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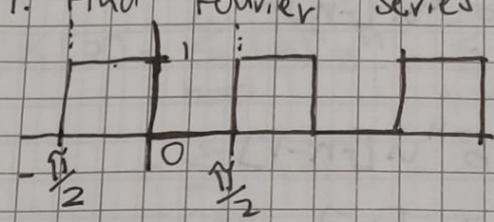
1. Find Fourier Series



$P = \frac{\pi}{2} + \frac{\pi}{2} = \pi \qquad L = \frac{\pi}{2}$

$= \frac{1}{2\frac{\pi}{2}} = \frac{1}{\pi}$

$$f(x) = a_0 + \sum_{n=1}^{N} \left[a_n \cos\left(\frac{n\pi}{L}x\right) + b_n \sin\left(\frac{n\pi}{L}\right) \right]$$

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(x)\,dx$$

$$= \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{0} 1\,dx + \cancel{\int_{0}^{\pi/2} 0\,dx} \to 0$$

$$= \frac{1}{\pi} (1 \cdot x)\Big|_{-\frac{\pi}{2}}^{0} = \frac{1}{\pi}\left(\frac{\pi}{2}\right)$$

$$\boxed{a_0 = \frac{1}{2}}$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right) dx \quad ; n = 1, 2, 3, \ldots$$

$$= \frac{2}{\pi} \int_{-\frac{\pi}{2}}^{0} 1 \cos\left(\frac{n\pi x}{\frac{\pi}{2}}\right) dx + \int_{0}^{\pi/2} 0 \cos\left(\frac{n\pi x}{\pi/2}\right) dx$$

$$a_n = \frac{2}{\pi}(0 + 0)$$

$\cancel{a_n = \frac{2}{\pi}}$

$\boxed{a_n = 0}$, integration of cos = sin

sin (n π) is always zero

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\left(\frac{n\pi}{L}x\right)dx, \quad n = 1, 2, 3, \ldots$$

$$= \frac{2}{\pi}\int_{-\pi/2}^{0} 1\sin\left(\frac{n\pi}{\pi/2}x\right)dx + \int_{0}^{\pi/2} \cancel{0 \sin\left(\frac{n\pi}{\pi/2}x\right)dx}$$

0

$$= \frac{2}{\pi}\int_{-\pi/2}^{0} \sin\left(\frac{n\pi}{\pi/2}x\right)dx$$

@ b_1

$$= \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{0} \sin\left(\frac{1\pi}{\pi/2}x\right)dx = -\frac{2}{\pi}$$

@ b_2

$$= \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{0} \sin\left(\frac{2\pi}{\frac{\pi}{2}}x\right)dx = 0$$

@ b_3

$$= \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{0} \sin\left(\frac{3\pi}{\frac{\pi}{2}}x\right)dx = -\frac{2}{3\pi}$$

@ b_4

$$= \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{0} \sin\left(\frac{4\pi}{\frac{\pi}{2}}x\right)dx = 0$$

@ b_5

$$= \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{0} \sin\left(\frac{5\pi}{\frac{\pi}{2}}x\right)dx = -\frac{2}{5\pi}$$

$$b_n = \begin{cases} 0 & \text{even} \\ \frac{1}{n}\left(-\frac{2}{\pi}\right) & \text{odd} \end{cases}$$

$$\boxed{f(x) = \sum_{n=1,3,5,\ldots}^{\infty}\left(\frac{1}{n}\right)\left(-\frac{2}{\pi}\right)\sin(nx)}$$

2. DTFT

a. $x[n] = 2^n u[-n]$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$u(n) = \begin{cases} 0 & t<0 \\ 1 & t \geq 0 \end{cases}$$

$$u(-n) = \begin{cases} 0 & t \geq 0 \\ 1 & t < 0 \end{cases}$$

$$= \sum_{n=-\infty}^{\infty} 2^n u[-n] e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{0} 2^n e^{-j\omega n}$$

$$\boxed{X(\omega) = \frac{1}{1-2e^{j\omega}} = \frac{1}{1-2e^{j2\pi f}}}$$

3. Z-transform and ROC

a) $x[n] = \begin{cases} (\frac{1}{2})^n - 2^n & n \geq 0 \\ 0 & n < 0 \end{cases}$

