DSP CLASSIFICATIONS OF SIGNALS

COE150

ENERGY SIGNALS AND POWER SIGNALS

The **ENERGY** of a signal x(n) is defined as

$$E = \sum_{n=-N_1}^{N_2 \; \alpha} |(x_n)|^2 \longrightarrow \text{infinite}$$

1, 2, 3, ...

The average power of a discrete time signal that is defined over the range N1 $\leq$n $\leq$ N2 is defined as

$$P = \frac{1}{\underset{+\alpha}{N_2} - \underset{-\alpha}{N_1} + 1} \sum_{n=-N_1 \; \alpha}^{N_2 \; \alpha} |(x_n)|^2$$

$x_n \; (2+j3)^2$

real imaginary

real value

PERIODIC AND APERIODIC SIGNALS

A discrete time signal is periodic, with period N, if and only if

$$x(n+N) = \boxed{x(n)} \qquad \boxed{\forall\ -\infty \le n \le \infty.}$$

$T =$

$N = 12$

CTS

$x(3) = x(3+12) \overset{N}{=} x(15)$

STATIC AND DYNAMIC SIGNALS

A discrete time system is called **static** if it is memoryless and depends only on the current input.

$y(n) = x(n)$

H

$x(n) \rightarrow \boxed{\text{system}} \rightarrow y(n)$

past / future values

dynamic system

$y(n) = 0.6\,x(n)$

$y(n) = \sum_{k=0} x(n)$

dynamic system $= 2x(n) + 4x^2(n)$

$y(n) = x(n-1) + 2x(n-2) \rightarrow$ dynamic

TIME INVARIANT VERSUS TIME VARIANT SYSTEMS

If the output is $y(n)$ for a relaxed, time invariant, or shift invariant system for the input $x(n)$, then the output is $y(n-m)$ for the shifted input $x(n-m)$.

$x(n) \rightarrow \boxed{} \rightarrow y(n)$

$x(n-1) \rightarrow \boxed{} \rightarrow y(n-1)$

1, 2, 3, 4 $\rightarrow$ 2 4 6 8

n

n n

$$y(n) = \sum_{k=0}^{K1} b(k)\, x(n-k) - \sum_{k=1}^{K2} a(k)\, y(n-k)$$

time invariant and

homogeneity

linearity

LINEAR VERSUS NONLINEAR SYSTEMS

A relaxed system is linear if and only if

$$\mathrm{T}[a_1 x_1(n) + a_2 x_2(n)] = a_1 \mathrm{T}[x_1(n)] + a_2 \mathrm{T}[x_2(n)]$$

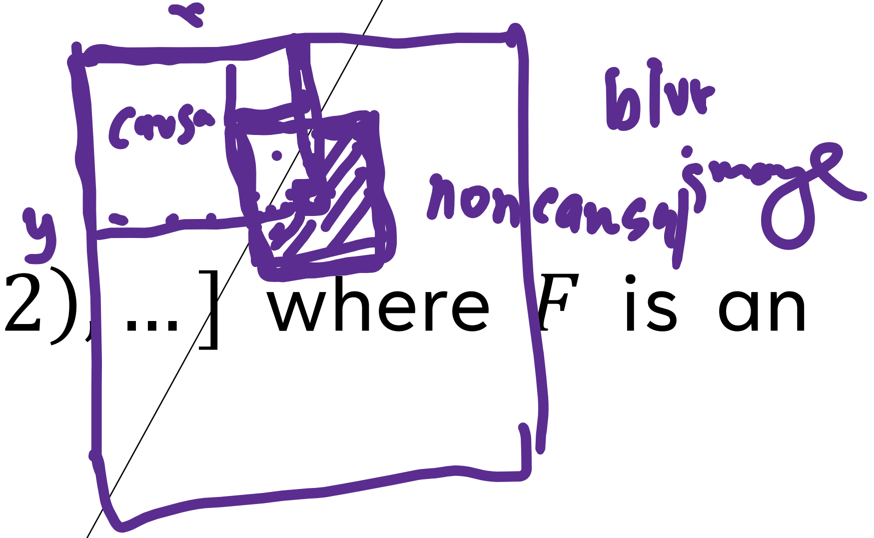
for arbitrary input sequences $x_1(n)$ and $x_2(n)$, and any arbitrary constants a_1 and a_2.

