evince /local/FSM/ex9_preliminary.pdf

$$\partial_t \rho + \nabla \cdot (\rho \vec{u}) = 0 \qquad \text{continuity}$$

$$\rho \left(\partial_t \vec{u} + (\vec{u} \cdot \nabla) \vec{u} \right) + \nabla P = 0 \qquad \text{Euler}$$

$$\partial_t (\rho e_{tot}) + \nabla \cdot [(\rho e_{tot} + P) \vec{u}] = 0 \qquad \text{Energy}$$

$$e_{tot} = e + \frac{\vec{u}^2}{2}$$

$$P = f(\rho, T) = \frac{c_s^2}{\gamma} \cdot \rho$$

cont: $\underline{\partial_t \rho + \partial_x (\rho \cdot u) = 0}$

Euler: $\underbrace{\rho (\partial_t u}_{} + u \cdot \partial_x u) + \partial_x P = 0$

$$\partial_t (\rho u) = \rho \partial_t u + u \underbrace{\partial_t \rho}_{= -\partial_x (\rho u)}$$

$$\rightarrow \rho \partial_t u = \partial_t (\rho u) + u \partial_x (\rho u)$$

$$\partial_t (\rho u) + u \partial_x (\rho u) + (\rho u) \partial_x u + \partial_x P = 0$$

$$\underline{\partial_t (\rho u) + \partial_x (\rho u^2 + P) = 0}$$

Energy: $\underline{\partial_t (e_{tot} \rho) + \partial_x [u(\rho e_{tot} + P)] = 0}$

$$\Rightarrow \quad \partial_t \underbrace{\begin{pmatrix} \rho \\ \rho u \\ \rho e_{tot} \end{pmatrix}}_{=: \vec{Q}} + \partial_x \underbrace{\begin{pmatrix} \rho u \\ \rho u^2 + P \\ (\rho e_{tot} + P) u \end{pmatrix}}_{= \vec{F}(\vec{Q})} = \vec{0}$$

$$q_1 = \rho \qquad q_2 = \rho u \qquad q_3 = \rho \cdot e_{tot}$$

$$f_1 = \rho u = q_2$$

$$P = \frac{c_s^2}{\gamma} \cdot \rho = (\gamma - 1) \rho e$$

$$= (\gamma - 1) \Big(\underbrace{\rho e + \rho \frac{u^2}{2}}_{= \rho \cdot e_{tot}} - \underbrace{\rho \frac{u^2}{2}}_{\frac{q_2^2}{2 q_1}} \Big)$$

$$= (\gamma - 1) \Big(\underbrace{\rho e_{tot}}_{q_3} - \frac{1}{2} \frac{q_2^2}{q_1} \Big)$$

$$f_2 = \rho u^2 + P = \frac{q_2^2}{q_1} + (\gamma - 1)\Big(q_3 - \frac{1}{2} \frac{q_2^2}{q_1} \Big)$$

$$= (\gamma - 1) q_3 + \frac{3 - \gamma}{2} \cdot \frac{q_2^2}{q_1}$$

$$f_3 = (\underbrace{\rho e_{tot}}_{= q_3} + P) \cdot \underset{\substack{\| \\ q_2/q_1}}{u} = \gamma \frac{q_3 q_2}{q_1} + \frac{(1-\gamma)}{2} \frac{q_2^3}{q_1^2}$$

$$\partial_t \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} + \underbrace{\partial_x \begin{pmatrix} f_1(q_1, q_2, q_3) \\ f_2(q_1, q_2, q_3) \\ f_3(q_1, q_2, q_3) \end{pmatrix}} = 0$$

$$= \partial_x F(Q) = \underbrace{\frac{\partial F}{\partial Q}}_{\text{Jacobian}} \cdot \partial_x Q = 0$$

$$\lambda_\pm = u \pm c_s$$
$$\lambda_0 = u$$

$$e_0 = \begin{pmatrix} 1 \\ u \\ u^2/2 \end{pmatrix} \qquad e_\pm = \begin{pmatrix} 1 \\ u \pm c_s \\ h_{tot} \pm c_s \cdot u \end{pmatrix}$$

