CHAPTER 8 Active Filters

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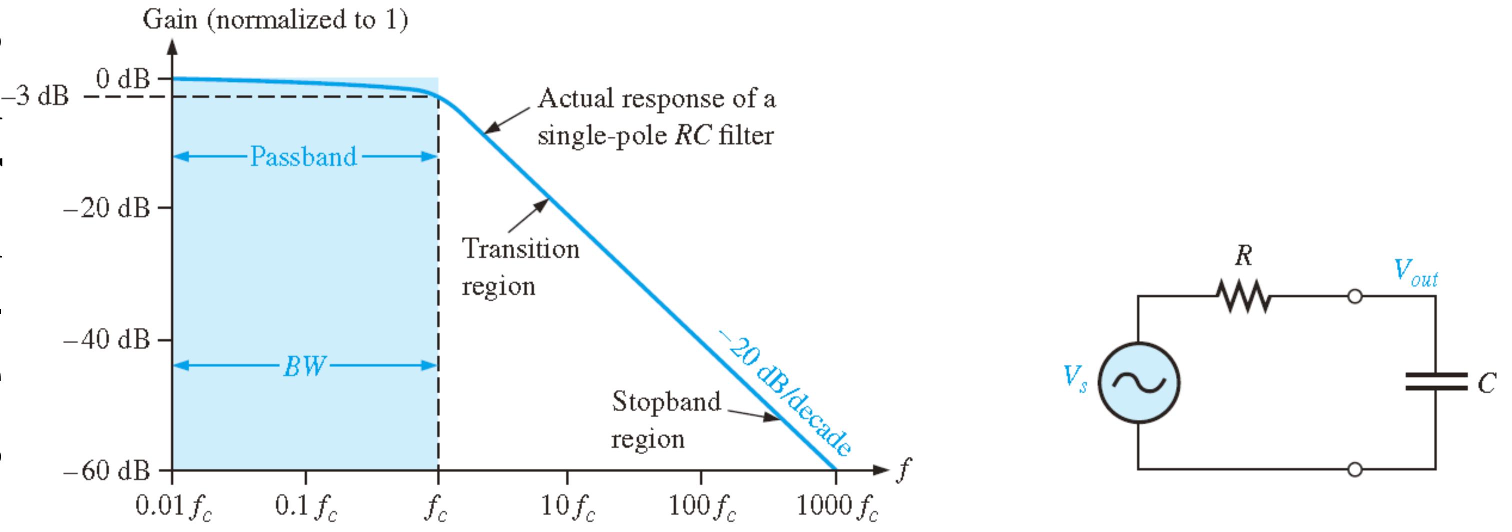
Filter

A **filter** is a circuit that passes certain frequencies and attenuates or rejects all other frequencies. The **passband** of a filter is the range of frequencies that are allowed to pass through the filter with minimum attenuation (usually defined as less than -3 dB of attenuation). The **critical frequency**, f_c, (also called the *cutoff frequency*) defines the end of the passband and is normally specified at the point where the response drops -3 dB (70.7%) from the passband response. Following the passband is a region called the *transition region* that leads into a region called the *stopband*. There is no precise point between the transition region and the stopband.

Low-pass Filter

- A **low-pass filter** is one that passes frequencies from dc to *fc* and significantly attenuates all other frequencies. The passband of the ideal low-pass filter is shown in the blue-shaded area of Figure 8–1(a); the response drops to zero at frequencies beyond the passband.



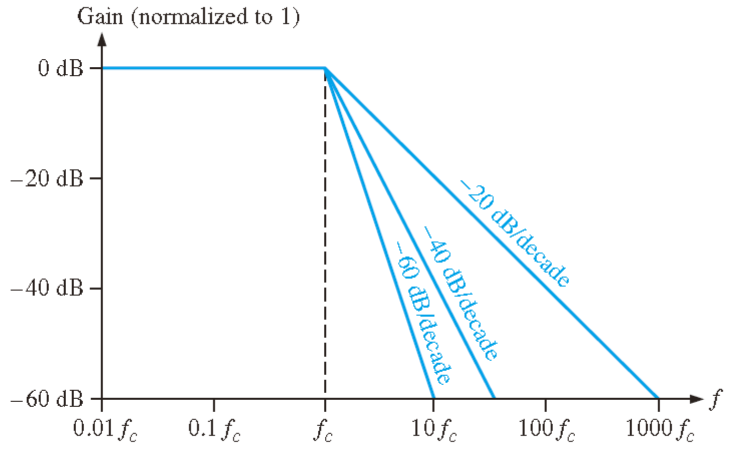
(a) Comparison of an ideal low-pass filter response (blue area) with actual response.

(b) Basic low-pass circuit

- The bandwidth of an ideal low-pass filter is equal to *fc*.

$$BW = f_c$$

- The most basic low-pass filter is a simple *RC* circuit consisting of just one resistor and one capacitor; This basic *RC* filter rolls off at -20 dB/decade beyond the critical frequency.



(c) Idealized low-pass filter responses

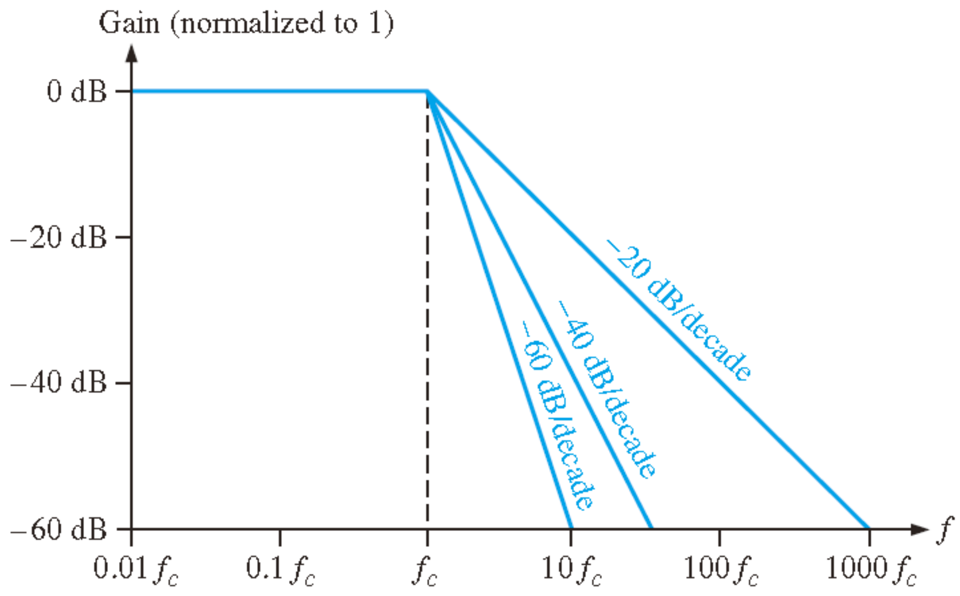
FIGURE 8-1 Low-pass filter responses.

