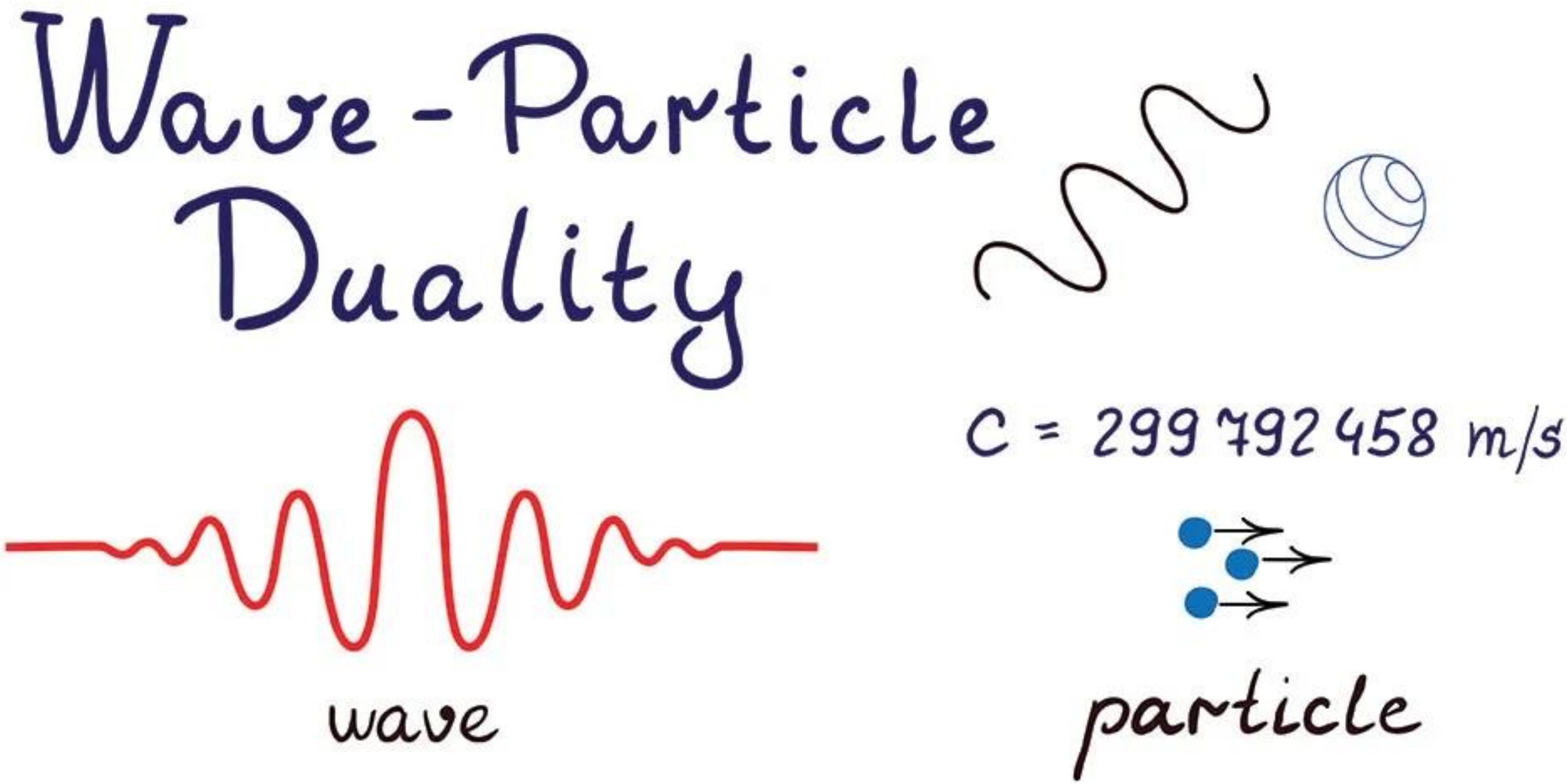
PHYS1001B College Physics IB

Modern Physics II Particles Behaving as Waves (Ch. 39)


Wave-Particle Duality
c = 299 792 458 m/s
wave
particle

39.1 Louis-Victor de Broglie, the seventh Duke de Broglie (1892–1987), broke with family tradition by choosing a career in physics rather than as a diplomat. His revolutionary proposal that particles have wave characteristics—for which de Broglie won the 1929 Nobel Prize in physics—was published in his doctoral thesis.

Nature loves symmetry
Light have duality, how about particles?

Outline

- 39-1 Electron Waves
- 39-2 The Nuclear Atom and Atomic Spectra
- 39-3 Energy Levels and the Bohr Model of the Atom
- 39-4 The Laser
- 39-5 Continuous Spectra
- 39-6 The Uncertainty Principle Revisited

39-1 Electron Waves

If a particle acts like a wave, it should have a wavelength and a frequency. De Broglie postulated that a free particle with rest mass m, moving with nonrelativistic speed v, should have a wavelength λ related to its momentum $p = mv$

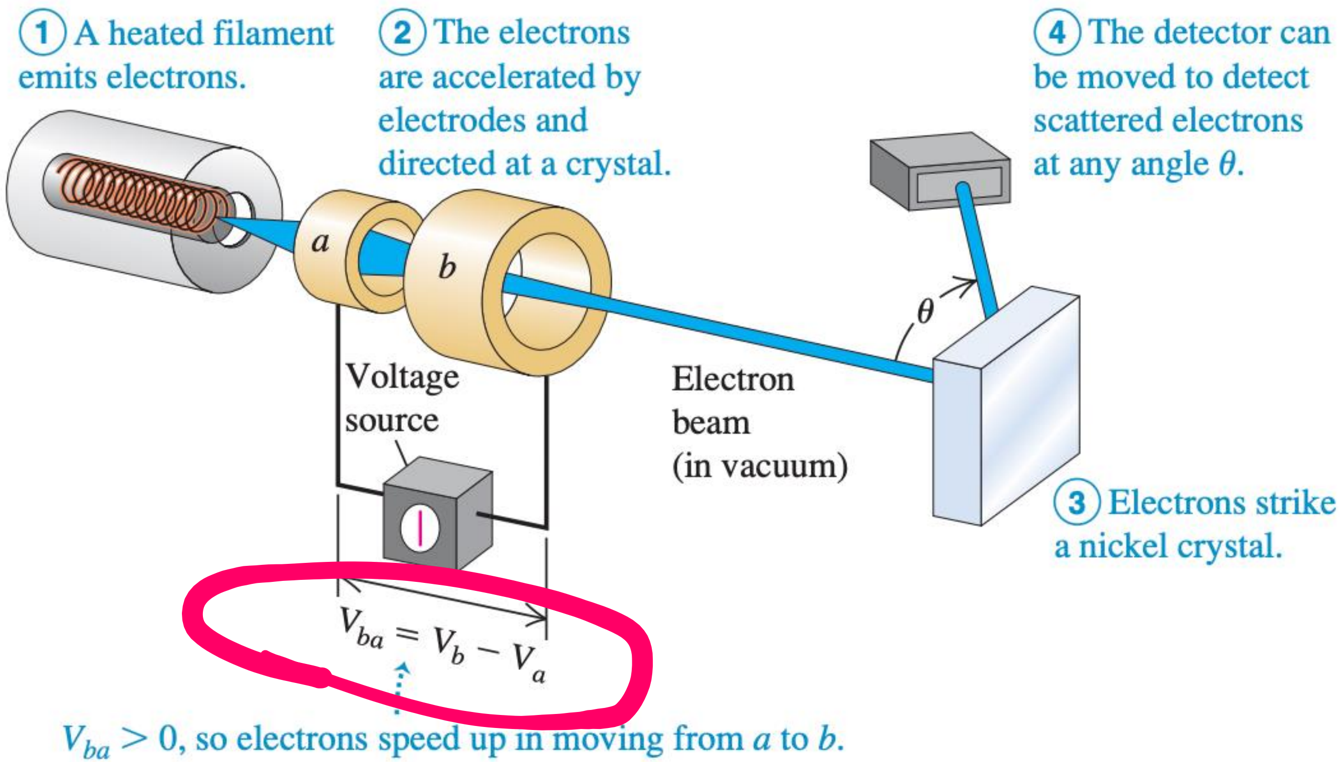
$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad \text{(de Broglie wavelength of a particle)}$$

where h is Planck's constant.