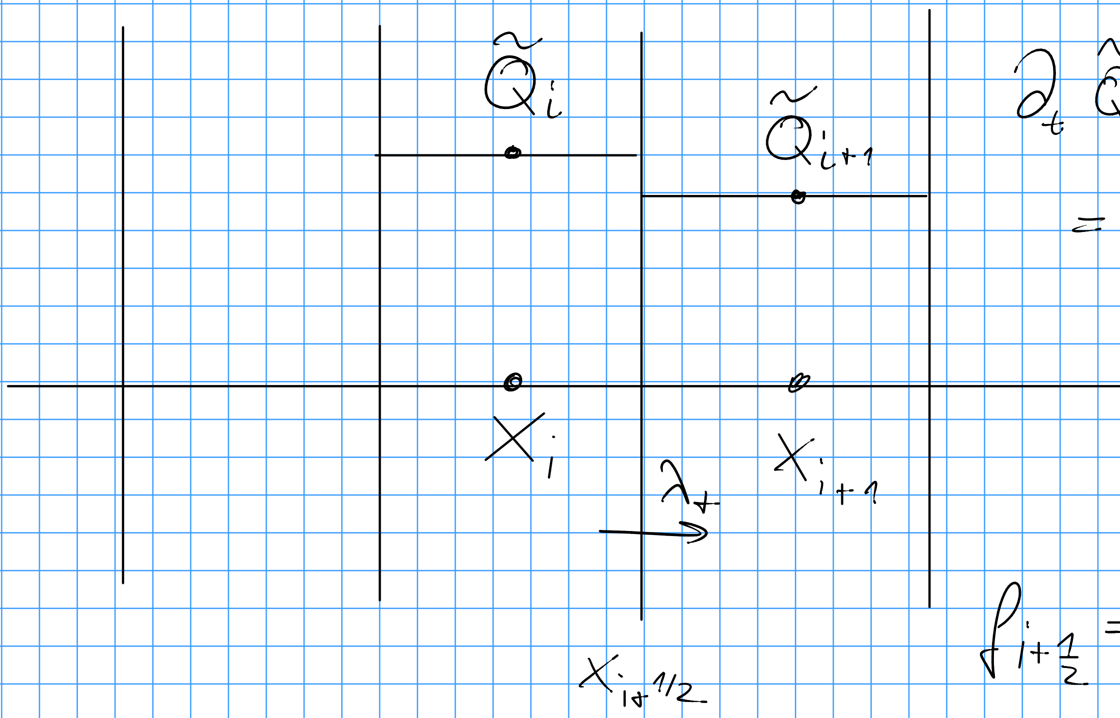
$$h_{tot} = \text{total enthalpy}$$
$$= e_{tot} + \frac{P}{\rho}$$

transform to the eigenspace of the Jacobian : $Q \longrightarrow \tilde{Q}$

$$\partial_t \begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \\ \tilde{q}_3 \end{pmatrix} + \begin{pmatrix} \lambda_+ & & 0 \\ & \lambda_0 & \\ 0 & & \lambda_- \end{pmatrix} \partial_x \begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \\ \tilde{q}_3 \end{pmatrix} = 0$$

$$\partial_t \tilde{q}_1 + \lambda_+ \partial_x \tilde{q}_1 = 0$$

$\vdots$



$$\partial_t \tilde{Q} + \begin{pmatrix} \lambda_+ & & \\ & \lambda_0 & \\ & & \lambda_- \end{pmatrix} \partial_x \tilde{Q} = 0$$

$$f_{i+\frac{1}{2}} = \begin{cases} \lambda_+ > 0: & \tilde{q}_i^{(1)} \cdot \lambda_+ \\ \lambda_+ < 0: & \tilde{q}_{i+1}^{(1)} \cdot \lambda_+ \\ \lambda_0 > 0: & \tilde{q}_i^{(2)} \cdot \lambda_0 \\ \lambda_0 < 0: & q_{i+1}^{(2)} \cdot \lambda_0 \end{cases}$$

$$\frac{\tilde{Q}_i^{n+1} - \tilde{Q}_i^{n}}{\Delta t} + \frac{f^n_{i+1/2} + f^n_{i-1/2}}{\Delta x} = 0$$

$$\rho, u, P, e_{tot} \longrightarrow \vec{Q}$$

on the grid: $\rho_i; u_i; P_i; e_{tot,i}$

$$\hookrightarrow \vec{Q}_i$$

$$\hookrightarrow \vec{\tilde{Q}}_i = \underset{\uparrow\, f(e_+, e_0, e_-)}{T} \cdot Q_i$$