$x[n] = (\frac{1}{2})^n - 2^n$

$\boxed{ROC = |Z| > \frac{1}{2}}$ $\boxed{ROC = |Z| > 2}$

$$X(\cancel{z}) = \sum_{n=0}^{\infty} (\tfrac{1}{2})^n - 2^n z^{-n}$$

$$= \sum_{n=0}^{\infty} [\tfrac{1}{2}$$

no overlap between
2 regions
Z-transfor D.N.E.

b) $x[n] = \{ \overset{0}{\underset{\uparrow}{1}}, \overset{1}{2}, \overset{2}{-1}, \overset{3}{0}, \overset{4}{0}, \overset{5}{0}, \overset{6}{-1} \}$

$N=6$

$x[n] \rightleftharpoons X[z]$

$$X[z] = \sum_{n=-\infty}^{\infty} x[n] z^{-n} = \sum_{n=0}^{6} x[n] z^{-n}$$

$$= x(0) z^{-0} + x(1) z^{-1} + x(2) z^{-2} + \cancel{x(3)} z^{-3} + \cancel{x(4)} z^{-4} + \cancel{x(5)} z^{-5} + x(6) z^{-6}$$

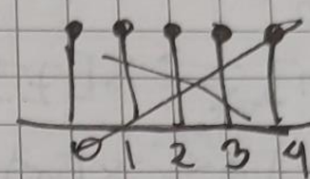
$$\boxed{= 1 + \frac{2}{z} - \frac{1}{z^2} - \frac{1}{z^6}}$$

ROC = entire z-plane except $z = 0$; finite causal

4. convolution of 2 signals

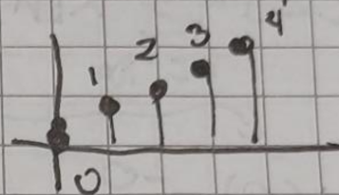
a) $x_1[n] = \{ \overset{-1}{1}, \underset{\uparrow}{\overset{0}{2}}, \overset{1}{-1} \}$

$x_2[n] = \begin{cases} n, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$



$x(n) = x_1(n) * x_2(n)$

$X_1(z) \cdot X_2(z)$



$= (1z + 2 - z^{-1})$

$(0z^0 + 1z^{-1} + 2z^{-2} + 3z^{-3} + 4z^{-4})$

$= 1z + 1 + \frac{2}{z} + \frac{3}{z^2} + \frac{4}{z^3} +$

$2 + 2z^{-1} + 4z^{-2} + 6z^{-3} + 8z^{-4} +$

$- \left(z^{-1} + \frac{1}{z^2} + \frac{2}{z^3} + \frac{3}{z^4} + \frac{4}{z^5} \right)$

$$\boxed{= z + \frac{3}{z} + \frac{6}{z^2} + \frac{8}{z^3} + \frac{5}{z^4} + \frac{4}{z^5} + 3}$$

b) $x_1[n] = u(n) \xrightarrow{} $ unit step
$$x_2[n] = \delta[n] + \left(\frac{1}{2}\right)^n u[n]$$

$$x(n) = x_1(n) * x_2(n)$$
$$= x_1(z) \cdot \cancel{x_2}(z)$$

$x_1[n] \Rightarrow$ unit step $\Rightarrow \cancel{X_1[z]} = \cancel{\Delta}$
$$\hookrightarrow x[\cancel{z}] = \sum_{n=0}^{\infty} a^n = \boxed{\frac{1}{1-a}}$$

$$x_2[n] = \delta[n] + \left(\frac{1}{2}\right)^{n} u[n]$$
$\hookrightarrow$ impulse $= 1$
$$X[z] = \sum_{n=\infty}^{\infty} \delta(n) z^{-n} \boxed{= 1}$$

$$Z\left(\left(\frac{1}{2}\right)^n u[n]\right) = \frac{1}{1-\frac{1}{2}z^{-1}}$$

$$\boxed{x(\cancel{z}) = \left(\frac{1}{1-a}\right)\left(1 + \frac{1}{1-\frac{1}{2}z^{-1}}\right)}$$