 = \frac{\sin 2(\pi/2)}{\pi 2} = 0$

Fourier series

GIBBS PHENOMENA



- Convergence in error can have some interesting characteristics

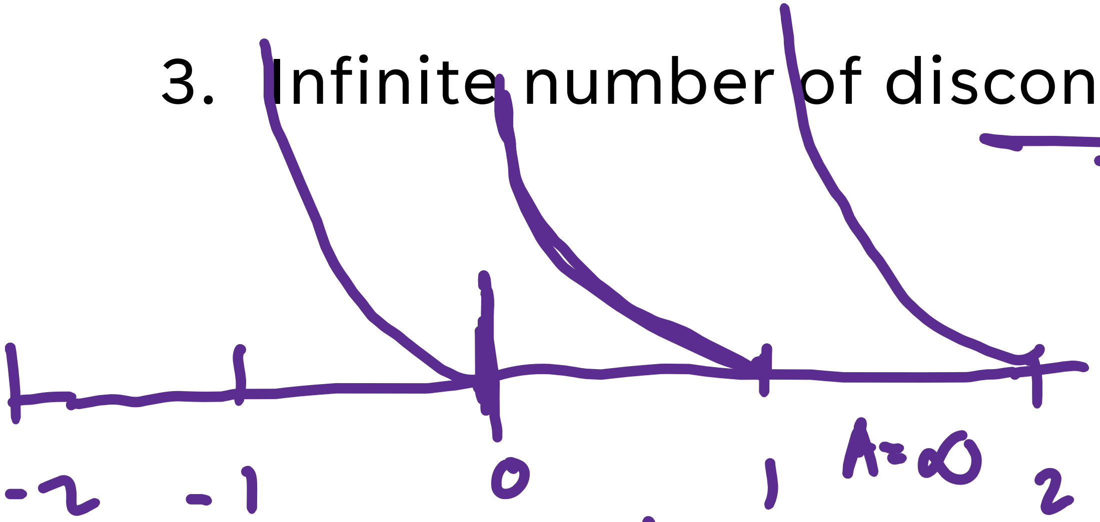
$$\boxed{x(t) = \sum_{k=-N}^{N} c_k e^{jk\omega_0 t}}$$

- This is known as Gibbs phenomena and was first observed by Albert Michelson in 1898.

CONDITIONS FOR WHICH THE ERROR IN THIS APPROXIMATION WILL TEND TO ZERO

fourier series

1. Infinite area under the curve
2. Infinite number of maxima and minima
3. Infinite number of discontinuities



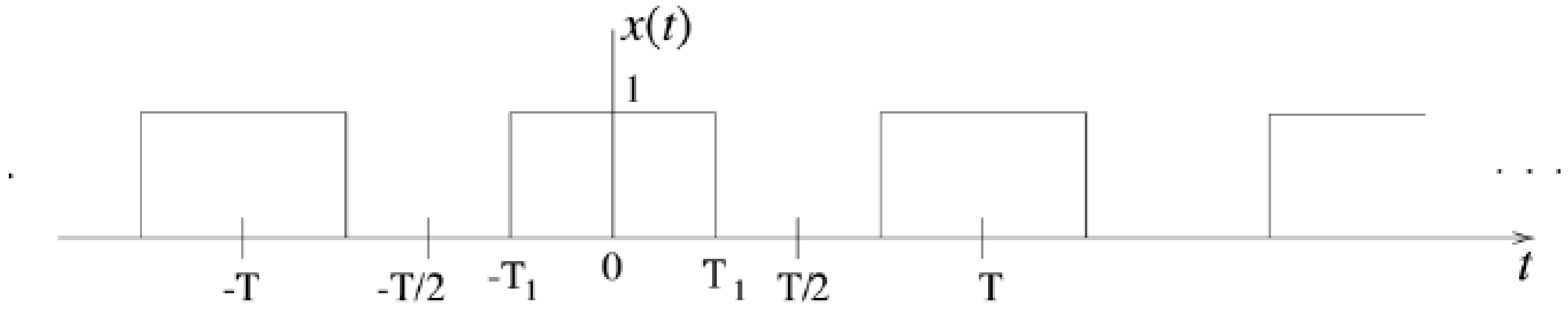
magnitude is ∞ ×

infinite

✓ direchlet conditions

× fourier series

MATLAB/SCILAB ACTIVITY



T = 2*pi

k=0, k=5, k=10