Energy of the particle $E = hf$

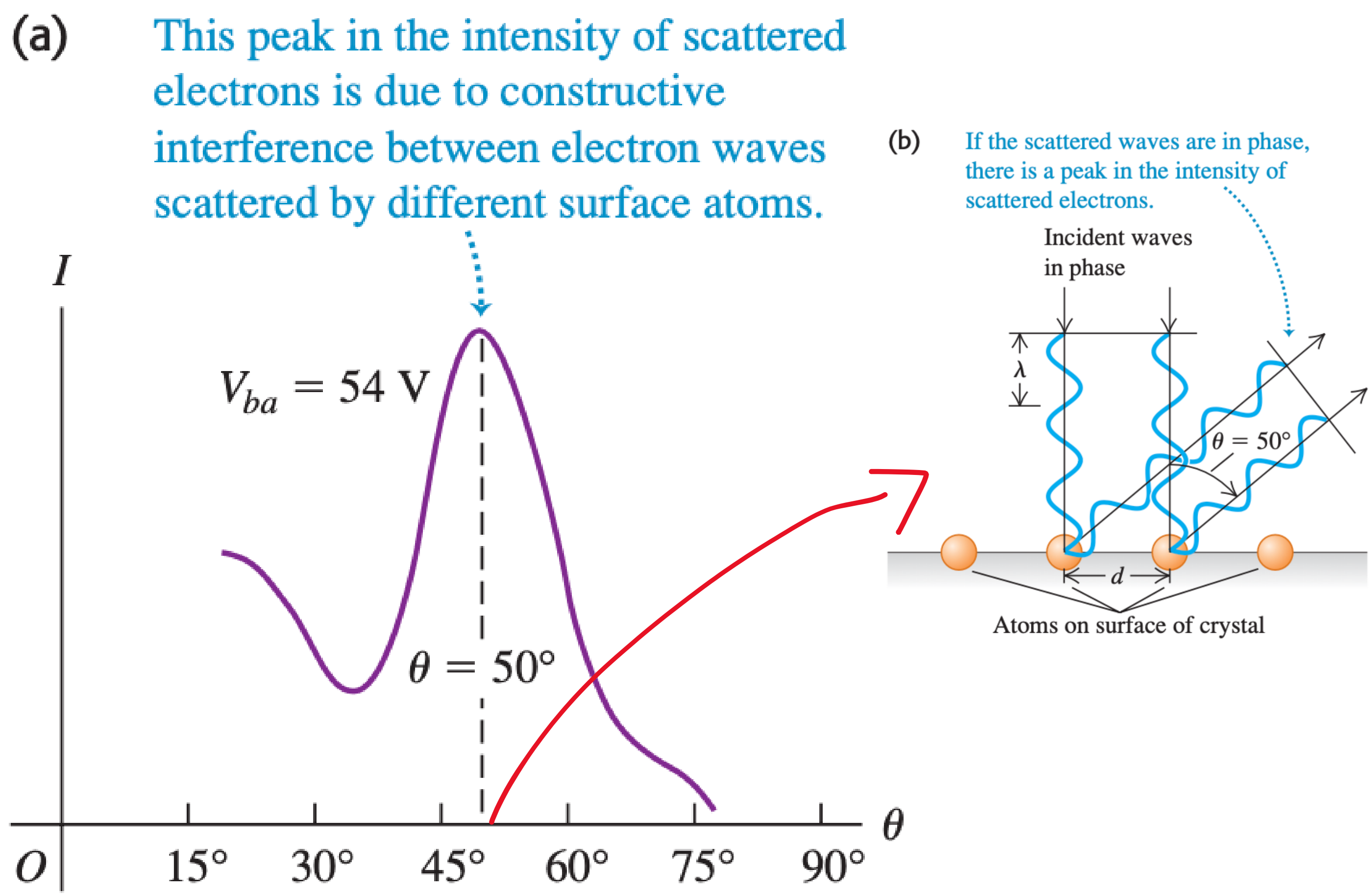
39-1 Electron Waves

Observing the Wave Nature of Electrons



$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV_{ba}}} \qquad \text{(de Broglie wavelength of an electron)}$$

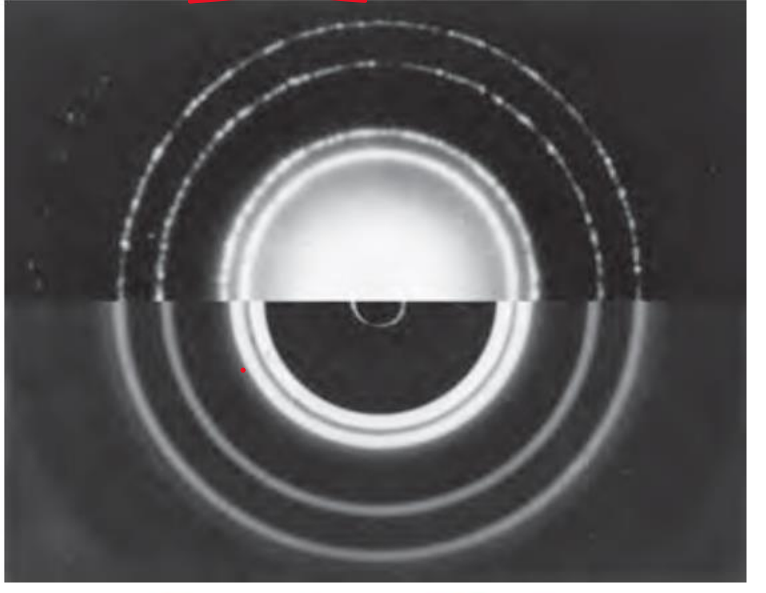
39-1 Electron Waves



39-1 Electron Waves

39.4 X-ray and electron diffraction. The upper half of the photo shows the diffraction pattern for 71-pm x rays passing through aluminum foil. The lower half, with a different scale, shows the diffraction pattern for 600-eV electrons from aluminum. The similarity shows that electrons undergo the same kind of diffraction as x rays.

Top: x-ray diffraction



Bottom: electron diffraction

$$d \sin\theta = m\lambda \qquad (m = 1, 2, 3, \dots)$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV_{ba}}} \qquad \text{(de Broglie wavelength of an electron)}$$

Sample Problem

Example 39.1 **An electron-diffraction experiment**

In an electron-diffraction experiment using an accelerating voltage of 54 V, an intensity maximum occurs for $\theta = 50°$ (see Fig. 39.3a). X-ray diffraction indicates that the atomic spacing in the target is $d = 2.18 \times 10^{-10}\ \text{m} = 0.218$ nm. The electrons have negligible kinetic energy before being accelerated. Find the electron wavelength.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV_{ba}}} \qquad \text{(de Broglie wavelength of an electron)}$$

$$\lambda = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{\sqrt{2(9.109 \times 10^{-31}\ \text{kg})(1.602 \times 10^{-19}\ \text{C})(54\ \text{V})}}$$

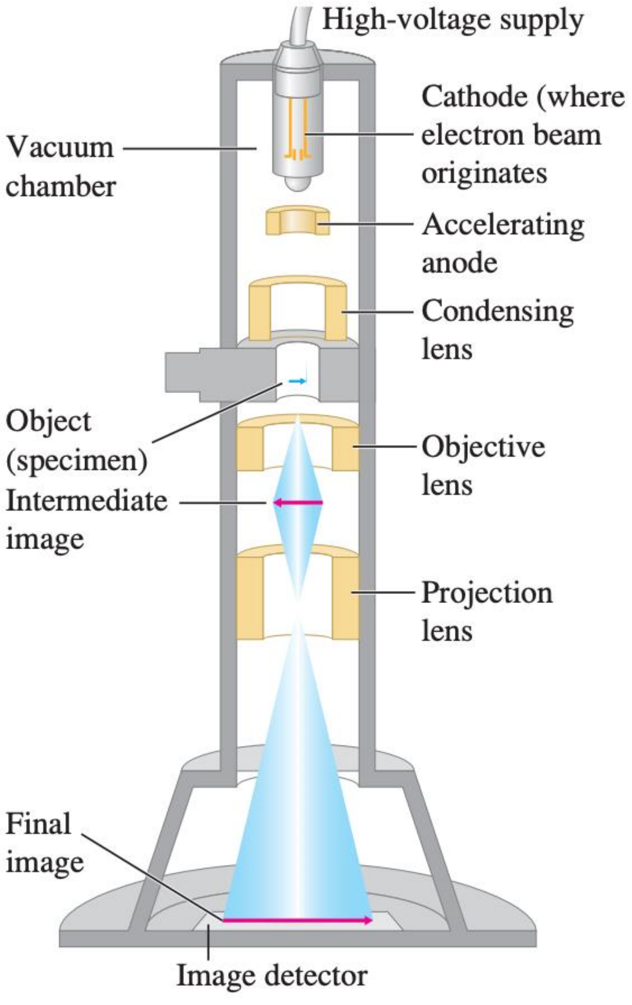
$$= 1.7 \times 10^{-10}\ \text{m} = 0.17\ \text{nm}$$

Alternatively, using Eq. (39.4) and assuming $m = 1$,

$$\lambda = d\sin\theta = (2.18 \times 10^{-10}\ \text{m})\sin 50° = 1.7 \times 10^{-10}\ \text{m}$$

39-1 Electron Waves

transmission electron microscope



39-1 Electron Waves

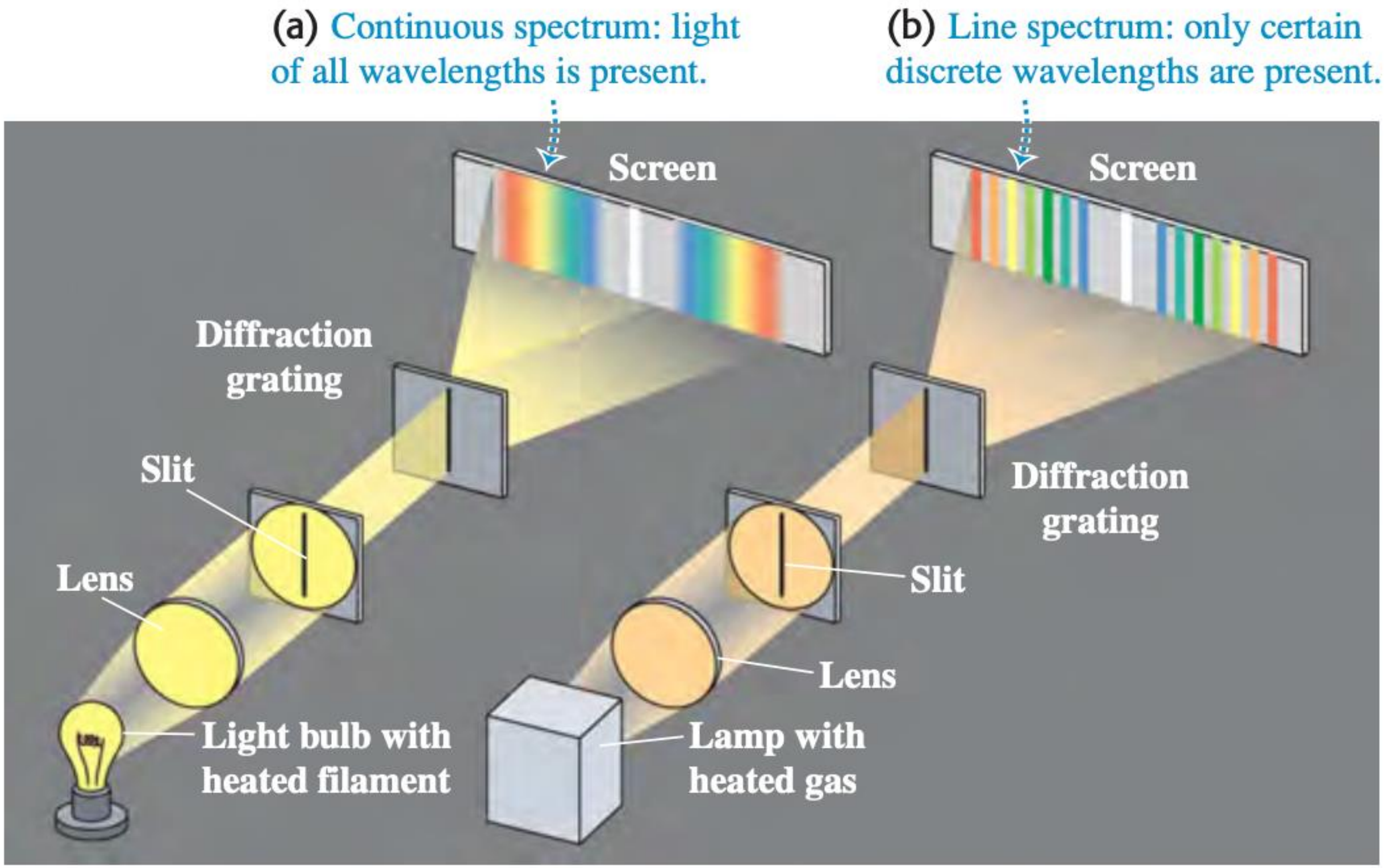
De Broglie Waves and the Macroscopic World

small, and the wave effects are unimportant. For instance, what is the wavelength of a falling grain of sand? If the grain's mass is 5×10^{-10} kg and its diameter is $0.07\ \text{mm} = 7 \times 10^{-5}$ m, it will fall in air with a terminal speed of about 0.4 m/s. The magnitude of its momentum is then $p = mv = (5 \times 10^{-10}\ \text{kg}) \times (0.4\ \text{m/s}) = 2 \times 10^{-10}\ \text{kg} \cdot \text{m/s}$. The de Broglie wavelength of this falling sand grain is then

