Forward Kinematics

Daniel Stangeland
s331489

September 25, 2020

1 Exercise 1

1.1 a) Assign reference frames to each link according to DH-convention.

There are four rules that guide the drawing of the DH (Denavit-Hartenberg) coordinate frames

1. The z-axis is the axis of rotation for a revolute joint or the axis of positive displacement for a prismatic joint
2. The x-axis must be perpendicular to both the current z-axis and the previous z-axis
   - $x_i \perp z_{i-1} \wedge x_i \perp z_i$
3. The y-axis is determined form the x-axis and z-axis by using the right-hand coordinate system
4. The x-axis must intersect the previous z-axis
   - rule does not apply to frame 0. This means that x_1 does not have to intersect z_0.

Using these rules we can make a step-by-step procedure on how to assign reference frames:

1. Locate and label the joint axes, $z_0, \dots, z_{n-1}$
2. Establish the base frame. Set the origin anywhere on the z_0-axis.

   For $i = 1, \dots, n-1$ perform step 3 to 5
3. Locate the origin o_i where the common normal to z_i and z_i.
   - If z_i intersects z_{i-1}, locate o_i at this intersection.
   - If z_i and z_{i-1} are parallel, locate o_i in any convenient position along z_i
4. Establish x_i along the common normal between z_{i-1} and z_i through o_i, or in the direction normal to the $z_{i-1} - z_i$ if z_{i-1} and z_i intersect.
5. Establish y_i to complete a right-handed frame
6. Establish the end-effector fram $o_n x_n y_n z_n$
   - Assuming the n^{th} joint is revolute, set $z_n = a$ parallel to z_{n-1}.

   Establish the origin o_n conveniently along z_n,
   - Preferably at the centre of the gripper or at the tip of any tool that the manipulator may be carrying.

   Set $y_n = s$ in the direction of the gripper closure and set $x_n = n$ as $s \times a$
   - If the tool is not a simple gripper set x_n and y_n conveniently to form a right-handed frame.
7. Create a table of DH parameters $\theta_i, d_i, a_i, \alpha_i$

Link	$\boldsymbol{\theta_i}$	d_i	$\boldsymbol{a_i}$	α_i
1	θ_1	d_1	a_1	α_1
2	θ_2	d_2	a_2	α_2
3	θ_3	d_3	a_3	α_3

   $\theta_i =$ The angle from x_{i-1} to x_i measured about z_{i-1}.
   - If joint i is revolute θ_i is variable.

   $d_i =$ distance along z_{i-1} from o_{i-1} to the intersection of x_i and z_{i-1} axes.
   - If join i is prismatic d_i is variable.

a_i = distance along x_i from the intersection of the x_i and z_{i-1} to o_i.

α_i = the angle from z_{i-1} to z_i measured about x_i

8. Form the homogeneous transformation matrices A_i by substituting parameters found in step 7 into

$$A_i = Rot_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} Rotx_{\alpha_i} = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

9. Form ${}^0T_n = A_1 \dots A_n$. This gives the pose of the tool framed expressed in base coordinates.

Rules from Alex's lecture

1. Assign z_i axis: z_i is parallel with joint $i+1$
2. Assign x_i (and origin of i^{th} reference frame)
   - *z_i and z_{i-1} are not co-planar* (This is the most unusual case):

     Then choose x_i in the common normal, the shortest line between z_i and z_{i-1} (which is perpendicular to both)
   - *z_i and z_{i-1} are parallel:*

     Then choose x_i in the common normal that goes through origin of frame $i-1$
   - *z_i and z_{i-1} intersect* (x_i normal to the plane):

     Choose origin of frame i at intersection of z_i and z_{i-1}

Using these tools combined with the rules for DH convention the reference frames are as follows:

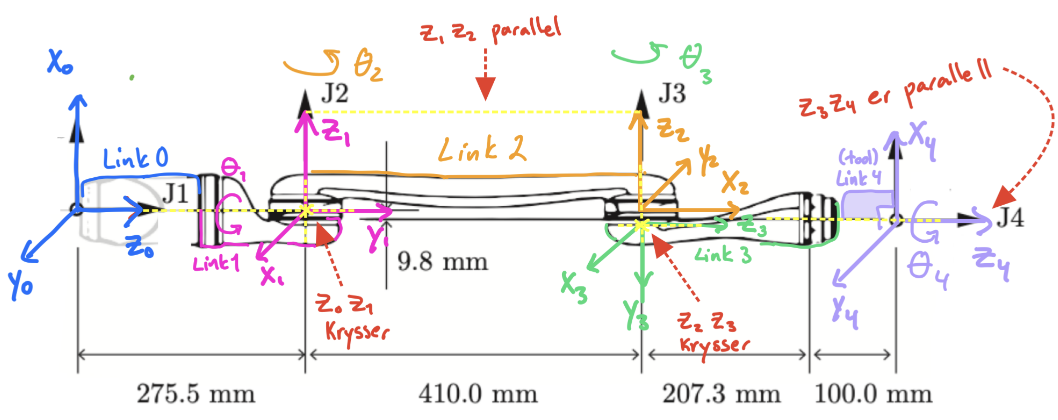

Figure 1: The KINOVA ultra light 4-dof robot arm at its home position with reference reference frames

1.2 b) Determine the homogeneous transform ${}^0T_4$ in the home configuration.

The homogeneous transform ${}^0T_4$ can determined by looking at figure 1 and comparing {4} to {0}.

First by looking at the reference frames we observe that there has been no rotation from {0} to {4} since the axes have the same orientation.

On the other hand there has been a translation, both in the x-direction and z-direction, respectively $-0.0098m$ and $0.9928m$.

This gives the following matrix:

$$
{}^0T_4 = \begin{bmatrix} {}^0\boldsymbol{R}_4 & {}^0\boldsymbol{t}_4 \\ 0_{1X3} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -0.0098 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.9928 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

1.3 c) Determine Denavit Hartneberg parameters

We can determine the Denavit-Hartneberg parameters ($\theta_i, d_i, a_i, \alpha_i$) by analysing the figure 1. The definition of each parameter can be found in Exercise 1a, step 7.

Link	$\boldsymbol{\theta_i}$	d_i [m]	$\boldsymbol{a_i}[m]$	α_i
1	$\frac{\pi}{2} + \theta_1^*$	0.2755	0	$\frac{\pi}{2}$
2	$\frac{\pi}{2} + \theta_2^*$	0	0.410	0
3	$-\frac{\pi}{2} + \theta_3^*$	-0.0098	0	$-\frac{\pi}{2}$
4	$-\frac{\pi}{2} + \theta_4^*$	0.307.3	0	0

1.4 d) Determine forward kinematics for generic joint angles ($\theta_1, \theta_2, \theta_3, \theta_4$)

Skipped this exercise since its to much algebra and the risk of making a mistake while computing for hand is too high.