Low-pass Filter

The critical frequency of a low-pass *RC* filter occurs when $X_C = R$, where

$$f_c = \frac{1}{2\pi RC}$$

- The simple *RC* filter in Figure 8–1 is a passive filter because it is composed only of passive components.
- Filters that include one or more op-amps in the design are called **active filters.** These filters can optimize the roll-off rate or other attribute (such as phase response) with a particular filter design.

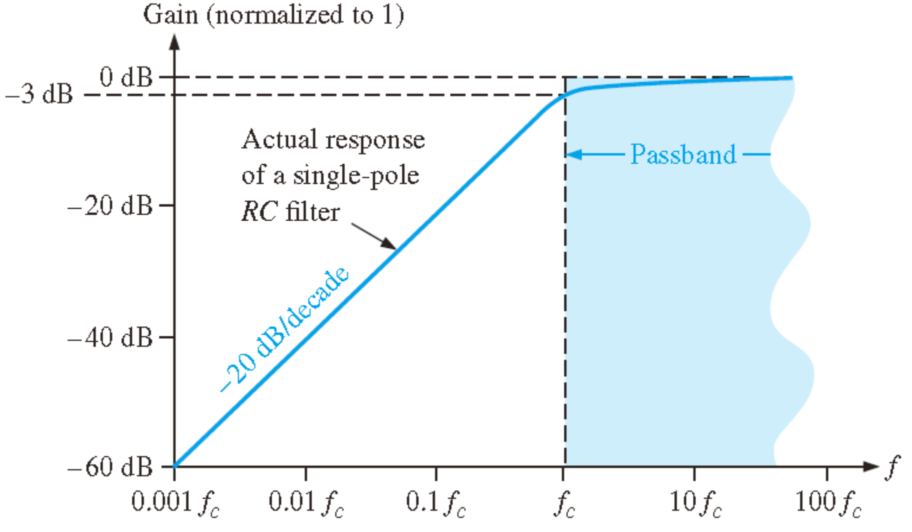


(c) Idealized low-pass filter responses

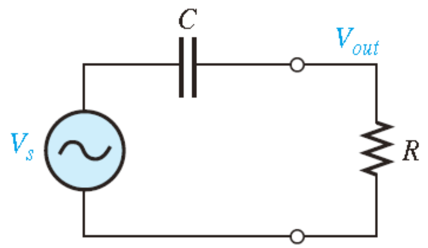
FIGURE 8-1 Low-pass filter responses.

High-pass Filter

- A **high-pass filter** is one that significantly attenuates or rejects all frequencies below *fc* and passes all frequencies above *fc*. The critical frequency is, again, the frequency at which the output is 70.7% of the input (or -3 dB) as shown in Figure 8–2(a).

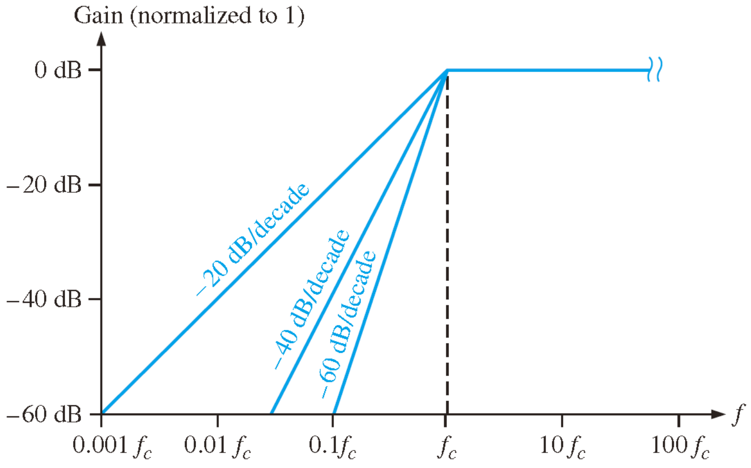


(a) Comparison of an ideal high-pass filter response (blue area) with actual response



(b) Basic high-pass circuit

- A simple *RC* circuit consisting of a single resistor and capacitor can be configured as a high-pass filter by taking the output across the resistor as shown in Figure 8–2(b).
- As in the case of the low-pass filter, the basic *RC* circuit has a roll-off rate of -20 dB/decade, as indicated by the blue line in Figure 8–2(a).



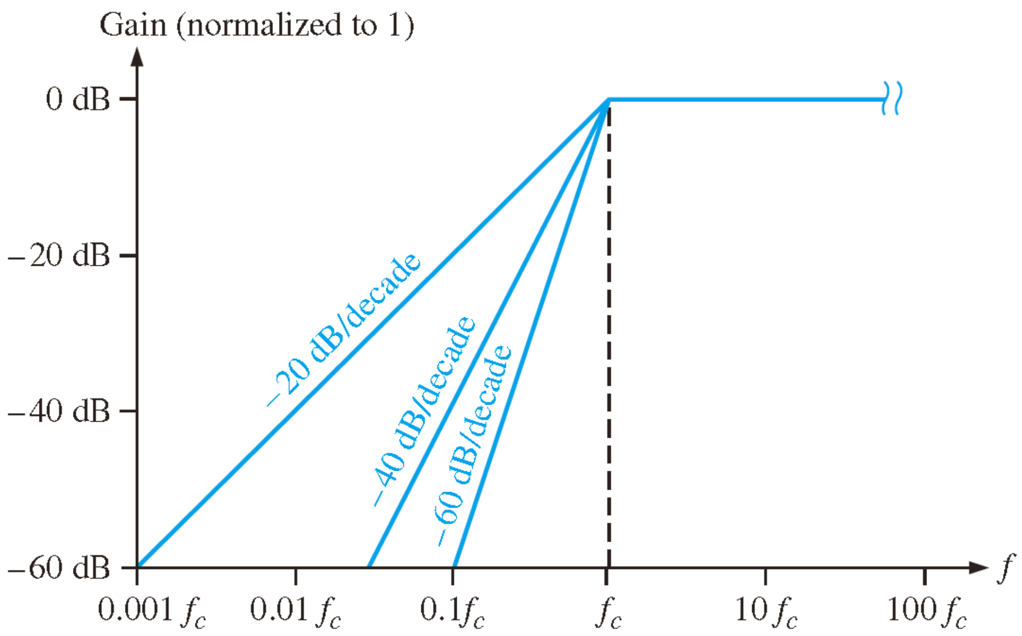
(c) Idealized high-pass filter responses

FIGURE 8-2 High-pass filter responses.

