

Confidence Interval (CI)

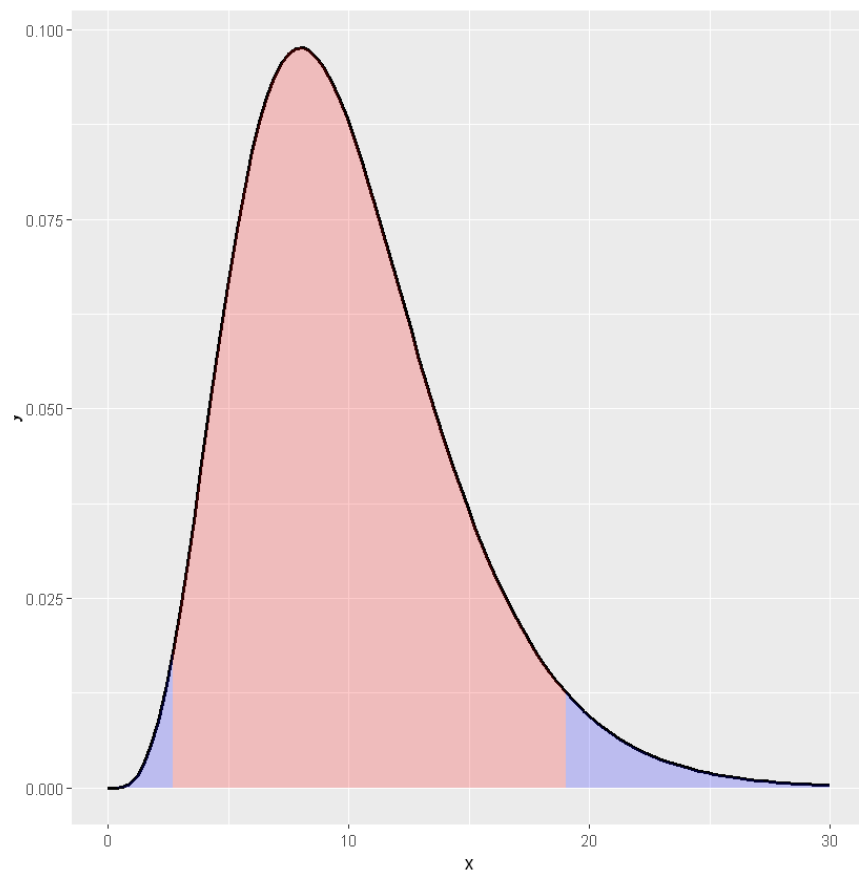
By D4R6

- The confidence interval of known the population mean case

$$\sum_{i=1}^n \frac{(X_i - \mu)^2}{\sigma^2} \sim \chi^2(n) \quad \frac{\frac{V_1}{r_1}}{\frac{V_2}{r_2}} \sim F(r_1, r_2), V_i \sim \chi^2(r_i) \text{ where } i = 1, 2 \text{ and } V_1, V_2 = i.i.d. \quad \frac{\sum_{i=1}^n \frac{(X_i - \mu_x)^2}{n\sigma_x^2}}{\sum_{i=1}^m \frac{(Y_i - \mu_y)^2}{m\sigma_y^2}} \sim \frac{\frac{\chi^2(n)}{n}}{\frac{\chi^2(m)}{m}} \sim F(n, m)$$

The diagram illustrates the derivation of the confidence interval for the ratio of variances. It starts with the F-distribution definition: $F_{(1-\frac{\alpha}{2})}(n, m) < \frac{\sum_{i=1}^n \frac{(X_i - \mu_x)^2}{n\sigma_x^2}}{\sum_{i=1}^m \frac{(Y_i - \mu_y)^2}{m\sigma_y^2}} < F_{(\frac{\alpha}{2})}(n, m)$. A blue arrow points from the right side of this inequality to a box containing $\frac{1}{F_{(\frac{\alpha}{2})}(m, n)} < \frac{\sum_{i=1}^n \frac{(X_i - \mu_x)^2}{n\sigma_x^2}}{\sum_{i=1}^m \frac{(Y_i - \mu_y)^2}{m\sigma_y^2}} < \frac{1}{F_{(1-\frac{\alpha}{2})}(m, n)}$. Another blue arrow points from the left side of the first inequality to a box containing $F_{(1-\frac{\alpha}{2})}(m, n) < \frac{\sum_{i=1}^m \frac{(Y_i - \mu_y)^2}{m\sigma_y^2}}{\sum_{i=1}^n \frac{(X_i - \mu_x)^2}{n\sigma_x^2}} < F_{(\frac{\alpha}{2})}(m, n)$. A purple box below the first inequality states $F_{(1-\frac{\alpha}{2})}(n, m) = \frac{1}{F_{(\frac{\alpha}{2})}(m, n)}$. A green line connects the first inequality to the final result.

$$\frac{\sum_{i=1}^n \frac{(X_i - \mu_x)^2}{n}}{\sum_{i=1}^m \frac{(Y_i - \mu_y)^2}{m}} \times F_{(1-\frac{\alpha}{2})}(m, n) < \frac{\sigma_x^2}{\sigma_y^2} < \frac{\sum_{i=1}^n \frac{(X_i - \mu_x)^2}{n}}{\sum_{i=1}^m \frac{(Y_i - \mu_y)^2}{m}} \times F_{(\frac{\alpha}{2})}(m, n)$$



In [20]:

```
n <- 10; s_sq <- 4.5

# 모평균이 알려져 있지 않은 경우의 신뢰 구간
CI_L <- (s_sq * (n - 1)) / qchisq(p = 0.05/2, df = n - 1, lower.tail = FALSE)
CI_R <- (s_sq * (n - 1)) / qchisq(p = 0.05/2, df = n - 1, lower.tail = TRUE)

sprintf("%s%.5f%s%.5f%s", '(' , CI_L , ',' , CI_R , ')')

significant_left <- qchisq(p = 0.05/2, df = n - 1, lower.tail = TRUE)
significant_right <- qchisq(p = 0.05/2, df = n - 1, lower.tail = FALSE)

ggplot(data.frame(x = c(0,30)), aes(x = x)) +
  stat_function(fun = dchisq, args = list(df = n), size = 1) +
  geom_area(stat = "function", fun = dchisq, args = list(df = n), fill = "red", alpha = 0.2
    , xlim = c(significant_left, significant_right)) +
  geom_area(stat = "function", fun = dchisq, args = list(df = n), fill = "blue", alpha = 0.2
    , xlim = c(0, significant_left)) +
  geom_area(stat = "function", fun = dchisq, args = list(df = n), fill = "blue", alpha = 0.2
    , xlim = c(significant_right, 30))

'(2.12903,14.99784)'
```