COE150.1

DIGITAL SIGNAL PROCESSING LABORATORY

ACTIVITY #6

Z Transform

I. Z Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

II. Introduction to Poles and Zeros of the Z-Transform

Once the Z-transform of a system has been determined, one can use the information contained in function's polynomials to graphically represent the function and easily observe many defining characteristics. The Z-transform will have the below structure, based on Rational Functions:

$$X(z) = \frac{P(z)}{Q(z)}$$

The two polynomials, $P(z)$ and $Q(z)$, allow us to find the poles and zeros of the Z-transform.

Zeros – The value/s for z where $P(z) = 0$. The complex frequencies that make the overall gain of the filter transfer function zero.

Poles – The value/s for z where $Q(z) = 0$. The complex frequencies that make the overall gain of the filter transfer function infinite.

Example 1: $\dfrac{z+1}{(z-\frac{1}{2})(z+\frac{3}{4})}$

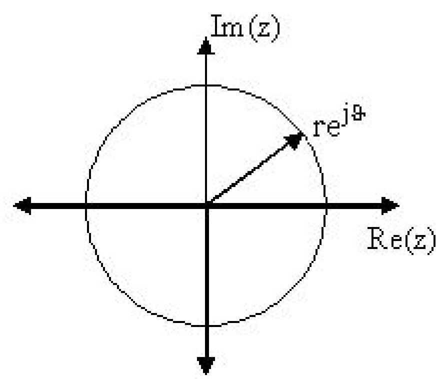
The zeros are: $\{-1\}$
The poles are: {1/2, -3/4}

The reason it is helpful to understand and create these pole/zero plots is due to their ability to help us easily design a filter. Based on the location of the poles and zeros, the magnitude response of the filter can be quickly understood. Also, by starting with the pole/zero plot, one can design a filter and obtain its transfer function very easily.

III. Z – Plane

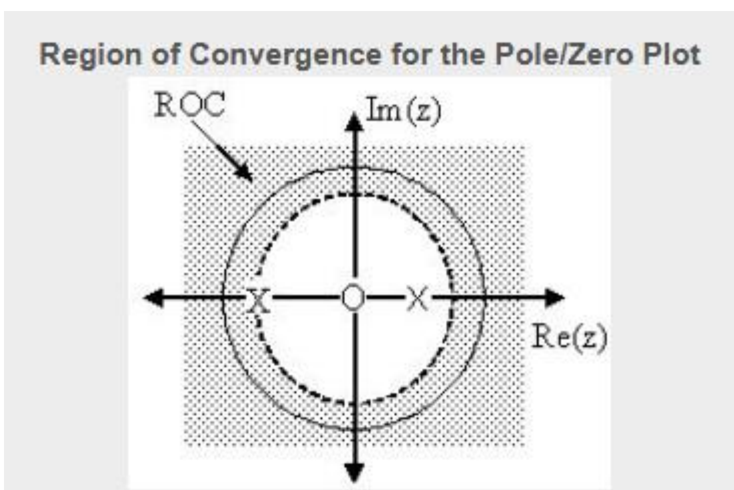
Once the poles and zeros have been found for a given Z-Transform, they can be plotted onto the Z-Plane. The Z-plane is a complex plane with an imaginary and real axis referring to the complex-valued variable z. The position on the complex plane is given by $re^{i\theta}$ and the angle from the positive, real axis around the plane is denoted by θ. When mapping poles and zeros onto the plane, poles are denoted by an "x" and zeros by an "o". The below figure shows the Z-Plane, and examples of plotting zeros and poles onto the plane can be found in the following section.

Z-Plane



Example 2: $H(z) = \frac{z}{(z-1/2)(z+3/4)}$

Zeros: {0}
Poles: {1/2, -3/4}



Region of Convergence for the Pole/Zero Plot

Example 3: Determine the pole-zero plot for $x(n) = a^n u(n)$.

$$X(z) = \frac{1}{1 - az^{-1}}$$

$$X(z) = \frac{z}{z - a}$$

Zero: {0}
Pole: {a}

IV. MATLAB Z Transform Command:

zpk('z') – specifies X(z) = z of the system.