High-pass Filter

The critical frequency for the basic high pass filter occurs when $X_C = R$, where

$$f_c = \frac{1}{2\pi RC}$$



(c) Idealized high-pass filter responses

FIGURE 8-2 High-pass filter responses.

Band-pass Filter

A **band-pass filter** passes all signals lying within a band between a lower-frequency limit and an upper-frequency limit and essentially rejects all other frequencies that are outside this specified band. A generalized band-pass response curve is shown in FIGURE 8-3 . The bandwidth (BW) is defined as the difference between the upper critical frequency (f_{c2}) and the lower critical frequency (f_{c1}).

$$BW = f_{c2} - f_{c1}$$

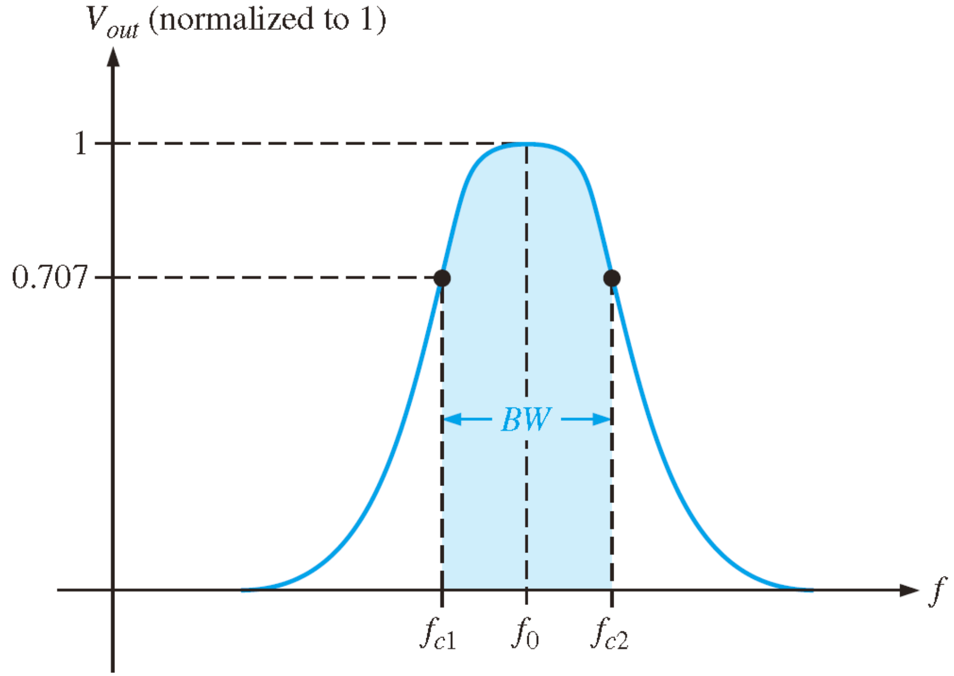


FIGURE 8-3 General band-pass response curve.

Band-pass Filter

- The critical frequencies are, of course, the points at which the response curve is 70.7% of its maximum.
- These critical frequencies are also called *3 dB frequencies*. The frequency about which the passband is centered is called the *center frequency*, f_0, defined as the geometric mean of the critical frequencies.

$$f_0 = \sqrt{f_{c1} f_{c2}}$$

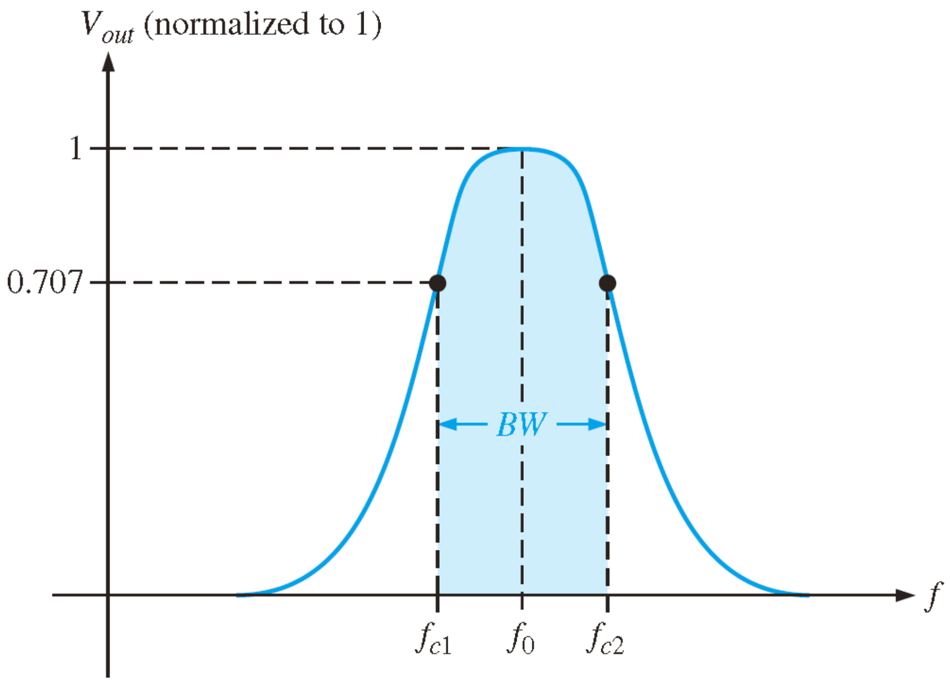


FIGURE 8-3 General band-pass response curve.

Band-Stop Filter

Another category of active filter is the **band-stop filter**, also known as *notch*, *band-reject*, or *band-elimination* filter. You can think of the operation as opposite to that of the band-pass filter because frequencies within a certain bandwidth are rejected, and frequencies outside the bandwidth are passed. A general response curve for a band-stop filter is shown in **FIGURE 8-4** Notice that the bandwidth is the band of frequencies between the 3 dB points, just as in the case of the band-pass filter response.

