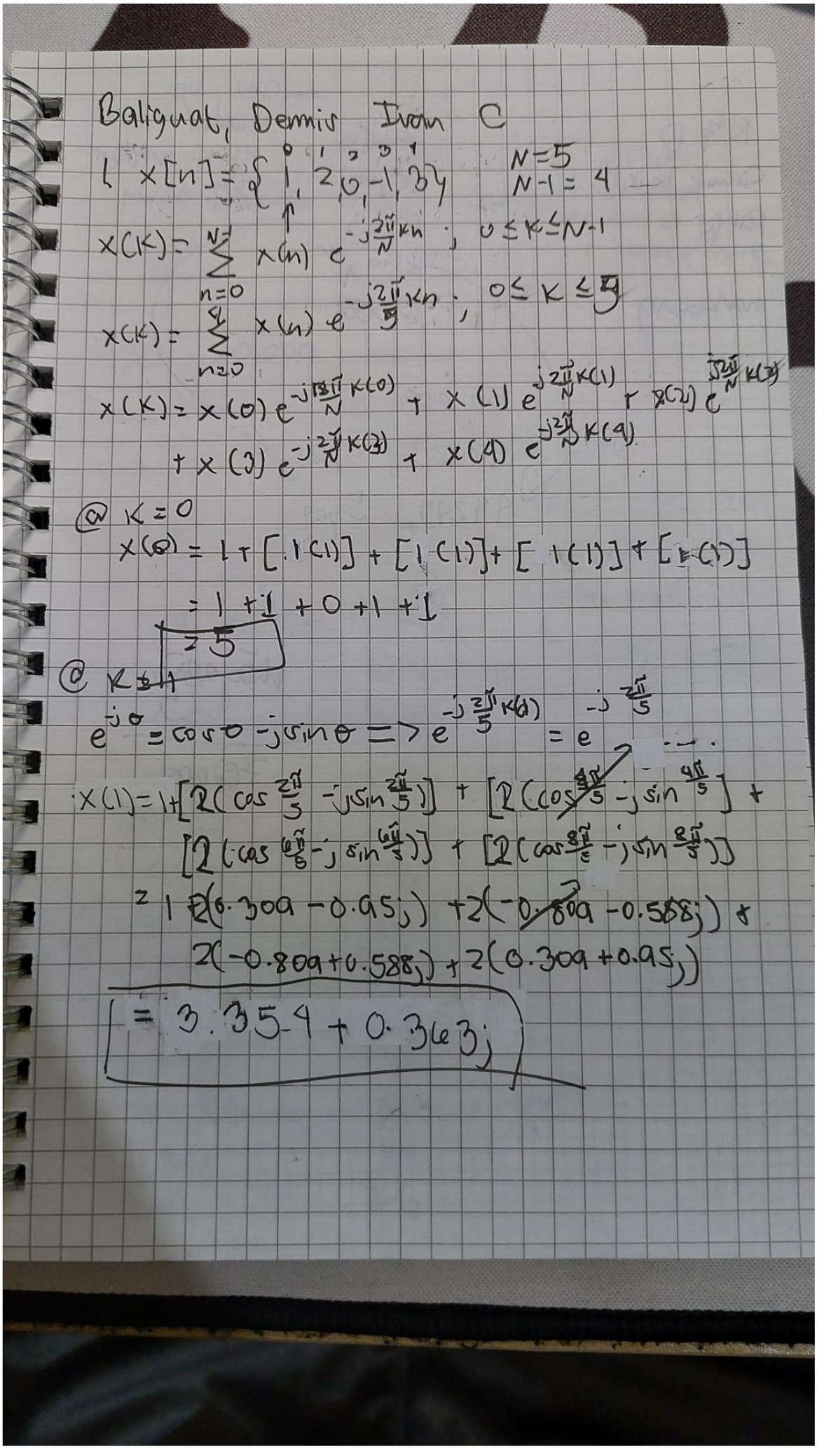

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1. $x[n] = \{1, 2, 0, -1, 3\}$ $N = 5$, $N-1 = 4$

$$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}Kn}; \quad 0 \le K \le N-1$$

$$X(K) = \sum_{n=0}^{4} x(n)\, e^{-j\frac{2\pi}{5}Kn}; \quad 0 \le K \le 4$$

$$X(K) = x(0)\, e^{-j\frac{2\pi}{N}K(0)} + x(1)\, e^{-j\frac{2\pi}{N}K(1)} + x(2)\, e^{-j\frac{2\pi}{N}K(2)} + x(3)\, e^{-j\frac{2\pi}{N}K(3)} + x(4)\, e^{-j\frac{2\pi}{N}K(4)}$$

