PHYS1001B College Physics IB

Optics II Geometric Optics (Ch. 34)

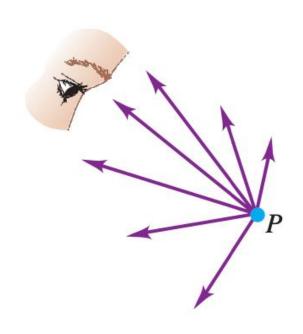
Introduction

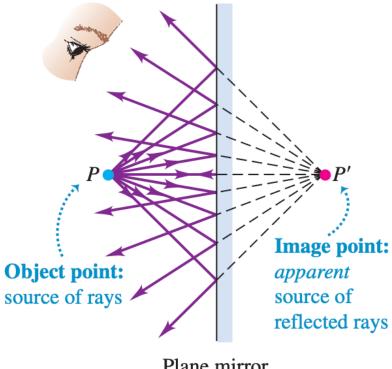


How do magnifying lenses work? At what distance from the object being examined do they provide the sharpest view?

Outline

- 34-1 Reflection and Refraction at a Plane Surface
- 34-2 Reflection at a Spherical Surface
- ▶ 34-4 Thin Lenses
- ▶ 34-5 Cameras
- 34-6 The Eye

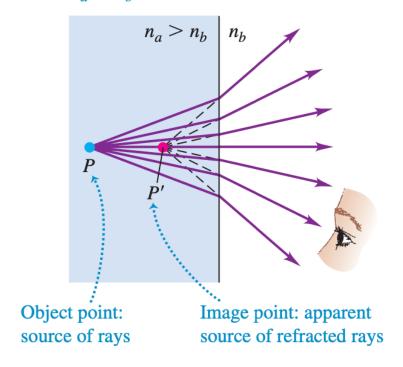




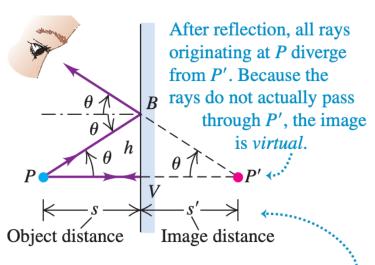
Plane mirror

34.3 Light rays from the object at point P are refracted at the plane interface. The refracted rays entering the eye look as though they had come from image point P'.

When $n_a > n_b$, P' is closer to the surface than P; for $n_a < n_b$, the reverse is true.



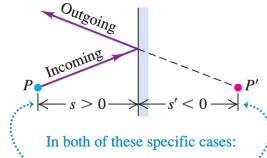
34.4 Construction for determining the location of the image formed by a plane mirror. The image point P' is as far behind the mirror as the object point P is in front of it.



Triangles *PVB* and *P'VB* are congruent, so |s| = |s'|.

34.5 For both of these situations, the object distance s is positive (rule 1) and the image distance s' is negative (rule 2).

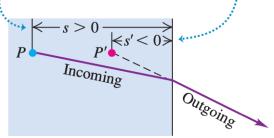
(a) Plane mirror



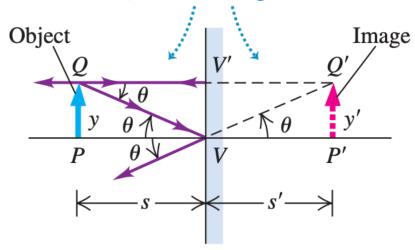
Object distance s is positive because the object is on the same side as the incoming light.

Image distance s' is negative because the image is NOT on the same side as the outgoing light.