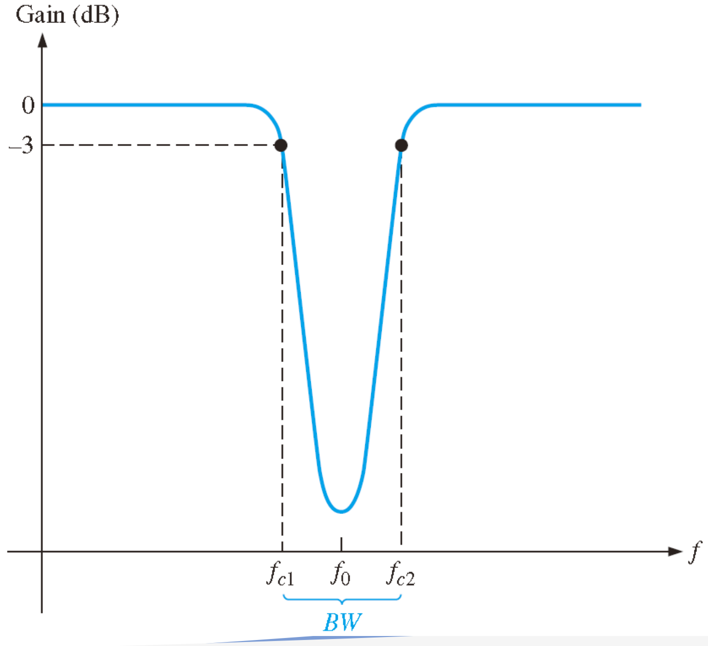


FIGURE 8-4 General band-stop filter response.

Active Low-Pass Filters

FIGURE 8-5 (a) shows an active filter with a single low-pass *RC* frequency-selective circuit that provides a roll-off of −20 dB/decade above the critical frequency, as indicated by the response curve in **FIGURE 8-5** (b). The critical frequency of the single-pole filter is $f_c = 1/(2\pi RC)$. The op-amp in this filter is connected as a noninverting amplifier with the closed-loop voltage gain in the passband set by the values of R_1 and R_2.

$$A_{cl(NI)} = \frac{R_1}{R_2} + 1$$

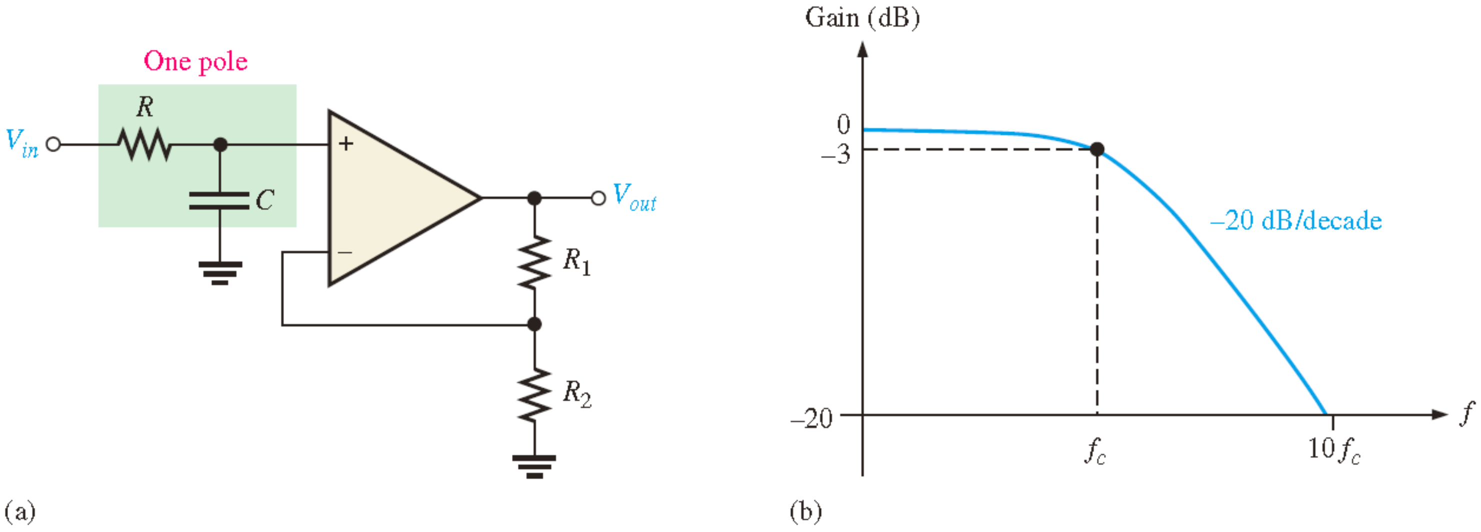


FIGURE 8-5 Single-pole active low-pass filter and response curve.

The Sallen-Key Low-Pass Filter

The Sallen-Key is one of the most common configurations for a second-order (two-pole) filter. It is also known as a VCVS (voltage-controlled voltage source) filter. A low-pass version of the Sallen-Key filter is shown in FIGURE 8-6 Notice that there are two low-pass RC circuits that provide a roll-off of -40 dB/decade above the critical frequency (assuming a Butterworth characteristic). One RC circuit consists of R_A and C_A, and the second circuit consists of R_B and C_B. A unique feature of the Sallen-Key low-pass filter is the capacitor C_A that provides feedback for shaping the response near the edge of the pass-band. The critical frequency for the Sallen-Key filter is

$$f_c = \frac{1}{2\pi\sqrt{R_A R_B C_A C_B}}$$

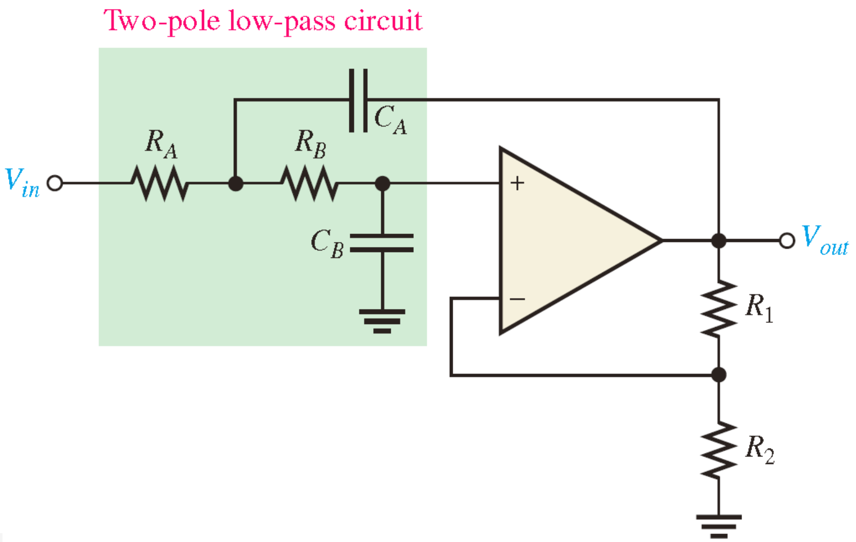


FIGURE 8-6 Basic Sallen-Key low-pass filter.