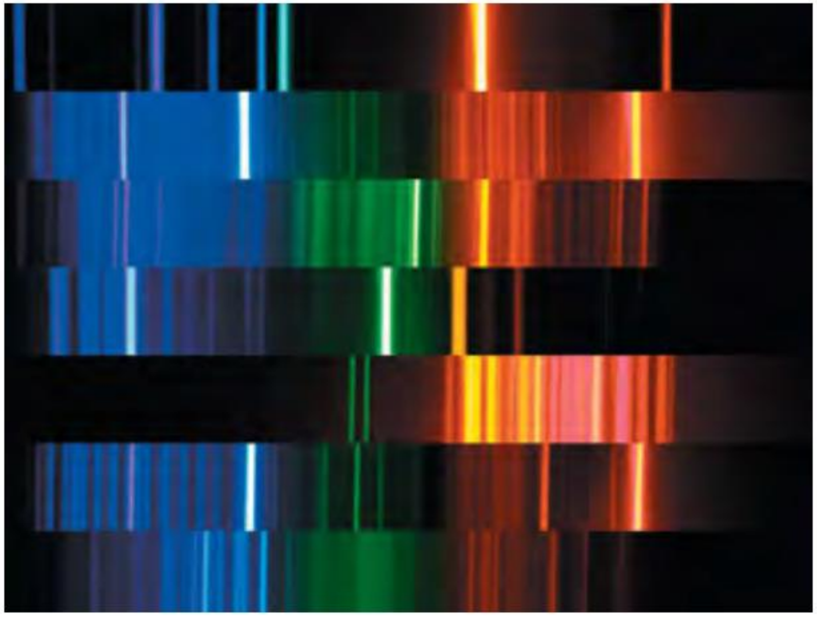
$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{2 \times 10^{-10}\ \text{kg} \cdot \text{m/s}} = 3 \times 10^{-24}\ \text{m}$$

Not only is this wavelength far smaller than the diameter of the sand grain, but it's also far smaller than the size of a typical atom (about 10^{-10} m). A more mas-

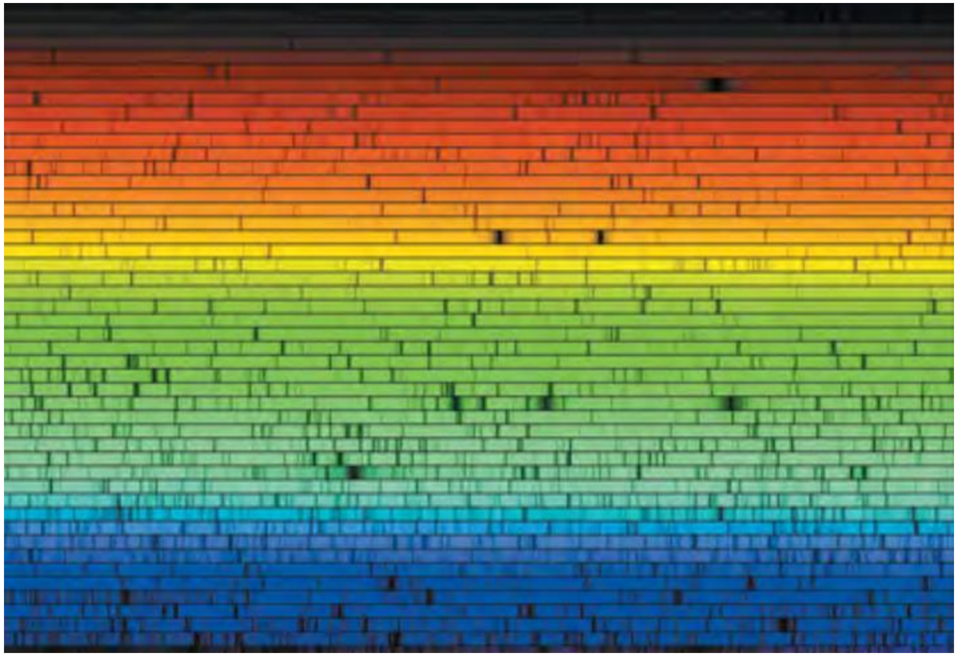
39-2 The Nuclear Atom and Atomic Spectra



39-2 The Nuclear Atom and Atomic Spectra

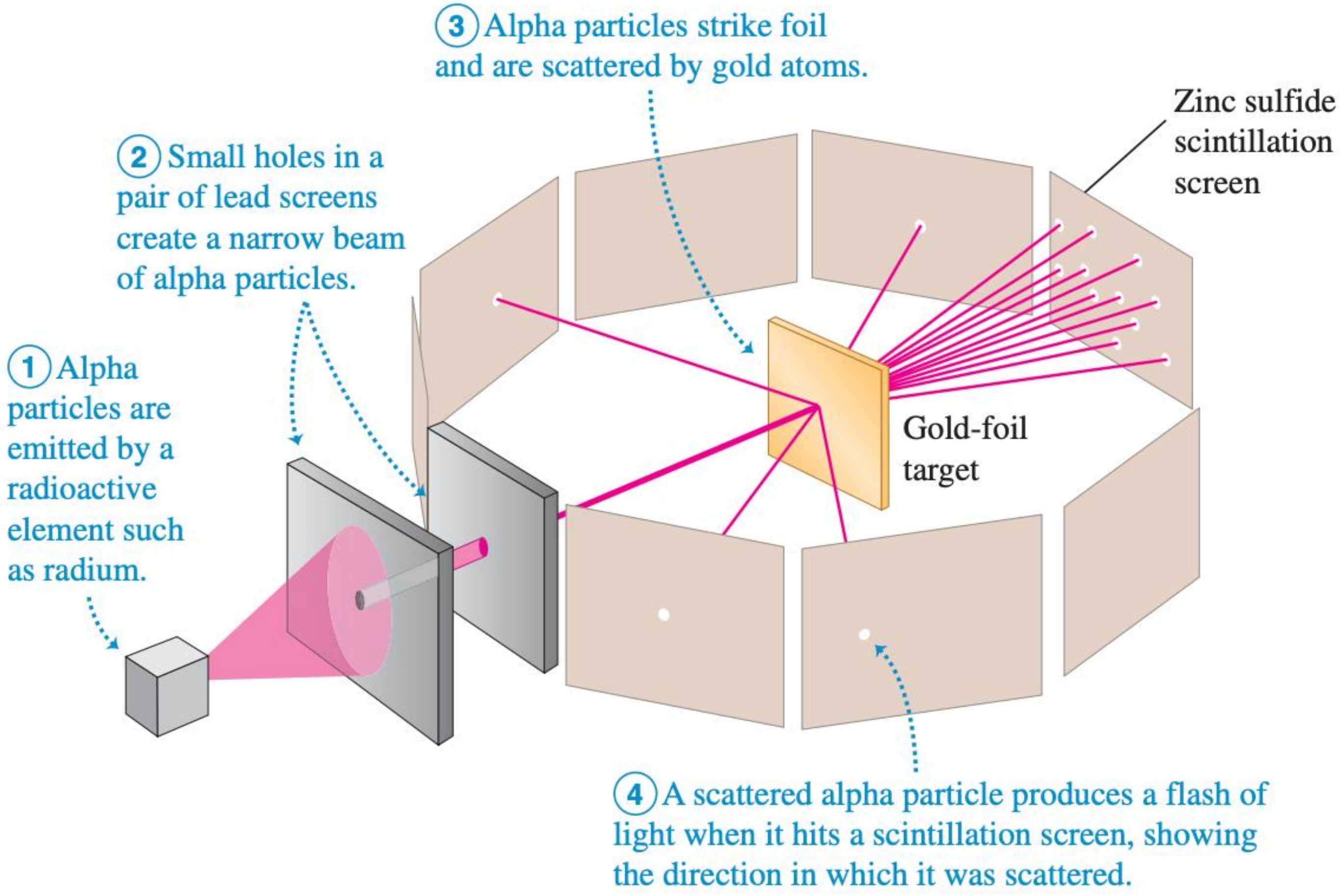


39-2 The Nuclear Atom and Atomic Spectra



39-2 The Nuclear Atom and Atomic Spectra

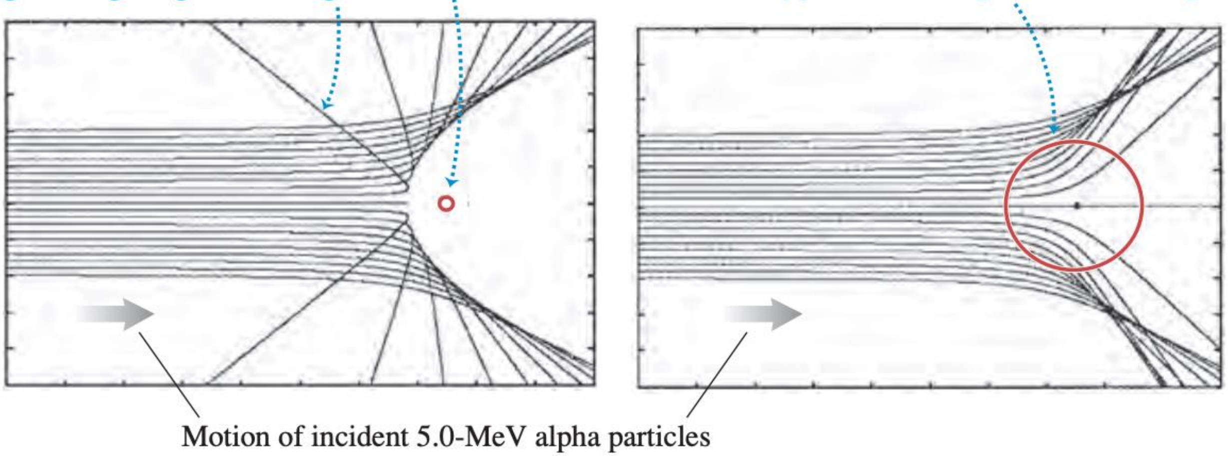
Exploration of the atom: Rutherford scattering experiments



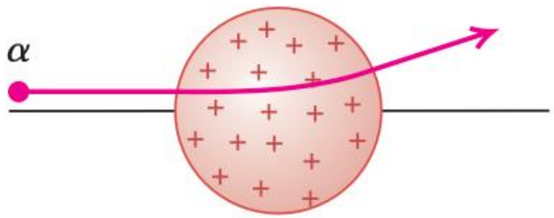
39-2 The Nuclear Atom and Atomic Spectra

(a) A gold nucleus with radius 7.0×10^{-15} m gives large-angle scattering.