(b) Plane refracting interface



For a plane mirror, PQV and P'Q'V are congruent, so y = y' and the object and image are the same size (the lateral magnification is 1).



$$m = \frac{y'}{y}$$
 (lateral magnification)

m>0 erect m<0 inverted

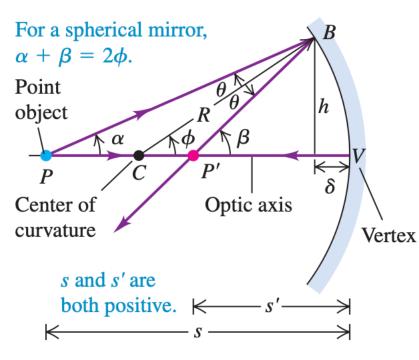
An image made by a plane mirror is reversed back to front: the image thumb P'R' and object thumb PR point in opposite directions (toward each other).

34.8 The image formed by a plane mirror is reversed; the image of a right hand is a left hand, and so on. (The hand is resting on a horizontal mirror.) Are images of the letters H and A reversed?



Object

(a) Construction for finding the position P' of an image formed by a concave spherical mirror



$$\phi = \alpha + \theta$$
 $\beta = \phi + \theta$

$$\alpha + \beta = 2\phi$$

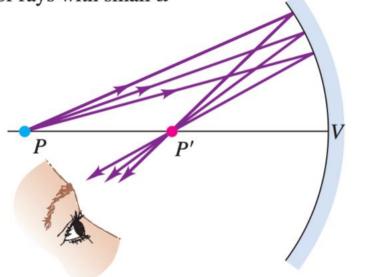
$$\tan \alpha = \frac{h}{s - \delta}$$
 $\tan \beta = \frac{h}{s' - \delta}$ $\tan \phi = \frac{h}{R - \delta}$

Small angle and delta

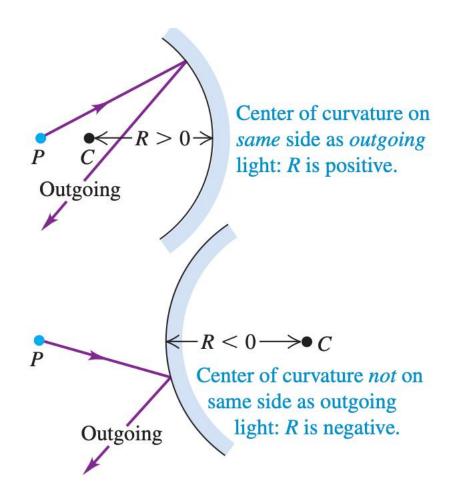
$$\alpha = \frac{h}{s}$$
 $\beta = \frac{h}{s'}$ $\phi = \frac{h}{R}$

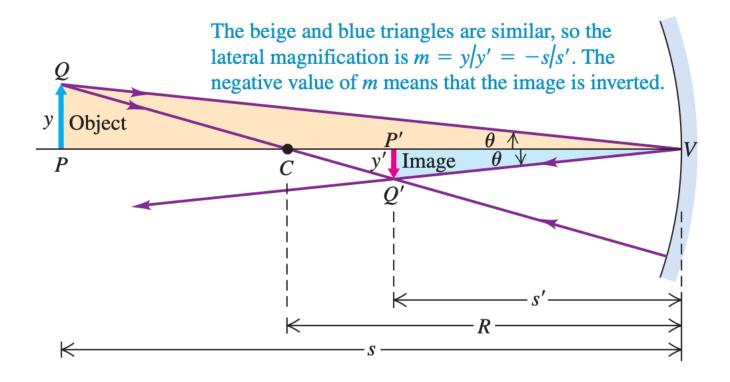
$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R}$$
 (object-image relationship, spherical mirror)

(b) The paraxial approximation, which holds for rays with small α



All rays from P that have a small angle α pass through P', forming a real image.





$$m = \frac{y'}{y} = -\frac{s'}{s}$$
 (lateral magnification, spherical mirror)

Example 34.1 Image formation by a concave mirror I

A concave mirror forms an image, on a wall 3.00 m in front of the mirror, of a headlamp filament 10.0 cm in front of the mirror. (a) What are the radius of curvature and focal length of the mirror?

(b) What is the lateral magnification? What is the image height if the object height is 5.00 mm?

