$$f(x) = \frac{x + e^{-x} - 1}{x^2}$$

limit :
$$\lim_{x \to 0} f(x) \underset{\uparrow}{=} \lim_{x \to 0} \frac{1 - e^{-x}}{2x} = \lim_{x \to 0} \frac{e^{-x}}{2} = \underline{\underline{\frac{1}{2}}}$$

l'Hospitals rule

<u>or</u>
$$f(x) = \left(x + \sum_{n=0}^{\infty} (-1)^n \frac{x^n}{n!} - 1 \right) \cdot \frac{1}{x^2}$$

$$= \left(\underbrace{-1 + x + (-1)^0 \frac{1}{1} + (-1)^1 \frac{x}{1}}_{= 0} + \sum_{n=2}^{\infty} (-1)^n \frac{x^n}{n!} \right) \frac{1}{x^2}$$

$$= \frac{1}{x^2} \cdot \sum_{n=2}^{\infty} (-1)^n \frac{x^n}{n!} = \sum_{n=2}^{\infty} (-1)^n \frac{x^{n-2}}{n!}$$

$$= (-1)^2 \cdot \frac{1}{2!} + (-1)^3 \frac{x}{3!} + (-1)^4 \frac{x^2}{4!} + (-1)^5 \frac{x^3}{5!}$$

$$+ (-1)^6 \frac{x^4}{6!} + \sum_{n=7}^{\infty} (-1)^n \frac{x^n}{n!}$$

for $x \to 0$ we see : $f(x) \to \frac{1}{2}$

size of the summands @ $x = 10^{-5}$

$$\frac{1}{2} \longrightarrow \frac{1}{2}$$

$$-\frac{x}{6} \longrightarrow -1,\overline{6} \cdot 10^{-6}$$

$$+\frac{x^2}{24} \longrightarrow 4,1\overline{6} \cdot 10^{-12}$$

$$-\frac{x^3}{120} \longrightarrow -8,\overline{3} \cdot 10^{-18}$$

$$+\frac{x^4}{720} \longrightarrow 1,3\overline{8} \cdot 10^{-24}$$

$$\vdots$$

beyond double point precision!

for $x \in [0, 10^{-5}]$

$$f(x) = \frac{1}{2} - \frac{x}{6} + \frac{x^2}{24}$$ to double precision