The Sallen-Key Low-Pass Filter

The component values can be made equal so that $R_A = R_B = R$ and $C_A = C_B = C$. In this case, the expression for the critical frequency simplifies to

$$f_c = \frac{1}{2\pi RC}$$

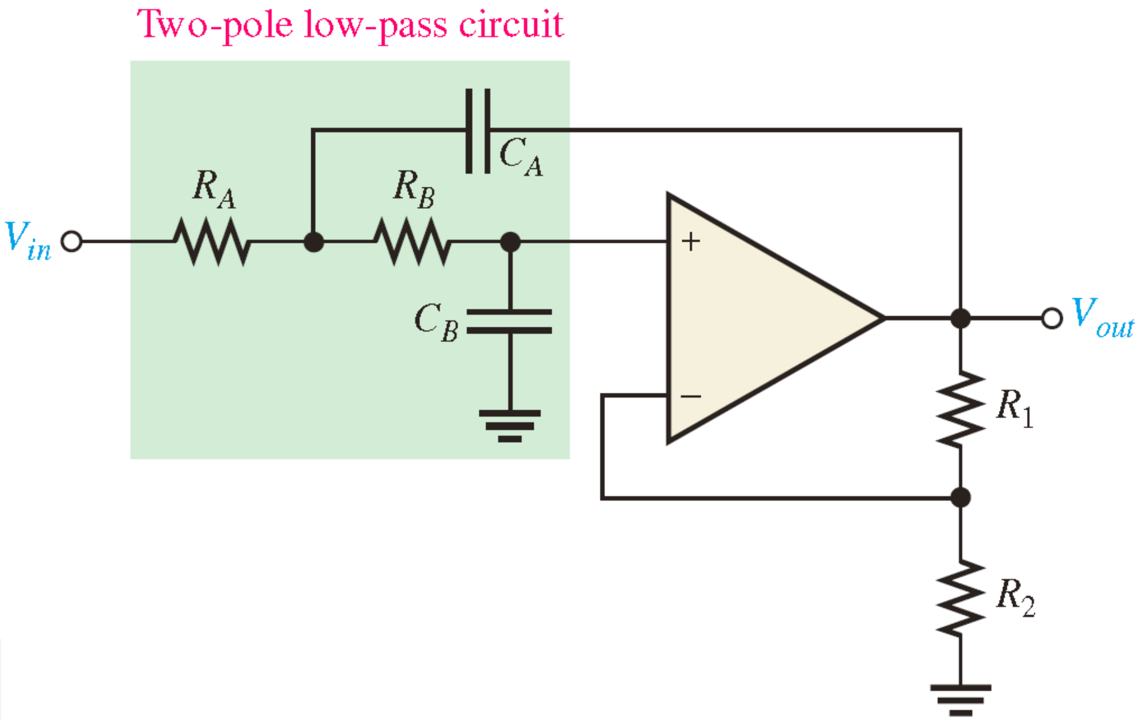
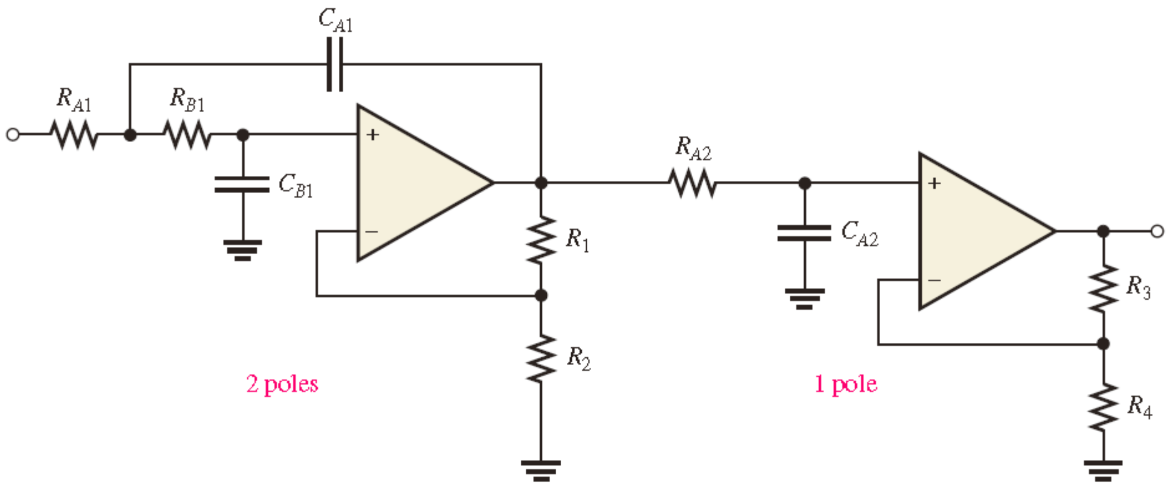
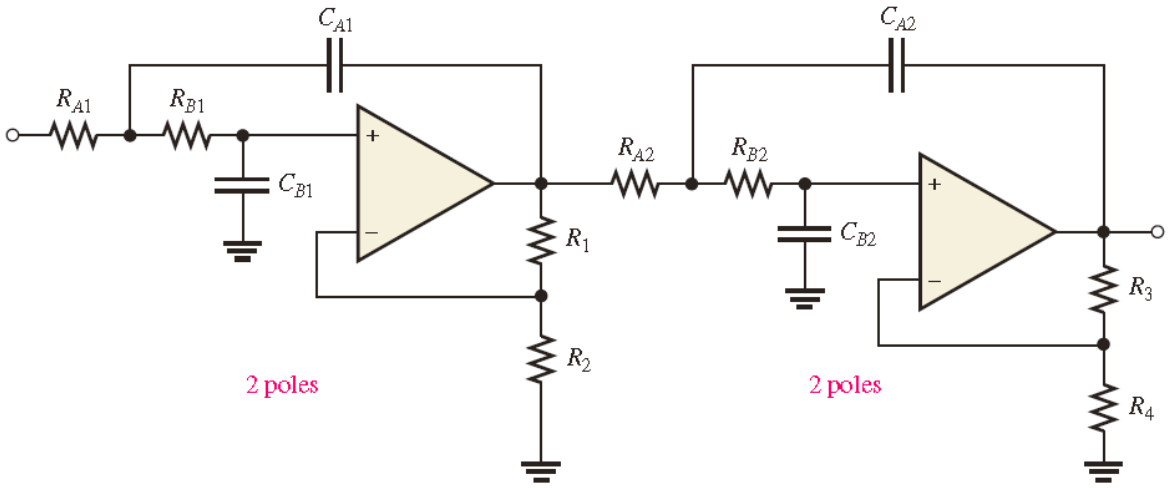


FIGURE 8-6 Basic Sallen-Key low-pass filter.