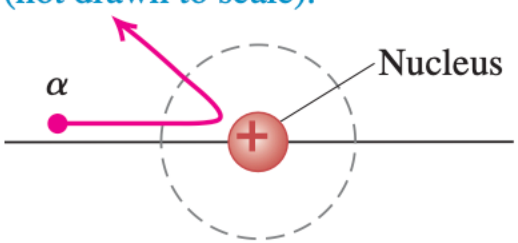
(b) A nucleus with 10 times the radius of the nucleus in **(a)** shows *no* large-scale scattering.



(a) Thomson's model of the atom: An alpha particle is scattered through only a small angle.



(b) Rutherford's model of the atom: An alpha particle can be scattered through a large angle by the compact, positively charged nucleus (not drawn to scale).



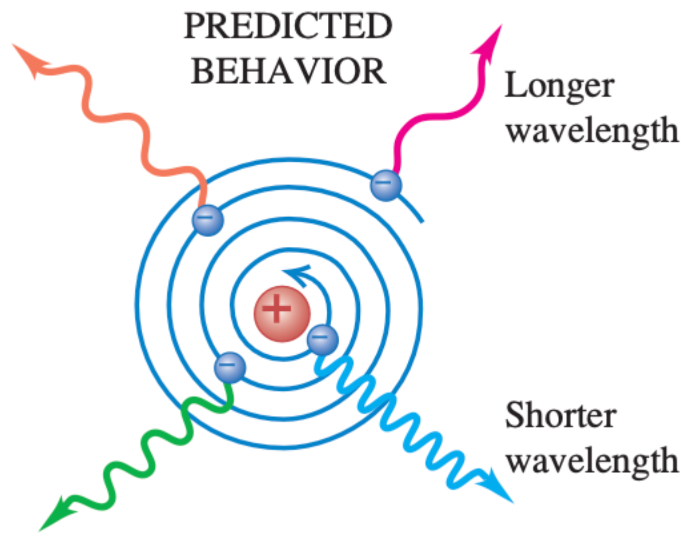
39-2 The Nuclear Atom and Atomic Spectra

Failure of classical physics

ACCORDING TO CLASSICAL PHYSICS:

- An orbiting electron is accelerating, so it should radiate electromagnetic waves.
- The waves would carry away energy, so the electron should lose energy and spiral inward.
- The electron's angular speed would increase as its orbit shrank, so the frequency of the radiated waves should increase.

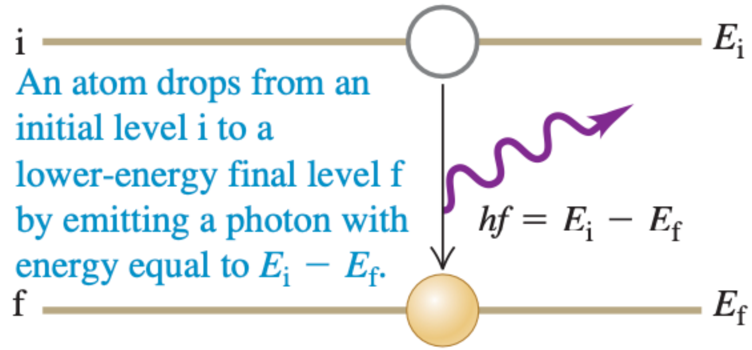
Thus, classical physics says that atoms should collapse within a fraction of a second and should emit light with a continuous spectrum as they do so.



IN FACT:

- Atoms are stable.
- They emit light only when excited, and only at specific frequencies (as a line spectrum).

39-3 Energy Levels and the Bohr Model of the Atom



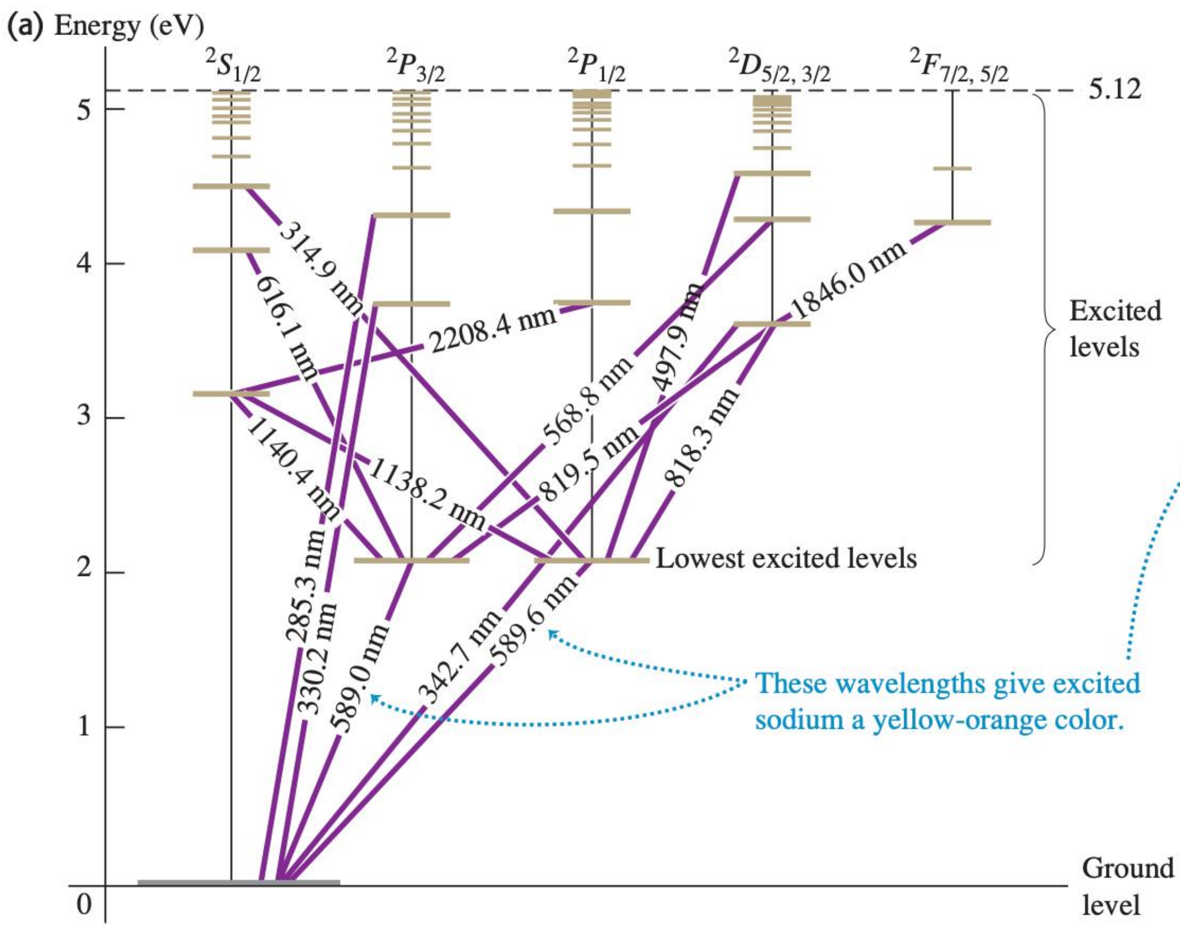
$$hf = \frac{hc}{\lambda} = E_i - E_f \qquad \text{(energy of emitted photon)}$$

For example, an excited lithium atom emits red light with wavelength $\lambda = 671$ nm. The corresponding photon energy is

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \mathrm{J \cdot s})(3.00 \times 10^{8}\ \mathrm{m/s})}{671 \times 10^{-9}\ \mathrm{m}}$$

$$= 2.96 \times 10^{-19}\ \mathrm{J} = 1.85\ \mathrm{eV}$$

39-3 Energy Levels and the Bohr Model of the Atom



(b)

Sample Problem

Example 39.5 Emission and absorption spectra

A hypothetical atom (Fig. 39.20a) has energy levels at 0.00 eV (the ground level), 1.00 eV, and 3.00 eV. (a) What are the frequencies and wavelengths of the spectral lines this atom can emit when excited? (b) What wavelengths can this atom absorb if it is in its ground level?

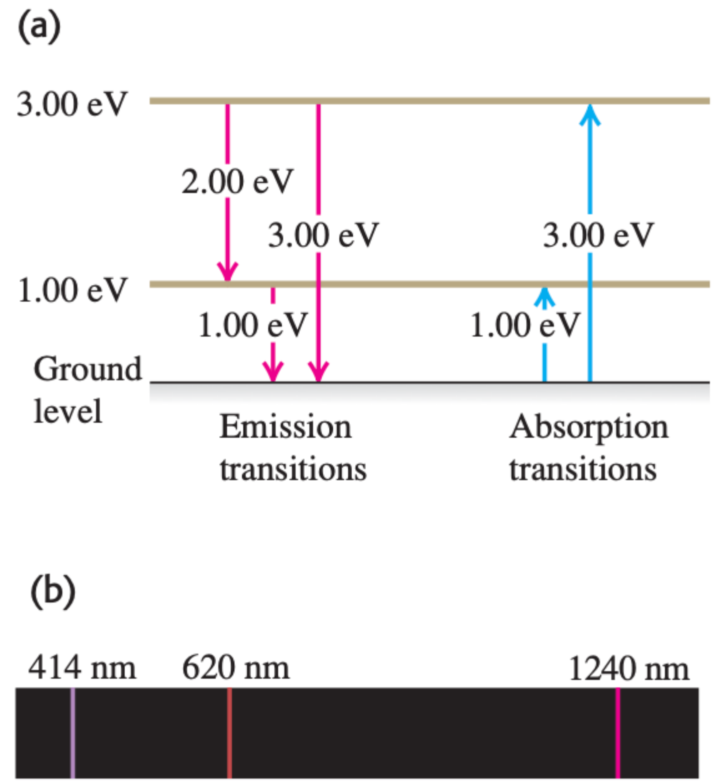
EXECUTE: (a) The possible energies of emitted photons are 1.00 eV, 2.00 eV, and 3.00 eV. For 1.00 eV, Eq. (39.2) gives

$$f = \frac{E}{h} = \frac{1.00\ \text{eV}}{4.136 \times 10^{-15}\ \text{eV} \cdot \text{s}} = 2.42 \times 10^{14}\ \text{Hz}$$

For 2.00 eV and 3.00 eV, $f = 4.84 \times 10^{14}$ Hz and 7.25×10^{14} Hz, respectively. For 1.00-eV photons,

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8}\ \text{m/s}}{2.42 \times 10^{14}\ \text{Hz}} = 1.24 \times 10^{-6}\ \text{m} = 1240\ \text{nm}$$

This is in the infrared region of the spectrum (Fig. 39.20b). For 2.00 eV and 3.00 eV, the wavelengths are 620 nm (red) and 414 nm (violet), respectively.

