Confidence Interval

By D4R6

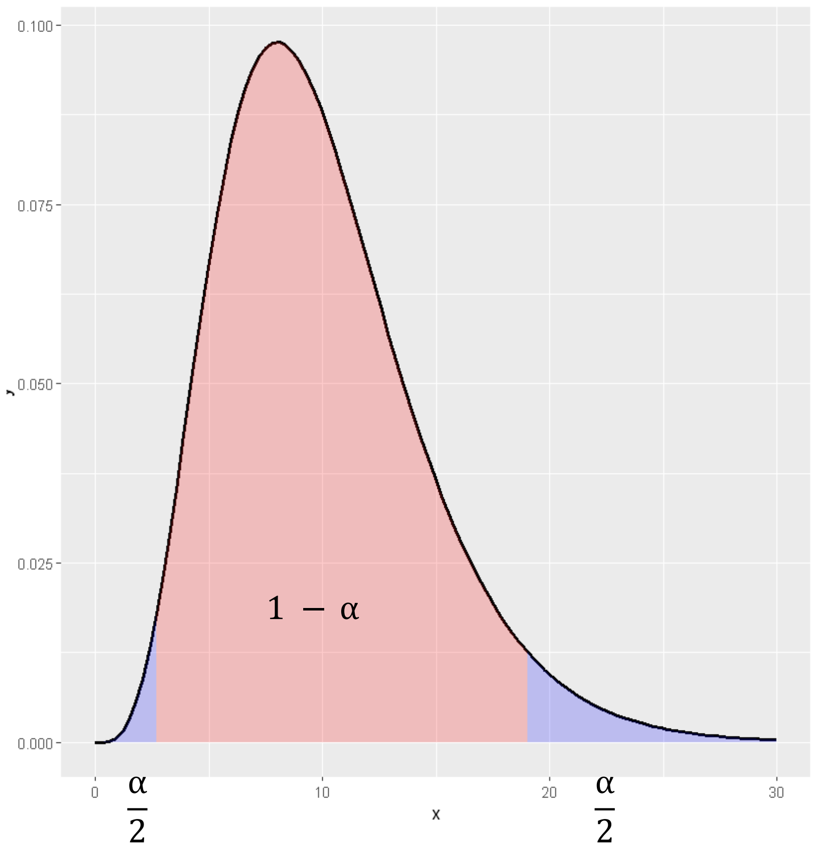
- The confidence interval of known the population mean case

$$\sum_{i=1}^{n}\frac{(X_i-\mu)^2}{\sigma^2}\sim\chi^2(n) \qquad \frac{\frac{V_1}{r_1}}{\frac{V_2}{r_2}}\sim F(r_1,r_2)\ ,V_i\sim\chi^2(r_i)\ \ where\ \ i=1,2\ and\ \ V_1,V_2=i.i.d.$$

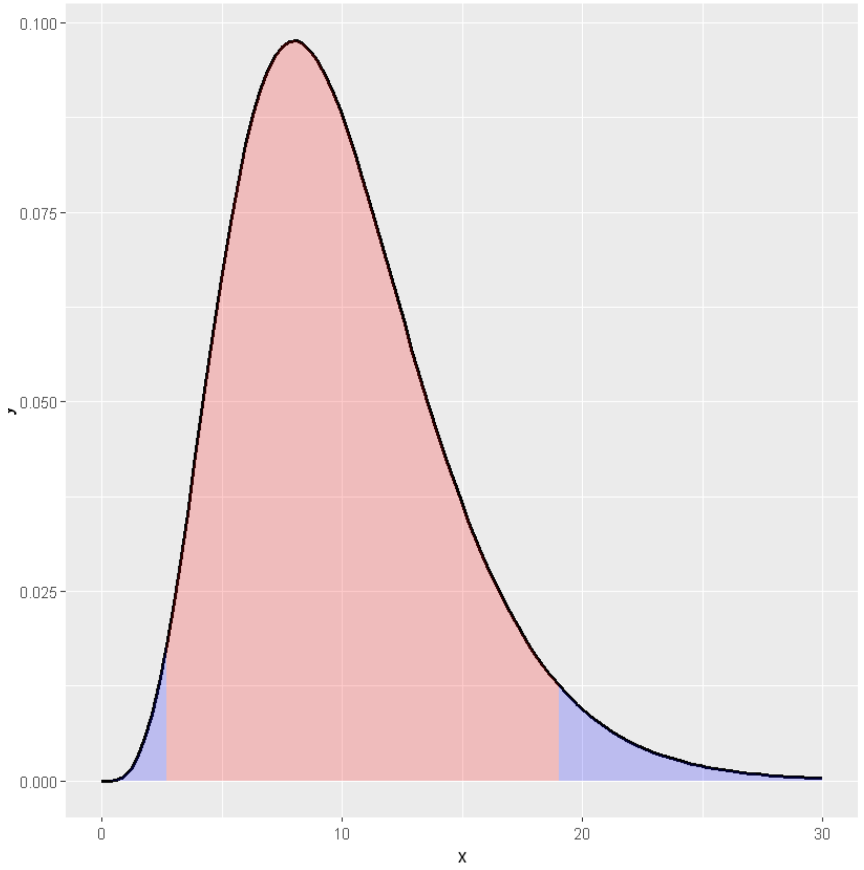
$$\frac{\sum_{i=1}^{n}\frac{(X_i-\mu_x)^2}{n\sigma_x^2}}{\sum_{i=1}^{m}\frac{\left(Y_i-\mu_y\right)^2}{m\sigma_y^2}}\sim\frac{\frac{\chi^2(n)}{n}}{\frac{\chi^2(m)}{m}}\sim F(n,m)$$