EXECUTE: (a) Both the object and the image are on the concave (reflective) side of the mirror, so both s and s' are positive; we have s = 10.0 cm and s' = 300 cm. We solve Eq. (34.4) for R:

$$\frac{1}{10.0 \text{ cm}} + \frac{1}{300 \text{ cm}} = \frac{2}{R}$$

$$R = 2\left(\frac{1}{10.0 \text{ cm}} + \frac{1}{300 \text{ cm}}\right)^{-1} = 19.4 \text{ cm}$$

The focal length of the mirror is f = R/2 = 9.7 cm.

(b) From Eq. (34.7) the lateral magnification is

$$m = -\frac{s'}{s} = -\frac{300 \text{ cm}}{10.0 \text{ cm}} = -30.0$$

Because m is negative, the image is inverted. The height of the image is 30.0 times the height of the object, or (30.0)(5.00 mm) = 150 mm.

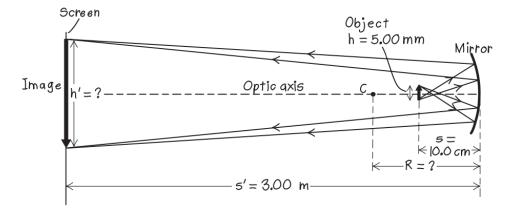
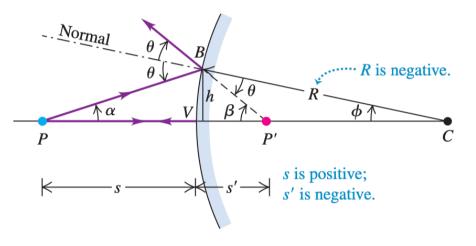
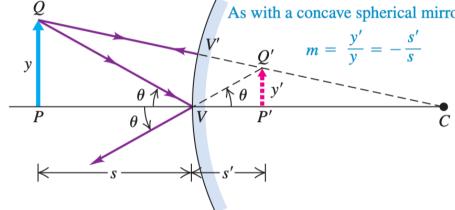


Image formation by a convex mirror

(a) Construction for finding the position of an image formed by a convex mirror

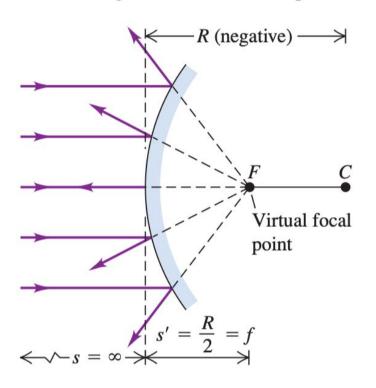


(b) Construction for finding the magnification of an image formed by a convex mirror

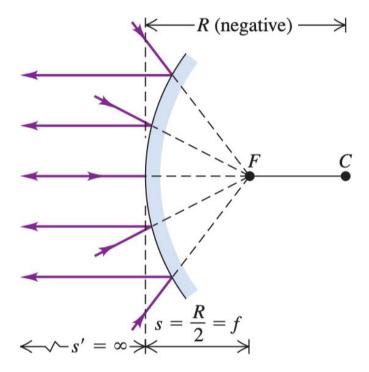


Focal point and focal length

(a) Paraxial rays incident on a convex spherical mirror diverge from a virtual focal point.



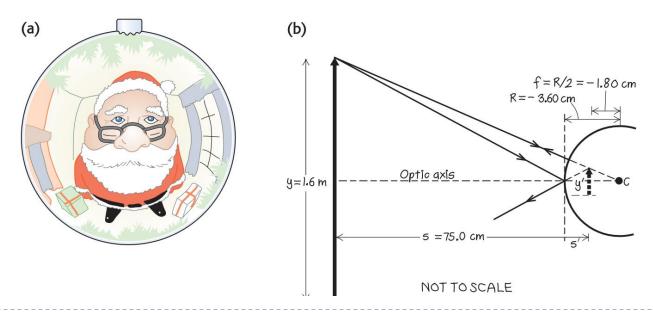
(b) Rays aimed at the virtual focal point are parallel to the axis after reflection.



