$$X < \begin{matrix} 0 \\ 1 \end{matrix}$$

Date.....................

Probability Distribution $\Rightarrow$ sum of all probabilities is 1 and the individual probabilities are ≥ 0

$P\{X=1\} = p = 1-q$

$P\{X=0\} = q = 1-p$

eg throw of coins

Consider a sequence of independent and identically distributed Bernoulli random variable, each with probability p of success

$P_r\{X_i = 1\} = p$

$P_r\{X_i = 0\} = 1-p$

$S_n = X_1 + X_2 + \dots + X_n$ be the number of successes in n bernoulli trials

$0 \leq S_n \leq n$

S_1 = # successes in 1 trial

:

Process S_n is defined as a bernoulli process

Index set $\rightarrow$ All natural numbers

State space $\rightarrow$ Non negative integers

$S_n = S_{n-1} + X_n$

$P\{S_n = K \mid S_{n-1} = K-1\}$ — Conditional prob

$\hookrightarrow = P(X_n = 1) = p$

$P\{S_n = K \mid S_{n-1} = K\} = P(X_n = 0) = 1-p$

$\rightarrow$ Now $S_{n-1} = K-1$ or $K-2$ $\rightarrow$ But this does not affect the conditional probability of S_n.

Future depends only on the present and not on the past.

$P\{S_n = K \mid S_{n-1} = K-1, S_{n-2} = K-1\} = P\{S_n = K \mid S_{n-1} = K-1, S_{n-2} = K-2\}$

Spiral

Date....................

A stochastic process in which a future state depends only on the present and not on the past is called a Markovial process and this property is called memory less probability

Non markovial → future depends on present and past

S_n → Binomial distribution with parameter n and p

Expected value of $S_n = np$
Variance of $S_n = npq$] Binomial distribution
Moment generation function $= (q+pe^t)^n$

Starting at particular bernoulli trial
The number of succeeding trials T before the next success occurs

$$P[T=K] = (1-p)^K \cdot p = \underline{q^K p} \quad \text{where} \quad K=0,1,2\ldots$$

$$p + qp + q^2p + q^3p + \ldots = p\underbrace{(1+q+q^2+q^3+\ldots)}_{\text{Geometric series}}$$

$$= p \cdot \frac{1}{1-q} = p \cdot \frac{1}{p} = 1$$

Hence, yes

Since it is after K trials,
~~It is at the~~ K+1 th trial.

In a bernoulle (process) variable, the number of trials having failures between two successive successes is distributed like a geometric random variable.

$$E(T) = \sum_{K=0}^{\infty} K q^K p = \frac{q}{p}$$

$$Var(T) = \frac{q}{p^2}$$

Moment $(T) = qe^t$

Spiral

Date....................

Q) What is a generating function? Give examples. Specifically, what is a moment generating function in case of a probability distribution

Expectation $\Rightarrow$ mean = $E(x) = \sum_{i=1}^{n} x_i p_i$

$E(x^2)$ = second moment about origin = $\sum_{i=1}^{n} x_i^2 p_i$

$E(x-a)^r$ = rth moment about the point a.

$E(x-\bar{x})^r$ - Central moment [replacing a with $\bar{x}$]

$M_x(t) = E[e^{tx}]$

$= E[1 + tx + \frac{t^2 x}{2!} + \frac{t^3 x}{3!} + \ldots]$

$= 1 + t u_1' t + \frac{u_2' t^2}{2!} + \frac{u_3' t^3}{3!} + \ldots$

Differentiate this thrice wrt t and put t=0 to get third moment

Moment generating function $\Rightarrow$ used to find all the moments from a single function.

$\frac{1}{1-t} = 1 + t + t^2 + t^3 + \ldots$

↳ differentiating thrice and putting t=0 gives 3!

↳ This generating function generates n!

If the Bernoulli trials are non homogeneous i.e

$P\{x_i = 1\} = p_i$

$P\{x_i = 0\} = 1 - p_i$ for i = 1, 2 . . n, . . .

Then find the expression for the probability that out of n trials, there are k successes. $P_x\{S_n = k\}$

Lab $\rightarrow$ Simulation of ~~Bio~~ ~~binomial~~ ~~program~~. bernoulli process

Write a program to find the number of successes in n trials in case of a bernoulli process

i) Homogeneous ii) Non-homogeneous

Date....................

Q. Write a program to find the number of failures between two successive successes in case of a bernoulli process (homogeneous)

POISSON PROCESS – basically counting in an interval

continuous parameter – discrete state process.

Number of customers arriving in interval $(0, t]$

Index set ⇒ time (continuous)

Event occurs at random times within the interval

Def. Let $N(t)$ denote the number of events that occur in the time interval $(0, t]$. These events are said to constitute a Poisson process with rate λ where $\lambda > 0$, if the following conditions are satisfied.

i) $N(0) = 0$ Number of events at time $t=0$ are 0

ii) Number of events that occur in disjoint time intervals are independent.

iii) The distribution of the number of events that occur in a given interval of time depends only on the length of the interval and not the location

iv) $P[N(\Delta t) = 1] = \lambda \Delta t + \cancel{\alpha \Delta t}\ o(\Delta t)$

$P[N(\Delta t) \geq 2] = o(\Delta t)$

~~Need to know these~~ ~~4 properties~~

Spiral