Cascaded Low-Pass Filters



(a) Third-order configuration



(b) Fourth-order configuration

Active High-Pass Filters

A high-pass active filter with a −20 dB/decade roll-off is shown in FIGURE 8-7 (a). Notice that the input circuit is a single high-pass *RC* circuit. The negative feedback circuit is the same as for the low-pass filters previously discussed. The high-pass response curve is shown in FIGURE 8-7

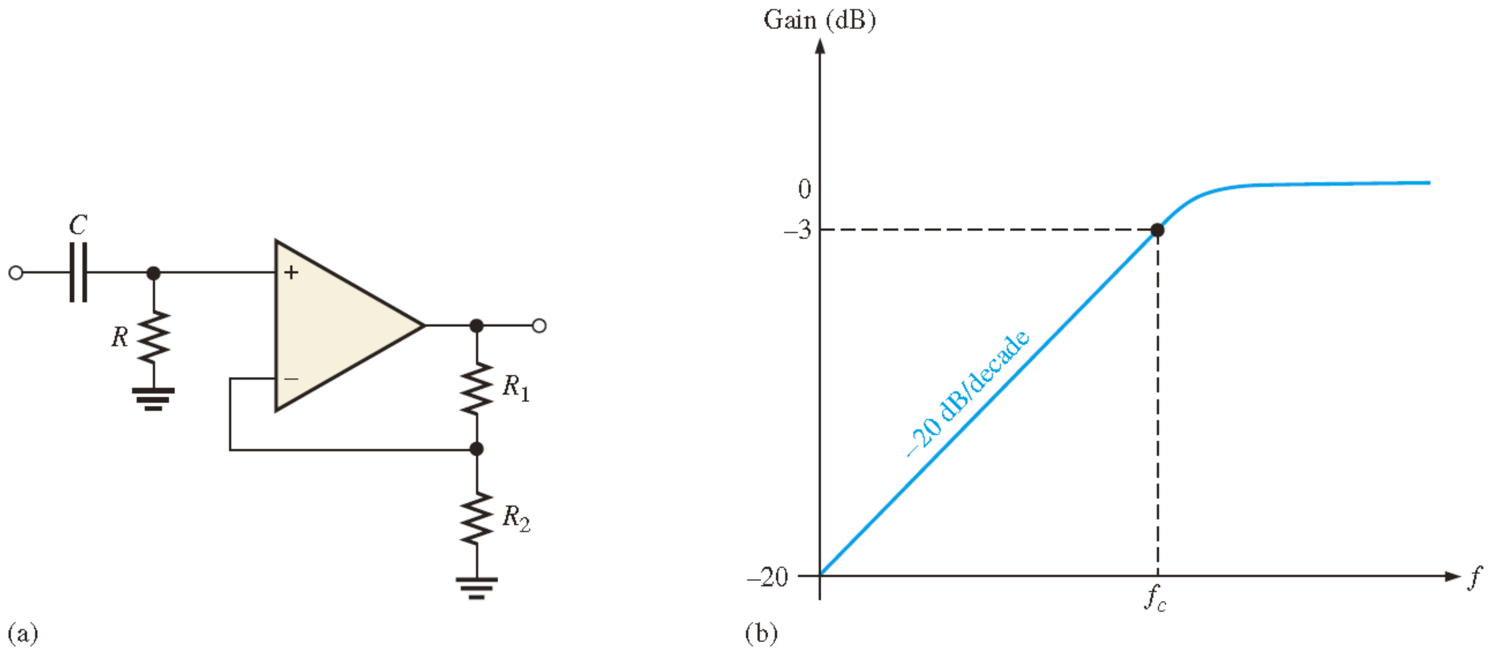
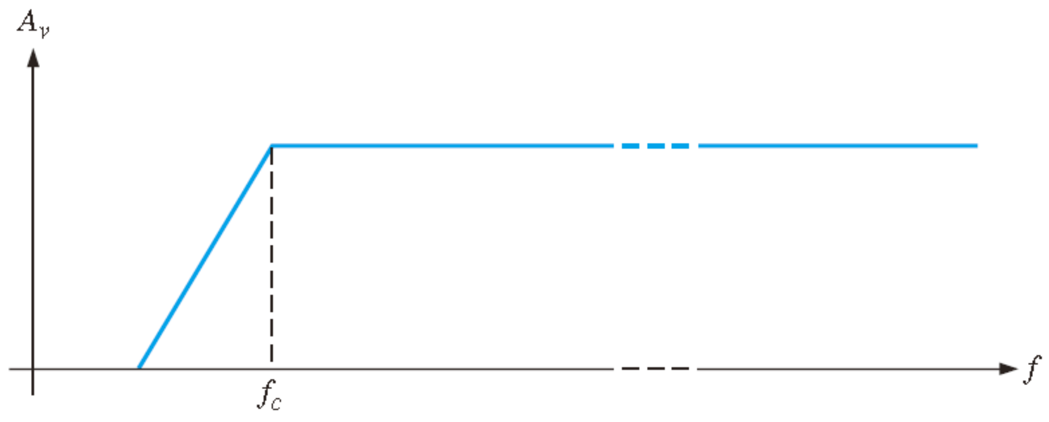


FIGURE 8-7 Single-pole active high-pass filter and response curve.

