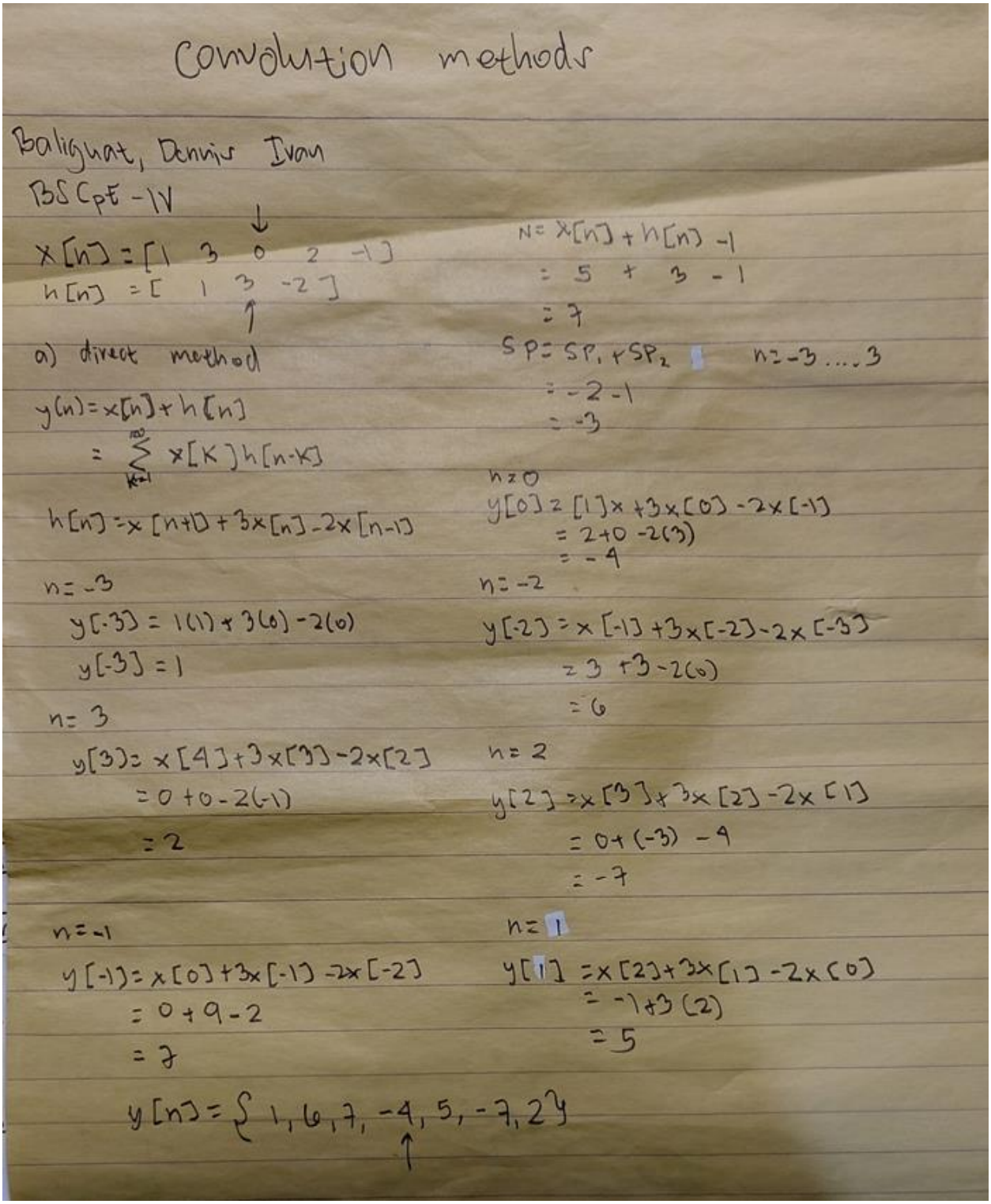


Convolution methods

Baliguat, Dennis Ivan
BSCpE - IV

$x[n] = [1 \quad 3 \quad \underset{\downarrow}{0} \quad 2 \quad -1]$

$h[n] = [1 \quad \underset{\uparrow}{3} \quad -2]$

a) direct method

$y(n) = x[n] * h[n]$

$= \sum_{k=1}^{\infty} x[K]\,h[n-K]$

$h[n] = x[n+1] + 3x[n] - 2x[n-1]$

$n = -3$

$y[-3] = 1(1) + 3(0) - 2(0)$

$y[-3] = 1$

$n = 3$

$y[3] = x[4] + 3x[3] - 2x[2]$

$= 0 + 0 - 2(-1)$

$= 2$

$n = -1$

$y[-1] = x[0] + 3x[-1] - 2x[-2]$

$= 0 + 9 - 2$

$= 7$

$N = x[n] + h[n] - 1$

$= 5 + 3 - 1$

$= 7$

$SP = SP_1 + SP_2$ $\quad n = -3 \ldots 3$

$= -2 - 1$

$= -3$

$n = 0$

$y[0] = [1]x + 3x[0] - 2x[-1]$

$= 2 + 0 - 2(3)$

$= -4$

$n = -2$

$y[-2] = x[-1] + 3x[-2] - 2x[-3]$

$= 3 + 3 - 2(0)$

$= 6$

$n = 2$

$y[2] = x[3] + 3x[2] - 2x[1]$

$= 0 + (-3) - 4$

$= -7$

$n = 1$

$y[1] = x[2] + 3x[1] - 2x[0]$

$= -1 + 3(2)$

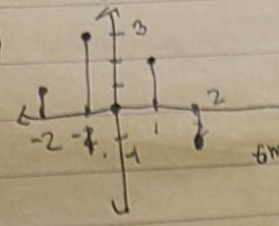
$= 5$

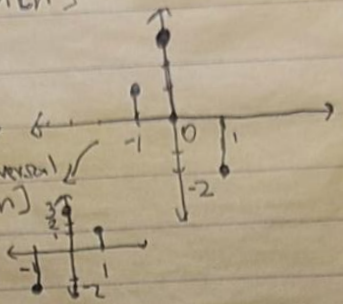
$y[n] = \{1, 6, 7, \underset{\uparrow}{-4}, 5, -7, 2\}$