Example 34.3 Santa's image problem

Santa checks himself for soot, using his reflection in a silvered Christmas tree ornament 0.750 m away (Fig. 34.18a). The diameter of the ornament is 7.20 cm. Standard reference texts state that he is a "right jolly old elf," so we estimate his height to be 1.6 m. Where and how tall is the image of Santa formed by the ornament? Is it erect or inverted?

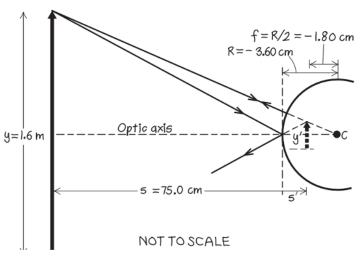
(a) The ornament forms a virtual, reduced, erect image of Santa. (b) Our sketch of two of the rays forming the image.



EXECUTE: The radius of the mirror (half the diameter) is R = -(7.20 cm)/2 = -3.60 cm, and the focal length is f = R/2 = -1.80 cm. From Eq. (34.6),

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{-1.80 \text{ cm}} - \frac{1}{75.0 \text{ cm}}$$
$$s' = -1.76 \text{ cm}$$

Because s' is negative, the image is behind the mirror—that is, on the side opposite to the outgoing light (Fig. 34.18b)—and it is virtual.



The image is about halfway between the front surface of the ornament and its center.

From Eq. (34.7), the lateral magnification and the image height are

$$m = \frac{y'}{y} = -\frac{s'}{s} = -\frac{-1.76 \text{ cm}}{75.0 \text{ cm}} = 0.0234$$

$$y' = my = (0.0234)(1.6 \text{ m}) = 3.8 \times 10^{-2} \text{m} = 3.8 \text{ cm}$$

Example 34.4

Concave mirror with various object distances

A concave mirror has a radius of curvature with absolute value 20 cm. Find graphically the image of an object in the form of an arrow perpendicular to the axis of the mirror at object distances of (a) 30 cm, (b) 20 cm, (c) 10 cm, and (d) 5 cm. Check the construction by *computing* the size and lateral magnification of each image.

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Measurements of the figures, with appropriate scaling, give the $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ following approximate image distances: (a) 15 cm; (b) 20 cm; (c) ∞ or $-\infty$ (because the outgoing rays are parallel and do not approximate distance): (d) -10 cm. To compute these converge at any finite distance); (d) -10 cm. To compute these distances, we solve Eq. (34.6) for s' and insert f = 10 cm:

(a)
$$\frac{1}{30 \text{ cm}} + \frac{1}{s'} = \frac{1}{10 \text{ cm}}$$
 $s' = 15 \text{ cm}$
(b) $\frac{1}{20 \text{ cm}} + \frac{1}{s'} = \frac{1}{10 \text{ cm}}$ $s' = 20 \text{ cm}$
(c) $\frac{1}{10 \text{ cm}} + \frac{1}{s'} = \frac{1}{10 \text{ cm}}$ $s' = \infty \text{ (or } -\infty)$
(d) $\frac{1}{5 \text{ cm}} + \frac{1}{s'} = \frac{1}{10 \text{ cm}}$ $s' = -10 \text{ cm}$

The signs of s' tell us that the image is real in cases (a) and (b) and virtual in case (d).

$$m=\frac{y'}{y}=-\frac{s'}{s}$$

The lateral magnifications measured from the figures are approximately (a) $-\frac{1}{2}$; (b) -1; (c) ∞ or $-\infty$; (d) +2. From Eq. (34.7),

(a)
$$m = -\frac{15 \text{ cm}}{30 \text{ cm}} = -\frac{1}{2}$$

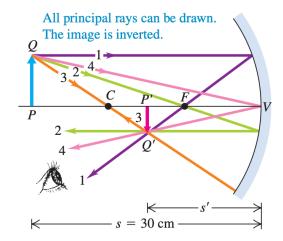
(b)
$$m = -\frac{20 \text{ cm}}{20 \text{ cm}} = -1$$