CAUSAL AND NONCAUSAL SYSTEMS

A system is said to be causal if the output of the system at any time n [i.e. $y(n)$] depends only on present and past inputs [i.e. $x(n), x(n-1), (n-2), \dots$], but does not depend on future inputs [i.e. $x(n+1), (n+2), \dots$].

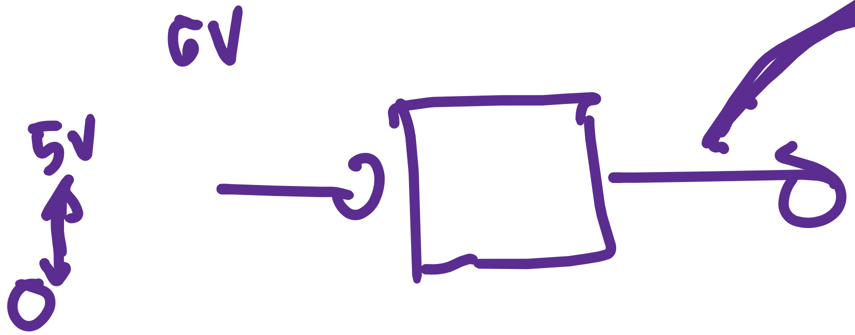
In mathematical terms,

$y(n) = F[x(n), x(n-1), (n-2), \dots]$ where F is an arbitrary function. Causal



STABLE AND UNSTABLE SYSTEMS

An arbitrary relaxed system is said to be bounded input and bounded output (BIBO) stable if and only if every bounded input produces a bounded output.



TRANSFORMATION OF SIGNALS

$$y(n) = x(\alpha n + \beta)$$

1. First, delay or advance the signal in accordance with the value of β.

2. Perform scaling of the signal in accordance with the magnitude of α.

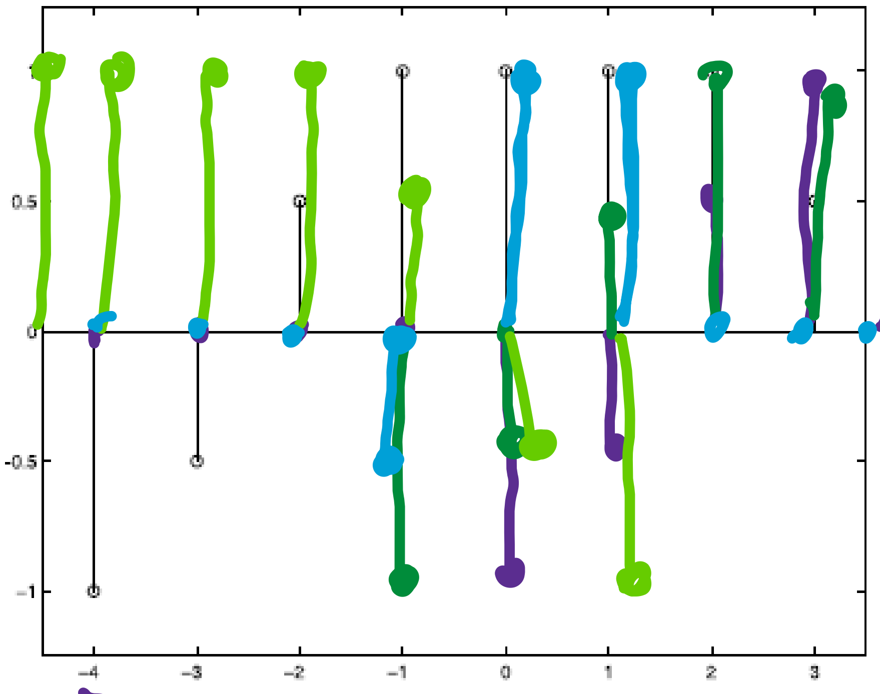
3. If α < 0, perform time reversal.