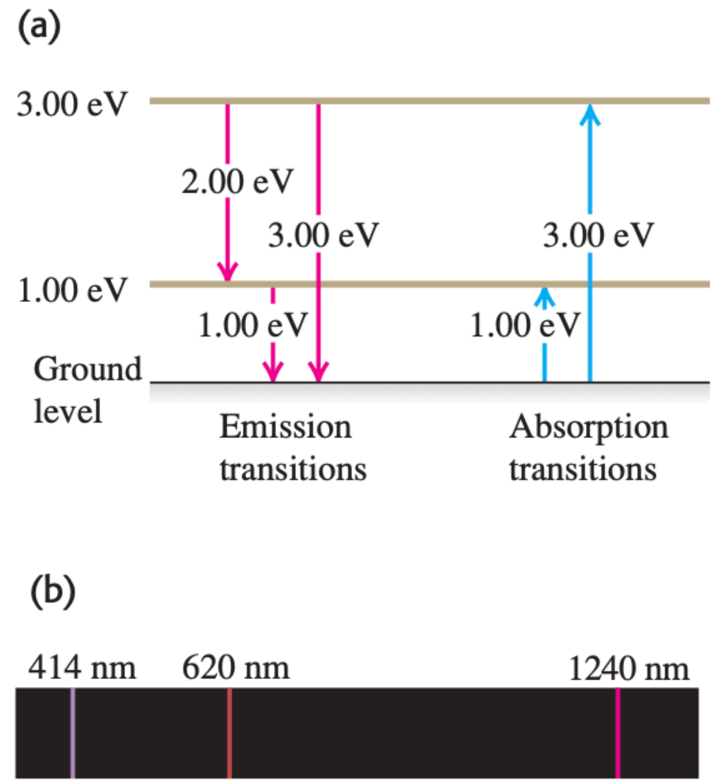


Sample Problem

Example 39.5 Emission and absorption spectra

A hypothetical atom (Fig. 39.20a) has energy levels at 0.00 eV (the ground level), 1.00 eV, and 3.00 eV. (a) What are the frequencies and wavelengths of the spectral lines this atom can emit when excited? (b) What wavelengths can this atom absorb if it is in its ground level?

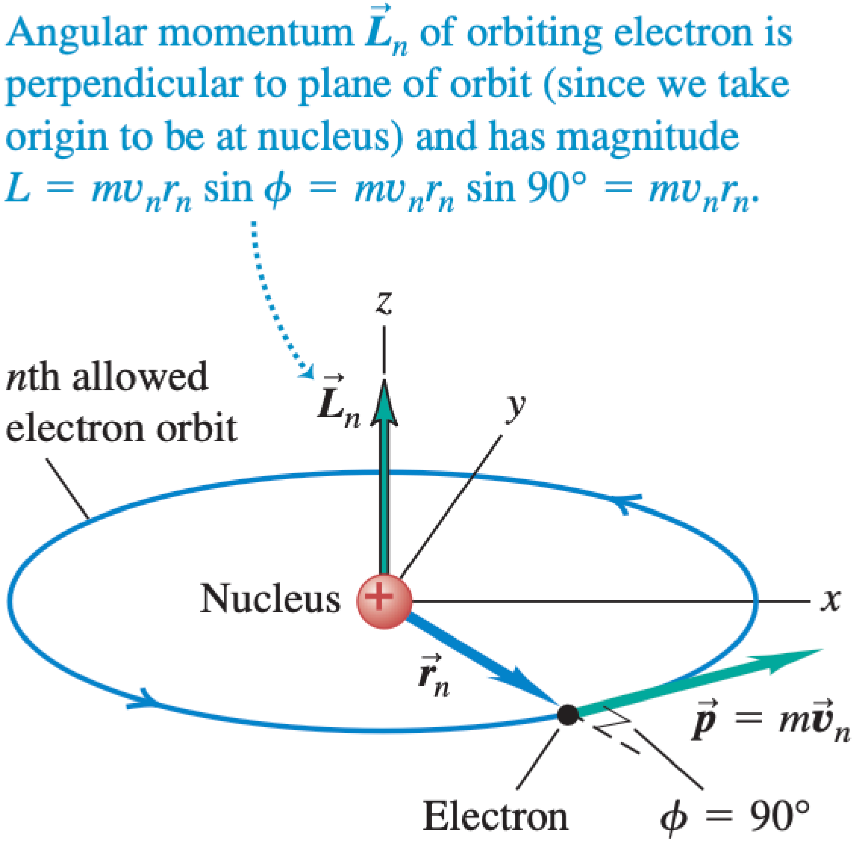
(b) From the ground level, only a 1.00-eV or a 3.00-eV photon can be absorbed (Fig. 39.20a); a 2.00-eV photon cannot be absorbed because the atom has no energy level 2.00 eV above the ground level. Passing light from a hot solid through a gas of these hypothetical atoms (almost all of which would be in the ground state if the gas were cool) would yield a continuous spectrum with dark absorption lines at 1240 nm and 414 nm.

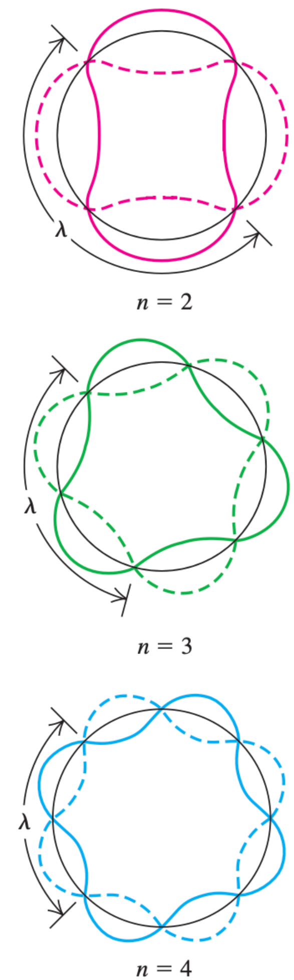


39-3 Energy Levels and the Bohr Model of the Atom

Fluorescence

39-3 Energy Levels and the Bohr Model of the Atom

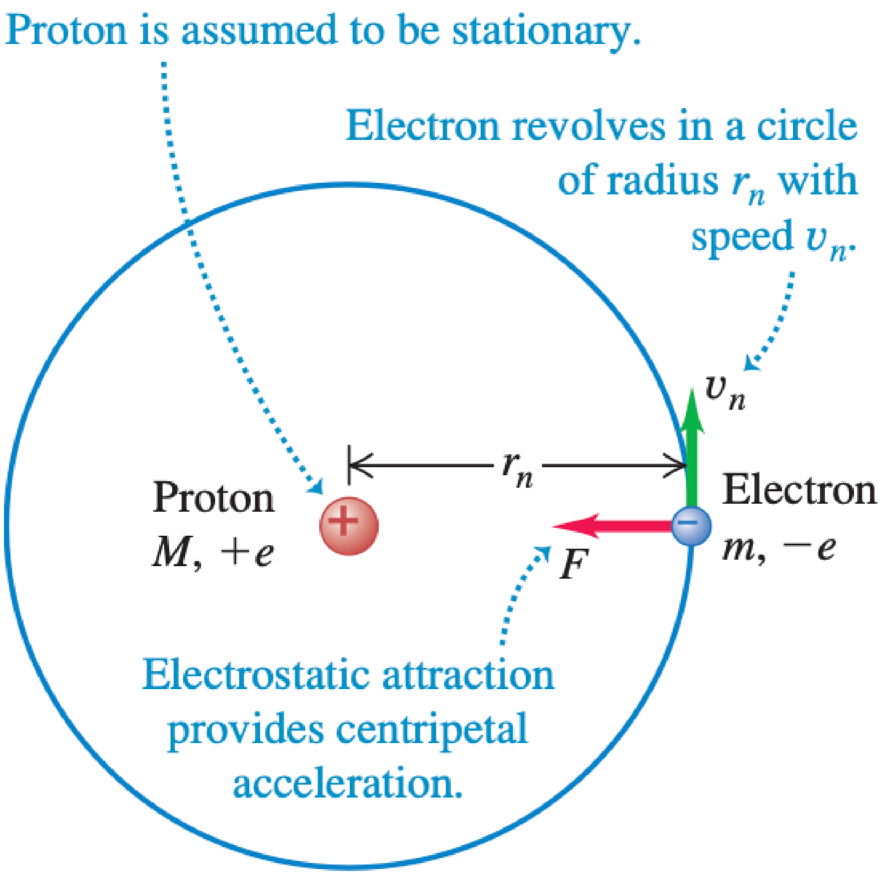




$$L_n = mv_n r_n = n\frac{h}{2\pi} \qquad \text{(quantization of angular momentum)}$$

39-3 Energy Levels and the Bohr Model of the Atom

Bohr model of the hydrogen atom



$$mv_n r_n = n\frac{h}{2\pi} \qquad \frac{1}{4\pi\epsilon_0}\frac{e^2}{{r_n}^2} = \frac{m{v_n}^2}{r_n}$$