@ $K = 0$

$$X(0) = 1 + [1(1)] + [1(1)] + [1(1)] + [1(1)]$$

$$= 1 + 1 + 0 + 1 + 1$$

$$= 5$$

@ $K = 1$

$$e^{-j\theta} = \cos\theta - j\sin\theta \Rightarrow e^{-j\frac{2\pi}{5}K(1)} = e^{-j\frac{2\pi}{5}}$$

$$X(1) = 1 + [2(\cos\tfrac{2\pi}{5} - j\sin\tfrac{2\pi}{5})] + [2(\cos\tfrac{4\pi}{5} - j\sin\tfrac{4\pi}{5})] + [2(\cos\tfrac{6\pi}{5} - j\sin\tfrac{6\pi}{5})] + [2(\cos\tfrac{8\pi}{5} - j\sin\tfrac{8\pi}{5})]$$

$$= 1 + 2(0.309 - 0.95j) + 2(-0.809 - 0.588j) + 2(-0.809 + 0.588j) + 2(0.309 + 0.95j)$$

$$= 3.354 + 0.363j$$

@ K=2

$X(2) = 1 + [(\cos \frac{4\pi}{5} - j\sin \frac{4\pi}{5}) + (\cos \frac{8\pi}{5} - j\sin \frac{8\pi}{5})] + [(\cos \frac{12\pi}{5} - j\sin \frac{12\pi}{5}) + (\cos \frac{16\pi}{5} - j\sin \frac{16\pi}{5})]$

$= -3.354 + 1.539j$

@ K=3

$X(3) = -3.354 - 1.539j$

@ K=4

$X(4) = 3.354 - 0.363j$

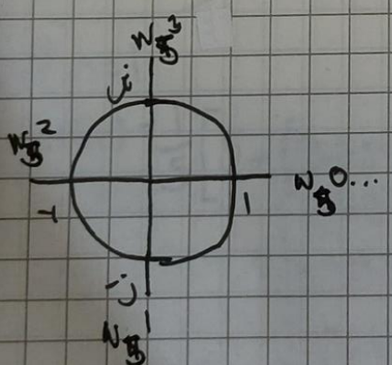
$X(K) = \{5, -3.354 + 6.363j, -3.354 + 1.539j, 3.354 - 1.539j, 3.354 - 0.363j\}$

Magnitude: $\{5, 3.37, 3.69, 3.69, 3.37\}$

phase: $\{0, 0.108, 2.71, -2.71, -0.37\}$

2. $x[n] = \{1, 2, 0, -1, 3\}$ $N=5$

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \end{bmatrix} = \begin{bmatrix} W_5^0 & W_5^0 & W_5^0 & W_5^0 & W_5^0 \\ W_5^0 & W_5^1 & W_5^2 & W_5^3 & W_5^4 \\ W_5^0 & W_5^2 & W_5^4 & W_5^6 & W_5^8 \\ W_5^0 & W_5^3 & W_5^6 & W_5^9 & W_5^{12} \\ W_5^0 & W_5^4 & W_5^8 & W_5^{12} & W_5^{16} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \\ 3 \end{bmatrix}$$



$W_N = e^{-j\frac{2\pi}{5}} \Rightarrow W_5^0 = 1 \Rightarrow W_N^0 = 1$

$W_5^1 = e^{-j\frac{2\pi}{5} \cdot 1} = \cos\frac{2\pi}{5} - j\sin\frac{2\pi}{5} = 0.309 - 0.951j$