$x(2n+3)$

TRANSFORMATION OF SIGNALS

Example

$$x(n) = -\delta(n+4) - 0.5\delta(n+3) + 0.5\delta(n+2) + \delta(n+1) + \delta(n) + \delta(n-1) + \delta(n-2) + 0.5\delta(n-3).$$



a.) $y_1(n) = x(n-4)$

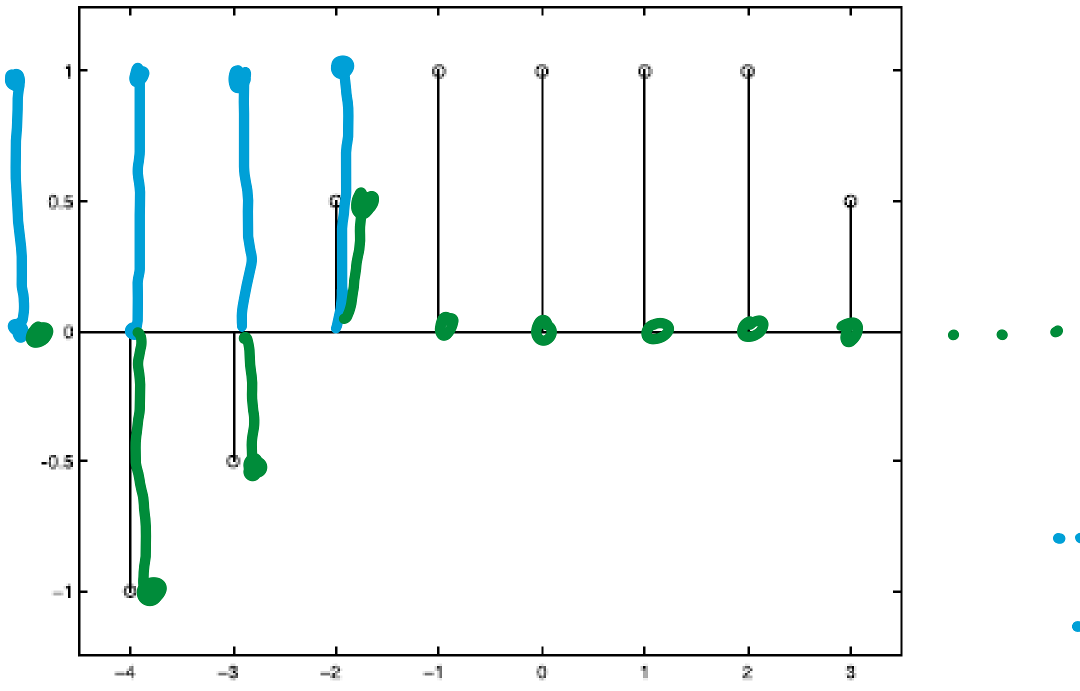
b.) $y_2(n) = y(3-n) = y(-(n-3))$

c.) $y_3(n) = x(3n)$

d.) $y_4(n) = x(3n+1)$

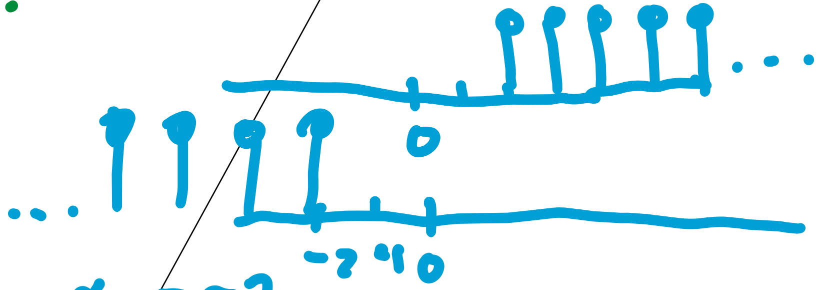
e.) $y_5(n) = x(n)\,u(2-n)$

$$y_5(n) = x(n)\, u(2 - n)$$



$$y(n) = \begin{cases} 0 & n < 0 \\ 1 & n \geq 0 \end{cases}$$

$$u(2-n) = u(-(\underline{n-2}))$$



$-\infty \to 2$