Active High-Pass Filters

Ideally, a high-pass filter passes all frequencies above f_c without limit, as indicated in FIGURE 8-8 (a), although in practice, this is not the case. As you have learned, all op-amps inherently have internal *RC* circuits that limit the amplifier's response at high frequencies. Therefore, there is an upper-frequency limit on the high-pass filter's response which, in effect, makes it a band-pass filter with a very wide bandwidth. In the majority



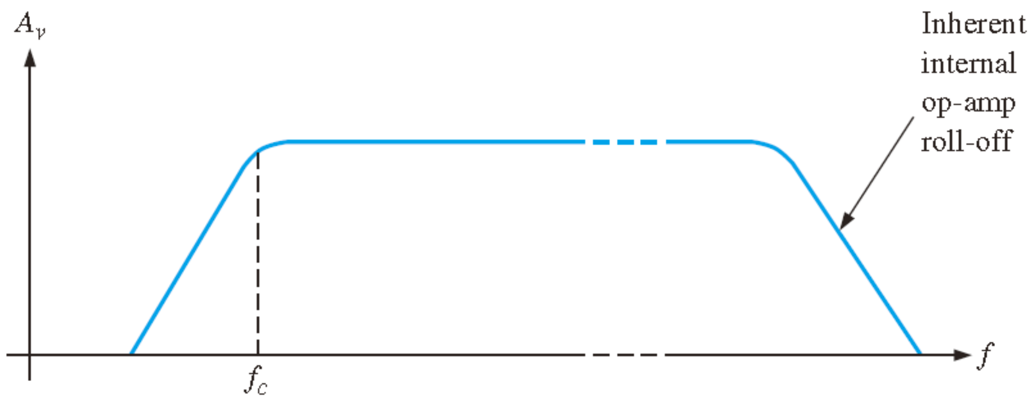


FIGURE 8-8 High-pass filter response.

The Sallen-Key High-Pass Filter

A high-pass Sallen-Key configuration is shown in FIGURE 8-9 . The components R_A, C_A, R_B, and C_B form the two-pole frequency-selective circuit. Notice that the positions of the resistors and capacitors in the frequency-selective circuit are opposite to those in the low-pass configuration. As with the other filters, the response characteristic can be optimized by proper selection of the feedback resistors, R_1 and R_2.

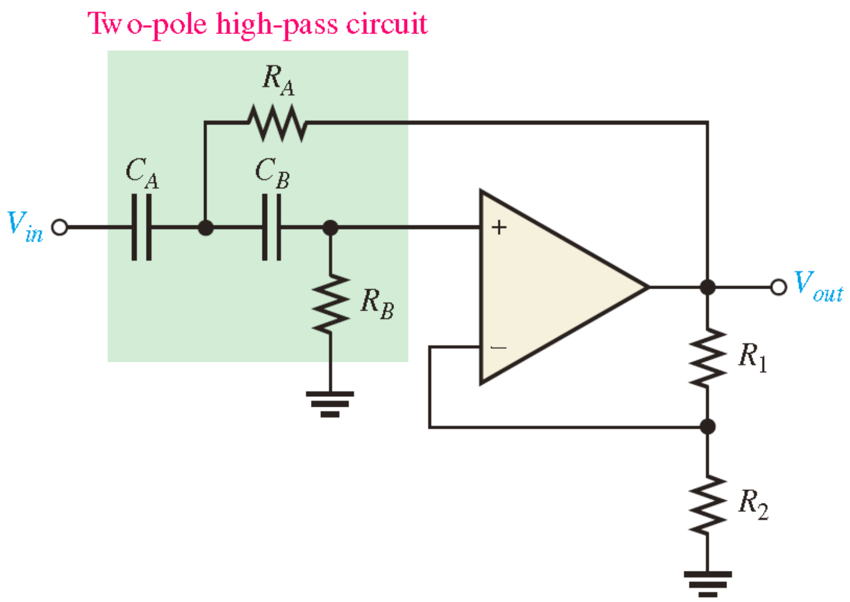
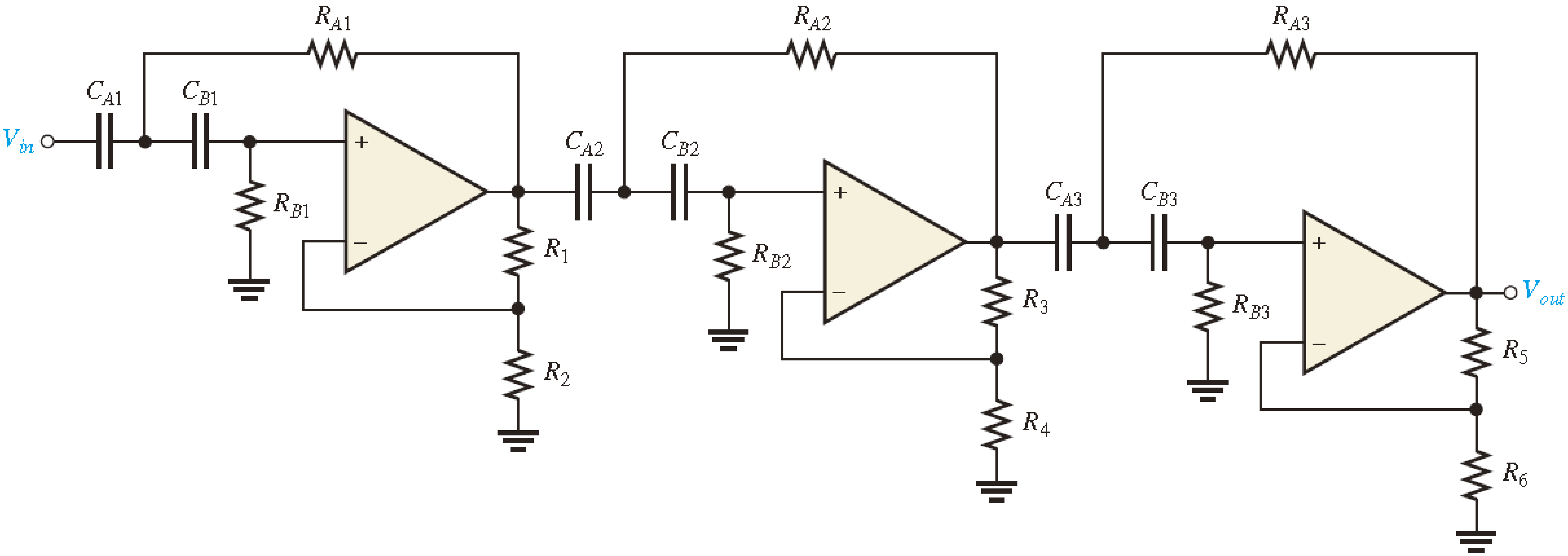


FIGURE 8-8 Basic Sallen-Key high-pass filter.