(c)
$$m = -\frac{\infty \text{ cm}}{10 \text{ cm}} = -\infty \text{ (or } +\infty \text{)}$$

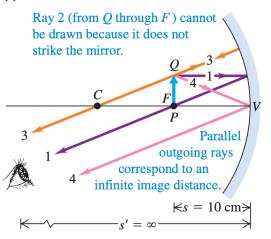
(d)
$$m = -\frac{-10 \text{ cm}}{5 \text{ cm}} = +2$$

The signs of m tell us that the image is inverted in cases (a) and (b) and erect in case (d).

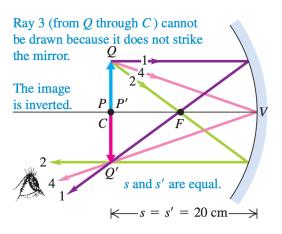
(a) Construction for s = 30 cm



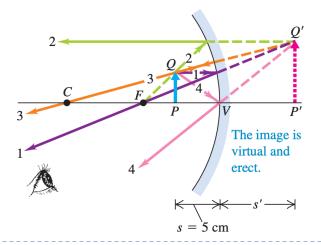
(c) Construction for s = 10 cm



(b) Construction for s = 20 cm



(d) Construction for s = 5 cm



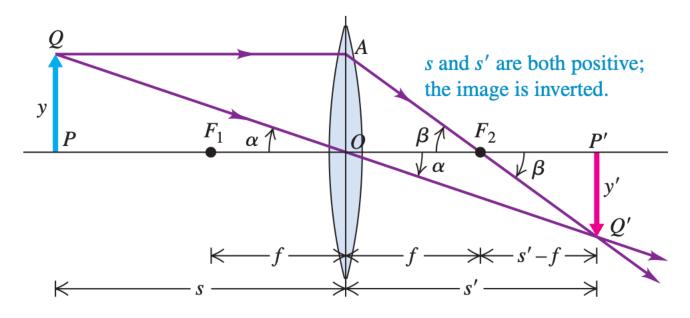
34-4 Thin Lenses

Optic axis (passes Second focal point: through centers of the point to which incoming parallel curvature of both lens surfaces) rays converge Focal length ···

- Measured from lens center
- Always the same on both sides of the lens
- Positive for a converging thin lens

34-4 Thin Lenses

Image of an Extended Object: Converging Lens



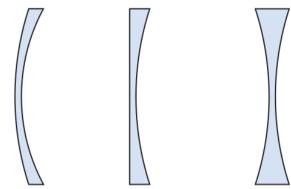
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$
 (object-image relationship, thin lens)

$$m = -\frac{s'}{s}$$
 (lateral magnification, thin lens)

34-4 Thin Lenses

(a) Converging lenses Meniscus Planoconvex Double convex

(b) Diverging lenses



Planoconcave

Double concave

The Lensmaker's Equation

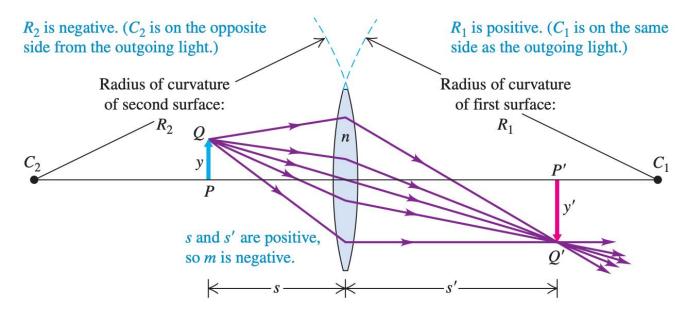
$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

the focal length f of a lens in terms of its index of refraction n and the radii of curvature R1 and R2 of its surfaces

Meniscus

Example 34.8 Determining the focal length of a lens

(a) Suppose the absolute values of the radii of curvature of the lens surfaces in Fig. 34.35 are both equal to 10 cm and the index of refraction of the glass is n = 1.52. What is the focal length f of the lens? (b) Suppose the lens in Fig. 34.31 also has n = 1.52 and the absolute values of the radii of curvature of its lens surfaces are also both equal to 10 cm. What is the focal length of this lens?