Orbital radii

$$r_n = \epsilon_0 \frac{n^2h^2}{\pi m e^2}$$

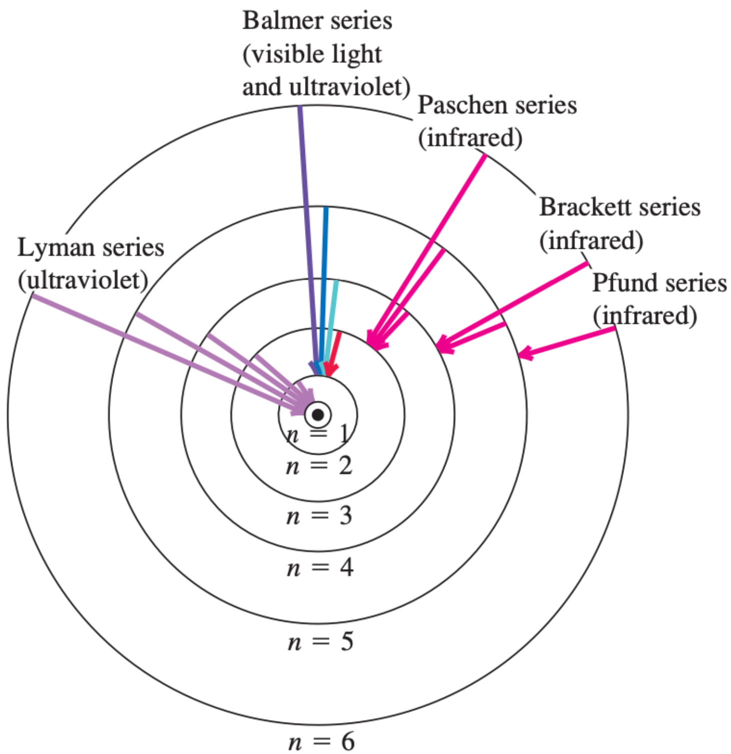
$$a_0 = \epsilon_0 \frac{h^2}{\pi m e^2} \qquad \text{(Bohr radius)}$$

$$r_n = n^2 a_0$$

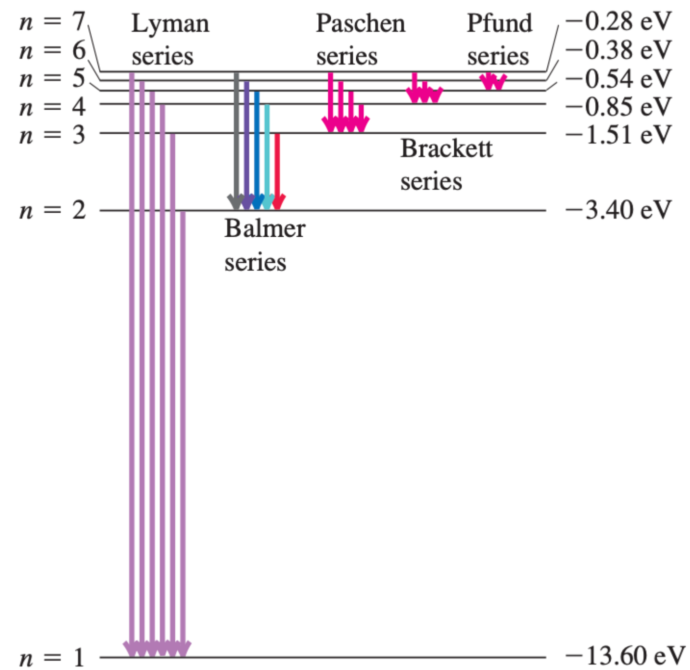
Orbital speeds

$$v_n = \frac{1}{\epsilon_0}\frac{e^2}{2nh}$$

39-3 Energy Levels and the Bohr Model of the Atom



(a) Permitted orbits of an electron in the Bohr model of a hydrogen atom (not to scale). Arrows indicate the transitions responsible for some of the lines of various series.

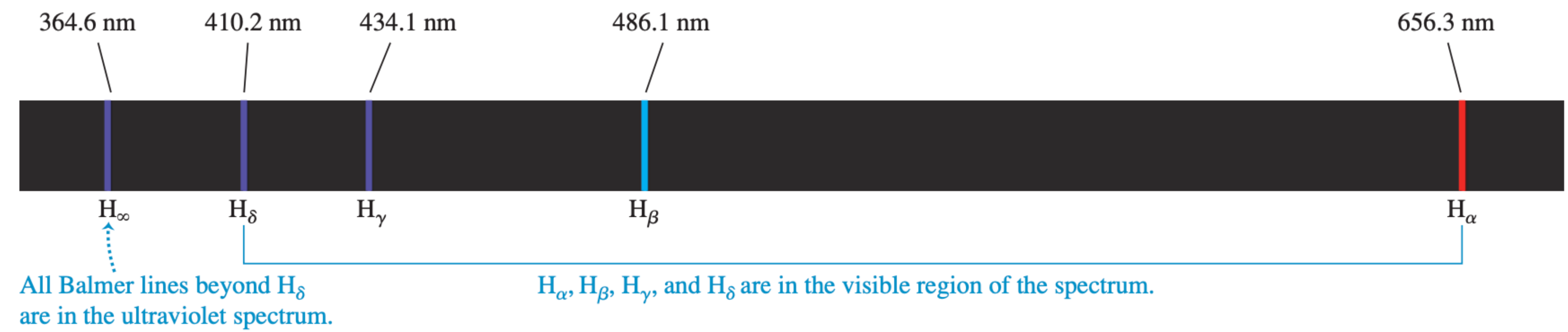


(b) Energy-level diagram for hydrogen, showing some transitions corresponding to the various series

$$E_n = -\frac{hcR}{n^2}, \qquad \text{where} \qquad R = \frac{me^4}{8\epsilon_0^2 h^3 c}$$

39-3 Energy Levels and the Bohr Model of the Atom

39.25 The Balmer series of spectral lines for atomic hydrogen. You can see these same lines in the spectrum of *molecular* hydrogen (H_2) shown in Fig. 39.8, as well as additional lines that are present only when two hydrogen atoms are combined to make a molecule.



Sample Problem

Example 39.6 Exploring the Bohr model

Find the kinetic, potential, and total energies of the hydrogen atom in the first excited level, and find the wavelength of the photon emitted in a transition from that level to the ground level.

EXECUTE: We could evaluate Eqs. (39.12), (39.13), and (39.14) for the *n*th level by substituting the values of m, e, ϵ_0, and h. But we can simplify the calculation by comparing with Eq. (39.15), which shows that the constant $me^4/8\epsilon_0^2h^2$ that appears in Eqs. (39.12), (39.13), and (39.14) is equal to hcR:

$$\begin{aligned}\frac{me^4}{8\epsilon_0^2h^2} &= hcR \\ &= (6.626 \times 10^{-34}\ \mathrm{J \cdot s})(2.998 \times 10^{8}\ \mathrm{m/s}) \\ &\quad \times (1.097 \times 10^{7}\ \mathrm{m^{-1}}) \\ &= 2.179 \times 10^{-18}\ \mathrm{J} = 13.60\ \mathrm{eV}\end{aligned}$$

This allows us to rewrite Eqs. (39.12), (39.13), and (39.14) as

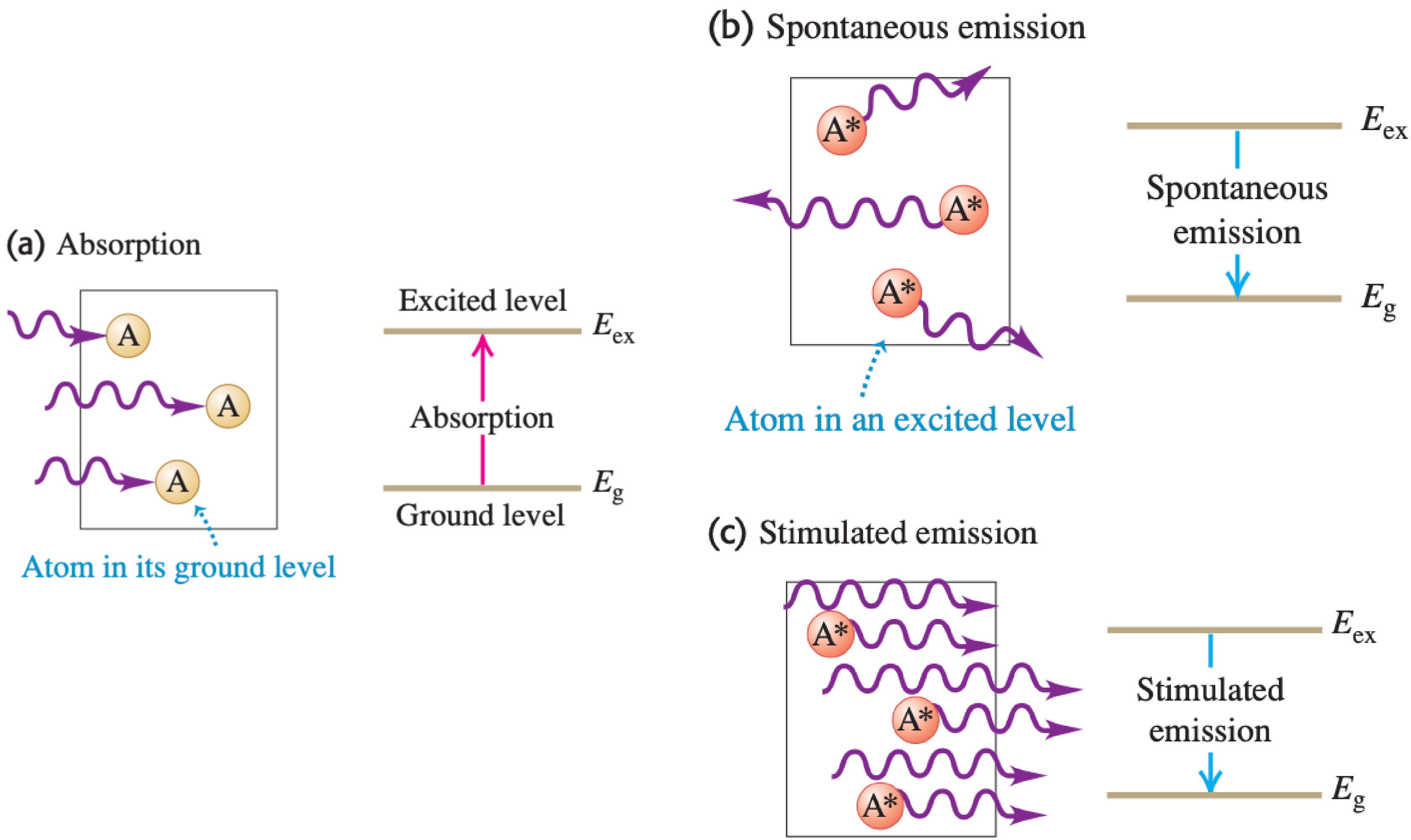
$$K_n = \frac{13.60\ \mathrm{eV}}{n^2} \qquad U_n = \frac{-27.20\ \mathrm{eV}}{n^2} \qquad E_n = \frac{-13.60\ \mathrm{eV}}{n^2}$$