b. graphical method

$x[n] = [\underset{-2}{1}\ \underset{-1}{3}\ 0\ 2\ -1]$

$h[n] = [\underset{-1}{1}\ \underset{1}{3}\ -2]$

$x[n]$ 

$h[n]$ 

time reversal $h[-n]$

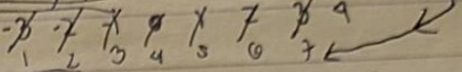
Length$_1$ = 5

Length$_2$ = 3

result length = 5 + 3 − 1 = 7

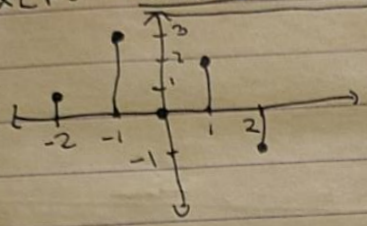
starting point
$x = -2$
$h = -1$

starting point = (x) + (h) = (−2) + (−1) = −3

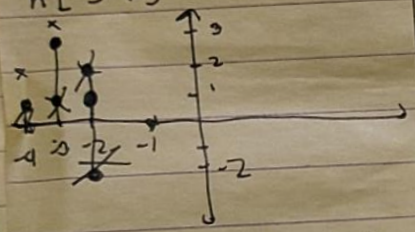
ending point 

= 3

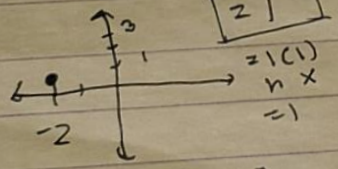
∴ from −3 to 3

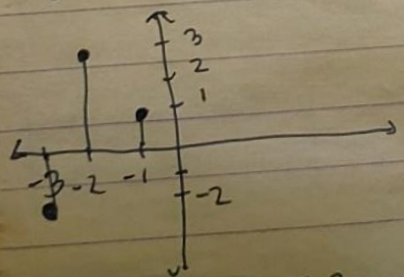
$x[k]$ → constant all throughout 

@ $q = -3$

$h[-3-k]$ 

$x[n] * h[-3-k] = y[n]$ = 1

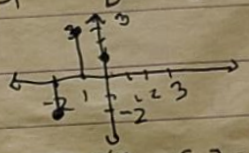


@ $q = -2$ $h[-2-k]$ 

$x[n] * h[-2-k] = y[n]$

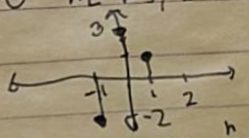
$= 3(1) + 1(3)$

= 6

@ $q = -1$ $h[-1-k]$ 

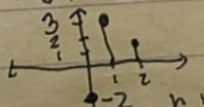
$x[n] * h[-1-k] = y[n] = -2(1) + 3(3) + 1(0)$

= 7

@ $q = 0$ $h[-k]$, 

$x[n] * h[-k] = y[n] = -2(3) + 3(0) + 1(2)$

$= -6 + 2$ = −4

@ $q = 1$ $h[1-k]$ 

$x[n] * h[1-k] = y[n] = -2(0) + 3(2) + 1(-1)$

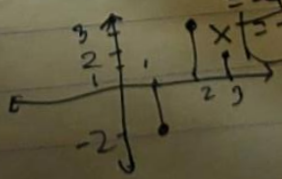
$= 0 + 6 - 1$

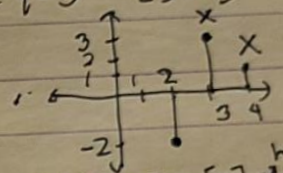
= 5

@ $q = 2$ $h[2-k]$

$x[n] * h[2-k] = y[n] = (-2)(2) + (3)(-1)$

$= -4 + -3$ = −7



@ $q = 3$ $h[3-k]$ 

$x[n] * h[3-k] = y[n] = (-2)(-1)$

= 2

$y[n] = \{1, 6, 7, -4, 5, -7, 2\}$

$x[n] = [1 \quad 3 \quad \underset{\uparrow}{0} \quad 2 \quad -1]$

$h[n] = [1 \quad 3 \quad -2]$

convolution sum

	1	3	-2	
	1	3	0	2	-1
		↑			

1	1	3	-2				
3		3	9	-6			
0			0	0	0		
2				2	6	-4	
-1					-1	-3	2
	1	6	+7	-4	5	-7	2

convolution array

$x[n]$

$h[n]$	1	3	0	2	-1
1	1	3	0	2	-1
3	3	9	0	6	-3
-2	-2	-6	0	-4	2

$y[n] = [1 \quad 6 \quad 7 \quad -4 \quad 5 \quad -7 \quad 2]$

Matrix by vector

$x[n]$ $\qquad$ $h[n]$ $\qquad\qquad$ $y[n]$

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 3 & 1 \\ 2 & 0 & 3 \\ -1 & 2 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 0 & 9 & -2 \\ 2 & 0 & -6 \\ -1 & 6 & 0 \\ 0 & -3 & -4 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 6 \\ 7 \\ -4 \\ 5 \\ -7 \\ 2 \end{bmatrix}$$