$$P\left(\chi^2_{\left(1-\frac{\alpha}{2}\right)}(n)<\sum_{i=1}^{n}\frac{(X_i-\mu)^2}{\sigma_x^2}<\chi^2_{\left(\frac{\alpha}{2}\right)}(n)\right)=P\left(\frac{\sum_{i=1}^{n}(X_i-\mu)^2/n}{\chi^2_{\left(1-\frac{\alpha}{2}\right)}(n)/n}<\sigma_x^2<\frac{\sum_{i=1}^{n}(X_i-\mu)^2/n}{\chi^2_{\left(\frac{\alpha}{2}\right)}(n)/n}\right)$$

$$P\left(\chi^2_{\left(1-\frac{\alpha}{2}\right)}(m)<\sum_{i=1}^{m}\frac{(Y_i-\mu)^2}{\sigma_y^2}<\chi^2_{\left(\frac{\alpha}{2}\right)}(m)\right)=P\left(\frac{\sum_{i=1}^{m}(Y_i-\mu)^2/m}{\chi^2_{\left(1-\frac{\alpha}{2}\right)}(m)/m}<\sigma_y^2<\frac{\sum_{i=1}^{m}(Y_i-\mu)^2/m}{\chi^2_{\left(\frac{\alpha}{2}\right)}(m)/m}\right)$$



$$P\left(\frac{\sum_{i=1}^{n}\frac{(X_i-\mu)^2}{n}}{\sum_{i=1}^{m}\frac{(Y_i-\mu)^2}{m}}\times\frac{\frac{\chi^2_{\left(1-\frac{\alpha}{2}\right)}(m)}{m}}{\frac{\chi^2_{\left(1-\frac{\alpha}{2}\right)}(n)}{n}}<\frac{\sigma_x^2}{\sigma_y^2}<\frac{\sum_{i=1}^{n}\frac{(X_i-\mu)^2}{n}}{\sum_{i=1}^{m}\frac{(Y_i-\mu)^2}{m}}\times\frac{\frac{\chi^2_{\left(\frac{\alpha}{2}\right)}(m)}{m}}{\frac{\chi^2_{\left(\frac{\alpha}{2}\right)}(n)}{n}}\right)\sim P\left(\frac{\sum_{i=1}^{n}\frac{(X_i-\mu)^2}{n}}{\sum_{i=1}^{m}\frac{(Y_i-\mu)^2}{m}}\times F_{1-\frac{\alpha}{2}}(m,n)<\frac{\sigma_x^2}{\sigma_y^2}<\frac{\sum_{i=1}^{n}\frac{(X_i-\mu)^2}{n}}{\sum_{i=1}^{m}\frac{(Y_i-\mu)^2}{m}}\times F_{\frac{\alpha}{2}}(m,n)\right)$$



In [20]:

```r
n <- 10; s_sq <- 4.5

# 모평균이 알려져 있지 않은 경우의 신뢰 구간
CI_L <- (s_sq * (n - 1)) / qchisq(p = 0.05/2, df = n - 1, lower.tail = FALSE)
CI_R <- (s_sq * (n - 1)) / qchisq(p = 0.05/2, df = n - 1, lower.tail = TRUE)

sprintf("%s%.5f%s%.5f%s",'(' ,CI_L ,',' ,CI_R ,')')

significant_left <- qchisq(p = 0.05/2, df = n - 1, lower.tail = TRUE)
significant_right <- qchisq(p = 0.05/2, df = n - 1, lower.tail = FALSE)

ggplot(data.frame(x = c(0,30)), aes(x = x)) +
    stat_function(fun = dchisq, args = list(df = n), size = 1) +
    geom_area(stat = "function", fun = dchisq, args = list(df = n), fill = "red", alpha = 0.2
              , xlim = c(significant_left, significant_right)) +
    geom_area(stat = "function", fun = dchisq, args = list(df = n), fill = "blue", alpha = 0.2
              , xlim = c(0, significant_left)) +
    geom_area(stat = "function", fun = dchisq, args = list(df = n), fill = "blue", alpha = 0.2
              , xlim = c(significant_right, 30))
```

'(2.12903,14.99784)'