For the first excited level ($n = 2$), we have $K_2 = 3.40$ eV, $U_2 = -6.80$ eV, and $E_2 = -3.40$ eV. For the ground level ($n = 1$), $E_1 = -13.60$ eV. The energy of the emitted photon is then $E_2 - E_1 = -3.40\ \mathrm{eV} - (-13.60\ \mathrm{eV}) = 10.20$ eV, and

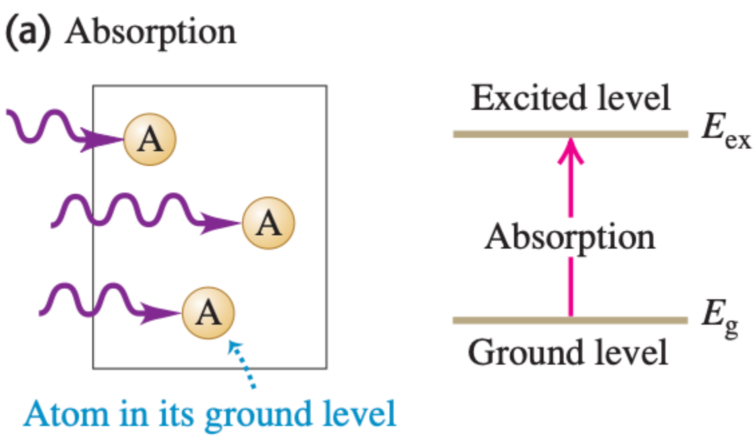
$$\begin{aligned}\lambda &= \frac{hc}{E_2 - E_1} = \frac{(4.136 \times 10^{-15}\ \mathrm{eV \cdot s})(3.00 \times 10^{8}\ \mathrm{m/s})}{10.20\ \mathrm{eV}} \\ &= 1.22 \times 10^{-7}\ \mathrm{m} = 122\ \mathrm{nm}\end{aligned}$$

This is the wavelength of the Lyman-alpha (L_α) line, the longest-wavelength line in the Lyman series of ultraviolet lines in the hydrogen spectrum (see Fig. 39.24).

39-4 The Laser



39-4 The Laser



$$\frac{n_{ex}}{n_g} = \frac{Ae^{-E_{ex}/kT}}{Ae^{-E_g/kT}} = e^{-(E_{ex}-E_g)/kT}$$

For example, suppose $E_{ex} - E_g = 2.0\text{ eV} = 3.2 \times 10^{-19}\text{ J}$, the energy of a 620-nm visible-light photon. At $T = 3000\text{ K}$ (the temperature of the filament in an incandescent light bulb),

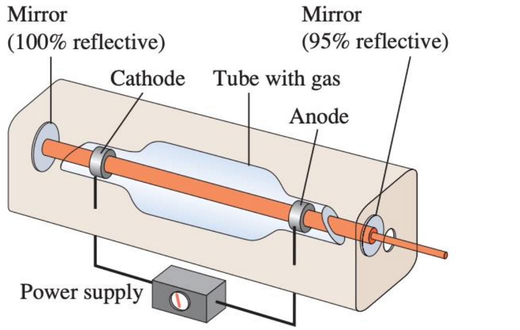
$$\frac{E_{ex} - E_g}{kT} = \frac{3.2 \times 10^{-19}\text{ J}}{(1.38 \times 10^{-23}\text{ J/K})(3000\text{ K})} = 7.73$$

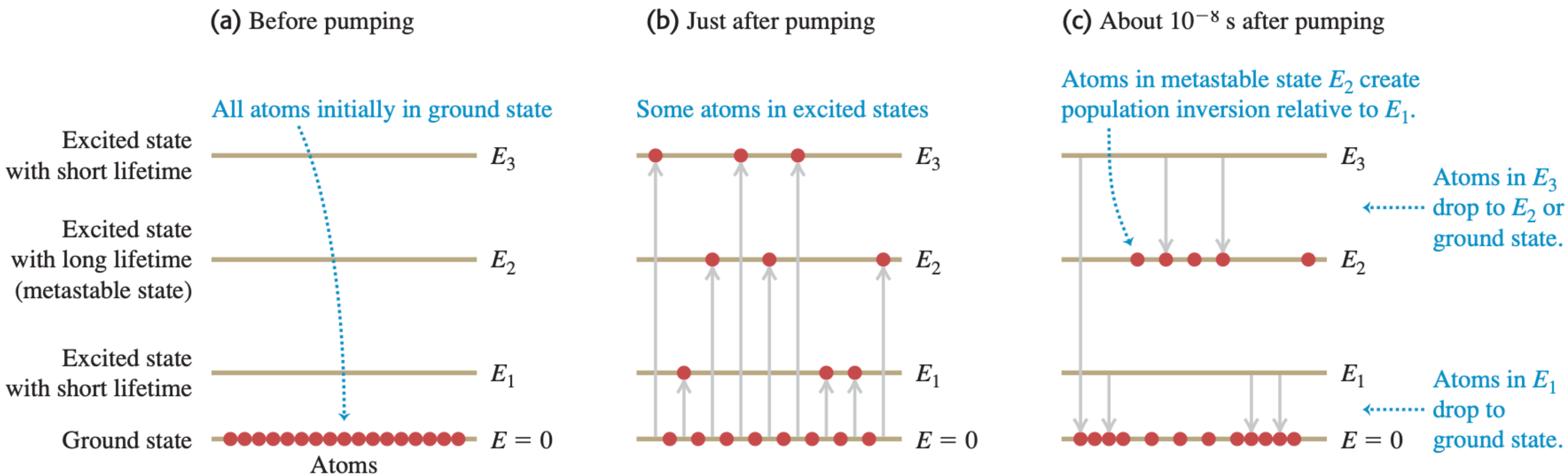
and

$$e^{-(E_{ex}-E_g)/kT} = e^{-7.73} = 0.00044$$

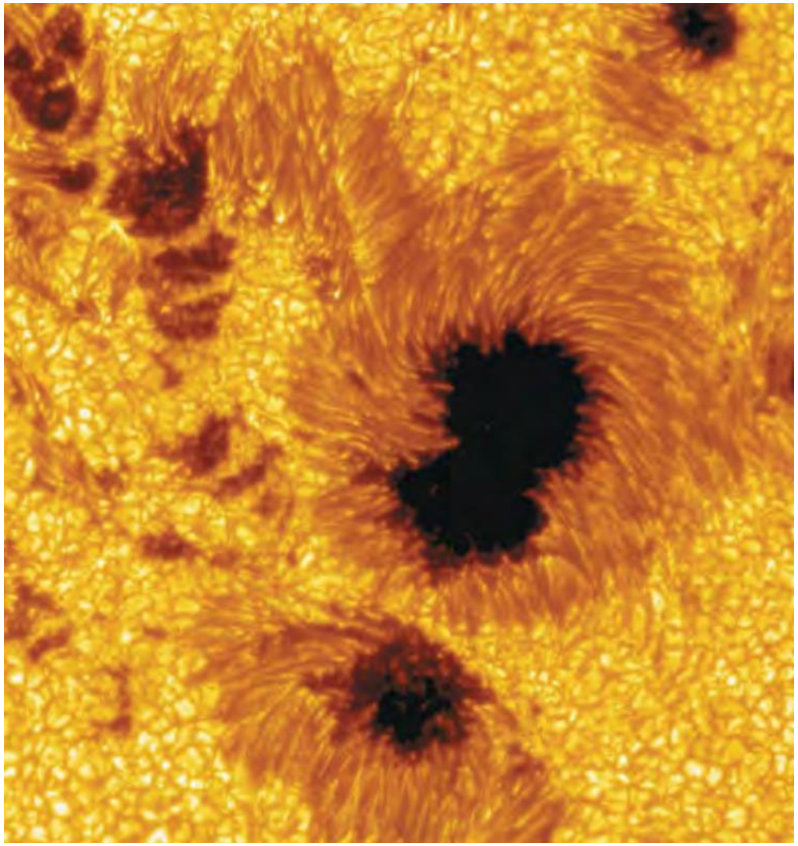
39-4 The Laser

Laser: a *nonequilibrium* situation in which the number of atoms in a higher-energy state is greater than the number in a lower-energy state, **population inversion**





39-5 Continuous Spectra



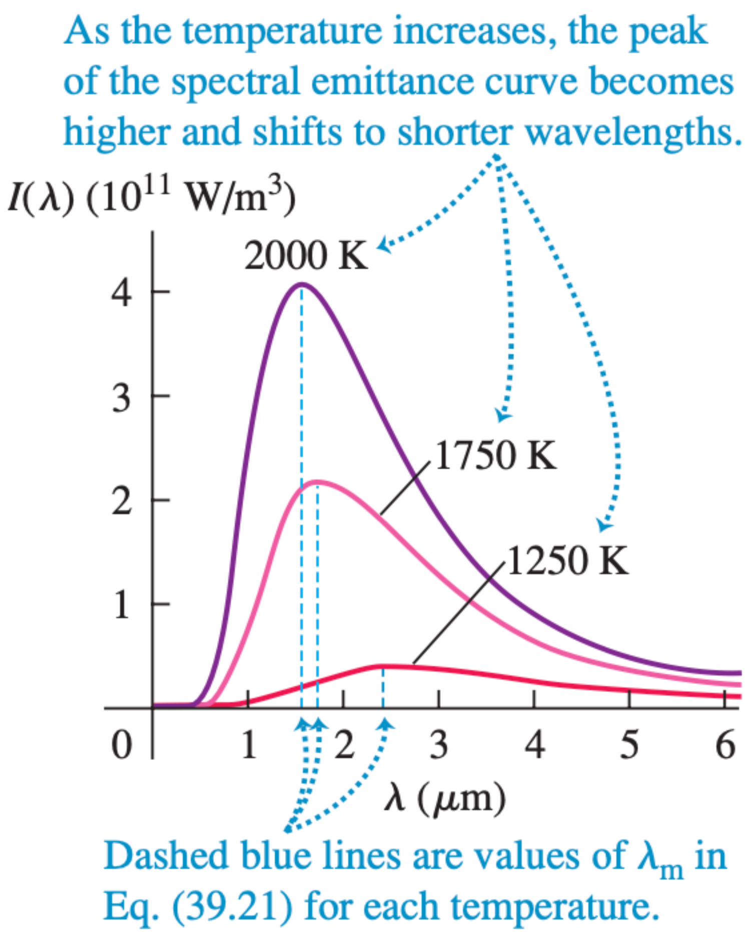
Stefan–Boltzmann law for a blackbody

$$I = \sigma T^4$$

$$\sigma = 5.670400(40) \times 10^{-8} \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}^4}$$