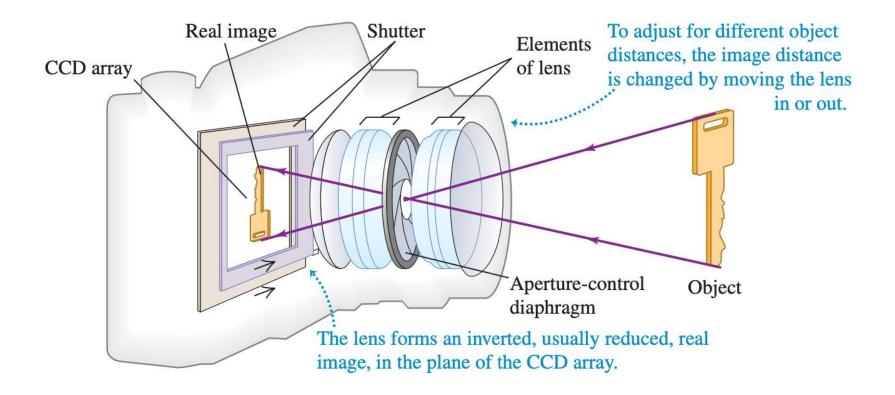


EXECUTE: (a) The lens in Fig. 34.35 is *double convex:* The center of curvature of the first surface (C_1) is on the outgoing side of the lens, so R_1 is positive, and the center of curvature of the second surface (C_2) is on the *incoming* side, so R_2 is negative. Hence $R_1 = +10$ cm and $R_2 = -10$ cm. Then from Eq. (34.19),

$$\frac{1}{f} = (1.52 - 1) \left(\frac{1}{+10 \text{ cm}} - \frac{1}{-10 \text{ cm}} \right)$$
$$f = 9.6 \text{ cm}$$

(b) The lens in Fig. 34.31 is *double concave*: The center of curvature of the first surface is on the *incoming* side, so R_1 is negative, and the center of curvature of the second surface is on the outgoing side, so R_2 is positive. Hence in this case $R_1 = -10$ cm and $R_2 = +10$ cm. Again using Eq. (34.19),

$$\frac{1}{f} = (1.52 - 1) \left(\frac{1}{-10 \text{ cm}} - \frac{1}{+10 \text{ cm}} \right)$$
$$f = -9.6 \text{ cm}$$



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(c) f = 300 mm

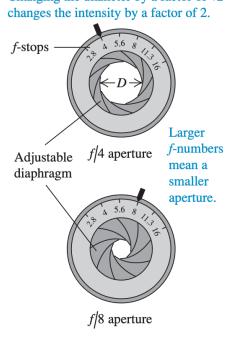


- 25° (105 mm)75° (28 mm)
- long focal length, called a *telephoto* lens, gives a small angle of view and a large image of a distant object
- a lens of short focal length gives a small image and a wide angle of view and is called a *wide-angle* lens.

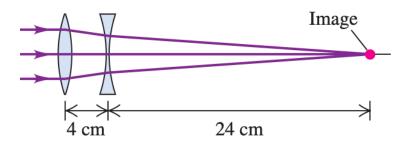
- For the film to record the image properly, the total light energy per unit area reaching the film (the "exposure") must fall within certain limits.
- This is controlled by the *shutter* and the *lens aperture*. The shutter controls the time interval during which light enters the lens.