$W_5^2 = -0.809 - 0.588j$

$W_5^3 = -0.809 + 0.58j$

$W_5^4 = 0.309 + 0.95j$

$W_5^6 =$

$W_5^8 =$

$W_5^9 =$

$W_5^{12} =$

$W_8^{16} =$

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & a & b & c & d \\ 1 & b & d & a & c \\ 1 & c & a & d & b \\ 1 & d & c & b & a \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \\ 3 \end{bmatrix}$$

let $a = 0.309 - 0.95j$

$b = -0.809 - 0.588j$

$c = -0.809 + 0.588j$

$d = 0.309 + 0.95j$

$$\begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \end{bmatrix} = \begin{bmatrix} 5 \\ 3.354 + 0.363j \\ -3.354 + 1.539j \\ -3.354 - 1.539j \\ 3.354 - 0.363j \end{bmatrix}$$

3. IDFT

$N=5$

$X(k) = \{5,\ 3.354+0.363j,\ -3.354+1.539j,\ -3.354-1.539j,\ 3.354-0.363j\}$

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}kn},\quad 0 \le n \le N-1$$

$$x(n) = \frac{1}{5}\sum_{k=0}^{4} X(k)\, e^{j\frac{2\pi}{5}kn},\quad 0 \le n \le N-1$$

$$= \frac{1}{5}\left[X(0) + X(1)e^{j\frac{2\pi}{5}n} + X(2)e^{j\frac{2\pi}{5}2n} + X(3)e^{j\frac{2\pi}{5}3n} + X(4)e^{j\frac{2\pi}{5}4n}\right]$$

@ $n=0$

$$x(0) = \frac{1}{5}[5 + (3.354+0.363j) + (-3.354+1.539j) + (-3.354-1.539j) + (3.354-0.363j)]$$

$$x(0) = \frac{1}{5}(5+0)$$

$$\boxed{x(0)=1}$$

$e^{j\theta} = \cos\theta + j\sin\theta$

@ $n=1$

$$x(1) = \frac{1}{5}\left[5 + \left[(3.354+0.363j)\left(\cos\left(\tfrac{2\pi}{5}\right) - j\sin\left(\tfrac{2\pi}{5}\right)\right]\right. +$$
$$\left[(-3.354+1.539j)\left(\cos\left(\tfrac{2\pi}{5}\right) - j\sin\left(\tfrac{2\pi}{5}\right)\right] +$$
$$\left[(-3.354-1.539j)\left(\cos\left(\tfrac{2\pi}{5}\right) - j\sin\left(\tfrac{2\pi}{5}\right)\right] +$$
$$\left.\left[(3.354-0.363j)\left(\cos\left(\tfrac{2\pi}{5}\right) - j\sin\left(\tfrac{2\pi}{5}\right)\right]\right]$$

$$\boxed{x(1)=2}$$

@ n=2

$$x(2) = \frac{1}{5}\left[5 + (3.354+0.363j)\left(\cos\frac{4\pi}{5} - j\sin\frac{4\pi}{5}\right) + (-3.354+1.539j)\left(\cos\frac{8\pi}{5} - j\sin\frac{8\pi}{5}\right) + (-3.354-1.539j)\left(\cos\frac{12\pi}{5} - j\sin\frac{12\pi}{5}\right) + (3.354-0.363j)\left(\cos\frac{16\pi}{5} - j\sin\frac{16\pi}{5}\right)\right]$$

$$\boxed{x(2) = 0}$$

@ n=3

$$\boxed{x(3) = -1}$$

@ n=4

$$\boxed{x(4) = 3}$$

$$\boxed{x(n) = \{1, 2, 0, -1, 3\}}$$

4.

$$x[n] = \frac{1}{5} W_5[k]$$

$$= \left(\frac{1}{5}\right) \begin{bmatrix} W_5^0 & W_5^0 & W_5^0 & W_5^0 & W_5^0 \\ W_5^0 & W_5^{-1} & W_5^{-2} & W_5^{-3} & W_5^{-4} \\ W_5^0 & W_5^{-2} & W_5^{-4} & W_5^{-6} & W_5^{-8} \\ W_5^0 & W_5^{-3} & W_5^{-6} & W_5^{-9} & W_5^{-12} \\ W_5^0 & W_5^{-4} & W_5^{-8} & W_5^{-12} & W_5^{-16} \end{bmatrix} \begin{bmatrix} 5 \\ 3.354+0.363j \\ -3.354+1.539j \\ -3.354-1.539j \\ 3.354-0.363j \end{bmatrix}$$

$$= \{1, 2, 0, -1, 3\}$$