$$\partial_t \tilde{q}^{(1)} + \lambda_+ \partial_x \tilde{q}^{(1)} = 0$$

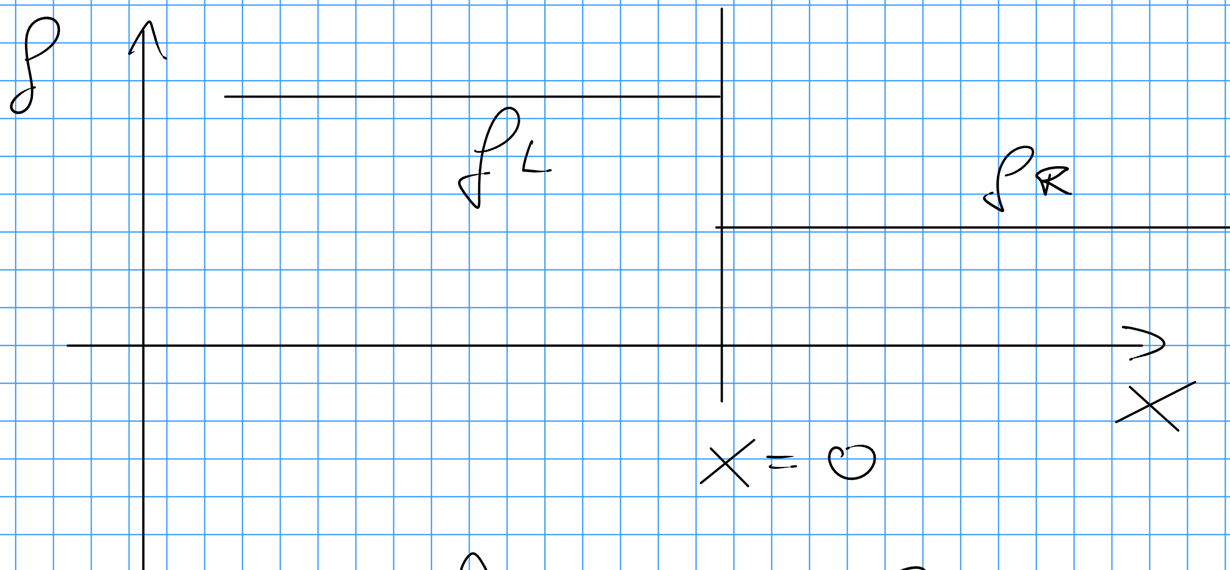
$$\partial_t \tilde{q}^{(2)} + \lambda_0 \partial_x \hat{q}^{(2)} = 0$$

$$\partial_t \tilde{q}^{(3)} + \lambda_- \partial_x \hat{q}^{(3)} = 0$$

$$\frac{\tilde{q}_i^{n+1\,(1)} - \tilde{q}_i^{n\,(1)}}{\Delta t} + \frac{f_{i+1/2}^{n\,(1)} - f_{i-1/2}^{n\,(1)}}{\Delta x} = 0$$

$$\tilde{q}_i^{n+1\,(1)} = \dots$$

$$\hookrightarrow \tilde{Q}_i^{n+1} \longrightarrow Q_i^{n+1}$$



$$\lambda_\pm = u \pm c_s$$

$$\frac{d}{dt} x = \lambda_\pm \qquad \partial_t s = 0$$

$$\partial_t \left(u \pm \frac{2}{\gamma - 1} c_s \right) = 0$$

$$\frac{d}{dt} x = \lambda_0 \qquad \partial_t u = 0$$

$$\partial_t P = 0$$

<u>3D</u>:

$$\partial_t \vec{Q} + \nabla \cdot \vec{F} = 0$$

$$Q = \begin{pmatrix} \rho \\ \rho u_x \\ \rho u_y \\ \rho u_z \\ \rho e_{tot} \end{pmatrix}$$

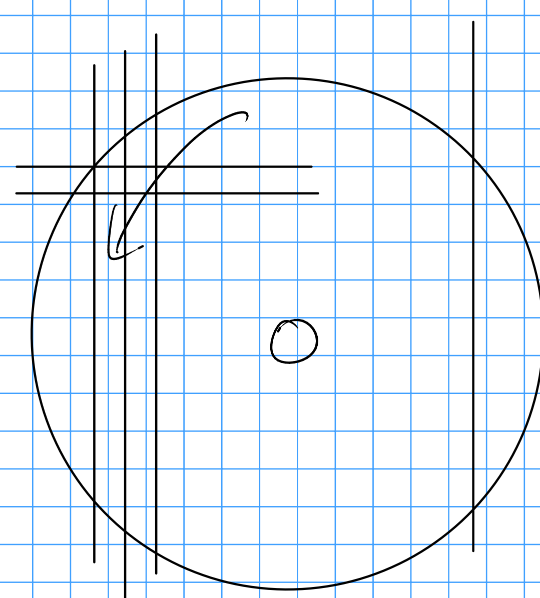
$$\nabla \cdot \vec{F} = \partial_x F_x + \partial_y F_y + \partial_z F_z$$