Example 4:

```
>> z = zpk('z');
>> H = (z+.1)*(z+.2)/(z^2+.6*z+.09)

H =

  (z+0.1) (z+0.2)
  ---------------
     (z+0.3)^2

Sample time: unspecified
Discrete-time zero/pole/gain model.
```

pole(SYS) – returns the poles P of the dynamic system SYS as a column vector.

Example 5:

```
>> pole(H)

ans =

  -0.3000 + 0.0000i
  -0.3000 - 0.0000i
```

zero(SYS) – returns the zeros of the dynamic system SYS.

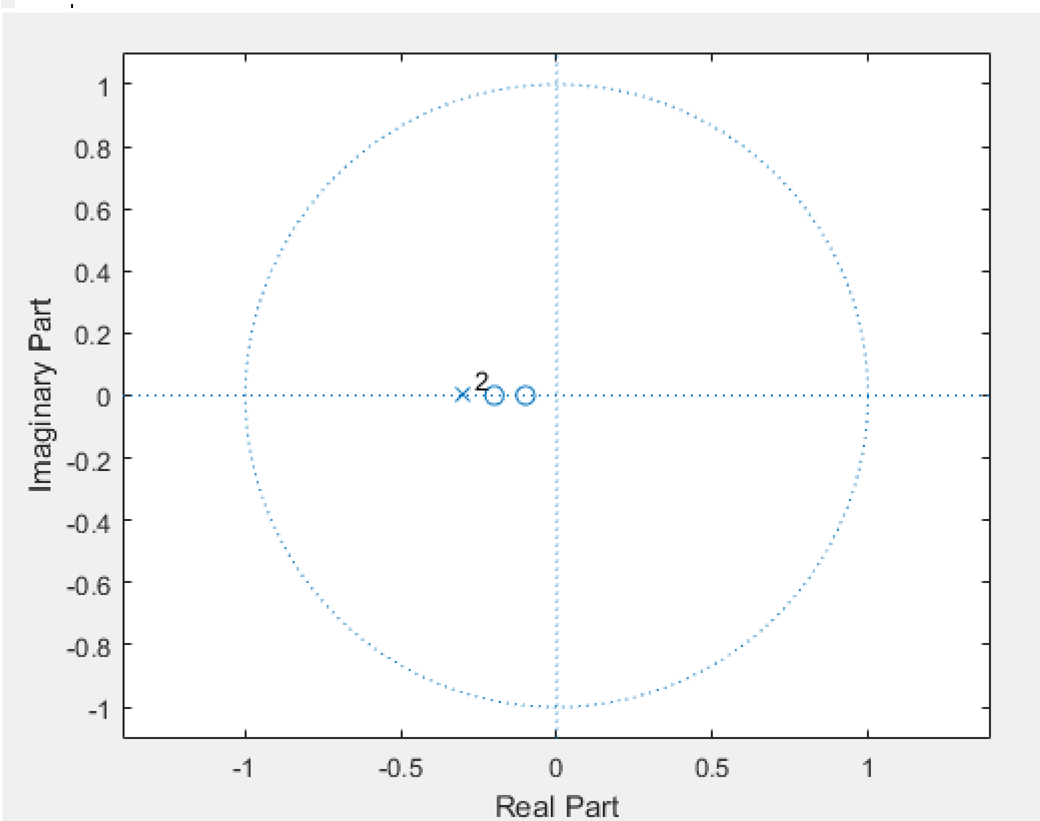
Example 6:

```
>> zero(H)

ans =

   -0.1000
   -0.2000
```

zplane(num, den) – used to plot the z-transform in matlab.

Example 7:

```
>> zplane(zero(H), pole(H))
```



V. Exercises

Determine the zeros and poles of the signal and plot the zero-pole using MATLAB/SCILAB:

a. $X(Z) = (z + 3)/(5z^3 + 3z^2 - 2.5)$

b. $X[Z] = \frac{3+3z^{-1}+3z^{-2}}{1+0.6z^{-1}+0.81z^{-2}}$

c. $x[n] = n^2 u[n]$