 Changing the diameter by a factor of $\sqrt{2}$

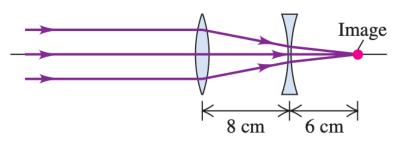
$$f$$
-number = $\frac{\text{Focal length}}{\text{Aperture diameter}} = \frac{f}{D}$



(a) Zoom lens set for long focal length



(b) Zoom lens set for short focal length



(c) A practical zoom lens



Example 34.12 Photographic exposures

A common telephoto lens for a 35-mm camera has a focal length of 200 mm; its f-stops range from f/2.8 to f/22. (a) What is the corresponding range of aperture diameters? (b) What is the corresponding range of image intensities on the film?

EXECUTE: (a) From Eq. (34.20), the diameter ranges from

$$D = \frac{f}{f\text{-number}} = \frac{200 \text{ mm}}{2.8} = 71 \text{ mm}$$

to

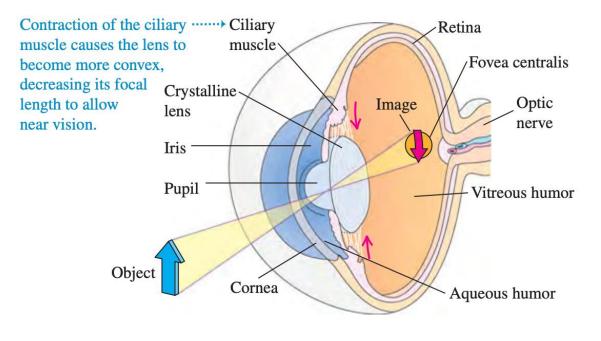
$$D = \frac{200 \text{ mm}}{22} = 9.1 \text{ mm}$$

(b) Because the intensity is proportional to D^2 , the ratio of the intensity at f/2.8 to the intensity at f/22 is

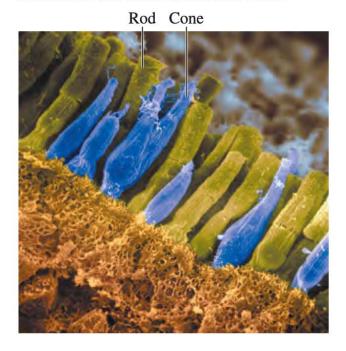
$$\left(\frac{71 \text{ mm}}{9.1 \text{ mm}}\right)^2 = \left(\frac{22}{2.8}\right)^2 = 62$$
 (about 2⁶)

34-6 The Eye

(a) Diagram of the eye

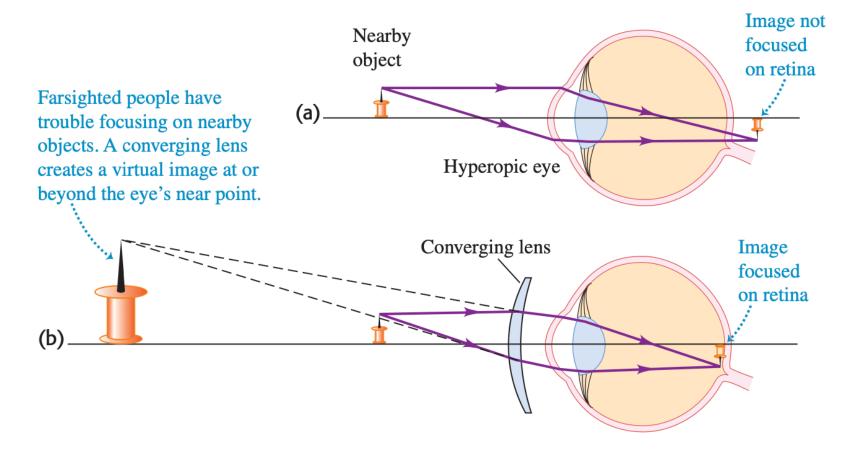


(b) Scanning electron micrograph showing retinal rods and cones in different colors



34-6 The Eye

Hyperopic (farsighted) eye



34-6 The Eye

Myopic (nearsighted) eye