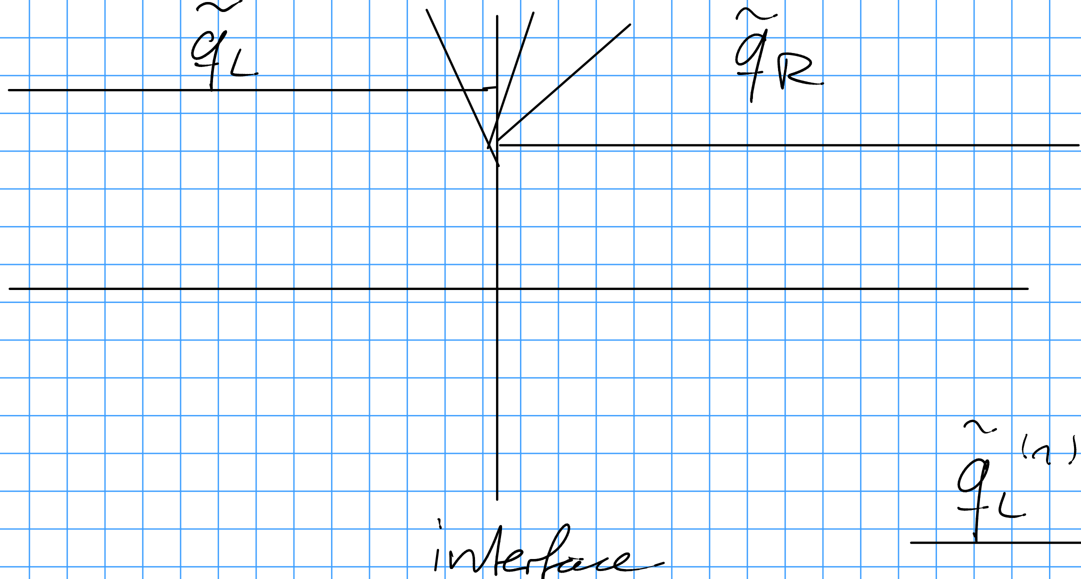
$$F_x = \begin{pmatrix} \rho u_x \\ \rho u_x^2 + P \\ \rho u_x u_y \\ \rho u_x u_z \\ \rho h_{tot} u_x \end{pmatrix} \qquad F_y = \begin{pmatrix} \rho u_y \\ \rho u_x u_y \\ \rho u_y^2 + P \\ \rho u_y u_z \\ \rho h_{tot} u_y \end{pmatrix}$$

$$\partial_t \tilde{Q} + \partial_x \tilde{F}_x = 0$$

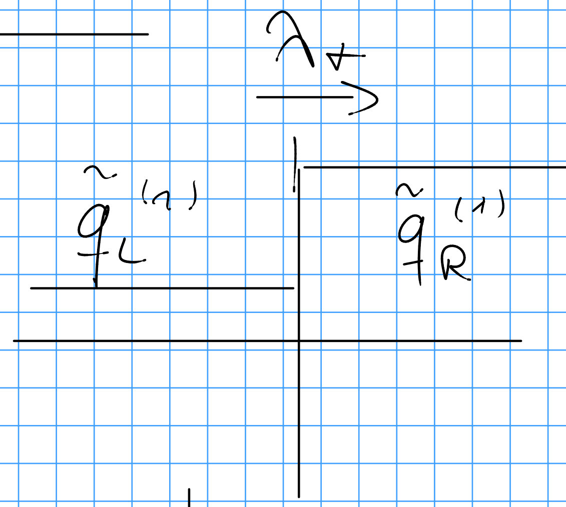
$$\partial_t \tilde{Q} + \partial_y \tilde{F}_y = 0$$

$$\partial_t \tilde{Q} + \partial_z \tilde{F}_z = 0$$





$t = 0$



$\tilde{q}_{interface}$ for $t > 0$