Cascaded High-Pass Filters



Active Band-Pass Filters

One way to implement a band-pass filter is a cascaded arrangement of a high-pass filter and a low-pass filter, as shown in **FIGURE 8-9** (a), as long as the critical frequencies are sufficiently separated. Each of the filters shown is a Sallen-Key Butterworth configuration so that the roll-off rates are −40 dB/decade, indicated in the composite response curve of

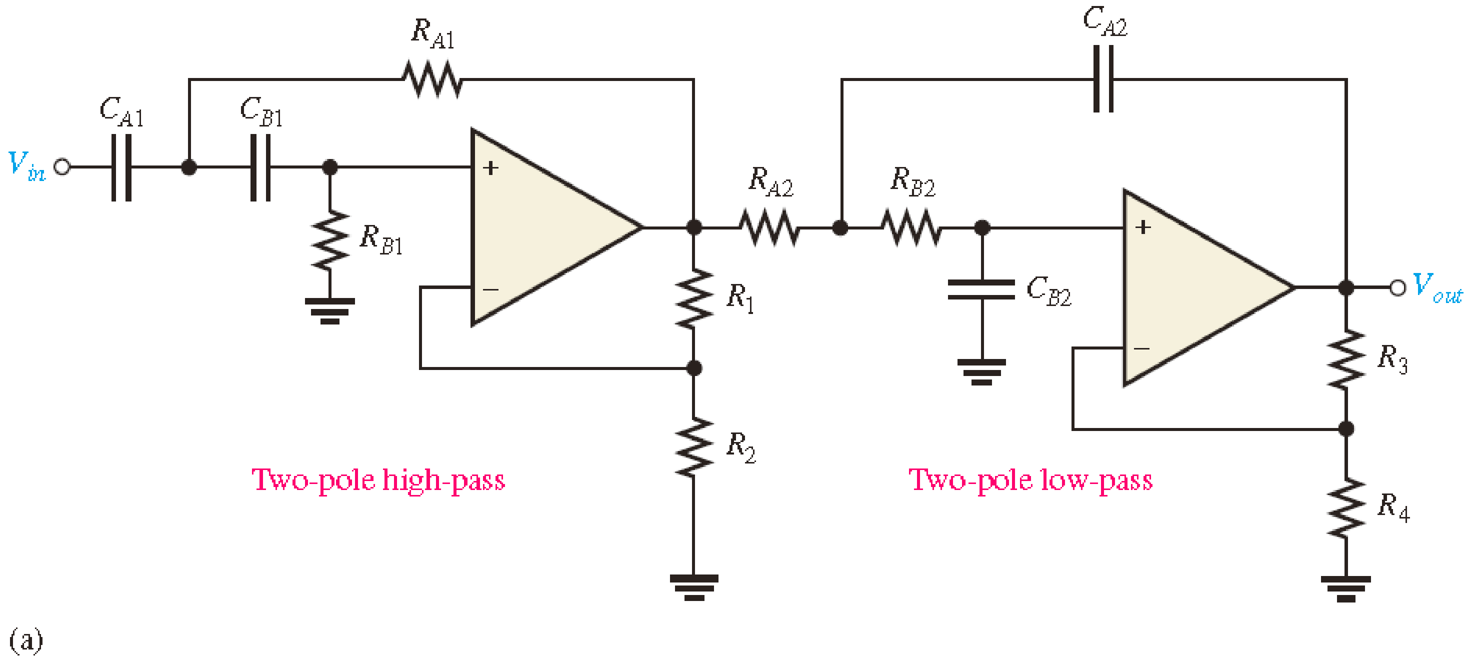


FIGURE 8-9 Band-pass filter formed by cascading a two-pole high-pass and a two-pole low-pass filter (it does not matter in which order the filters are cascaded).

Active Band-Pass Filters

The lower frequency f_{c1} of the passband is the critical frequency of the high-pass filter. The upper frequency f_{c2} is the critical frequency of the low-pass filter. Ideally, as discussed earlier, the center frequency f_0 of the passband is the geometric mean of f_{c1} and f_{c2}. The following formulas express the three frequencies of the band-pass filter in Of course, if equal-value components are used in implementing each filter, the critical frequency equations simplify to the form $f_c = 1/(2\pi RC)$.

$$f_{c1} = \frac{1}{2\pi\sqrt{R_{A1}R_{B1}C_{A1}C_{B1}}}$$

$$f_{c2} = \frac{1}{2\pi\sqrt{R_{A2}R_{B2}C_{A2}C_{B2}}}$$

$$f_0 = \sqrt{f_{c1}f_{c2}}$$

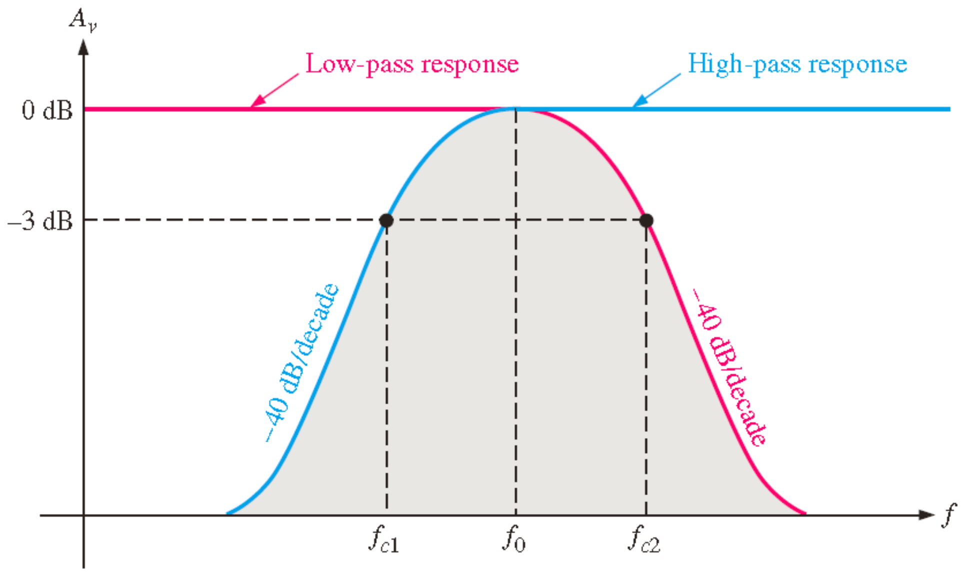


FIGURE 8-9 Band-pass filter formed by cascading a two-pole high-pass and a two-pole low-pass filter