Dark sunspots $(4000\ \mathrm{K}/5800\ \mathrm{K})^4 = 0.23$

39-5 Continuous Spectra



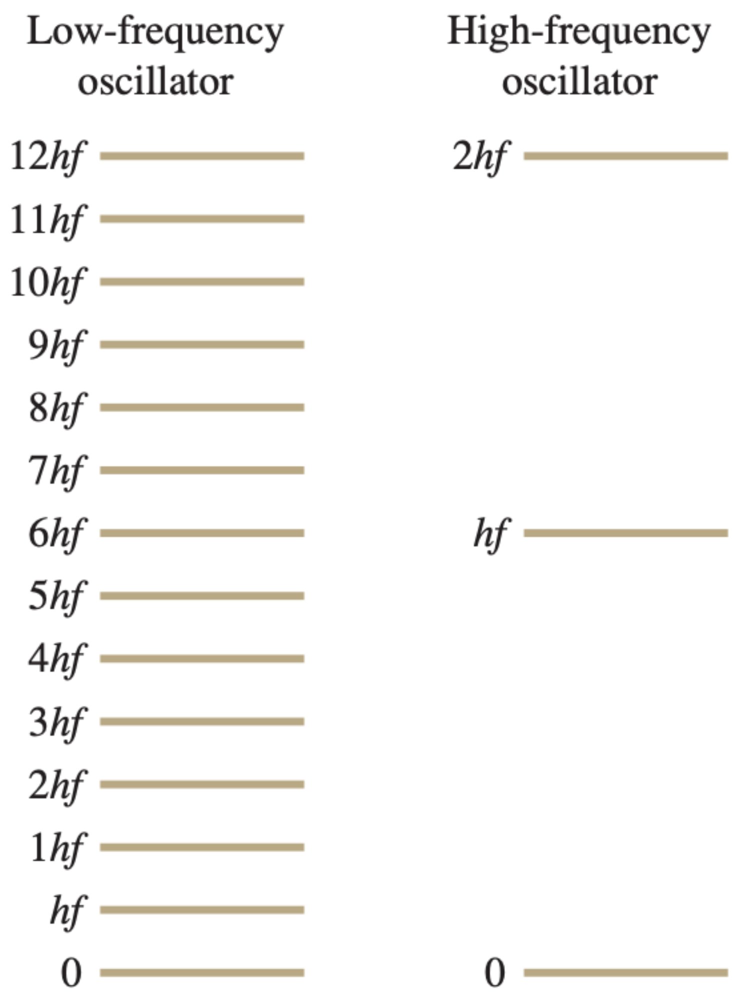
Wien displacement law

$$\lambda_m T = 2.90 \times 10^{-3}\ \text{m} \cdot \text{K}$$

Total intensity

$$I = \int_0^\infty I(\lambda)\, d\lambda$$

39-5 Continuous Spectra



Planck radiation law

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)}$$

Resulting in Stefan–Boltzmann law

$$I = \int_0^\infty I(\lambda)\, d\lambda = \frac{2\pi^5 k^4}{15c^2 h^3} T^4 = \sigma T^4$$

$$\sigma = \frac{2\pi^5 k^4}{15c^2 h^3}$$

Sample Problem

Example 39.7 **Light from the sun**

To a good approximation, the sun's surface is a blackbody with a surface temperature of 5800 K. (We are ignoring the absorption produced by the sun's atmosphere, shown in Fig. 39.9.) (a) At what wavelength does the sun emit most strongly? (b) What is the total radiated power per unit surface area?

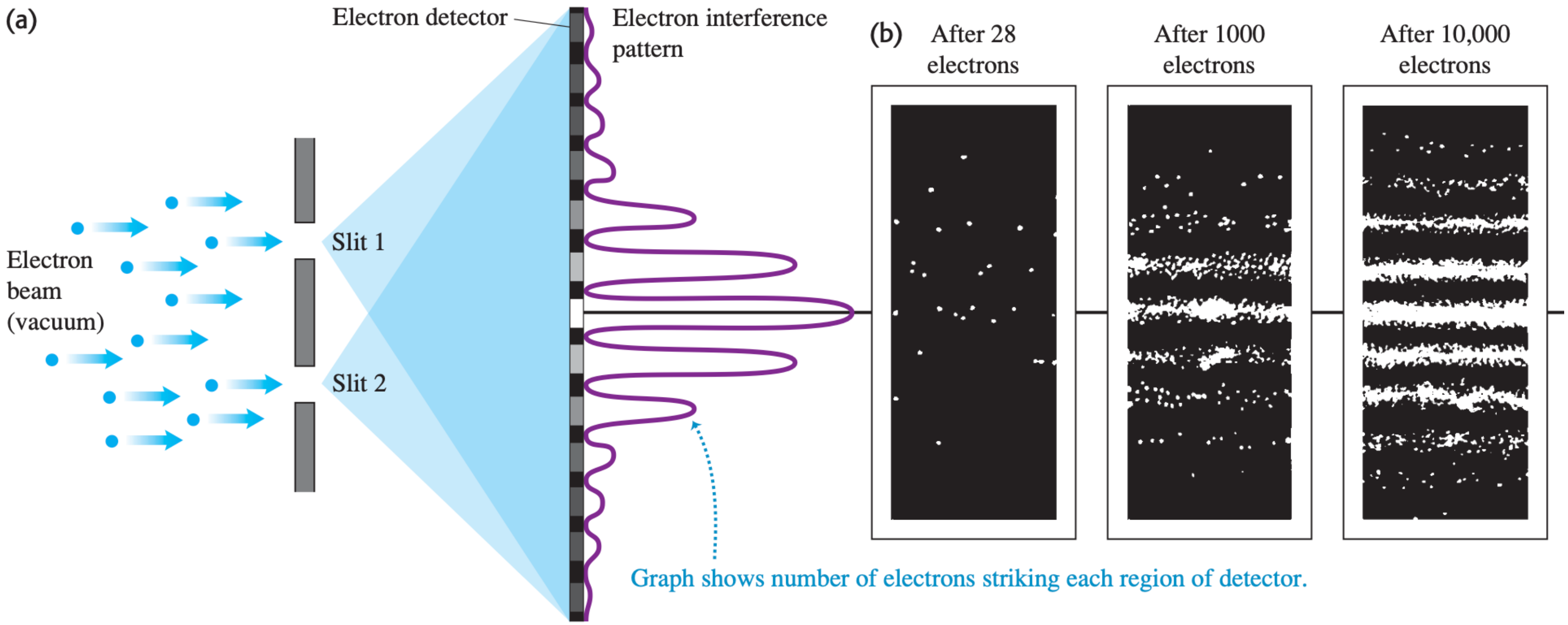
EXECUTE: (a) From Eq. (39.21),

$$\lambda_m = \frac{2.90 \times 10^{-3}\ \mathrm{m \cdot K}}{T} = \frac{2.90 \times 10^{-3}\ \mathrm{m \cdot K}}{5800\ \mathrm{K}}$$
$$= 0.500 \times 10^{-6}\ \mathrm{m} = 500\ \mathrm{nm}$$

(b) From Eq. (39.19),

$$I = \sigma T^4 = (5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4})(5800\ \mathrm{K})^4$$
$$= 6.42 \times 10^7\ \mathrm{W/m^2} = 64.2\ \mathrm{MW/m^2}$$

39-6 The Uncertainty Principle Revisited



$$\Delta x \Delta p_x \geq \hbar/2$$
$$\Delta y \Delta p_y \geq \hbar/2$$
$$\Delta z \Delta p_z \geq \hbar/2$$

(Heisenberg uncertainty principle for position and momentum)

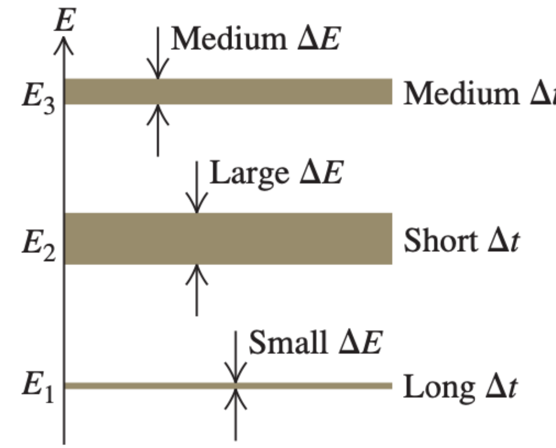
$$\Delta t \, \Delta E \geq \hbar/2$$

(Heisenberg uncertainty principle for energy and time interval)

Sample Problem

Example 39.10 The uncertainty principle: energy and time

A sodium atom in one of the states labeled "Lowest excited levels" in Fig. 39.19a remains in that state, on average, for 1.6×10^{-8} s before it makes a transition to the ground level, emitting a photon with wavelength 589.0 nm and energy 2.105 eV. What is the uncertainty in energy of that excited state? What is the wavelength spread of the corresponding spectral line?



EXECUTE: From Eq. (39.30),

$$\Delta E = \frac{\hbar}{2\Delta t} = \frac{1.055 \times 10^{-34}\ \mathrm{J \cdot s}}{2(1.6 \times 10^{-8}\ \mathrm{s})}$$
$$= 3.3 \times 10^{-27}\ \mathrm{J} = 2.1 \times 10^{-8}\ \mathrm{eV}$$

The atom remains in the ground level indefinitely, so that level has *no* associated energy uncertainty. The fractional uncertainty of the *photon* energy is therefore

$$\frac{\Delta E}{E} = \frac{2.1 \times 10^{-8}\ \mathrm{eV}}{2.105\ \mathrm{eV}} = 1.0 \times 10^{-8}$$

You can use some simple calculus and the relation $E = hc/\lambda$ to show that $\Delta\lambda/\lambda \approx \Delta E/E$, so that the corresponding spread in wavelength, or "width," of the spectral line is approximately

$$\Delta\lambda = \lambda\frac{\Delta E}{E} = (589.0\ \mathrm{nm})(1.0 \times 10^{-8}) = 0.0000059\ \mathrm{nm}$$